A review of converted wave AVO analysis

Jian Sun and Kris Innanen

ABSTRACT

The AVO analysis of converted wave developed rapidly since multi-component seismic data was introduced to industry. PP-PS simultaneous AVO analysis and inversion may improve estimation of rock properties by combining extracted attributes to yield fractional contrasts in P-wave and S-wave velocities and density, and discriminate between reservoir and non-productive lithology. In this paper, we review many of the milestones in the development of converted wave AVO methodology and classify those approximations of P-SV wave reflection coefficient in the light of the way of derivation and their characteristics.

INTRODUCTION

In recent years, amplitude versus offset (AVO) technique is widely applied in seismic exploration and AVO analysis can be used as a direct indicator in hydrocarbon detection, lithology identification. Considering the types of seismic data applied, AVO analysis can be separated into conventional P-wave AVO analysis and converted P-SV wave AVO analysis. The former one has been quite successfully performance in seismic data processing and benefited seismic interpretation in past few years.

Due to less attenuation of S-wave than P-wave when propagating in unsaturated porosity of rocks, converted P-SV wave might have a greater signal to noise ratio (Mavko et al., 1998; Thomsen, 1999; Caldwell, 1999). Therefore, P-SV wave AVO analysis might provide more information than that obtained from P-wave AVO analysis. The gradual mature of multicomponent acquisition makes it possible for converted P-SV wave AVO analysis.

Since the first approximation for P-SV wave reflection coefficient was introduced by Bortfeld (1961), many of research on converted wave AVO have been studied, such as Bortfeld (1961), Aki and Richards (1980), Zhou (1993), Donati (1998), Larsen (1999), Wang (2011). In this paper, we review those approximations for converted wave reflection coefficients in sight of their properties and the way of their derivation. On account of space constraints, we aren't able to look back to each moment of the growth of the converted wave AVO analysis.

THEORY

When an incident P-wave wave strikes a boundary between two elastic media at an angle greater than zero, the energy will be separated for reflected and transmitted P-wave and S-wave on both sides of the boundary, as shown in Figure 1.

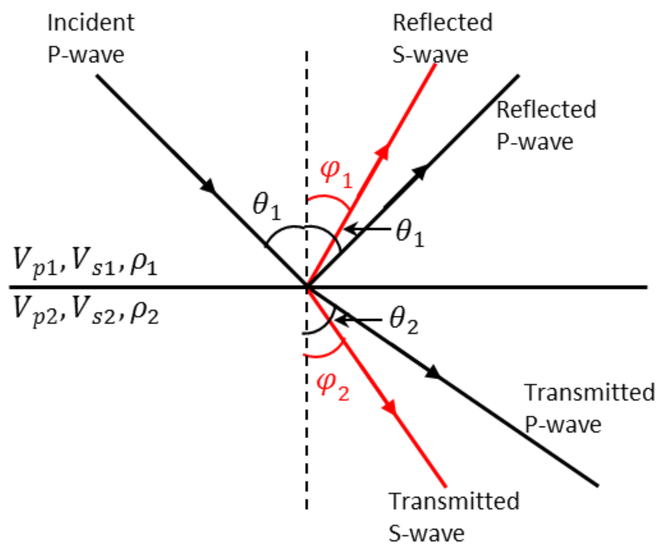

FIG.1. Reflections and transmissions between two elastic media for an incident P-wave

The incident, reflected and transmitted angles can be related to P-wave and S-wave velocities using Snell's Law:

$$p = \frac{\sin\theta_1}{V_{p1}} = \frac{\sin\theta_2}{V_{p2}} = \frac{\sin\varphi_1}{V_{s1}} = \frac{\sin\varphi_2}{V_{s2}}. \tag{1}$$

Reflection coefficients of the reflected and transmitted waves for both PP-wave and P-SV wave can be calculated by solving the exact following matrix equation (Zoeppritz, 1919):

$$\begin{bmatrix} -sin\theta_1 & -cos\varphi_1 & sin\theta_2 & cos\varphi_2 \\ cos\theta_1 & -sin\varphi_1 & cos\theta_2 & -sin\varphi_2 \\ sin2\theta_1 & \frac{V_{p1}}{V_{s1}}cos2\varphi_1 & \frac{\rho_2 V_{s2}^2 V_{p1}}{\rho_1 V_{s1}^2 V_{p2}}cos2\varphi_1 & \frac{\rho_2 V_{s2} V_{p1}}{\rho_1 V_{s1}^2}cos2\varphi_2 \\ -cos2\varphi_1 & \frac{V_{s1}}{V_{p1}}sin2\varphi_1 & \frac{\rho_2 V_{p2}}{\rho_1 V_{p1}}cos2\varphi_2 & -\frac{\rho_2 V_{s2}}{\rho_1 V_{p1}}sin2\varphi_2 \end{bmatrix} \begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pp} \\ T_{ps} \end{bmatrix} = \begin{bmatrix} sin\theta_1 \\ cos\theta_1 \\ sin2\theta_1 \\ cos2\varphi_1 \end{bmatrix}. \tag{2}$$

where V_{p1} and V_{p2} are P-wave velocities for both sides of the interface, V_{s1} and V_{s2} are S-wave velocities across the interface, ρ_1 and ρ_1 are densities of two elastic media. θ_1, θ_2 φ_1, φ_2 are shown in Figure 1.

