Locating microseismic events and traveltime mapping using locally spherical wavefronts

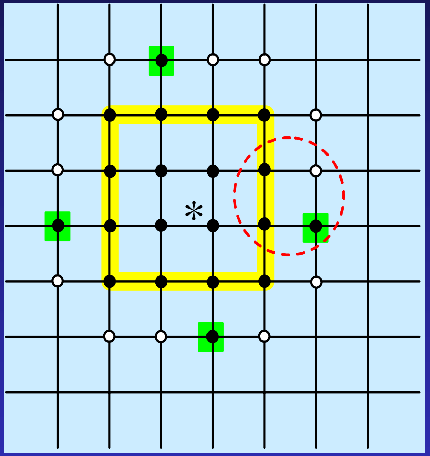
John C. Bancroft and Xiang Du

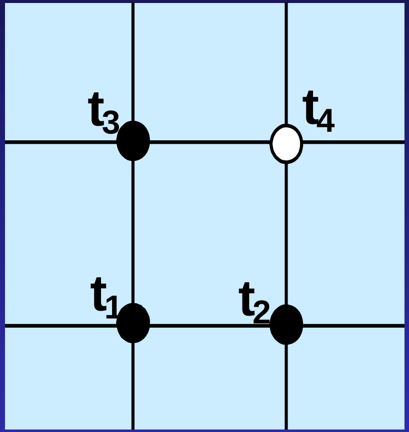
CREWES/University of Calgary

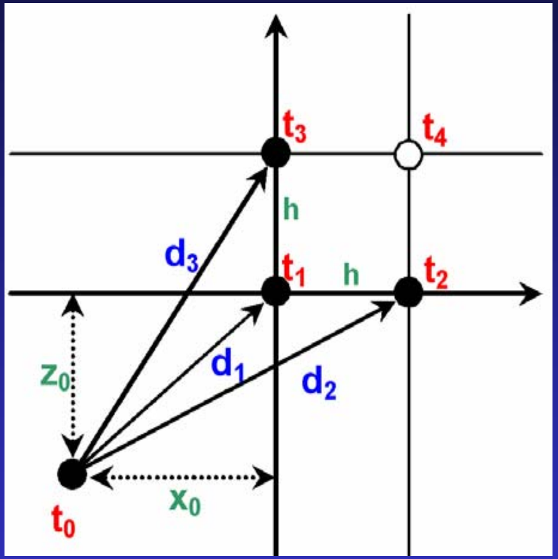
CREWES 2006

NSERC

Motivation

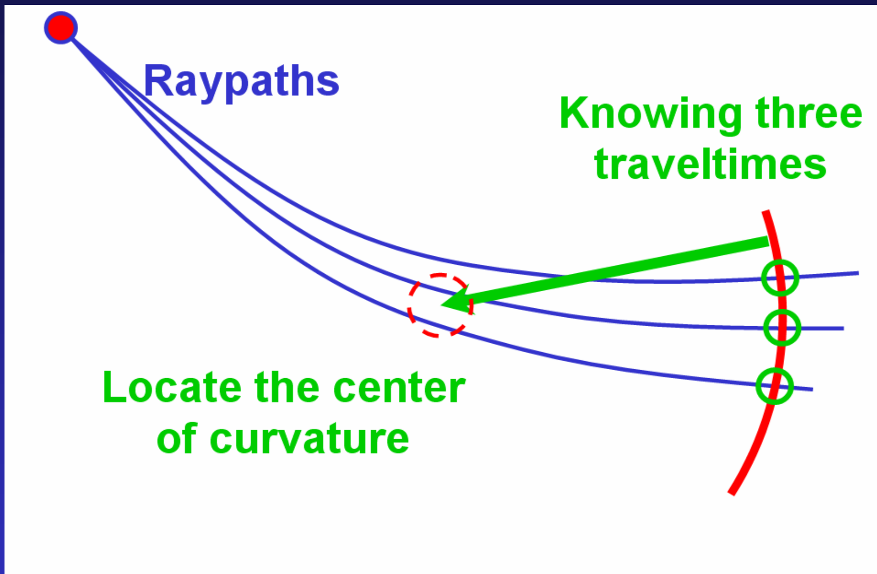


