# The FOCI method versus other wavefield extrapolation methods

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# Motivation

To compare the *forward operator and conjugate inverse* (FOCI) method for calculating wavefield extrapolators with

- the Hale (1991) method
- the *weighted least square* (WLSQ) method (Thorbecke et al., 2004)

# Outline

- Brief review of the theory of Hale’s extrapolator
- Brief review of the theory of  WLSQ’s extrapolator
- Comparisons of the three extrapolators:
    - Amplitude spectra
    - Phase errors
    - Impulse responses
    - and prestack depth migrations of the Marmousi dataset using Hale’s, WLSQ’s, and FOCI’s extrapolators

# Wavefield extrapolation methods:

- Are more powerful in handling strong lateral velocity variations than ray theory based methods

- Have two major problems:

  - Computationally expensive
  - Instability of the extrapolation operator

# Wavefield extrapolation methods

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\frac{\partial^2 \bar{\tilde{\psi}}}{\partial z^2} = -k_z{}^2 \bar{\tilde{\psi}}$$

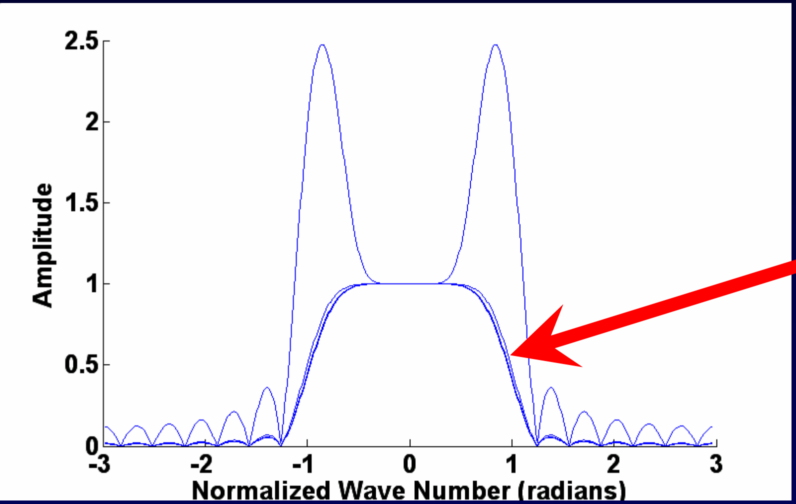
where

$$k_z{}^2 = \frac{\omega^2}{v^2} - k_x{}^2$$

$$\bar{\tilde{\psi}}\left(k_x, z, \omega\right) = \bar{\tilde{\psi}}\left(k_x, z=0, \omega\right) e^{ik_z z}$$

# Hale's extrapolator (Hale, 1991)

$$\mathbf{D(k) = e^{ik_z \Delta z}},$$




$$\tilde{\mathbf{W}}(\mathbf{k}) = \sum_{m=0}^{M-1} \mathbf{c}_m \mathbf{B}_m(\mathbf{k})$$

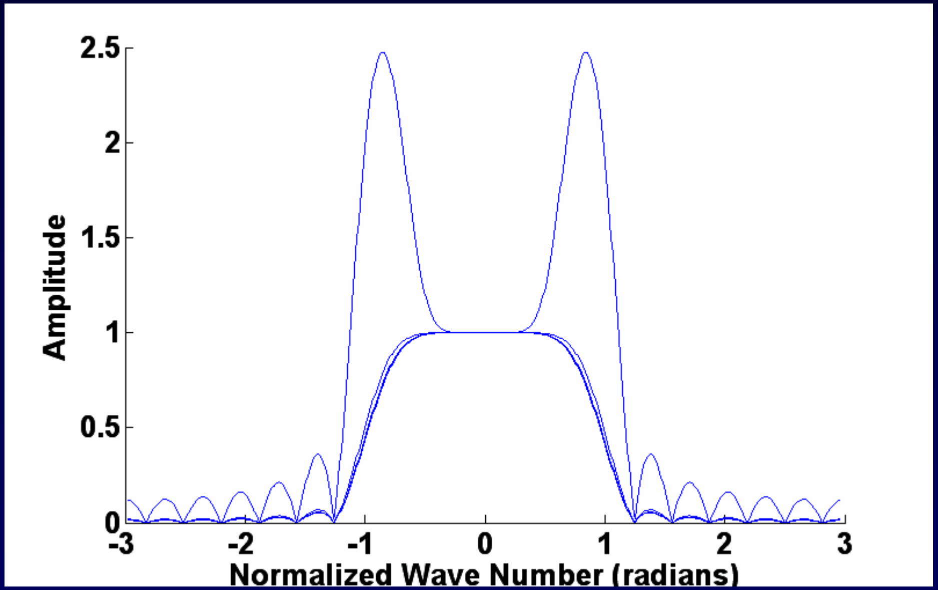
Basis function

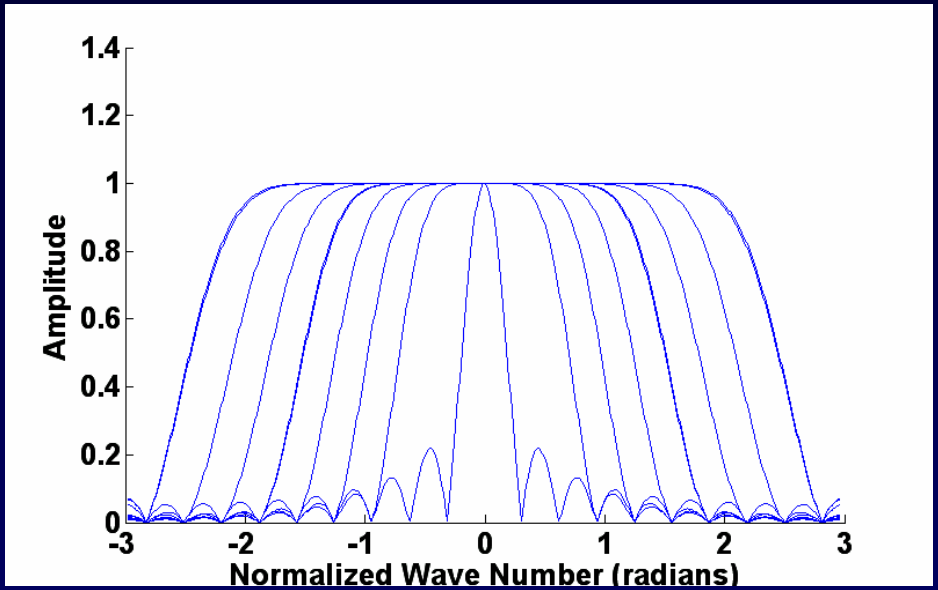
$$M < (N+1)/2$$

**N operator length**

**M number of basis functions**

# Hale’s extrapolator



# WLSQ's extrapolator (Thorbecke et al., 2004)

$$\tilde{\mathbf{W}}\left(\mathbf{m}\Delta\mathbf{k_x}\right) = \Delta\mathbf{x}\sum_{n=-N}^{n=N}\mathbf{exp}\left(\mathbf{im}\Delta\mathbf{k_x}\mathbf{n}\Delta\mathbf{x}\right)\mathbf{W}\left(\mathbf{n}\Delta\mathbf{x}\right)$$