Equation (1) can provide precise values of the amplitudes of the reflected and transmitted waves, but it does not provide an intuitive understanding of the effects of the parameter changes on the amplitudes. Also, underlying elastic parameters, which caused amplitude changes, are difficult to be inverted using equation (1). For these reasons, much AVO work has been done based on approximations to Zoeppritz equation (1919). We review and classify those approximations according to the properties and the way of the derivation, such as Aki-Richards's (1980) approximation, power series approximations, approximation using shear modulus, approximation as a function of impedances, normalized approximations. And basic characteristics of those approximations will be described in this paper.

Bortfeld approximation (1961)

Bortfeld (1961) derived several formulae for calculating P-SV wave reflection coefficient as a function of the incident angle. He claimed that his approximation of Zoeppritz equations were accurate to within a few degrees of the critical angle. His formulae for P-SV reflection coefficient is

$$
\begin{aligned}
R_{ps} = b\sin\theta_1 \sqrt{\frac{V_{p1}q_2\rho_1\cos\theta_1}{V_{p2}q_1\rho_1\cos\varphi_1}} \Bigg\{ \frac{1}{q_1}F(q_1) - \frac{V_{p2}}{V_{p1}q_2}\sqrt{\frac{q_1}{q_2}}F(q_2) \\
-\sin^2\theta_1 \left[\frac{1}{q_1^3}G(q_1) - \frac{V_{p2}^3}{V_{p1}^3 q_2^3}\sqrt{\frac{q_1}{q_2}}G(q_2) \right] \Bigg\}.
\end{aligned}
\tag{3}
$$

where

$$
q_1 = \frac{V_{p1}}{V_{s1}},\; q_2 = \frac{V_{p2}}{V_{s2}},\; ,q_2 = \frac{V_{p2}}{V_{s2}}. \tag{4}
$$

$$
a = \frac{\ln(\rho_2/\rho_1)}{\ln(V_{s2}/V_{s1})},\; b = \frac{\ln(V_{s2}/V_{s1})}{\ln(V_{p2}/V_{p1})}. \tag{5}
$$

and

$$
F(q_X) = \frac{4+2a}{3b-1} + \frac{aq_X}{b+1}. \tag{6}
$$

$$
G(q_X) = \frac{2+a}{14b-2} \pm \frac{4+\frac{7}{4}a}{5b+1}q_X + \frac{2+a}{6b+6}q_X^2 \mp \frac{a/4}{b+5}q_X^3. \tag{7}
$$

This approximation is complicated but an important aspect of this equation is the insight that it gives the interpreter into predicting how amplitude varies with offset as a function of rock properties. And it is the first approximation of Zoeppritz equations to P-SV wave reflection coefficient.

Aki-Richards approximation

Richards and Frasier (1976) derived the formulae of P-SV wave reflection coefficient as a function of normalized velocity and density when they were studying scattered pulse shapes by modeling in homogeneities as a sequence of infinitesimally thin homogeneous layers. Based on that, Aki and Richards (1980) proposed an approximation for P-SV wave reflection coefficient with the assumption of small changes of elastic parameter across the interface:

$$R_{ps}(\theta) = -\frac{\sin\theta}{2\cos\varphi}[(1 - 2\frac{V_s^2}{V_p^2}\sin^2\theta + 2\frac{V_s}{V_p}\cos\theta\cos\varphi)\frac{\Delta\rho}{\rho} - (4\frac{V_s^2}{V_p^2}\sin^2\theta - 4\frac{V_s}{V_p}\cos\theta\cos\varphi)\frac{\Delta V_s}{V_s}]. \tag{8}$$