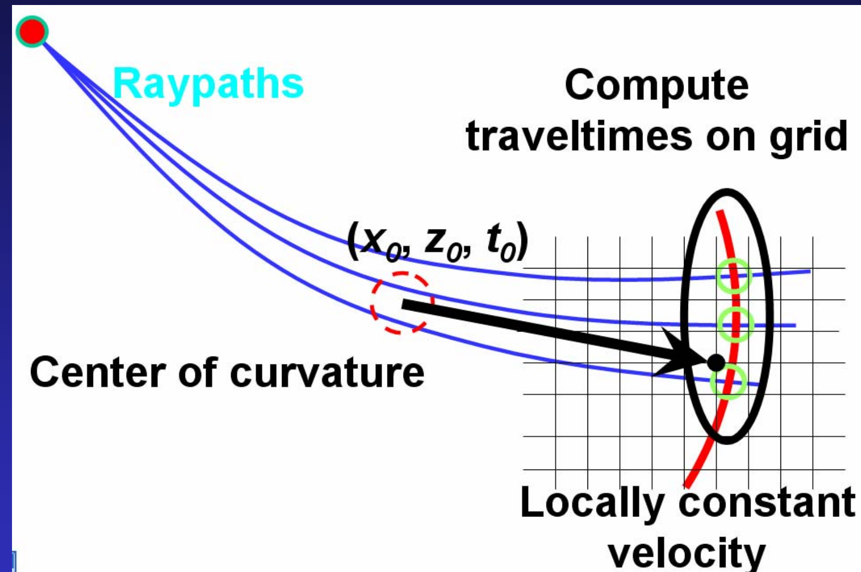




Traveltime grids

Motivation



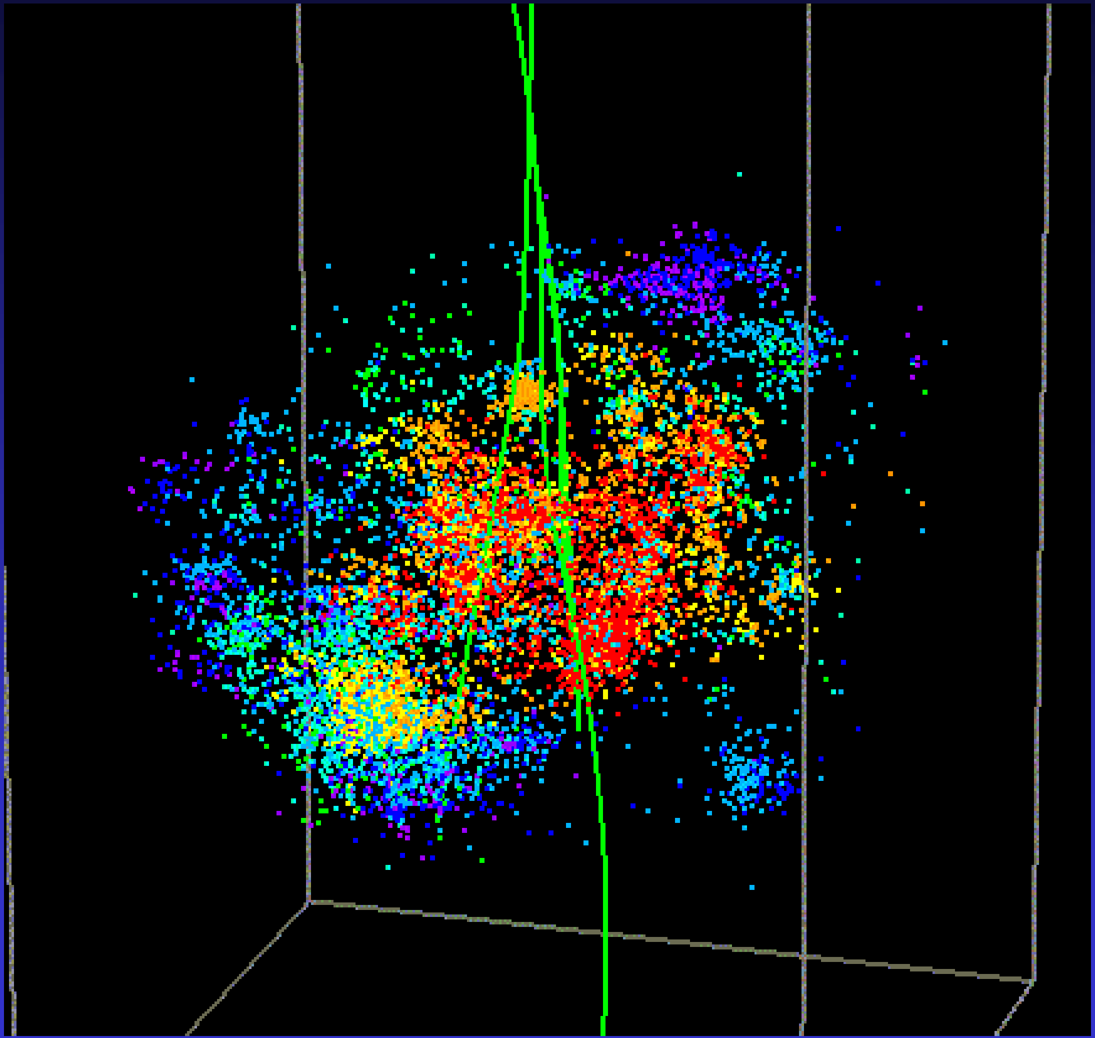


Traveltimes on raypaths to grids

Motivation

Well fraccing

CO_2 injection



Motivation

The main point

- Given three receiver locations (2D)
- Traveltimes of an event at those receiver locations

