Porous medium – the P-wave case

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ABSTRACT

A subset of the general equations used to describe seismic wave propagation in a poroviscoelastic medium is investigated using finite difference methods. Theoretically for an isotropic medium of this type there are four modes a propagation: a compressional (P) wave, a shear (S_V) wave and a shear (S_H) wave as well as what in termed a slow compressional (P_S) whose actual existence has, until fairly recently, been questioned. The scaled down version of the full poroviscoelastic equation set is one which is viscoelastic and but does not contain any shear type propagation. It is the acoustic analogue of the elastic wave equation, being a set of two coupled equations in the fast and slow compressional (P) wave modes, which may be further downgraded to a single equation in the fast compressional (P) wave mode as the slow compressional (P) wave mode is difficult to physically detect and as a consequence omitted, at least in this preliminary study. The relatively simple equation remaining was chosen so that a comparison with the seismic response of the acoustic wave could be done to ascertain the possible usefulness of pursuing this topic further. Apart from hydrocarbon related seismic applications, the use of this theory for near surface seismic or Ground Penetrating Radar (GPR) applications to locate toxic or hazardous waste sites and possibly be of assistance in delineating the extent of seepages either from actual dumping or deteriorating containers is a possibility.

INTRODUCTION

It is difficult to discuss the problem of seismic waves propagating in a porous viscoelastic medium without simultaneously introducing concepts, and thus equations related to the interaction between the fluid and the solid matrix, that is, without speaking to at least the basic concepts of Darcy's Law. There are numerous works which concentrate on developing the theory of seismic wave propagation within the framework of what could loosely be termed either reservoir geophysics or reservoir engineering. To list some of the more prominent papers and texts that deal with this topic presents another problem, as the notations are not standardized nor are the coordinate systems, or possibly more accurately, the relative coordinate systems employed. Further, many of the theoretical developments are problem specific. As for the theory and only the classic papers in this research area by Frankel (1944) and Biot (1956a, 1956b, 1956c, 1962a,1962b) will be referenced, leaving the investigation of matters related to Darcy's Law to the reader. For a marginally comprehensive knowledge of this area of study, topics, as mentioned in the previous sentence, must eventually be looked at in some depth.

The motivation for considering this problem is, as an example, that for a conventional hydrocarbon (oil and gas) production reservoir at some depth within the earth, fluids flow within the interstitial cavities of a matrix solid, driven conventionally, by hydrostatic pressure and is brought to the surface at specific locations (wells). To assume that the reservoir may be described by an acoustic or even elastic or anelastic medium could be considered presumptuous. Although history has shown that these are reasonable initial approximations, new technologies for monitoring the production history (time lapse seismology) and enhanced production, involving the injection of fluids either of necessity or convenience into the reservoir may indicate that a more relevant theory should be considered. Over the past few decades the technology associated with seismic data acquisition and primary and secondary recovery methods has increased enormously in complexity and utility. What have not kept pace, to the same extent, are fundamental seismic modeling capabilities.

In a general isotropic poroviscoelastic medium there are four modes of propagation, the conventional longitudinal P wave and the two shear modes S_V and S_H. As in the elastic case the S_H propagates independently of the P and S_V. The P wave is associated with wave propagation in the matrix. In addition, there is, at least theoretically, another P wave, associated with the liquid, propagating in the medium with a velocity that is usually less than the shear, S_V, wave. If the medium is anisotropic, it is deemed to be anisotropic in viscoelastic parameters as well as permeability. One of the basic premises of the governing theory is that there is an interaction between a wave propagating in the matrix and in the fluid. A problem of significance is determining the viscoelastic parameters that define the medium. It is for this reason only the longitudinal case will be considered here, a modified analogue of the elastodynamic problem. Further, it will be assumed that that the medium is dissipative, that is, the medium is viscoelastic. As a preliminary exercise, this was thought to be a reasonable starting point.