with

$$\begin{aligned} &\theta = (\theta_2 + \theta_1)/2\,,\ \varphi = (\varphi_2 + \varphi_1)/2\,, \\ &\Delta\rho = \rho_2 - \rho_1,\ \rho = (\rho_2 + \rho_1)/2\,, \\ &\Delta V_p = V_{p2} - V_{p1},\ V_p = (V_{p2} + V_{p1})/2\,, \\ &\Delta V_s = V_{s2} - V_{s1},\ V_s = (V_{s2} + V_{s1})/2\,, \\ &\left|\frac{\Delta\rho}{\rho}\right| \ll 1,\ \left|\frac{\Delta V_p}{V_p}\right| \ll 1,\ \left|\frac{\Delta V_s}{V_s}\right| \ll 1. \end{aligned} \tag{9}$$

where, θ is the average between the incident angle θ_1 (also the reflected angle) and the transmitted angle θ_2 for P-wave. φ is the average between the reflected angle φ_1 and the transmitted angle φ_2 for P-SV wave. $\Delta\rho$ and ρ are the change and average of densities (ρ_1 and ρ_2) across the interface, respectively. ΔV_p and V_p are the change and average in P-wave velocities (V_{p1} and V_{p2}) on both sides of the interface. ΔV_s and V_s are the change and average of S-wave velocities (V_{s1} and V_{s2}) across the interface. Those parameters ($\theta, \theta_1, \theta_2, \varphi, \varphi_1, \varphi_2, \Delta\rho, \rho, \rho_1, \rho_2, \Delta V_p, V_p, V_{p1}, V_{p2}, \Delta V_s, V_s, V_{s1}, V_{s2}$) will be considered as global variables in this paper if there is no other declaration.

This approximation has indicated that all the conversions between P and S waves are insensitive to first-order changes in the P-wave velocity (Aki and Richards, 1980). And equation (8) gives some insight into the separate contributions made by $\Delta\rho$, ΔV_p, ΔV_s. But the approximate formulate (8) will fail if the angles ( ) are near 90 degree.

Power series approximations

1) Zhou's approximation (1993)

Similar to the compressional wave AVO analysis, Zhou (1993) analyzed basic AVO analysis of P-SV wave and SH wave on synthetic data. And based on Aki-Richard approximations and basic assumptions, Zhou (1993) proposed the P-SV wave reflection coefficient can be written as the cubic function of the incident angle:

$$R_{ps}(\theta) = S\sin\theta + C\sin^3\theta. \tag{10}$$

where

$$S = -\frac{1}{2}(1+\frac{2V_s}{V_p})\frac{\Delta\rho}{\rho} - \frac{2V_s}{V_p}\frac{\Delta V_s}{V_s}. \tag{11}$$

$$C = \frac{1}{2}\frac{V_s}{V_p}(1+\frac{3V_s}{2V_p})\frac{\Delta\rho}{\rho} + 2\frac{V_s}{V_p}(1+\frac{2V_s}{V_p})\frac{\Delta V_s}{V_s}. \tag{12}$$

Basic assumptions:

$$\frac{\sin\theta}{V_p} = \frac{\sin\varphi}{V_s}. \tag{13}$$

$$\frac{1}{\cos\varphi} \approx 1 + \frac{1}{2}\sin^2\varphi. \tag{14}$$

Zhou (1993) demonstrated that the seismic property of SH wave can be derived from P-SV wave. And seismic sections of reflection coefficient, S-wave velocity, and density can be inverted by P-SV wave AVO analysis using equation (10). More accuracy AVO analysis for modern converted wave interpretation are needed than the accuracy of result obtained from equation (10) , but equation (10) still give a good insight into P-SV wave AVO analysis.

2) Li's approximation (1996)

Based on the approximation of Bortfeld (1961), Li (1996) proposed the approximation of P-SV wave reflection coefficient by expanding odd terms of the sine's Taylor series,

$$\begin{aligned} R_{ps}(\theta) = &-\left(\frac{3\Delta\rho}{2\rho} + 2\frac{\Delta V_s}{V_s}\right)\sin\theta \\ &+\left[\frac{\Delta V_s}{V_s} + 2\left(\frac{V_s}{V_p}\right)\frac{\Delta V_s}{V_s} + \frac{1}{4}\left(\frac{V_s}{V_p}\right)^3\frac{\Delta\rho}{\rho} + 2\left(\frac{V_s}{V_p}\right)\frac{\Delta\rho}{\rho}\right]\sin^3\theta. \end{aligned} \tag{15}$$

The inversion for small angles of incidence case is facilitated because the ratio between P-wave and S-wave velocities is not included in the first term. And is we assume V_p/V_s =2, combining with P wave AVO analysis, cross-plot analysis can be described using this equation.

