# RefMod: software program for reflectivity modeling

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## ABSTRACT

RefMod is a software program designed to generate body-wave synthetic seismograms based on the reflectivity method. The model consists of flat elastic isotropic layers, with a point source (explosion) on the surface and receivers on the surface. The software generates two-component seismograms: vertical and horizontal components. The reflectivity method of modeling has a number of advantages. It generates high-frequency synthetics, and includes inter-bed multiples and mode-conversions. Attenuation is easily incorporated by using complex velocities. The method allows the user to turn 'On' and 'Off' the effects of multiples and mode-conversions, allowing easier interpretation of different contributions. The current implementation does not include free-surface effects.

The software is developed in JAVA programing language, which makes it platform independent: Windows, Mac OS, Linux, and UNIX. In order to run the software, the user must install Java Virtual Machine (JVM) from the Oracle website. It is multi-threaded, i.e. one can use the power of multicore processors. The generated vertical and horizontal component shot gathers can be exported as SEGY files.

### INTRODUCTION

The reflectivity method of seismic modeling (Kennett, 1983) is a powerful algorithm for synthetic seismogram generation in a stratified media. It has a number of advantages: generates a broad-band seismic seismograms; inter-bed multiples are included in the solution; mode-conversions are included in the solution; free-surface effects can be included; attenuation is easily incorporated by using complex velocities; effects like inter-bed multiples and mode-conversions can be turned 'On' and 'Off' for easier interpretation of the various contributions.

RefMod is a software program designed to generate body-waves synthetic seismograms, based on the reflectivity method. The software is developed in JAVA programing language, which makes it platform independent: Windows, Mac OS, Linux, and UNIX. In order to run the software, the user must install Java Virtual Machine (JVM) from the Oracle website. It is multi-threaded, i.e. one can use the power of multicore processors. The software generates vertical and horizontal component shot gathers, which can be exported as SEGY files.

# THEORY

The reflectivity method theory is discussed in details by Kennett (1983). The software development follows closely the tutorial published by Muller (1985).

The earth model consists of a stack of N flat, elastic layers (Figure 1). Each layer is described by its elastic properties: Vp, Vs, and density, and its thickness h. Attenuation is implemented by adding a constant Q and computing the corresponding complex

velocities (Muller, 1985). The source is a point source (explosion) located on the surface. The receivers are located on the surface as well. The layers are numbered from 0 to N-1.



Figure 1: The reflectivity modeling method computes synthetic seismogram in a stratified earth model.

At the center of the method is the computation of the overall *reflectivity matrix*  $\mathbf{R}$  of the stack of layers. The last layer N-1 is infinite (half space). It is assumed that there is no up-going wavefield at the last interface, between the half-space N-1 and layer N-2.

The computation of the reflectivity matrix **R** is done in frequency-slowness  $(\omega - p)$  domain. The basic steps of the algorithm are:

- For each frequency  $\omega$ 
  - For each slowness p
    - Compute  $\mathbf{R}(\omega, p)$
  - Integrate (sum) over *p*
  - Multiply with source spectrum S(ω)
- Inverse Fourier transform

The reflectivity matrix at the bottom of a particular layer is defined as a *base matrix* **BM**. It is a 4x4 matrix, containing the P-P, P-S, S-P, and S-S reflectivity.

$$\mathbf{BM} = \begin{bmatrix} BM - PP & BM - SP \\ BM - PS & BM - SS \end{bmatrix}$$

The reflectivity matrix at the top of a particular layer is defined as a *top matrix* **TM**. It is a 4x4 matrix, containing the P-P, P-S, S-P, and S-S reflectivity.

$$\mathbf{TM} = \begin{bmatrix} TM - PP & TM - SP \\ TM - PS & TM - SS \end{bmatrix}$$

The base matrix **BM** is propagated to the top of the layer, the top matrix **TM**, by a *phase shift matrix*  $\mathbf{E}$ , defined by:

$$\mathbf{E} = \begin{bmatrix} \exp(-iq_{p}\omega h) & 0\\ 0 & \exp(-iq_{s}\omega h) \end{bmatrix}$$

where:

 $q_p$  – P-wave vertical slowness

 $q_s$  – S-wave vertical slowness

$$h$$
 – layer thickness

The propagation equation is defined as:

$$[\mathbf{T}\mathbf{M}] = [\mathbf{E}][\mathbf{B}\mathbf{M}][\mathbf{E}]$$
(1)

At an interface between a layer n and n+1 one can compute the base matrix **BM** in layer n, knowing the top matrix **TM** in layer n+1 using the *potential reflection and transmission coefficients* at the interface. At the interface, there are two set of reflection and transmission coefficient: for the down-going wavefield and for the up-going wavefield. One can define the following 4x4 matrices:

• Down-going wavefield reflection coefficients, RD

$$\mathbf{RD} = \begin{bmatrix} RD \_ PP & RD \_ SP \\ RD \_ PS & RD \_ SS \end{bmatrix}$$