In addition, since many reservoir environments are partially saturated with one or more fluids, incorporation of partial saturation as well as multi-fluid interaction and dissipation due to absolute movements of the fluid is necessary. Development of a more realistic physical and mathematical model is essential in understanding the propagation of seismic waves in the real environment.

From Hassanzadeh (1991) "Synthetic seismograms computed using this method indicate that in the presence of heterogeneities the Biot mechanism (namely, the macroscopic differential fluid-solid movement, controlled primarily by hydraulic permeability) contributes considerably to the dispersion and dissipation of compressional waves. The dispersion and dissipation of the fast wave is due to the conversion of a small amount of its energy into the slow wave each time it crosses an interface between two dissimilar materials. In cases involving several layers and multiple reflections, as in the crosswell geometry, the cumulative effect of this kind of conversion is significant. The amount of energy lost as a result of this kind of conversion is influenced by the permeability of the medium. No equivalent single-phase model can adequately describe these effects on seismic waves propagating through fluid filled porous media."

BIOT-FRANKEL'S EQUATIONS

The Biot – Frankel equations governing compressional (P) wave propagation in a porous viscoacoustic (fast and slow $P-waves$) medium are given by

$$P\nabla^2 U + Q\nabla^2 V = \partial_t^2\left(\rho_{11}U + \rho_{12}V\right) + b\,\partial_t\left(U - V\right) + f_1\left(\mathbf{x},t\right) \quad (1)$$

$$Q\nabla^2 U + R\nabla^2 V = \partial_t^2\left(\rho_{12}U + \rho_{22}V\right) - b\,\partial_t\left(U - V\right) + f_2\left(\mathbf{x},t\right) \quad (2)$$

The damping coefficient b in equations (1-2) is related to Darcy's coefficient of permeability k, fluid viscosity, η, and porosity ϕ as $b = \eta\phi^2/k$. The quantity $U\left(\mathbf{x},t\right)$ is the dilation ($U\left(\mathbf{x},t\right) = \nabla\cdot\mathbf{u}\left(\mathbf{x},t\right)$) in the solid and $V\left(\mathbf{x},t\right)$ is the dilation ($V\left(\mathbf{x},t\right) = \nabla\cdot\mathbf{v}\left(\mathbf{x},t\right)$) in the fluid and it has been assumed that the source is located in some region or inclusion of convenience of the medium. Biot's coefficients, P, Q and R $\left(P = \lambda + 2\mu\right)$, which may be functions of position, must satisfy the inequality $PR - Q^2 > 0$. In the limit, $R,\ Q,\ \rho_{12},\ \rho_{22} \to 0$, a scalar compressional wave equation is all that remains.

In the "viscous" case, $b = \eta\phi^2/k$ where as previously stated η – viscosity, ϕ – porosity and k – permeability. Equations (1) and (2) for b are valid in the low-frequency range where the flow in the porous medium is of the Poiseuille[1] type. Most studies have indicated that for seismic frequencies, these equations are reasonably good approximations (Dutta and Ode, 1979; Bourbie et al., 1987). In the theory of poroelastic wave propagation, Biot (1956a) defined a characteristic frequency f_C given by the equation

$$f_C = \frac{\eta\phi}{2\pi\rho_f k} \quad (3)$$

where the quantities ϕ, η and k are defined above. The pore fluid density is denoted as ρ_f and is discussed below. Frequencies less than f_C, which include common seismic frequencies, are assumed to be satisfied by this relation.

The coefficients ρ_{11}, ρ_{12} and ρ_{22}, which have the dimensions of density and may also be spatially dependent, satisfy the inequalities

$$\rho_{11} > 0,\quad \rho_{12} > 0,\quad \rho_{22} > 0 \quad (4)$$

$$\rho_{11}\rho_{22} - \rho_{12}^2 > 0 \quad (5)$$

In the limit of the non-porous case, $\rho_{11} \to \rho$, where ρ is the density of an elastic medium

[1] Poiseuille flow means that any possible fluid flow within the medium is non-turbulent.