3) Gonzalez's approximation (2000)

Starting from Aki-Richards (1980) non-linear expression and the derivation proposed by Donati and Martin (1998) for P-S reflectivity, Gonzalez et al. (2000) derived a linear approximation, which is only valid for the small incident angles case, for P-S wave reflection coefficient,

$$R_{ps}(\theta) = E\sin\theta . \tag{16}$$

where

$$E = \left[-\frac{\triangle\rho}{2\rho} - \left(\frac{V_s}{V_p}\right)^2 \left(\frac{2\triangle V_s}{V_s} + \frac{\triangle\rho}{\rho}\right)\left(\frac{V_p}{V_s} - \frac{V_s}{2V_p}\right)\right]. \tag{17}$$

$$B = \left[\frac{\triangle V_p}{2V_p} + \frac{4V_pV_s}{2V_p{}^2 - V_s{}^2}\frac{\triangle\rho}{2\rho}\right] + \frac{4V_pV_s}{2V_p{}^2 - V_s{}^2}E . \tag{18}$$

Equation (16) proposed defines a P-SV gradient (E), which can be calculated from the converted wave CDP gathers, in the same way as the conventional P-P wave AVO attributes. And an approximate linear relation between P-P gradient (B) and P-SV AVO gradient (E) was obtained by in Gonzalez et al. (2000), shown in equation (18), which is very helpful for joint P-P and P-SV wave inversion.

4) Vant's approximation (2001)

Vant and Brown (2001) expanded the sine and cosine function from the Zoeppritz equations using the Taylor series and developed the first-order approximation for P-SV reflection coefficient, which can be expressed as

$$R_{ps} = \frac{-2\theta(V_{p_2}V_{s_2}\rho_2\triangle\rho + 2\rho_1\triangle\mu)}{(\rho_1V_{p_1} + \rho_2V_{p_2})(\rho_1V_{s_1} + \rho_2V_{s_2})} . \tag{19}$$

where $\triangle\rho = \rho_2 - \rho_1$ and $\triangle\mu = \mu_2 - \mu_1 = \rho_2V_{s_2}^2 - \rho_1V_{s_1}^2$.

The sign of P-SV reflection coefficient changes when the following condition met (Vant and Brown, 2001):

$$\frac{\triangle\mu}{\triangle\rho} = -\frac{V_{p_2}V_{s_2}\rho_2}{2\rho_1} . \tag{20}$$

If the notation of Aki and Richards (1980) are used, the further analysis of the conditions governing the polarities of R_{pp} and R_{ps} are opposite when $\triangle\rho/\rho$ and $\triangle V_s/V_s$ have contrary signs.

Vant's approximation (2001) only works for near-zero offset data, i.e., the incidence angle is less than 15 degree. And equations (19) and (20) indicate that the change in P-wave velocity across the interface does not influence the change in the sign of P-SV reflection coefficient at small offsets.

5) High-contrast approximation (Ramos and Castagna, 2001; Zheng, 1991)

From the exact P-SV reflectivity given by the well-known Zoeppritz equations, an accurate, high-contrast approximation for P-SV converted-wave reflection coefficient was derived by Zheng (1991) and further analyzed by Ramos and Castagna (2001).

Zheng (1991) expanded the Zoeppritz equation in Taylor series and kept the third-order term. Then the P-SV reflection coefficient can be obtained:

$$R_{ps}(\theta) = A\sin\theta + B\sin^3\theta + C\sin^5\theta. \quad (21)$$

where

$$A = \left[-2\frac{V_s}{V_p}\frac{\triangle V_s}{V_s}\right] - \left[\left(\frac{1}{2} + \frac{V_s}{V_p}\right)\frac{\triangle\rho}{\rho}\right]. \quad (22)$$

$$B = \left[\left(2\left(\frac{V_s}{V_p}\right)^2 + \left(\frac{V_s}{V_p}\right)\right)\frac{\triangle V_s}{V_s}\right] + \left[\left(\frac{3}{4}\left(\frac{V_s}{V_p}\right)^2 + \left(\frac{V_s}{2V_p}\right)\right)\frac{\triangle\rho}{\rho}\right]. \quad (23)$$

$$C = \left[\left(\frac{V_s}{V_p}\right)^4 \frac{\triangle V_s}{V_s}\right] + \left[\frac{1}{2}\left(\frac{V_s}{V_p}\right)^4 \frac{\triangle\rho}{\rho}\right]. \quad (24)$$

Compared to Aki-Richards's approximation, equation (21) can be propitious to any lithological interface because it is direct expansion of Zoeppritz equations. It shows the approximate relationship between reflection coefficient and incidence angle without any constraints. But its' complicated form and the uncertainly relationship of lithological parameters make it difficult to be applied in converted AVO inversion. Also, this approximation provided a new clue for calculating P-SV wave reflection coefficient since Zheng (1991) first introduced Taylor series expansion into approximation of converted wave reflection coefficient.