• Down-going wavefield transmission coefficients, **TD** 

$$\mathbf{TD} = \begin{bmatrix} TD \_ PP & TD \_ SP \\ TD \_ PS & TD \_ SS \end{bmatrix}$$

• Up-going wavefield reflection coefficients, RU

$$\mathbf{RU} = \begin{bmatrix} RU PP & RU SP \\ RU PS & RU SS \end{bmatrix}$$

• Up-going wavefield transmission coefficients, TU

$$\mathbf{TU} = \begin{bmatrix} TU \ PP & TU \ SP \\ TU \ PS & TU \ SS \end{bmatrix}$$

Muller (1985) provides the equations for computing the above matrix entries.

The base matrix is computed by the following equation:

$$[\mathbf{B}\mathbf{M}]_{n} = [\mathbf{R}\mathbf{D}] + [\mathbf{T}\mathbf{U}] \bullet ([\mathbf{I}] - [\mathbf{T}\mathbf{M}]_{n+1} \bullet [\mathbf{R}\mathbf{U}])^{-1} \bullet [\mathbf{T}\mathbf{M}]_{n+1} \bullet [\mathbf{T}\mathbf{D}]$$
(2)

where I is the identity matrix.

Assuming no up-going wavefield at the last interface:

 $TM_{N-1} = 0$ , then  $BM_{N-2}=RD$ .

Then by recursively applying equations (1) and (2) one can compute the reflectivity matrix at the top of the first layer,  $TM_0$ . The matrix is the overall reflectivity of the stratified media, i.e.  $R = TM_0$ .

The displacement amplitudes u of the horizontal and the vertical components as a function of  $\omega$  and offset x are computed by:

$$\binom{u_H}{u_V} = -2\omega^2 v_{p0} F(\omega) \left( \sum_p \mathbf{JURS}_1 + \sum_p \mathbf{JURS}_2 \right)$$
(3)

where:

-  $v_{p0}$  – P-wave velocity in the first layer

-  $F(\omega)$  – source spectrum

- 
$$\mathbf{J} = \begin{bmatrix} -J_1(p \, ox) & 0\\ 0 & iJ_0(p \, ox) \end{bmatrix}$$
 - matrix with Bessel function elements  
-  $\mathbf{U} = \begin{bmatrix} p & q_{s0}\\ q_{p0} & -p \end{bmatrix}$   
-  $\mathbf{S}_1 = \begin{bmatrix} (ip^3)/q_{p0}\\ -ip^2 \end{bmatrix}$ 

- 
$$\mathbf{S}_2 = \begin{bmatrix} ipq_{p0} \\ ip^2 \end{bmatrix}$$

Equation (3) is a modified version of equation (94) in Muller (1985), assuming an explosion point source on the surface.

The final step in the algorithm is inverse Fourier transform of the computed horizontal and vertical component values to obtain amplitude displacement synthetic seismograms.

## **IMPLEMENTATION**

RefMod is a software program which implements the reflectivity algorithm. It is developed in JAVA programing language. Figure 2 is a screen shot of the software interface.



Figure 2: RefMod software interface.

The *Import* button allows the user to import ASCII file well logs containing Vp, Vs, and Density elastic properties. Two wavelets are supported: zero phase Ricker and

minimum phase. The wavelet is chosen in the *Wavelet* tab. *NS Out* text field specifies the length (number of samples) of the output seismograms. The sampling rate is 1 ms. *Num w* text field is the number of frequencies from zero to Nyquist. *Num Offsets* and *Max Offset* specify the offset range. *Num p* and *Max p* is the range of the slowness parameter. *Qp* and *Qs* are the P-wave and S-wave attenuation factors. *Max Freq* check box allows the user to limit the computation to the specified maximum frequency in Hz. *Mult Interbed* and *Mode Conv* check boxes allow the user to turn "On" and "Off" the inter-bed multiples and mode conversions. The *Phase Rot* check box applies a 90 degree phase shift to the data.

*Color Scaler* controls the intensity of the color plot. The maximum amplitude of the generated synthetics is computed and scaler\*MaxAmplitude is used for display clipping. The use may apply  $t^n$  amplitude gain to the plot. The text field dx and dy control the plot width and height. The radio buttons *SynthV* and *SynthH* are used to switch the plot from vertical to horizontal components. The *ScrShot* button allows the user to save a screen shot as a PNG file.

The *Export* button allows the user to export the generated synthetic seismograms as SEGY files, one containing the vertical component and the other containing the horizontal component. The SEGY file format is IEEE.

# CONCLUSIONS

The reflectivity modeling method is a powerful algorithm for generating synthetic seismograms in stratified media. It has been implemented in a modern programing language, JAVA, which is platform independent and supports rich graphics user interface and multicore processing. Software development is an evolutionary process and new releases of improved new versions of the software are planned.

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### REFERENCES

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