# 1D internal multiple prediction in a multidimensional world: errors and recommendations

Pan Pan and Kris Innanen

## ABSTRACT

Internal multiples are more difficult to estimate and eliminate than free surface multiples. To eliminate these effects, internal multiple prediction becomes a necessity. In this paper, we employ the 1D internal multiple algorithm due to Weglein and collaborators in the 1990s. We review the basic principles of the internal multiple prediction algorithm. The key characteristic of the inverse scattering series based method is that information from the subsurface is not a requirement, because they are fully data-driven. Internal multiples from all possible generators are computed and shown in the output. Its performance is demonstrated using complex synthetic data sets. Then we systematically study how the presence of offset and the existence of dipping angle in the reflectors affect the 1D internal multiple prediction algorithm. Finally, we give some recommendations for the applications of this method.

## **INTRODUCTION**

For the exploration of oil and gas reservoirs, multiples can be one of the main issues in applying the seismic method. According to the classification of Xiao et al. (2003), there are three basic methods for suppressing multiples. The first type is deconvolution methods, which use the periodicity of multiples for suppression and are effective in suppressing short period free surface multiples generated at shallow reflectors. Secondly, filtering methods use differential moveout between primaries and multiples that are separate in the *f-k*,  $\tau$ -*p*, or Radon domains. These filtering methods can successfully suppress multiples generated at moderate to deep reflectors where multiples are wellseparated from their primaries. However, the above two methods assume that the earth is one dimensional with horizontal uniform layers. Dipping reflectors and variations in the overburden can cause serious problem in those methods. However, the third type of methods, wavefield prediction and subtraction, overcomes these problems. These methods are based on the wave equation and use recorded data to predict multiples by wave extrapolation and inversion procedures. These wavefield methods obtain multiple free data by subtracting the predicted multiples and can suppress all multiples generated by any complex system of reflectors.

The internal multiple prediction method that we use in this paper is derived from inverse scattering series, which belongs to wavefield methods. This method is capable of attenuating internal multiples without any a priori information about the medium. Furthermore, it can accommodate a multidimensional earth.

# THEORY

The detailed discussions of inverse scattering series theory and the derivations of the algorithm were given in previous work of Pan and Innanen (2013), the prediction algorithm in 1D normal incidence case can be written as,

$$b_{3IM}(k_z) = \int_{-\infty}^{\infty} dz e^{ik_z z} b_1(z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_z z'} b_1(z') \int_{z'+\epsilon}^{\infty} dz'' e^{ik_z z''} b_1(z''), \qquad (1)$$

 $k_z = 2\omega/c_0$  is the vertical wavenumber, which is the conjugate of the pseudo-depth  $(z = c_0 t/2)$ ,  $c_0$  is the constant reference velocity,  $b_{3IM}(k_z)$  is the prediction of the internal multiples. The  $b_1(z)$  entries are the input data traces in pseudo-depth.

The above algorithm is composed of three events that satisfy z'' > z' and z > z'. The travel time of the internal multiple is the sum of the travel times of the two lower events which are  $b_1(z)$  and  $b_1(z'')$ , minus the travel time of the higher one,  $b_1(z')$ . The parameter  $\epsilon$  is included in equation 1 to ensure that z'' > z' and z > z'. For band-limited data, this parameter is related to the width of the wavelet.

The inverse scattering series based internal multiple prediction algorithm (Araújo et al., 1994; Weglein et al., 1997) is a fully data-driven method. Any information about the reflectors that generate the internal multiples or the medium through which the multiples propagate is not required (Luo et al., 2011). It predicts all the internal multiples from all possible generators at once. Besides, it also predicts the correct travel times and approximate amplitudes of all the internal multiples which is demonstrated using the subevent interpretation (Pan and Innanen, 2013). Hernandez et al. (2011, 2012) have studied land and physical modeling applications of 1D prediction. In this paper, we continue to study the 1D algorithm.

## SYSTEMATIC STUDY OF PREDICTION ERRORS

In this part, we will systematically study the prediction errors. First of all, we need to build a velocity model and a shot record. The velocity model is shown in Figure 1 (a) and the shot record is shown in Figure 1 (b). The shot record we use is with the direct wave removed. If we do not remove the direct wave, lots of noise can be seen in the data set, especially in the zero offset trace, so removing it is crucial. The CREWES codes *afd\_vmodel.m* and *afd\_shotrec.m* are used to create the data. To avoid free surface multiples, we need to ensure that the boundary condition is absorbing on all four sides. Note that a frequency band will also be required to be chosen in the *afd\_shotrec.m*. We choose [10 20 80 100] as the frequency band in order to get a localized wavelet. All the parameters are listed in Table 1.