6) Sun's approximation (2003)

On basis of the P-S wave reflection coefficient approximate equation given by Aki-Richard (1980), Sun et al. (2003) presented an accurate approximation for the small incident angle in weak-contrast media. The equation can be expressed as:

$$R_{ps}(\theta) = A\sin\theta + B\sin^3\theta. \quad (25)$$

where

$$A = \frac{1}{4}\left[-\frac{\triangle\rho}{\rho}\left(2 + \frac{\triangle\rho}{\rho} + \frac{\triangle V_s}{V_s}\right) + 2\left(-2 + \frac{\triangle\rho}{\rho} + \frac{\triangle V_p}{V_p}\right)\left(\frac{\triangle\rho}{\rho} + 2\frac{\triangle V_s}{V_s}\right)\right]. \quad (26)$$

$$B = \frac{1}{8}\left(C + D\right). \quad (27)$$

$$C=-2\frac{\triangle\rho}{\rho}\frac{\triangle V_p}{V_p}-2\frac{V_s}{V_p}\left(-2+\frac{\triangle\rho}{\rho}+\frac{\triangle V_p}{V_p}\right)\left(\frac{\triangle\rho}{\rho}+2\frac{\triangle V_s}{V_s}\right). \tag{28}$$

$$\begin{aligned}D=&\left(\frac{V_s}{V_p}\right)^2\left[11\left(\frac{\triangle\rho}{\rho}\right)^2+4\frac{\triangle V_s}{V_s}\left(4-5\frac{\triangle V_s}{V_s}\right)+3\frac{\triangle\rho}{\rho}\left(2+9\frac{\triangle V_s}{V_s}\right)\right]\\&+4\frac{\triangle\rho}{\rho}\left(\frac{V_s}{V_p}\right)^3\left(4\frac{\triangle\rho}{\rho}-15\frac{\triangle V_s}{V_s}\right).\end{aligned} \tag{29}$$

Equations (25)-(29) can provide a more accuracy solutions because it remains the second terms in Taylor series for both velocity and density. Combining these equations with Gardner's equations and relationships among velocity, density and elasticity modulus, inversions of Poisson's ratio, Lame constant and fluid factors can be achieved.

7) Wang's approximation (2011)

To resolve the problems Ramos's approximation has, such as the poor adaptability for large incidence angle and less reliability of AVO attributes calculation with less than 30 degrees incidence angles from real seismic data, Wang (2011) derived a new approximation for the P-SV wave reflection coefficient in a similar way to Ramos's approximation. Wang's approximation can be written as

$$R_{ps}(\theta)=A\sin\theta+B\sin^3\theta+C_2\sin^5\theta. \tag{30}$$

where

$$A=\left[-2\frac{V_s}{V_p}\frac{\triangle V_s}{V_s}\right]-\left[\left(\frac{1}{2}+\frac{V_s}{V_p}\right)\frac{\triangle\rho}{\rho}\right]. \tag{31}$$

$$B=\left[\left(2\left(\frac{V_s}{V_p}\right)^2+\left(\frac{V_s}{V_p}\right)\right)\frac{\triangle V_s}{V_s}\right]+\left[\left(\frac{3}{4}\left(\frac{V_s}{V_p}\right)^2+\left(\frac{V_s}{2V_p}\right)\right)\frac{\triangle\rho}{\rho}\right]. \tag{32}$$

$$C_2=\left(\frac{V_s}{V_p}\right)^4\frac{\triangle V_s}{V_s}+\frac{5}{16}\left(\frac{V_s}{V_p}\right)^4\frac{\triangle\rho}{\rho}+\frac{1}{8}\frac{V_s}{V_p}\frac{\triangle\rho}{\rho}+\frac{1}{8}\frac{V_s}{V_p}\frac{\triangle V_s}{V_s}. \tag{33}$$

Compared to Ramos's approximation, equation (30) has more accuracy without increasing its complexity and works well for large incidence angles. And when the angle of incidence is less than 30 degrees, the fifth-power of $\sin\theta$ can be neglected. Then equation (30) can be simplified as

$$\begin{aligned}R_{ps}(\theta)&\approx A\sin\theta+B\sin^3\theta\\&=A(\sin\theta-\sin^3\theta)+P\sin^3\theta.\end{aligned} \tag{34}$$

where

$$P = A + B$$
$$= \left[2\left(\frac{V_s}{V_p}\right)^2 - \frac{V_s}{V_p}\right]\frac{\Delta V_s}{V_s} + \left[\frac{3}{4}\left(\frac{V_s}{V_p}\right)^2 - \frac{V_s}{2V_p} - \frac{1}{2}\right]\frac{\Delta\rho}{\rho}. \quad (35)$$

For the equation (35), typically, $V_p/V_s \approx 2$ makes the first term approach zero, and fractional changes of density are normally very small which makes the second term close to zero too. Therefore, when the incidence angle is less than 30 degree, only the first term is left in equation (34) and this simplification makes it easier to do an AVO analysis of converted wave.