In an infinite medium, apart from source excitation, the problem may be fully specified by the initial conditions

$$U\big|_{t=0} = \partial_t U\big|_{t=0} = V\big|_{t=0} = \partial_t V\big|_{t=0} = 0 \tag{7}$$

and assuming that the solution is such that radiation conditions are physically realizable.

The source, which will be chosen as a point source of P waves, is located within the medium, which may be taken to be a vertically and laterally inhomogeneous half space, or at the interface, $z = 0$.

At the surface, $z = 0$, the stress free boundary conditions are

$$\partial_z U\big|_{z=0} = \partial_z V\big|_{z=0} = 0\,. \tag{8}$$

In the latter case, the free surface boundary conditions are of the form

$$\partial_z U\big|_{z=0} = \delta(x)\gamma_S f(t) \tag{9}$$

$$\partial_z V\big|_{z=0} = \delta(x)\gamma_f f(t) \tag{10}$$

where γ_1 and γ_2 are constants are specified in terms of the porosity (ϕ) and effective density of the matrix solid, together with factors inherent to the fluid; $\gamma_S = (1-\phi)$ and $\gamma_f = \phi$ with $\gamma_S + \gamma_f = 1$. The above equations would be applicable to a layer at the earth's surface that is fully or partially fluid saturated. The reason for continuing to refer to this last type of problem is that it is familiar, and as the equations for wave propagation in this medium type is the same as for a hydrocarbon reservoir at depth, may serve as a point of reference.

The wave propagation will be assumed to be confined to a spatial plane defined as $(0 \le x \le a; 0 \le z \le b\ )$ and all of these four boundaries are initially assumed to be perfectly reflecting. These conditions require that some measures such as absorbing boundaries (Clayton and Engquist, 1977 and Reynolds, 1978) or attenuating boundaries (for example, Cerjan et al., 1985) be incorporated in the solution method so that spurious reflections from them will not contaminate the wavefield propagating within the spatial plane.

INERTIAL CONSTANTS

Inertial constants, which take into account the fact that the relative fluid flow through the pores is not uniform, can be expressed in terms of the actual mass densities of the matrix and fluid, ρ_S and ρ_f, respectively, plus an apparent mass density representing the inertial coupling:

$$\rho_{11} = \rho_S (1-\phi) + \rho_f \phi (T-1) \tag{11}$$

$$\rho_{12} = \rho_f \phi (1-T) \tag{12}$$

$$\rho_{22} = \rho_f \phi T \tag{13}$$

where ϕ is the porosity as defined before and T is called the tortuosity parameter. The tortuosity T, which must satisfy the condition $T \geq 1$, is related not only to porosity but also to the geometry of the medium where flow occurs. Berryman (1980) investigated the effect of porosity on this parameter, and by making an analogy to the mass effect added in the analysis of the movement of a solid obstacle in a fluid flow, proposed

$$T = 1 - r\left(1 - 1/\phi\right) \tag{14}$$

where r is a factor to be determined from a microscopic model of the frame moving in the fluid. For the case of solid spherical particles in a fluid, $r = 1/2$. Other investigators have related tortuosity T to the concept of formation factor (Archie, 1942; Johnson and Sen, 1981).

In order for the kinetic energy to be a positive definite quadratic form, the coefficients ρ_{11}, ρ_{12}, and ρ_{22} must satisfy the following conditions (Biot, 1956a):

$$\rho_{11} > 0, \ \ \rho_{22} > 0, \ \ \rho_{11}\rho_{22} - \rho_{12}^2 > 0 \tag{15}$$

The total density ρ, of the fluid-solid aggregate, can be expressed as

$$\rho = \rho_{11} + 2\rho_{12} + \rho_{22} \tag{16}$$

or alternatively as

$$\rho = \rho_S (1-\phi) + \rho_f \phi \tag{17}$$

which represents the weighted sum of the solid and fluid mass densities.