1. Locate the source and time of the event
2. Compute another traveltime at a nearby location

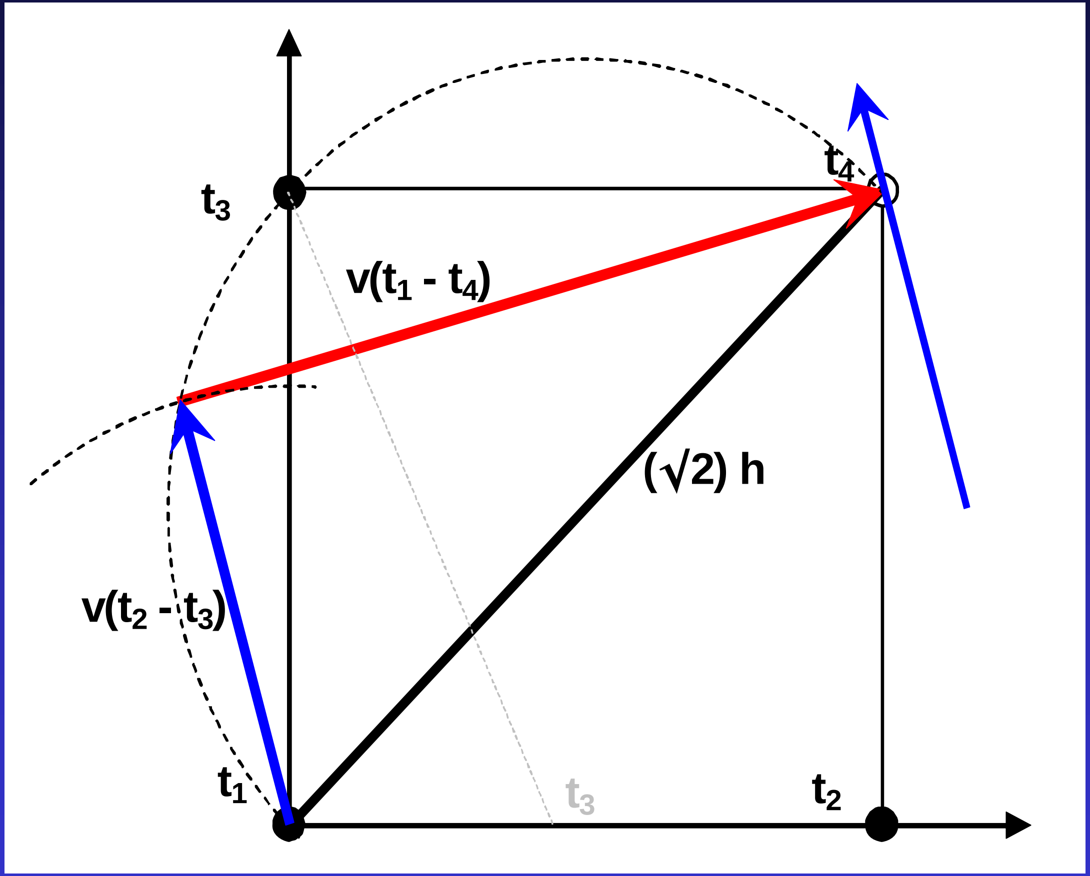
- Later, four receivers for 3D

Vidale's method (still plane waves)

$$\left(\frac{dt}{dx}\right)^2 + \left(\frac{dt}{dx}\right)^2 = \frac{1}{v^2}$$

$$t_4 = t_1 + \sqrt{\frac{2h^2}{v^2} - \left(t_3 - t_2\right)^2}$$

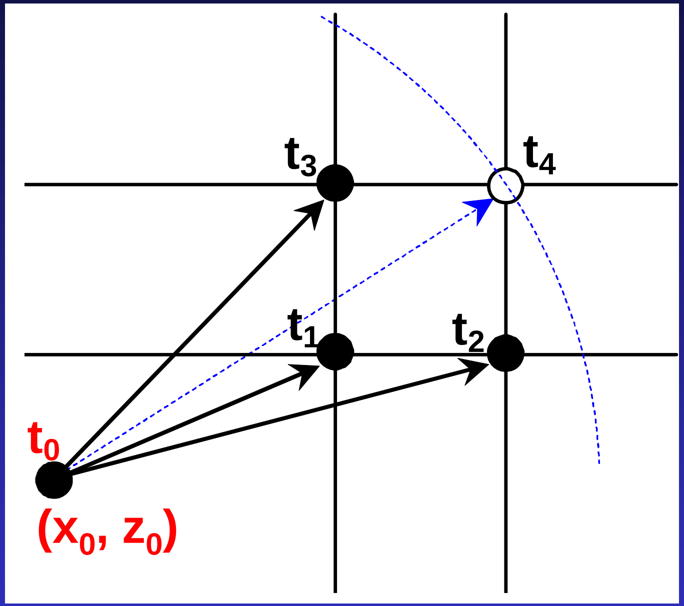
$$\left(t_4 - t_1\right)^2 + \left(t_3 - t_2\right)^2 = \frac{2h^2}{v^2}$$