Approximations using shear-modulus

Yang's approximation (1996)

According to the approximation given by Mallick (1993), Yang (1996) presented a new approximate expression as the function of ray parameters and the shear modulus.

$$R_{ps}(\theta) \approx \frac{-2q_{V_{p1}}}{(q_{V_{p1}}+q_{V_{p2}})(q_{V_{s1}}+q_{V_{s2}})}\, p\frac{V_{p_1}}{V_{s_1}}\frac{\Delta\rho}{\rho} - \frac{4q_{V_{p1}}q_{V_{p2}}q_{V_{s2}}}{(q_{V_{p1}}+q_{V_{p2}})(q_{V_{s1}}+q_{V_{s2}})}$$
$$\times\, p\frac{V_{p_1}}{V_{s_1}}\frac{\Delta\mu}{\rho} + \frac{2q_{V_{p1}}q_{V_{p2}}q_{V_{s2}}}{(q_{V_{p1}}+q_{V_{p2}})(q_{V_{s1}}+q_{V_{s2}})}\, p\frac{V_{p_1}}{V_{s_1}}\frac{\Delta\mu}{\rho}\frac{\Delta\rho}{\rho}. \quad (36)$$

where $p = \frac{\sin\theta_1}{V_{p1}} = \frac{\sin\theta_2}{V_{p2}} = \frac{\sin\varphi_1}{V_{s1}} = \frac{\sin\varphi_2}{V_{s2}}$ is the parameter, $\rho = \frac{\rho_1+\rho_2}{2}$ is the average density of two media across the interface and $\Delta\rho = \rho_2 - \rho_1$, $q_{V_{XY}} = \frac{\cos\theta_Y}{V_{XY}} = \sqrt{\frac{1}{V_{XY}^2} - p^2}$ are vertical slowness of P-wave and S-wave, $\mu = (\mu_1+\mu_2)/2$ is the average shear modulus and $\Delta\mu = \mu_2 - \mu_1 = \rho_2 V_{s2}^2 - \rho_1 V_{s1}^2$.

This approximate expression shows that, for non-normal incidence, P-SV wave reflection coefficient mainly depends on two elastic parameters: $\frac{\Delta\rho}{\rho}$-the ratio change of density and $\frac{\Delta\mu}{\rho}$ - shear modulus change and density contrast.

Sun's approximation (2006)

Sun et al. (2006) simplified Aki-Richards approximation into a new approximation in terms of shear modulus, ratios change of density and P-wave velocity. The equation can be written as

$$R_{ps}(\theta)=\frac{1}{2}\left(\frac{\triangle\rho}{\rho}+\frac{\triangle v_p}{v_p}\right)+\frac{1}{2}\tan^2\theta\frac{\triangle v_p}{v_p}-2\sin^2\theta\frac{\triangle\mu}{\lambda+2\mu}. \tag{37}$$

where μ is the shear modulus.

This approximation for P-SV wave reflection coefficient doesn't involve any term related to S-wave and provide more accuracy result for converted AVO analysis. Since equation (37) is based on Aki-Richards's approximation, it is still constrained to small changes in the elastic properties between interfaces.

Approximation with impedances

To the realization of impedance inversion, Larsen（1999）reformulated Aki-Richards approximation into the function of P-wave and S-wave Impedance,

$$\begin{aligned}R_{ps}(\theta)=\frac{-V_p\tan\varphi}{2V_s}\Bigg[&\left(1+2\sin^2\varphi-2\frac{V_s}{V_p}\cos\theta\cos\varphi\right)\frac{\triangle\rho}{\rho}\\&-\left(4\sin^2\varphi-4\frac{V_s}{V_p}\cos\theta\cos\varphi\right)\frac{\triangle J}{J}\Bigg].\end{aligned} \tag{38}$$

where $\varphi=(\varphi_1+\varphi_2)/2$ is the average between the incident angle and transmitted angle for P-SV wave, $J=\rho V_s$ （$\frac{\triangle J}{J}=\frac{\triangle V_s}{V_s}+\frac{\triangle\rho}{\rho}$） is the impedance for S-wave.