If the porosity of the material is assumed to remain constant, the parameters P, Q, and R in equations (1) and (2) can be expressed in terms of more familiar constants such as the bulk modulus of the fluid (K_f), the bulk modulus of the individual solid grains (K_s), the frame moduli (K_b and μ), and the porosity ϕ (Geertsma and Smit, 1961; Stoll, 1974):

$$P = \left[(1-\phi)(\alpha-\phi)K_S + \phi K_S K_b / K_f\right] / D + 4\mu/3 \qquad (18)$$

$$R = \phi^2 K_S / D \qquad (19)$$

$$Q = (\alpha-\phi)\phi K_S / D \qquad (20)$$

$$D = \alpha - \phi + \phi K_S / K_f \qquad (21)$$

$$\alpha = 1 - K_b / K_S \qquad (22)$$

However, some of the quantities, in particular, the frame bulk modulus (K_b) and shear modulus (μ), must be inferred.

THE 2.5D FINITE DIFFERENCE – FINITE INTEGRAL TRANSFORM PROBLEM

Equations (1) and (2) are considered in a $3D$ Cartesian space. The media parameters are assumed to vary in the (x,z) plane. They are taken as constant in the y direction. A finite Fourier cosine transform (Appendix A) with respect to the y coordinate is applied leaving problem that will be dealt with using a finite difference methods. It may have more instructive to assume that the medium is slightly varying in the y direction. However, this introduces another subtopic in this area of research which will be dealt with at a later time. It would also complicate the derivations to the point that the original intent could be lost. This initial problem is complicated enough. After applying the finite cosine transform equations (1) and (2) become

$$\nabla^2_{(x.z)}\begin{bmatrix} S \\ F \end{bmatrix} - \left(\frac{n\pi}{c}\right)^2 \begin{bmatrix} S \\ F \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} \ddot{S} \\ \ddot{F} \end{bmatrix} + \begin{bmatrix} b_1 & -b_1 \\ -b_2 & b_2 \end{bmatrix}\begin{bmatrix} \dot{S} \\ \dot{F} \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \qquad (23)$$

where the dot and double dot above a quantity indicate the first or second partial derivative with respect to time. $(U \to S, V \to F)$ $\left(S^{time}_{x,z}, F^{time}_{x,z}\right)$

The quantities which require definition in the above equations are given by the following sequence of equations. Many of the quantities are discussed in the previous sections.

$$A_{11} = \left[\frac{R\rho_{11} - Q\rho_{12}}{PR - Q^2}\right] \qquad (24)$$

$$A_{12} = \left[\frac{R\rho_{12} - Q\rho_{22}}{PR - Q^2}\right] \qquad (25)$$

$$A_{21} = \left[\frac{P\rho_{12} - Q\rho_{11}}{PR - Q^2}\right] \qquad (26)$$

$$A_{22} = \left[\frac{P\rho_{22} - Q\rho_{12}}{PR - Q^2} \right] \tag{27}$$

$$b_1 = \left[b\left(R+Q\right)\right] \big/ D \tag{28}$$

$$b_2 = \left[b\left(P+Q\right)\right] \big/ D \tag{29}$$

$$D = PR - Q^2 > 0 \tag{30}$$

In what follows, the following definitions will be required:

$$\delta = \delta t/h \quad , \quad h = \delta x = \delta z \tag{31}$$

After approximating the time derivatives equation (23) has the form

$$\begin{gathered}
\delta t^2 \nabla^2_{(x.z)} \begin{bmatrix} S \\ F \end{bmatrix} + \left[2\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} - \left(\frac{\delta t\, j\pi}{c} \right)^2 \right] \begin{bmatrix} S^{\ell}_{m,n} \\ F^{\ell}_{m,n} \end{bmatrix} = \\
\begin{bmatrix} A_{11} + b_1 \delta t/2 & A_{12} - b_1 \delta t/2 \\ A_{21} - b_2 \delta t/2 & A_{22} + b_2 \delta t/2 \end{bmatrix} \begin{bmatrix} S^{\ell+1}_{m,n} \\ F^{\ell+1}_{m,n} \end{bmatrix} + \\
\begin{bmatrix} A_{11} - b_1 \delta t/2 & A_{12} + b_1 \delta t/2 \\ A_{21} + b_2 \delta t/2 & A_{22} - b_2 \delta t/2 \end{bmatrix} \begin{bmatrix} S^{\ell-1}_{m,n} \\ F^{\ell-1}_{m,n} \end{bmatrix} + \delta t^2 \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}.
\end{gathered} \tag{32}$$

and upon introducing the spatial derivatives of the two dimensional Laplacian the following is obtained after some manipulations

$$\begin{gathered}
\delta^2 \begin{bmatrix} S^{\ell}_{m+1,n} + S^{\ell}_{m-1,n} + S^{\ell}_{m,n+1} + S^{\ell}_{m,n-1} \\ F^{\ell}_{m+1,n} + F^{\ell}_{m-1,n} + F^{\ell}_{m,n+1} + F^{\ell}_{m,n-1} \end{bmatrix} - \\
\begin{bmatrix} 4\delta^2 - 2A_{11} + \left(\delta t\, j\pi/c\right)^2 & 4\delta^2 - 2A_{12} + \left(\delta t\, j\pi/c\right)^2 \\ 4\delta^2 - 2A_{21} + \left(\delta t\, j\pi/c\right)^2 & 4\delta^2 - 2A_{22} + \left(\delta t\, j\pi/c\right)^2 \end{bmatrix} \begin{bmatrix} S^{\ell}_{m,n} \\ F^{\ell}_{m,n} \end{bmatrix} - \\
\begin{bmatrix} A_{11} - b_1 \delta t/2 & A_{12} + b_1 \delta t/2 \\ A_{21} + b_2 \delta t/2 & A_{22} - b_2 \delta t/2 \end{bmatrix} \begin{bmatrix} S^{\ell-1}_{m,n} \\ F^{\ell-1}_{m,n} \end{bmatrix} - \delta t^2 \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \\
\begin{bmatrix} A_{11} + b_1 \delta t/2 & A_{12} - b_1 \delta t/2 \\ A_{21} - b_2 \delta t/2 & A_{22} + b_2 \delta t/2 \end{bmatrix} \begin{bmatrix} S^{\ell+1}_{m,n} \\ F^{\ell+1}_{m,n} \end{bmatrix}
\end{gathered} \tag{33}$$

Introducing the next four matrix definitions

$$\mathbf{D} = \begin{bmatrix} A_{11} + b_1 \delta t/2 & A_{12} - b_1 \delta t/2 \\ A_{21} - b_2 \delta t/2 & A_{22} + b_2 \delta t/2 \end{bmatrix} \tag{34}$$

$$\boldsymbol{\mathcal{D}} = \mathbf{D}^{-1} = \frac{1}{DEN}\begin{bmatrix} A_{22} + b_2\delta t/2 & -\left(A_{12} - b_1\delta t/2\right) \\ -\left(A_{21} - b_2\delta t/2\right) & A_{11} + b_1\delta t/2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{D}}^{(1)} & \boldsymbol{\mathcal{D}}^{(3)} \\ \boldsymbol{\mathcal{D}}^{(2)} & \boldsymbol{\mathcal{D}}^{(4)} \end{bmatrix} \tag{35}$$

$$\hat{\boldsymbol{\mathcal{D}}} = \begin{bmatrix} 4\delta^2 - 2A_{11} + \left(\delta t\ j\pi/c\right)^2 & 4\delta^2 - 2A_{12} + \left(\delta t\ j\pi/c\right)^2 \\ 4\delta^2 - 2A_{21} + \left(\delta t\ j\pi/c\right)^2 & 4\delta^2 - 2A_{22} + \left(\delta t\ j\pi/c\right)^2 \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\mathcal{D}}}^{(1)} & \hat{\boldsymbol{\mathcal{D}}}^{(3)} \\ \hat{\boldsymbol{\mathcal{D}}}^{(2)} & \hat{\boldsymbol{\mathcal{D}}}^{(4)} \end{bmatrix} \tag{36}$$