Fast, OK accuracy, industry standard

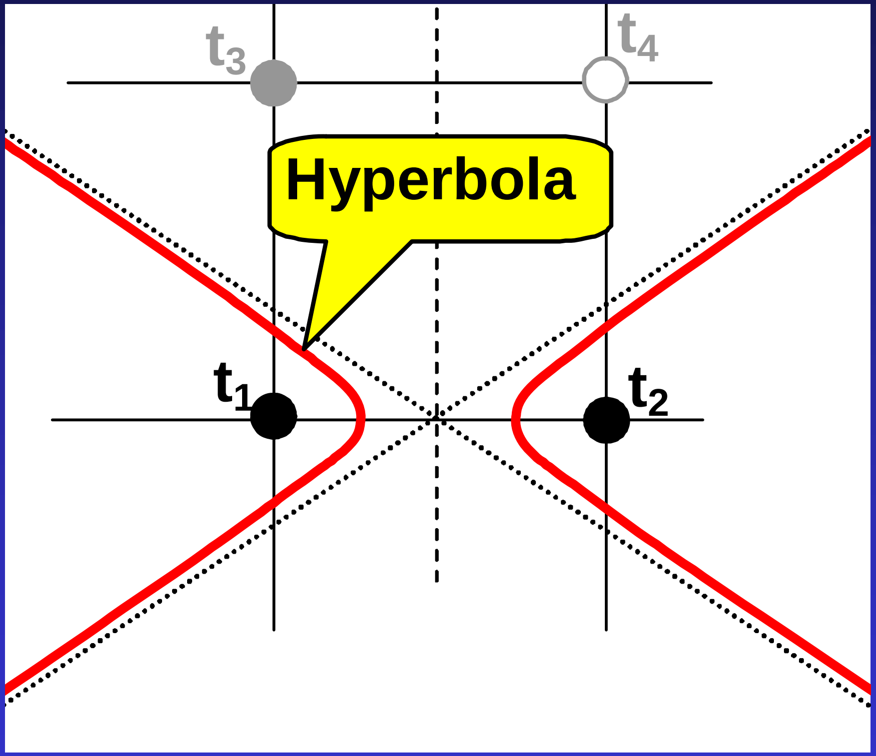
Curved wavefront assumption

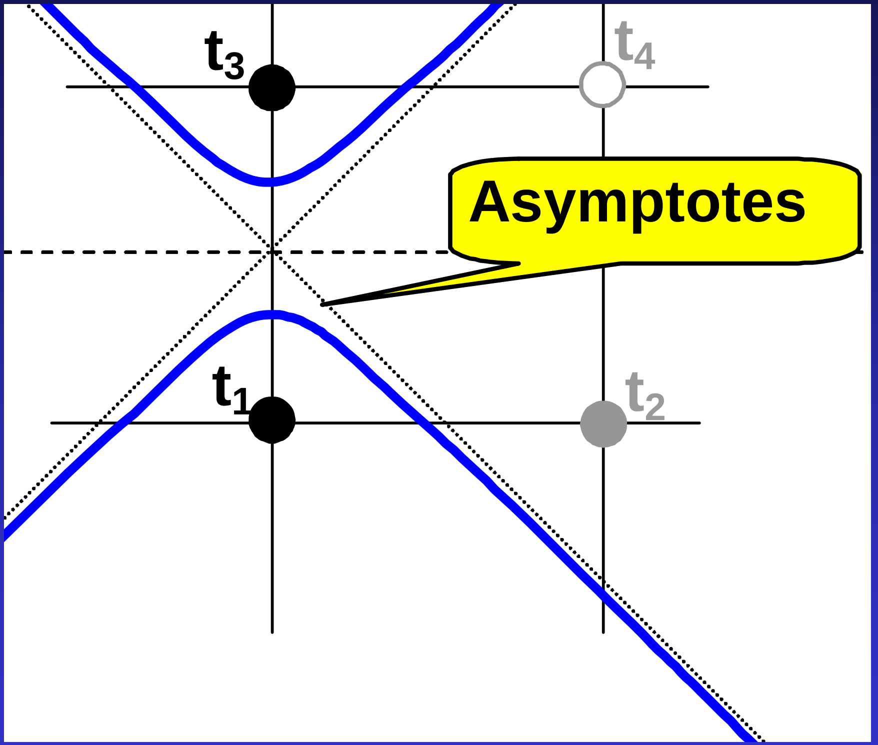
1. Locally constant velocity
2. Estimate center of curvature
3. Apparent source location (x_0, z_0)
4. Source time t_0
5. From apparent source
   - get time t_4