Finally, we put the shot record with the direct wave removed into the internal multiple prediction algorithm. We choose trace No.512 because this trace is zero offset. Several tests are made to choose the epsilon value with the optimal value determined to be 60. Artifacts from badly chosen epsilon values will often be seen at the arrival time of the primaries in the output data. For the synthetic data, artifacts are inevitable because of edge effects and noise, so we attempt using a larger medium to decrease edge effects and removing the direct wave to minimize noise. For the sake of simplicity, we only consider the two major internal multiples for which the arrival times are  $t_1 = 0.6659s$  and  $t_2 = 0.966s$ . The red circles indicate the positions of the internal multiples in the input data and output data. We can clearly see that the arrival times are correctly predicted, which means our IM prediction algorithm works well on the zero offset data. Through this we can confirm that the algorithm effectively predicts internal multiples based on the combination of the primaries, which satisfy the lower-higher-lower condition.



FIG. 1. (a) Three layered velocity model. (b) Shot record with primaries and internal multiples shown. Yellow lines indicate the locations of primaries, and red lines indicate the locations of internal multiples.



FIG. 2. (a) Shot record of the model. (b) Zero offset trace plotted in wiggle format with a scale of 5. (c) The same trace as (b) with a larger scale of 20. Yellow lines indicate the locations of primaries, and red lines indicate the locations of internal multiples.



FIG. 3. Application of the internal multiple prediction algorithm to the zero offset trace. (a) Input data. (b) Prediction output. The red circles indicate the positions of the internal multiples. The red arrows indicate the positions of the internal multiples in the input trace.

PARAMETER	VALUE
Number of <i>t</i>	1024
Number of <i>x</i>	1024
Number of z	1024
Interval sample time	3ms
Velocity and depth of the first interface	3000m/s at 200m
Velocity and depth of the second interface	2000m/s at 500m
Velocity and depth of the third interface	3000m/s at 800m
Wave speed of the source/ receiver medium	1500m/s
Time step	0.4ms
Maximum time of the shot record	3.07s
Location of the source	(1, 512)
Frequency band (Hz)	[10 20 80 100]
Epsilon	60

Table 1. Parameters of the velocity model and shot record

# The influence of the offset

Through Figure 3 we have confirmed that the internal multiple prediction algorithm on zero offset trace over layered media correctly predicts the arrival times of internal

multiples. Now, let us test what will happen to larger offset values. We will measure the prediction errors of internal multiples 1 and 2, respectively. In Figure 4, the blue line is the prediction error of the internal multiple 1 and the red line is the prediction error of the internal multiple 2.



FIG. 4. Prediction errors plotted against an increasing series of offset

Through Figure 4, we can see that as the offset becomes larger, prediction error increases. When the offset reaches 360 m, the prediction result becomes unacceptable. The error of the internal multiple 1 is 8.38% and the second internal multiple is 6.37%. After several tests, we can say that offset is a significant factor affecting the internal multiple prediction.

# The influence of dipping angle

Now, zero offset trace over dipping interface with gradually increasing dipping angles will be tested. We choose dipping angles from 0 to 15 degrees to analyze the result. Three cases will be shown here: where the generator becomes the dipping interface, where the second layer becomes the dipping interface, and where the third layer becomes the dipping interface. We will test each of the cases respectively.

Figure 5 is the velocity model and the shot record with the second layer's dipping angle at  $\theta=3^{\circ}$ . Through the shot record we can see that with the dipping interface, the travel times of the internal multiples are affected. First, we will study how the generator becoming the dipping interface affects the algorithm.



FIG. 5. (a) Three layered velocity model with the second layer's dipping angle  $\theta$ =3°. (b) Shot record of the model in (a).



FIG. 6. (a) Shot record of the model with the second layer's dipping angle  $\theta$ =3°. (b) Zero offset trace plotted in wiggle format with a scale of 5. (c) The same trace as (b) with a larger scale of 20. Yellow lines indicate the locations of primaries, and red lines indicate the locations of internal multiples.



FIG. 7. Application of the internal multiple prediction algorithm to zero offset trace with the second layer's dipping angle  $\theta$ =3°. (a) Input data. (b) Prediction output. The red arrows indicate the positions of the internal multiples in the input data.



FIG. 8. Prediction errors in the zero offset trace plotted against an increasing series of dipping angles, the generator is the dipping interface.

In Figure 8, the blue line is the prediction error of the internal multiple 1 and the red line is the prediction error of the internal multiple 2. The dipping interface is the generator. Error steadily decreases as dipping angle approaches 6 degrees, and steadily increases thereafter. This means that a dipping angle of 6 degrees is an optimal point for

predicting internal multiples. However, all error is within 1% in the range of 0-10 degrees, with a sharp increase starting after 10 degrees. This means that when the generator is the dipping interface, the algorithm will yield acceptable results in the range of 0-10 degrees.

Then, we will analyze the effect of the second layer which becomes the dipping interface.