$$\bar{\boldsymbol{\mathcal{D}}} = \begin{bmatrix} A_{11} - b_1\delta t/2 & A_{12} + b_1\delta t/2 \\ A_{21} + b_2\delta t/2 & A_{22} - b_2\delta t/2 \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{\mathcal{D}}}^{(1)} & \bar{\boldsymbol{\mathcal{D}}}^{(3)} \\ \bar{\boldsymbol{\mathcal{D}}}^{(2)} & \bar{\boldsymbol{\mathcal{D}}}^{(4)} \end{bmatrix} \tag{37}$$

an intermediate result is obtained as

$$\begin{aligned} \begin{bmatrix} S_{m,n}^{\ell+1} \\ F_{m,n}^{\ell+1} \end{bmatrix} = \delta^2\boldsymbol{\mathcal{D}}\begin{bmatrix} S_{m+1,n}^{\ell} + S_{m-1,n}^{\ell} + S_{m,n+1}^{\ell} + S_{m,n-1}^{\ell} \\ F_{m+1,n}^{\ell} + F_{m-1,n}^{\ell} + F_{m,n+1}^{\ell} + F_{m,n-1}^{\ell} \end{bmatrix} - \\ \boldsymbol{\mathcal{D}}\hat{\boldsymbol{\mathcal{D}}}\begin{bmatrix} S_{m,n}^{\ell} \\ F_{m,n}^{\ell} \end{bmatrix} - \boldsymbol{\mathcal{D}}\bar{\boldsymbol{\mathcal{D}}}\begin{bmatrix} S_{m,n}^{\ell-1} \\ F_{m,n}^{\ell-1} \end{bmatrix} - \delta t^2\boldsymbol{\mathcal{D}}\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \end{aligned} \tag{38}$$

from which after more algebra the following two equations result:

$$\begin{aligned} S_{m,n}^{\ell+1} = {} & \delta^2\mathcal{D}_{m,n}^{(1)}\left[S_{m+1,n}^{\ell} + S_{m-1,n}^{\ell} + S_{m,n+1}^{\ell} + S_{m,n-1}^{\ell}\right] + \\ & \delta^2\mathcal{D}_{m,n}^{(3)}\left[F_{m+1,n}^{\ell} + F_{m-1,n}^{\ell} + F_{m,n+1}^{\ell} + F_{m,n-1}^{\ell}\right] - \\ & \left[\mathcal{D}_{m,n}^{(1)}\hat{\mathcal{D}}_{m,n}^{(1)} + \mathcal{D}_{m,n}^{(3)}\hat{\mathcal{D}}_{m,n}^{(2)}\right]S_{m,n}^{\ell} - \left[\mathcal{D}_{m,n}^{(1)}\hat{\mathcal{D}}_{m,n}^{(3)} + \mathcal{D}_{m,n}^{(3)}\hat{\mathcal{D}}_{m,n}^{(4)}\right]F_{m,n}^{\ell} - \\ & \left[\mathcal{D}_{m,n}^{(1)}\bar{\mathcal{D}}_{m,n}^{(1)} + \mathcal{D}_{m,n}^{(3)}\bar{\mathcal{D}}_{m,n}^{(2)}\right]S_{m,n}^{\ell-1} - \left[\mathcal{D}_{m,n}^{(1)}\bar{\mathcal{D}}_{m,n}^{(3)} + \mathcal{D}_{m,n}^{(3)}\bar{\mathcal{D}}_{m,n}^{(4)}\right]F_{m,n}^{\ell-1} - \\ & \delta t^2\left[\mathcal{D}_{m,n}^{(1)}f_1 + \mathcal{D}_{m,n}^{(3)}f_2\right] \end{aligned} \tag{39}$$