Difference between traveltimes

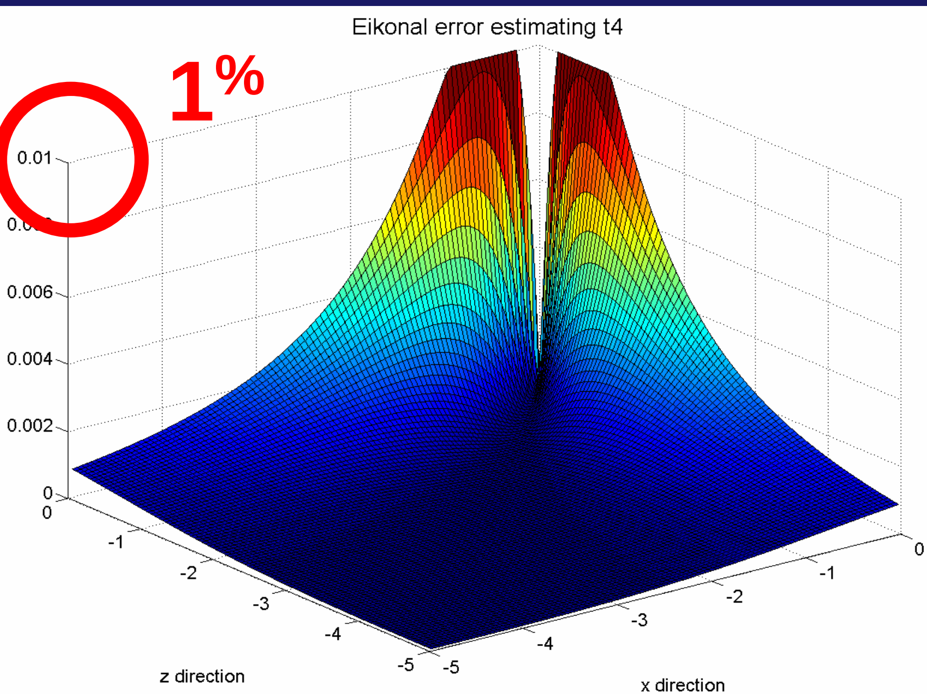
$\boxed{(t_2 - t_1)}$ and $\boxed{(t_3 - t_1)}$