FIG. 9. Prediction error in the zero offset trace plotted against an increasing series of dipping angles, the second layer is the dipping interface.

In Figure 9, the blue line is the prediction error of the internal multiple 1 and the red line is the prediction error of the internal multiple 2. The dipping interface is the second interface. The graph displays larger fluctuations in the first internal multiple than the second one, implying a greater effect on the first internal multiple. Error of internal multiple 1 is within 1% for the range of 0-11 degrees, and increases sharply thereafter. Error of internal multiple 2 remain within 1% for the range of 0-15 degrees.

Finally, we will study the third layer, which becomes the dipping interface this time. In Figure 10, the blue line is the prediction error of the internal multiple 1 and the red line is the prediction error of the internal multiple 2. The dipping interface is the third interface. The prediction error of the first internal multiple remains constant, implying that with the increases of the dipping angle of the third interface, the position of the first internal multiple is unaffected, as is in the prediction error decreases with increases in the dipping angle until 10 degrees, and sharply increases thereafter. However, all error is within 1% for the range of 0-15 degrees, indicating a negligible influence on the algorithm for this range of dipping angles.



FIG. 10. Prediction error in the zero offset trace plotted against an increasing series of dipping angles, the third layer is the dipping interface.

## RECOMMENDATIONS

We have employed internal multiple prediction algorithms based on the inverse scattering series theory for the estimation of internal multiples. The performance of the 1D algorithm was demonstrated with complex synthetic data sets with encouraging results. But still there's always error within the prediction algorithm. If we set the acceptable error range at 5%, then when the offset reaches 300m, the result becomes unacceptable. Thus we do not recommend applying this method when the offset is greater than 300m. As for the influence of dipping angles on the prediction algorithm, we only focus on the zero offset trace. When we have information about the subsurface, whichever layer becomes the dipping interface will yield a different result. When the generator becomes the dipping interface, it has the strongest effect on the algorithm. In this case, we recommend applying this method when the dipping angle is within 10 degrees. When the second layer becomes the dipping interface, error of internal multiple 1 exceeds 1% when dipping angle reaches 11 degrees. In this case, we recommend applying this method when the dipping angle is within 11 degrees. When the third layer becomes the dipping interface, it has negligible effects on the algorithm. In this case, we recommend applying this method at any dipping angle within 15 degrees. Therefore, even without any advanced knowledge of the multiple generators, this algorithm is able to predict correct travel times of internal multiples when the dipping angle is less than 10 degrees. In summary, we recommend using this algorithm to predict internal multiples in a multidimensional world.

## CONCLUSIONS

We employ the 1D internal multiple algorithm due to the work of Weglein and collaborators in the 1990s in MATLAB. We review the basic principles of the internal

multiple prediction algorithm. Internal multiples from all possible generators are computed and shown in the output. Its performance is demonstrated using complex synthetic data sets. Some recommendations for the applications of this method are given. We do not recommend applying this method when the offset is greater than 300m. Although this theory is based on a flat layered structure, the result of small dipping angle layers is acceptable. All results up and until dipping angle equals 10 degrees show good results.

# ACKNOWLEDGEMENTS

I thank the sponsors of CREWES for the financial support and all the staff and students. I am particularly grateful to Dr. Kris Innanen for his patient guidance.

## REFERENCES

- Araújo, F. V., Weglein, A. B., Carvalho, P. M., and Stolt, R. H., 1994, Inverse scattering series for multiple attenuation: an example with surface and internal multiples: SEG Expanded Abstracts, 13, 1039–1041.
- Hernandez, M., and Innanen, K. A., 2012, Application of internal multiple prediction: from synthetic to lab to land data: CREWES Annual Report, 24.
- Hernandez, M., Innanen, K. A., and Wong, J., 2011, Internal Multiple Attenuation based on inverse scattering: Theoretical review and implementation in synthetic data: CREWES Annual Report, 23.
- Hernandez, M., Innanen, K. A., and Wong, J., 2011, Internal Multiple Attenuation based on inverse scattering: Implementation in physical model seismic data: CREWES Annual Report, 23.
- Luo, Y., Kelamis, P. G., Fu, Q., Huo, S., Sindi, G., Aramco, S., Hsu, S., and Weglein, A. B., 2011, Elimination of land internal multiples based on the inverse scattering series: The Leading Edge, no. 884-889.

Pan, P., and Innanen, K. A., 2013, A review of internal multiple prediction: CREWES Annual Report, 25.

- Weglein, A. B., Gasparotto, F. A., Carvalho, P. M., and Stolt, R. H., 1997, An inverse-scattering series method for attenuating multiples in seismic reflection data: Geophysics, 62, No. 6, 1975–1989.
- Xiao, C., Bancroft, J., Brown, J., and Cao, Z., 2003, Multiple suppression: A literature review: CREWES Annual Report, 15.