$$\tilde{\mathbf{w}} = \boldsymbol{\Gamma}\mathbf{w}$$

$$\langle\mathbf{w}\rangle = \left[\boldsymbol{\Gamma}^{\mathbf{H}}\tilde{\boldsymbol{\Lambda}}\boldsymbol{\Gamma}\right]^{\mathbf{-1}}\boldsymbol{\Gamma}^{\mathbf{H}}\tilde{\boldsymbol{\Lambda}}\tilde{\mathbf{w}}$$

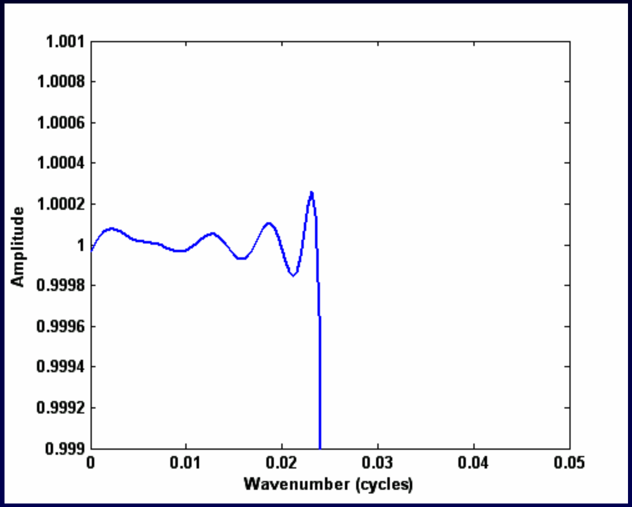
where,

$$\tilde{\Lambda} = \textbf{weight function}$$

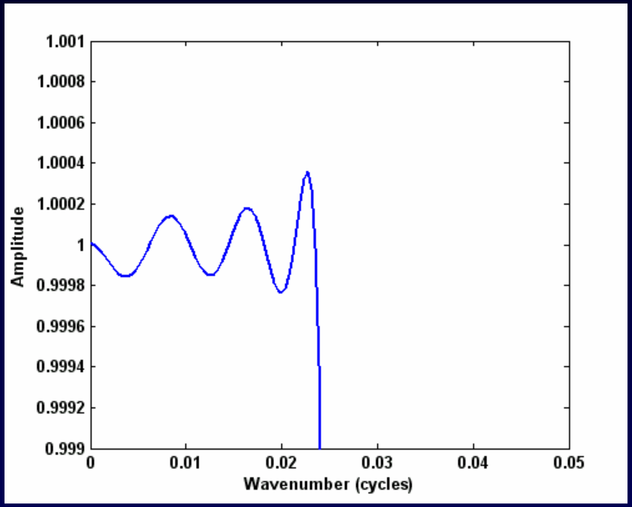
# WLSQ’s extrapolator

## v=2000 m/s and frequency=50 Hz

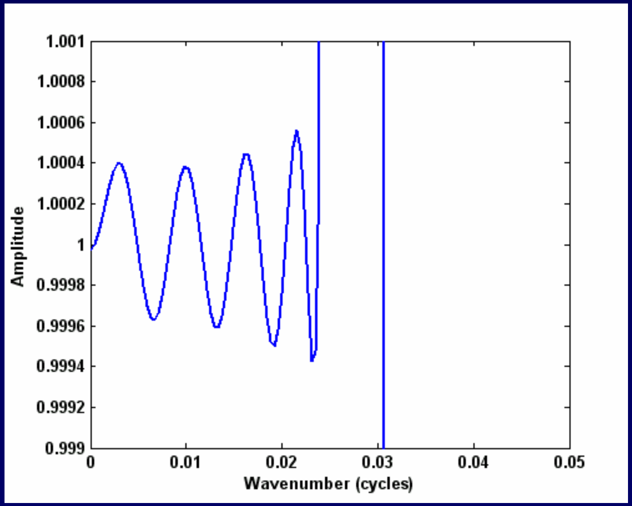
dx=10m, dz=2m, and N=25



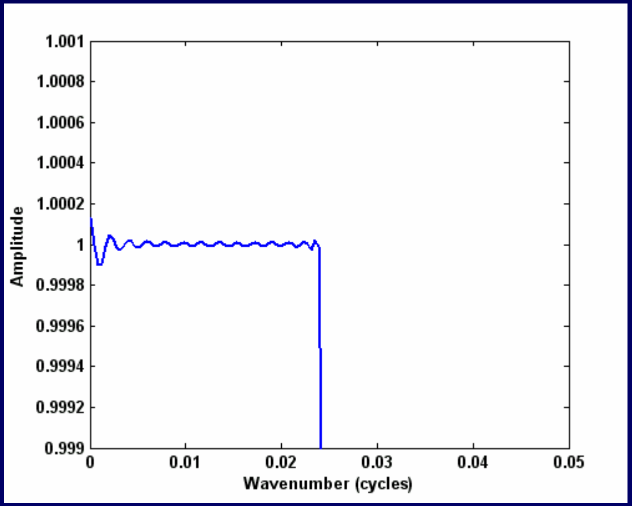

dx=10m, dz=2m, and N=19




dx=10m, dz=10m, and N=19



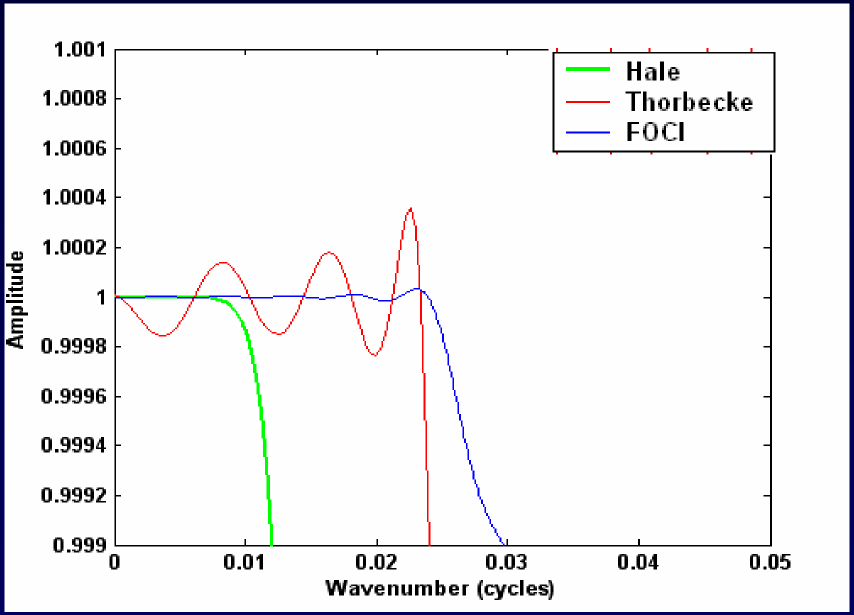

dx=10m, dz=10m, and N=101



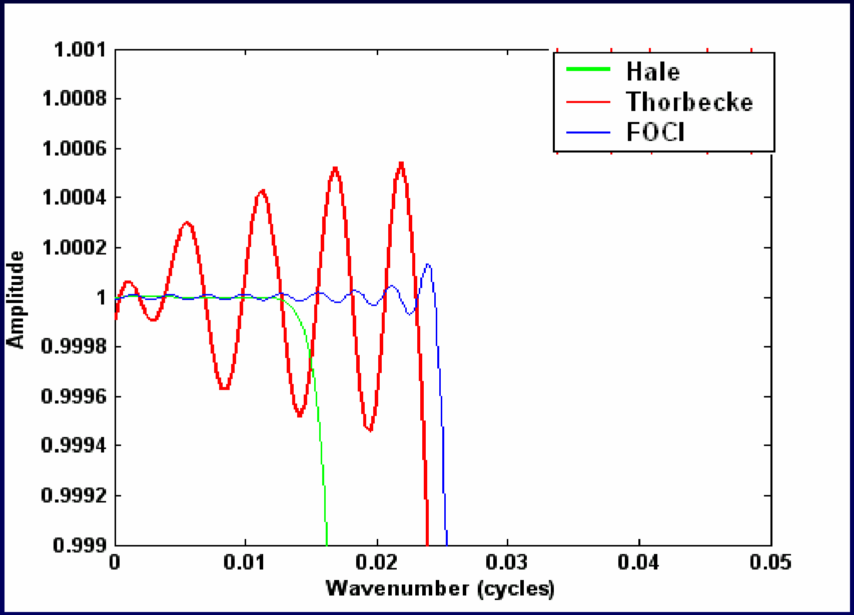
# Amplitude Spectra of Hale's, WLSQ's, and FOCI's extrapolators