$$\begin{aligned} F_{m,n}^{\ell+1} = {} & \delta^2\mathcal{D}_{m,n}^{(2)}\left[S_{m+1,n}^{\ell} + S_{m-1,n}^{\ell} + S_{m,n+1}^{\ell} + S_{m,n-1}^{\ell}\right] + \\ & \delta^2\mathcal{D}_{m,n}^{(4)}\left[F_{m+1,n}^{\ell} + F_{m-1,n}^{\ell} + F_{m,n+1}^{\ell} + F_{m,n-1}^{\ell}\right] - \\ & \left[\mathcal{D}_{m,n}^{(2)}\hat{\mathcal{D}}_{m,n}^{(1)} + \mathcal{D}_{m,n}^{(4)}\hat{\mathcal{D}}_{m,n}^{(2)}\right]S_{m,n}^{\ell} - \left[\mathcal{D}_{m,n}^{(2)}\hat{\mathcal{D}}_{m,n}^{(3)} + \mathcal{D}_{m,n}^{(4)}\hat{\mathcal{D}}_{m,n}^{(4)}\right]F_{m,n}^{\ell} - \\ & \left[\mathcal{D}_{m,n}^{(2)}\bar{\mathcal{D}}_{m,n}^{(1)} + \mathcal{D}_{m,n}^{(4)}\bar{\mathcal{D}}_{m,n}^{(2)}\right]S_{m,n}^{\ell-1} - \left[\mathcal{D}_{m,n}^{(2)}\bar{\mathcal{D}}_{m,n}^{(3)} + \mathcal{D}_{m,n}^{(4)}\bar{\mathcal{D}}_{m,n}^{(4)}\right]F_{m,n}^{\ell-1} - \\ & \delta t^2\left[\mathcal{D}_{m,n}^{(2)}f_1 + \mathcal{D}_{m,n}^{(4)}f_2\right] \end{aligned} \tag{40}$$

Although the above appear quite convoluted, the production of finite difference computer code is reasonably uncomplicated. The stability criterion is the same as that for an $3D$ acoustic wave equation.

CONCLUSIONS

The general equations used to describe the fast and slow $P-$wave seismic wave propagation in a poroviscoelastic medium is investigated using finite difference – finite integral transform methods. The $y-$component of the two coupled $3D$ equations is removed using a finite cosine transform. The resultant problem is a finite difference in (x,z,t). Finite difference analogues for the two equations are presented. A preliminary program has been written for the *coder* (C++) enhanced version of Matlab. This has been left in a beta state as the intent of this project was to introduce at least a mild variation of media parameters in the y direction. As previously mentioned, this extension produces extremely complicated computer code which is under development.

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APPENDIX A: FINITE COSINE TRANSFORM

If the function $\phi(y)$ satisfies the Dirichlet conditions in the interval $(0,c)$ and if in this interval the relation

$$\Phi(n) = \int_0^c \phi(y) \cos\left(\frac{n\pi y}{c}\right) dy \tag{A.1}$$

is valid at all points in the interval $(0,c)$, where the function $\phi(y)$ is continuous, the following equality

$$\phi(y) = \frac{\Phi(0)}{c} + \frac{2}{c}\sum_{n=1}^{\infty} \Phi(n)\cos\left(\frac{n\pi y}{c}\right) \equiv \frac{2}{c}\sum_{n=0}^{\infty} \Phi(n)\cos\left(\frac{n\pi y}{c}\right) \tag{A.2}$$

holds.

It is understood that the $n=0$ term has been included in the summation in the last term in equation (5) for convenience of notation. Some upper bound on the summation must be determined that adequately approximates the infinite series. This number has a linear dependence on the distance *c* and is dependent on the spectral content of the source wavelet.