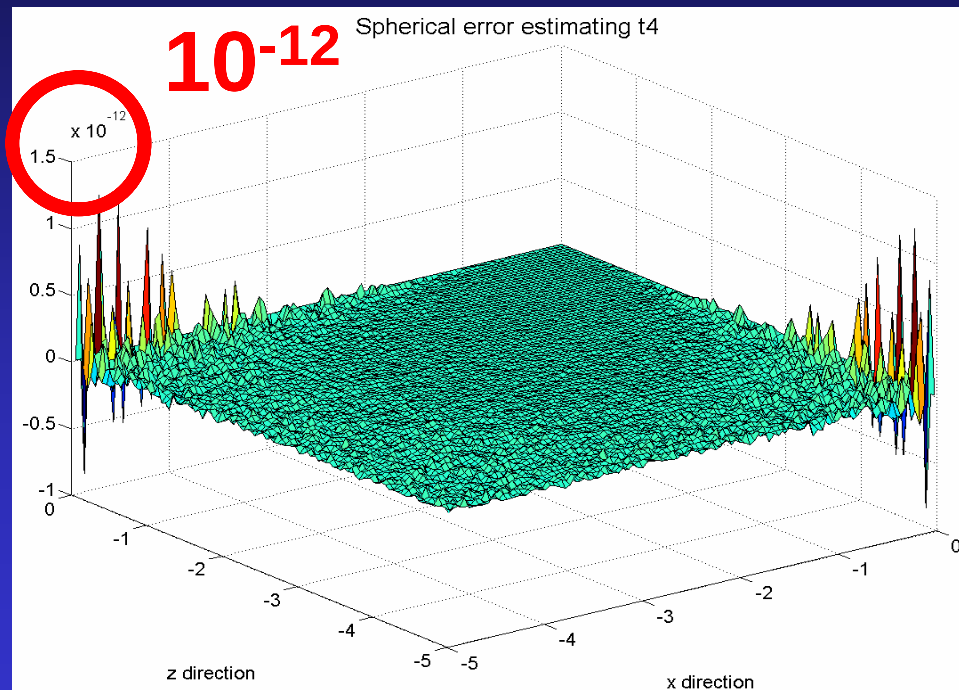




Loran C navigation system

Error in estimating t_4