v=2000 m/s and frequency=50 Hz

dx=10m, dz=2m, and N=19



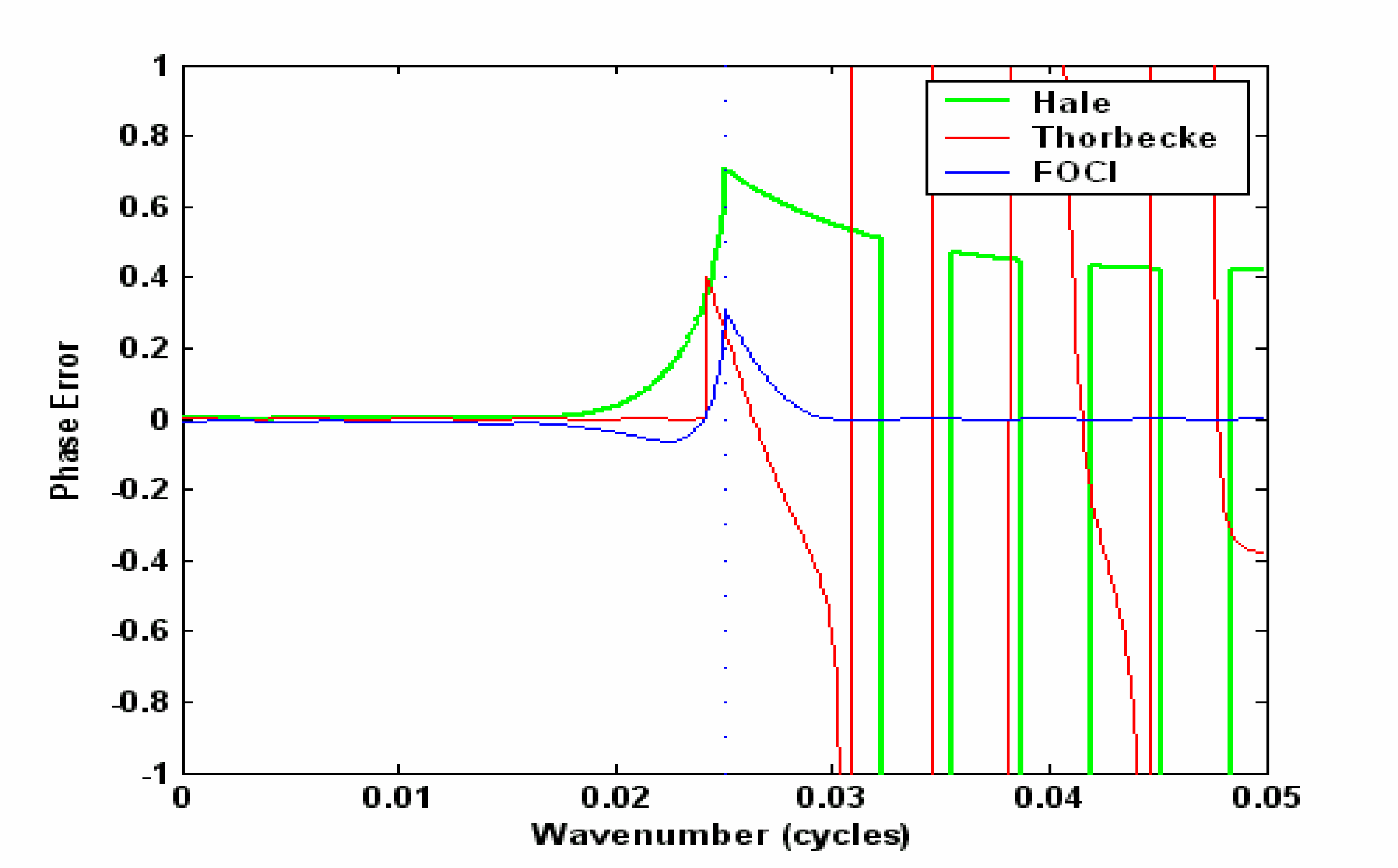

dx=10m, dz=10m, and N=31



# Phase error of Hale's, WLSQ's, and FOCI's extrapolators

v=2000 m/s and frequency=50 Hz

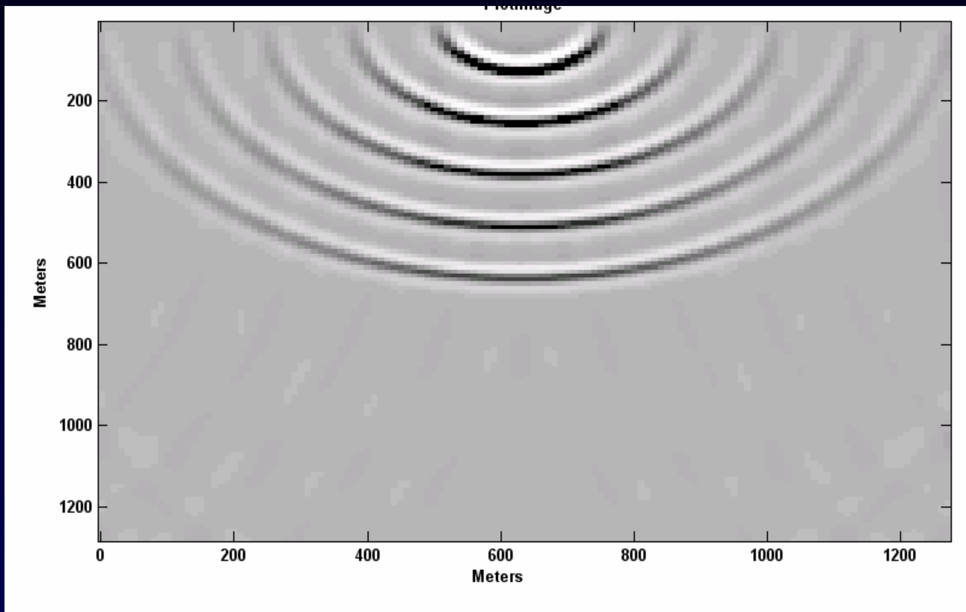