Considering the Gardner's equation $\frac{\triangle\rho}{\rho}=\frac{1}{4}\frac{\triangle V_p}{V_p}$, the ratios change of density and velocity can be expressed in terms of P-wave impedance, such as:

$$\frac{\triangle\rho}{\rho}=\frac{1}{5}\left(4\frac{\triangle\rho}{\rho}+\frac{\triangle\rho}{\rho}\right)=\frac{1}{5}\left(\frac{\triangle V_p}{V_p}+\frac{\triangle\rho}{\rho}\right)=\frac{1}{5}\frac{\triangle I}{I}. \tag{39}$$

where $I=\rho V_s$ （$\frac{\triangle I}{I}=\frac{\triangle V_p}{V_p}+\frac{\triangle\rho}{\rho}$） is the impedance for P-wave.

The approximation can also be written as a function of the ratio change of P-wave and S-wave impedances using this relationship.

Normalized approximations

Donati approximation (1998)

Considering the Snell's Law $\sin\varphi = \frac{V_s}{V_p}\sin\theta$, $\cos\varphi = 1-\frac{1}{2}\left(\frac{V_s}{V_p}\right)^2 \sin^2\theta$, and trigonometry $\tan\varphi \simeq \frac{V_s}{V_p}\sin\theta$ (when the incident angle is less than 30 degree), Donati (1998) demonstrated a normalized expression to P-SV wave reflection coefficient based on Aki-Richards approximation:

$$R'_{ps}(\theta) = \frac{R_{ps}(\theta)}{\sin\theta} \simeq A + B\cos\theta + C\cos^2\theta \tag{40}$$

where

$$A = -\frac{1}{2}\left(\left(1-2\left(\frac{V_s}{V_p}\right)^2\right)\frac{\Delta\rho}{\rho} - 4\left(\frac{V_s}{V_p}\right)^2\frac{\Delta V_s}{V_s}\right). \tag{41}$$

$$B = -\frac{1}{2}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_s}{V_s}\right)\left(2\frac{V_s}{V_p} - \left(\frac{V_s}{V_p}\right)^3\right). \tag{42}$$

$$C = -\left(\frac{V_s}{V_p}\right)^2\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_s}{V_s}\right). \tag{43}$$

The reflection coefficient for converted wave can be written as a linear equation if the truncation was considered from the second-order term,

$$R'_{ps}(\theta) = \frac{R_{ps}(\theta)}{\sin\theta} \simeq A + B\cos\theta . \tag{44}$$

where A and B terms are related to relative contrast of density ($\Delta\rho/\rho$) and rigidity ($\Delta\mu/\mu$), the velocity ratio between P-wave and S-wave (V_p/V_s) , and A can be simplified as (Donati et al., 1998):

$$A = \left(\frac{V_s}{V_p}\right)^2\frac{\Delta\mu}{\mu} - \frac{\Delta\rho}{\rho} . \tag{45}$$

Other than Aki-Richards's approximation (1980), Donati's approximation (1998) gives the emphasis on the influence of combination of elastic properties in successive ranges of incident angle. Two seismic sections, the ratio of rigidity and P-wave and S-wave velocities relative contrast, can be obtained using Donati's approximation (1998) for converted wave AVO analysis.

Sun's approximation with Intercept-gradient (2003)

Intercept and gradient might be a significant indicator in AVO interpretation, which was introduced into P-P wave AVO analysis by Castagna et al. (1998). Based on that, Sun et al. (2003) normalized the approximation for P-SV reflection coefficient derived by Zheng (1991) and proposed the intercept-gradient attribute of P-SV wave. Then, a normalization of P-SV wave reflection coefficient can be processed if the incident angle is not zero,

$$R'_{ps} = R_{ps} / \sin\theta = P_{ps} + G_{ps} \sin^2\theta . \tag{46}$$

where P_{ps} is the pseudo-intercept of P-SV wave and can be calculated by $P_{ps} = -2\gamma \frac{\Delta V_s}{V_s} - \left(\frac{1}{2} + \gamma\right)\frac{\Delta\rho}{\rho}$, G_{ps} is the pseudo-gradient of P-SV wave which can obtained by $G_{ps} = \left(2\gamma^2 + \gamma\right)\frac{\Delta V_s}{V_s} + \left(\frac{3}{4}\gamma^2 + \frac{1}{2}\gamma\right)\frac{\Delta\rho}{\rho}$, and $\gamma = \frac{V_s}{V_p}$.

There is much information we can obtain from equation (46). For example, if we assume the ratio change of density $\frac{\Delta\rho}{\rho}$ approaches zero, the ratio of gradient and intercept can be expressed as

$$\frac{G_{ps}}{P_{ps}} = -\frac{1+2\gamma}{2} \tag{47}$$

We can find that the ratio of gradient and intercept is the function of the ratio of P-wave and S-wave velocities. Then, it can be used to the inversion of $\frac{V_s}{V_p}$. And the intercept of P-SV wave is only influenced by the normalized S-wave velocity if the ratio of P-wave and S-wave velocities is considered as a constant.