dx=10m, dz=10m, and N=31



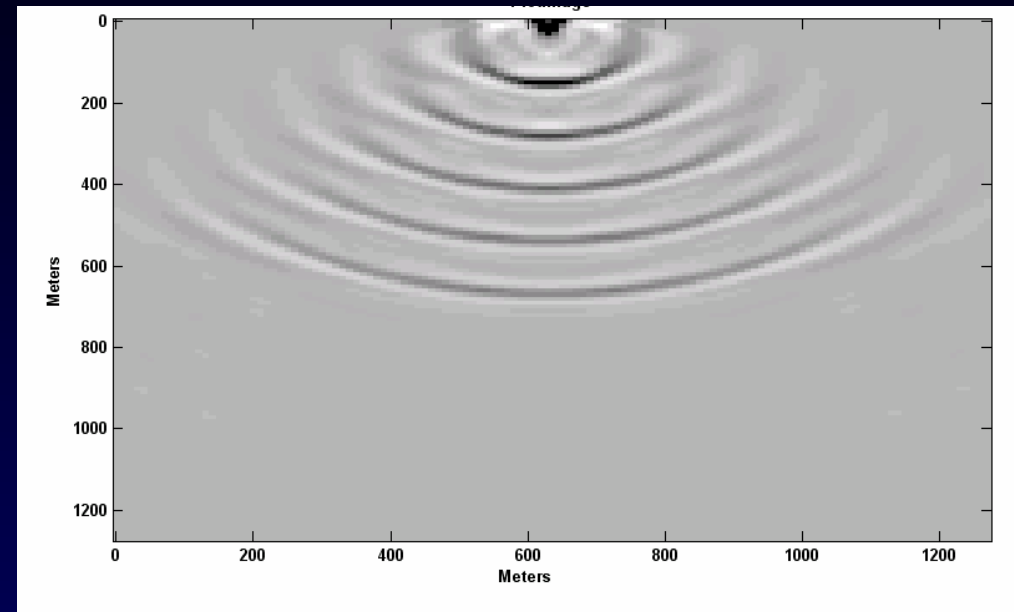
# Impulse responses

## N=31 velocity=2000 m/s

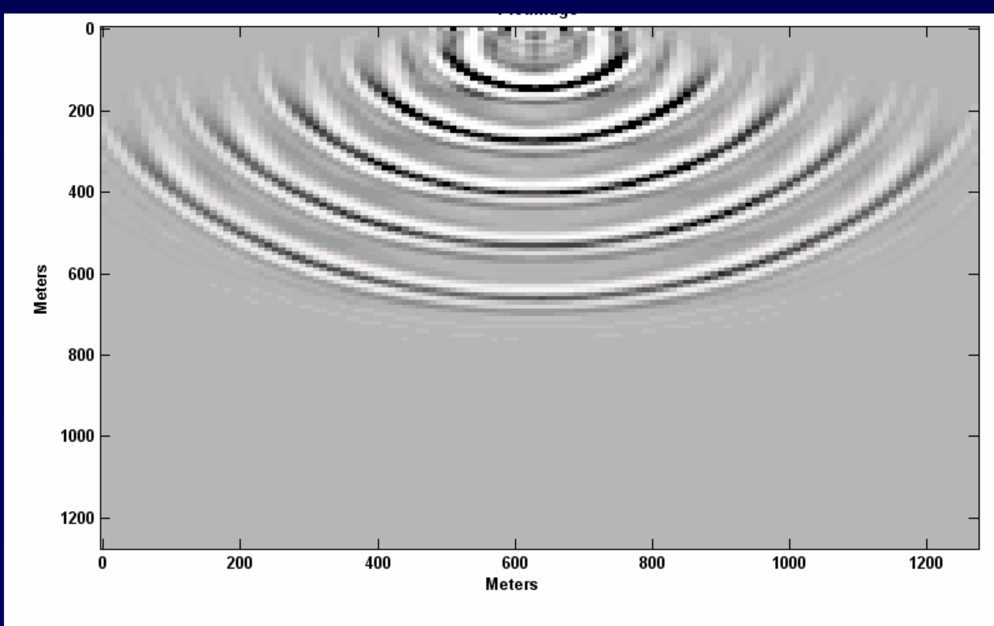
### Phase-shift



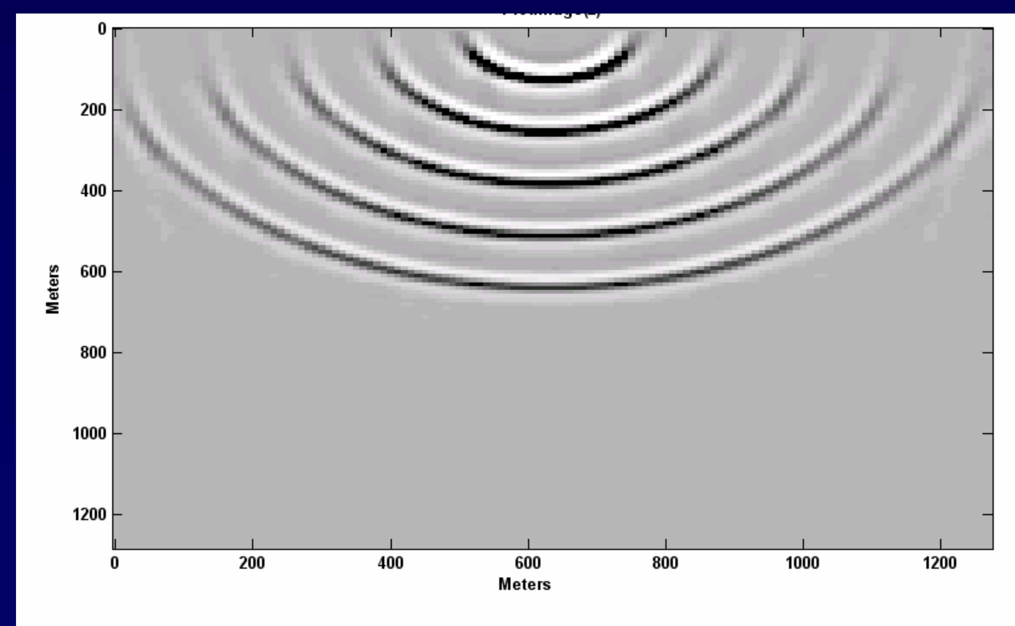

### Hale



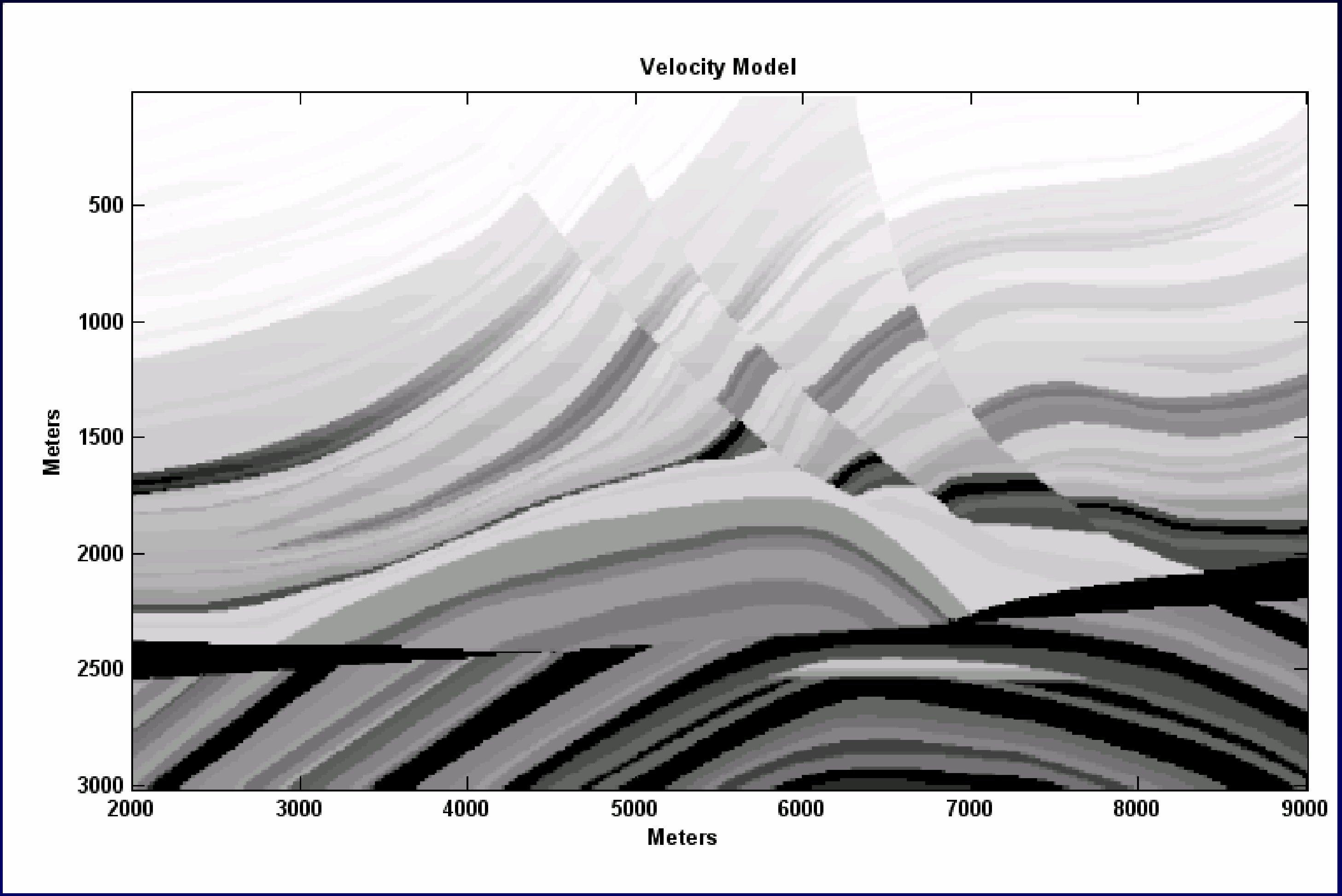

### WLSQ




### FOCI



# Marmousi Prestack Depth Migrations

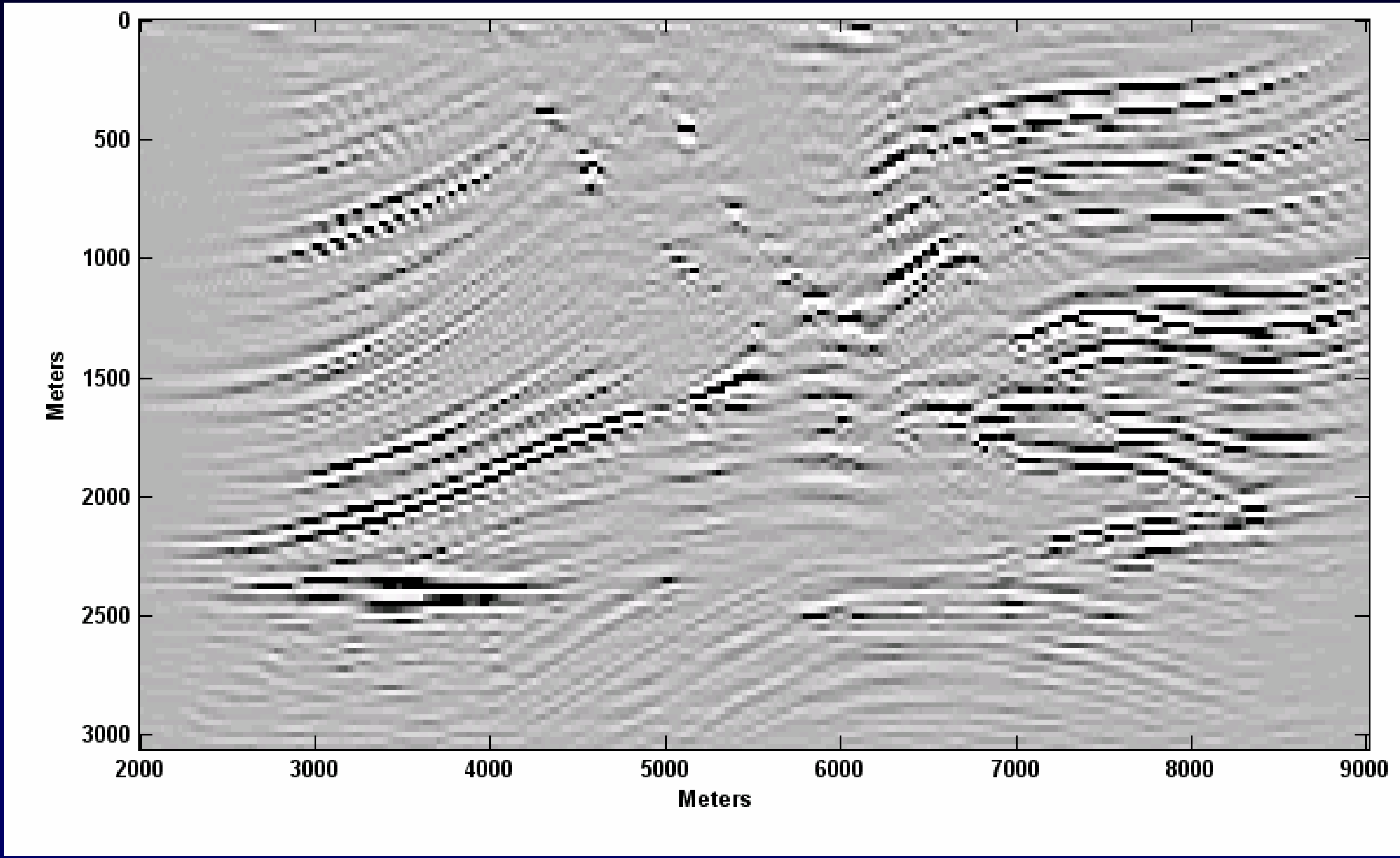


# Hale’s and FOCI’s extrapolators

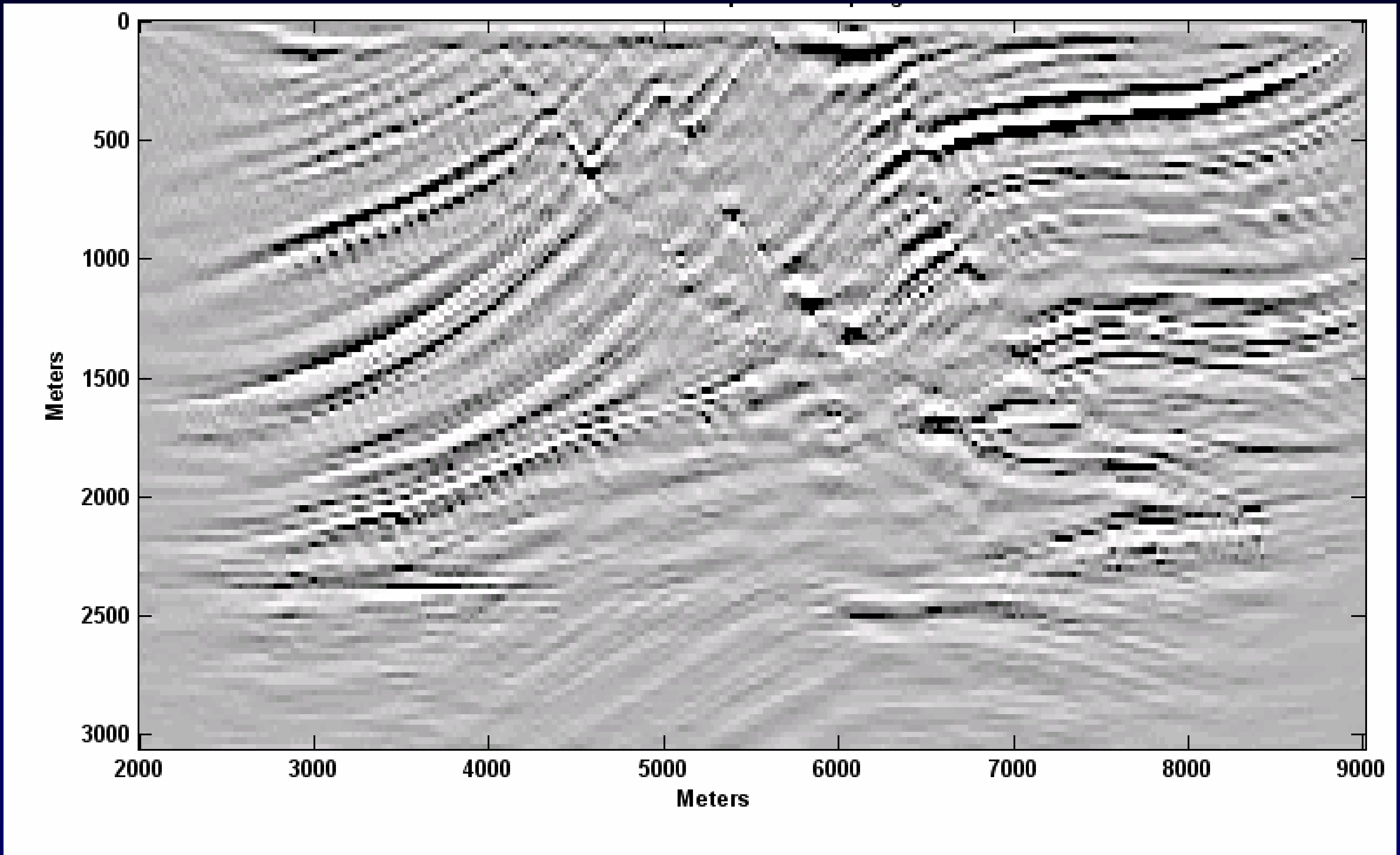
dx=25 m
dz=25 m

operator length= 19 points

# Hale's extrapolator
# run time=3.5 hours