CONCLUSIONS

Converted P-SV wave AVO analysis plays an increasingly important part in lithological exploration and might provide additional information because of better signal-to-noise ratio of S-wave at times. Beyond that, improved prediction can be achieved by combining P-P and P-SV wave inversions. In this paper, we reviewed many of most recent research accomplishments in AVO analysis of converted wave since the approximations were given by Bortfeld (1961) and Aki and Richards (1980), according to their properties and the approximate way. The key features of those approximations and the most current achievements in converted P-SV wave AVO analysis are summarized, but we make no claim that the review is exhaustive. However, as an introduction, this review could be helpful for understanding the theoretical developments of converted P-SV wave AVO analysis.

ACKNOWLEDGEMENTS

I would like to thank the sponsors of the CREWES Project for their continued support, and Dr. Innanen for his excellent suggestions and encouragement.

REFERENCES

Aki, K. and Richards, P.G., 1980, Quantitative seismology: Theory and methods: W.H. Freeman and Co., v.1.

Bortfeld, R., 1961, Approximation to the reflection and transmission coefficients of plane longitudinal and transverse wave: Geophysical Prospecting, **9**, No. 4, 485-502

Donati, M. and Martin, N., 1998, Making AVO analysis for converted waves a practical issue: 68th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

Caldwell, J., 1999, Marine multicomponent seismology: The Leading Edge, **18**, 1274-1282.

Gardner, G. H. F., Gardner, L. W., and Gregory, A. R., 1974, Formation velocity and density—The diagnostic basis for stratigraphic traps: Geophysics, **39**, 770–780.

Gonzalez E. F., 2000, Facies classification using P-to-P and P-to-S AVO attributes: 70th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1-14.

Li, Z. and Hu, G., 1996, P-SV wave AVO analysis: Journal of Chengdu Institute of Technology, **23**, No. 4, 73-79.

Larsen, J. A., 1999, AVO inversion by simultaneous PP and PS inversion: M.Sc. thesis, Univ. of Calgary.

Mallick, S., 1993, A simple approximation to the P-wave reflection coefficient and its implication in the inversion of amplitude variation with offset data: Geophysics, **58**, 544-552.

Mavko, G., Mukerji, T., and Dvorkin, J., 1998, The rock physics handbook: Tools for seismic analysis for porous media: Cambridge University Press.

Richards, P. G. and Frasier, C. W., 1976, Scattering of elastic waves from depth-dependent inhomogeneities: Geophysics, **41**, 441-458.

Ramos, A.C.B. and Castagna, J.P., 2001, Useful approximations of converted-wave AVO: Geophysics, **66**, 1721-1734.

Swan, H. W., 1993, Properties of direct AVO hydrocarbon indicators, *in* Castagna, J. P., and Backus, M. M., Eds., Offset-dependent reflectivity-Theory and practice of AVO analysis: Soc. Expl. Geophys., 78–92.

Stewart, R. R., Gaiser, J. E., Brown, R. J., and Lawton, D. C., 2003, Tutorial Converted-wave seismic exploration: Applications: Geophysics, **68**, No. 1, 40-57.

Sun, P., Sun, J. and Lu, X., 2003, Progress in research on the method of P-SV wave AVO: Progress In Geophysics, **18**, No. 4, 602-607.

Sun, P., Sun, J. and Lu, X., 2006, P-SV wave reflection coefficient approximations and characteristics of AVO attributes: ACTA Geoscientica Sinica, **27**, No. 1, 85-89.

Thomsen, L., 1999, Converted-wave reflection seismology over inhomogeneous anisotropic media: Geophysics, **64**, 678-690.

Vant, A. and Brown R.J., 2001, Approximating the P-S reflection coefficient for small incidence angles: CREWES Research Report, **13**, 159-179.

Wang, P. and Hu, T., 2011, AVO approximation for PS-wave and its amplification in PP/PS joint inversion: Applied Geophysics, **8**, No. 3, 189-196.

Yang, H. and Yin, K., 1996, P-SV reflection coefficient approximation using shear modulus and its significance in multiwave AVO inversion: Oil Geophysics Prospecting, **31**, No. 3, 374-381.

Zoeppritz, K., 1919, Erdbebenwellen VIII B, Uber Reflexion and Durchgang Seismischer Wellen Duch Unstetigkeitsflachen: Gottinger Nachr., **1**, 66-84.

Zheng, X., 1991, Approximations of Zoeppritz equations and its applications: Oil Geophysics Prospecting, **26**, No. 2, 129-144.

Zhou, Z., 1993, AVO analysis of P-SV and SH waves: Oil Geophysics Prospecting, **28**, No. 4, 430-438.