# FOCI's extrapolator run time=2.0 hours

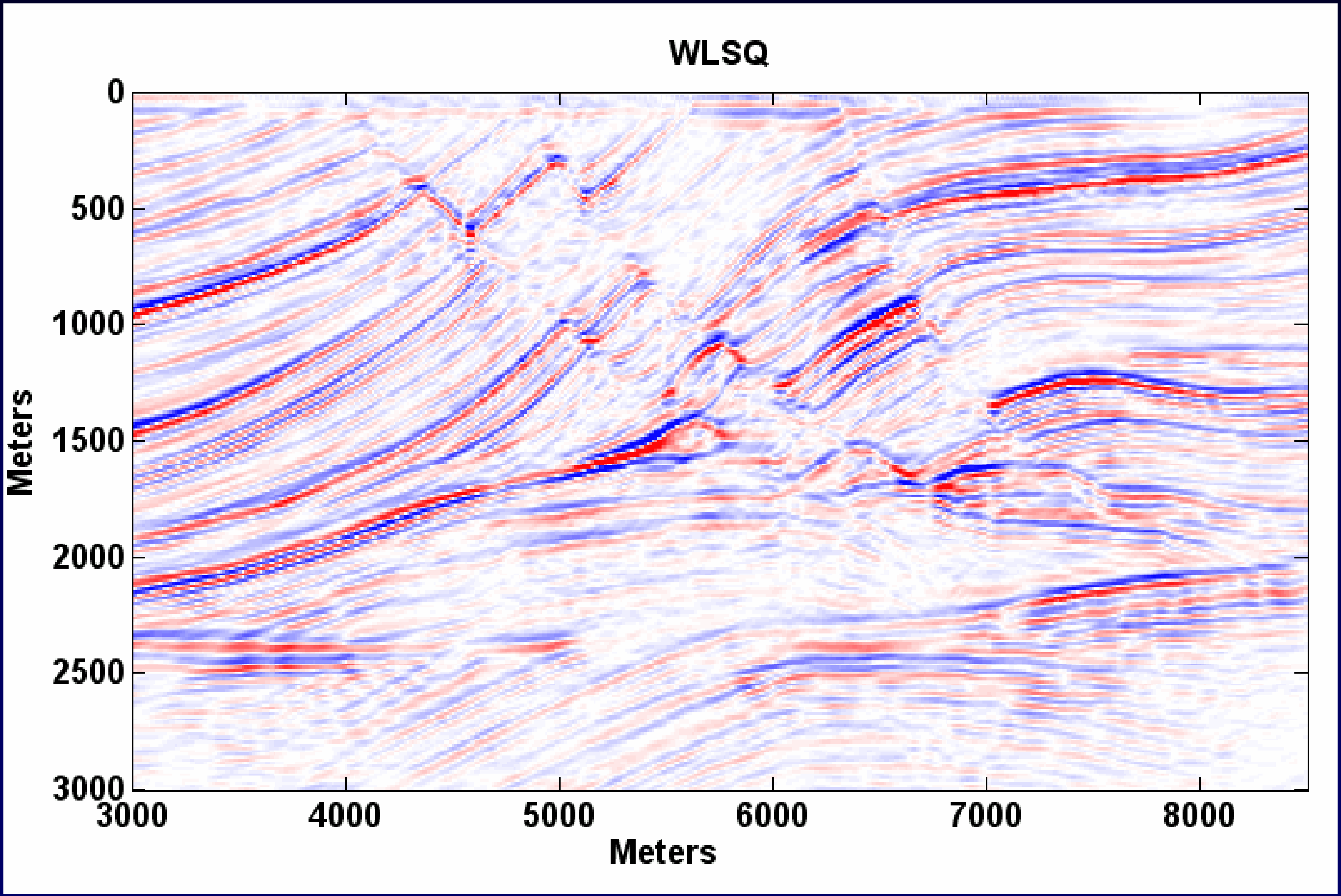


# WLSQ’s and FOCI’s extrapolators

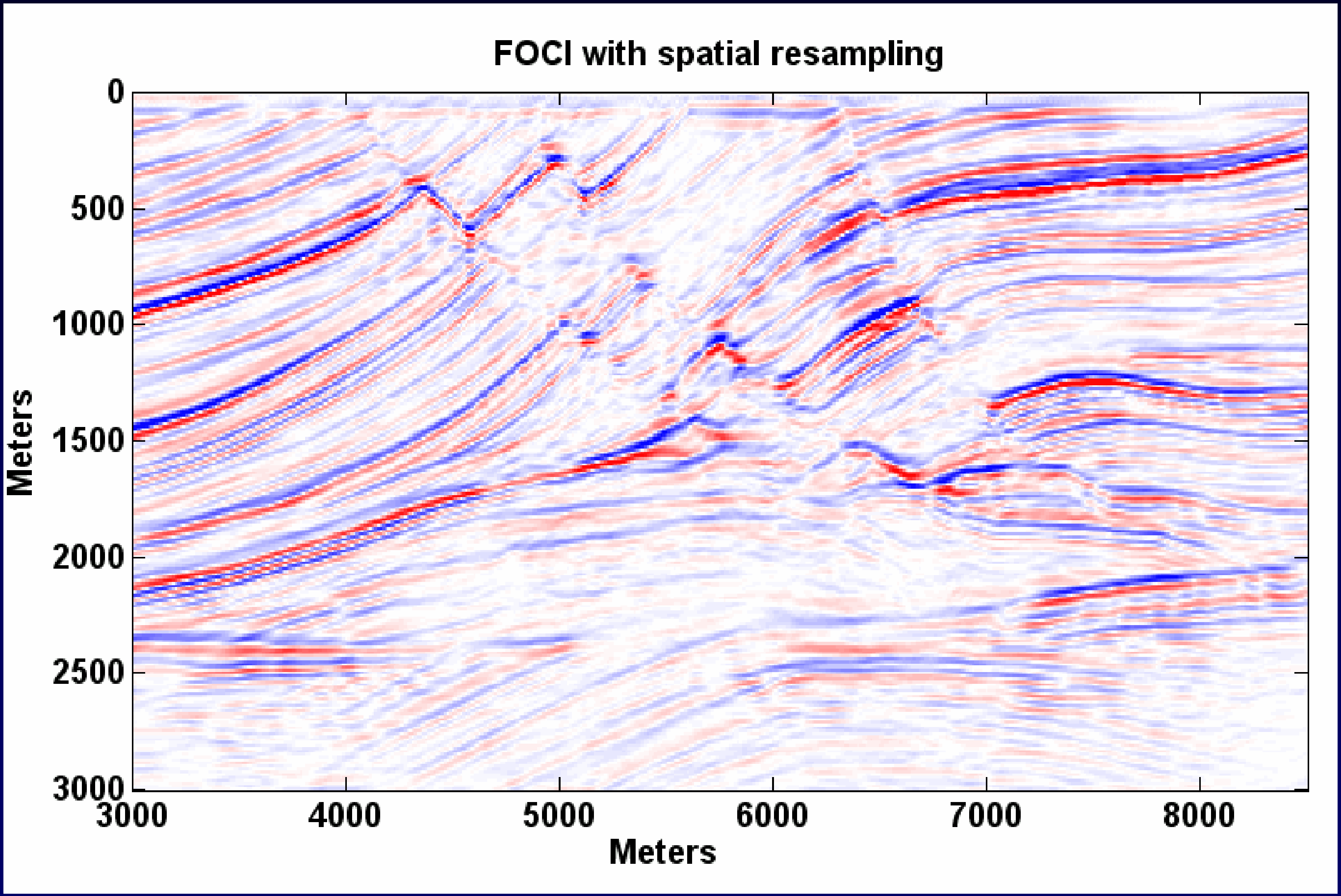
dx=12.5 m
dz=12.5 m

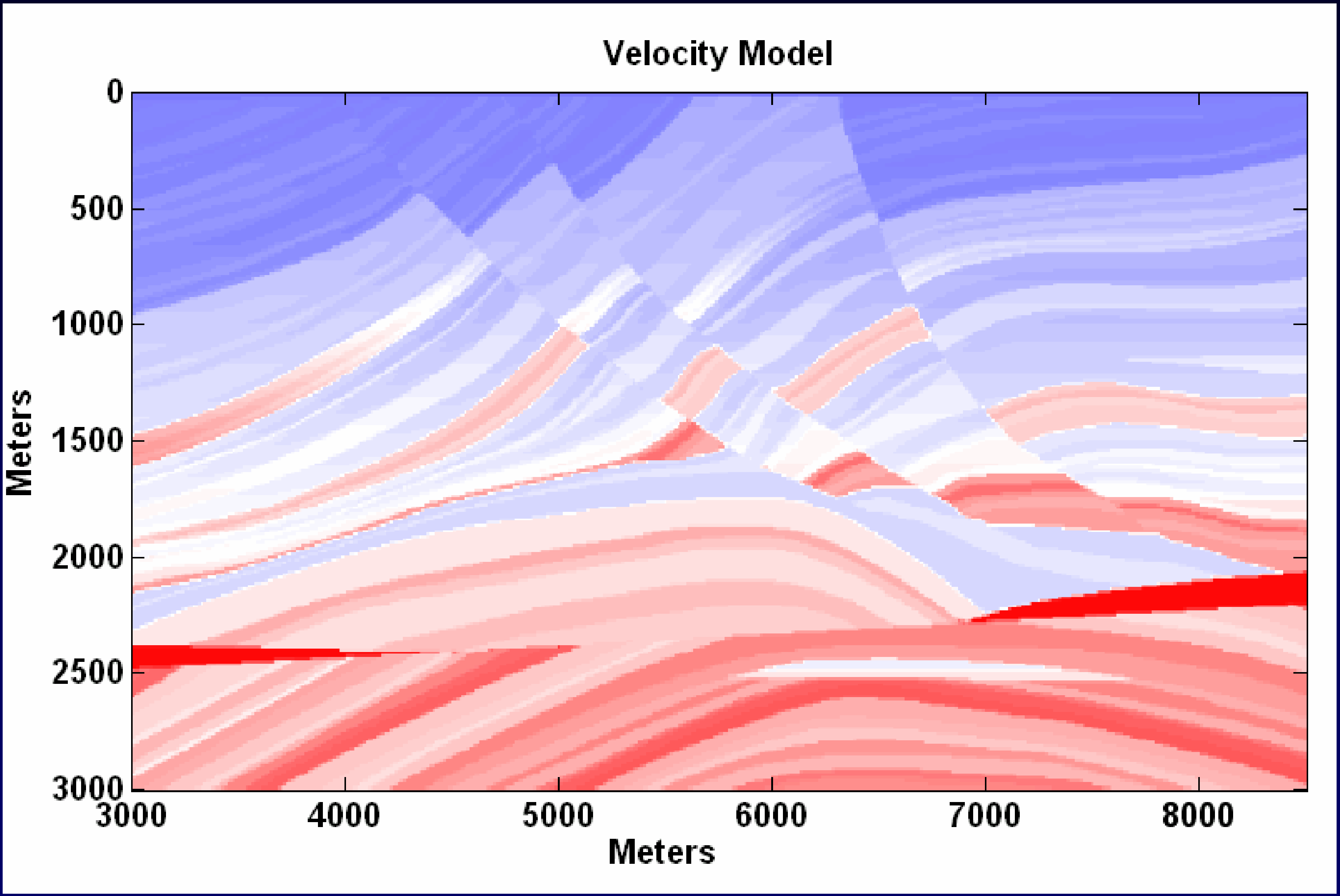
operator length= 51 points

# WLSQ's extrapolator
# Run time=16 hours



# FOCI's extrapolator
# Run time=12 hours





Velocity Model
0
500
1000
1500
2000
2500
3000
Meters
3000
4000
5000
6000
7000
8000
Meters

# Conclusions

- FOCI results are comparable with Hale's and WLSQ's results.
- FOCI is computationally more efficient than the other methods due to spatial resampling.
- Spatial resampling can not be easily implemented in the other methods.
- This new method is a promising technique for seismic imaging.

# Acknowledgments

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