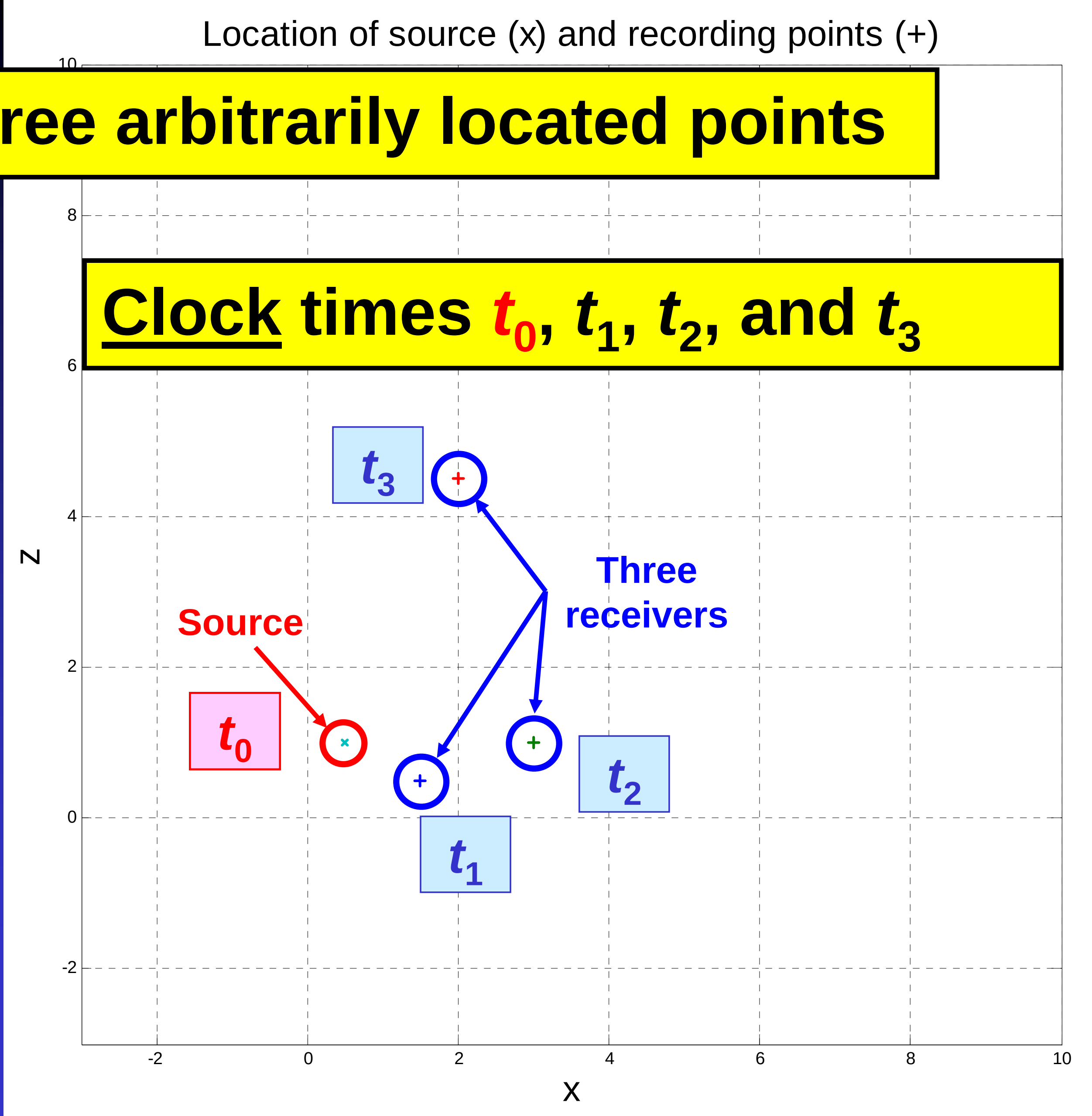


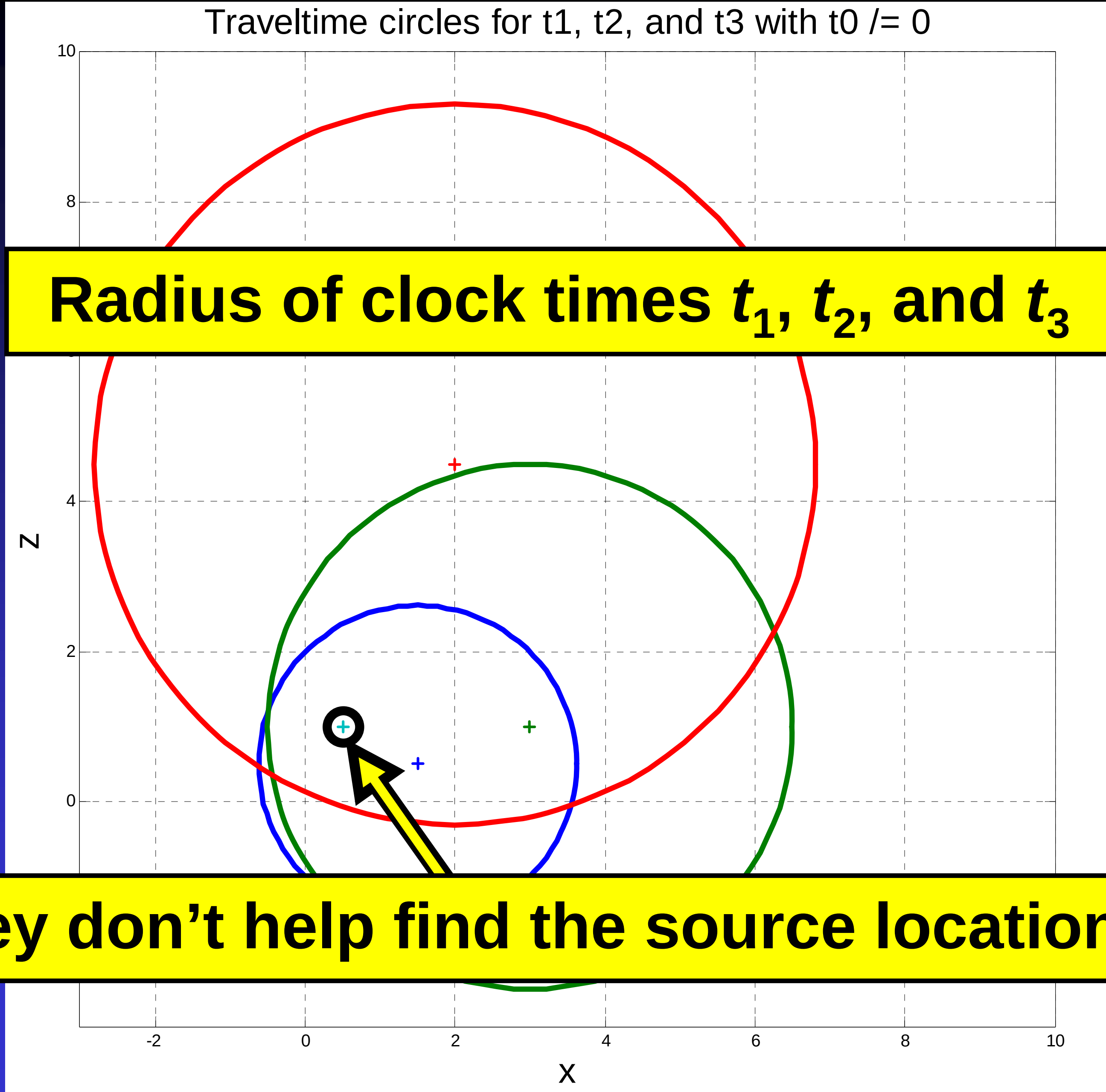


Vidale's solution **Iterative solution**

Now, three arbitrarily located points

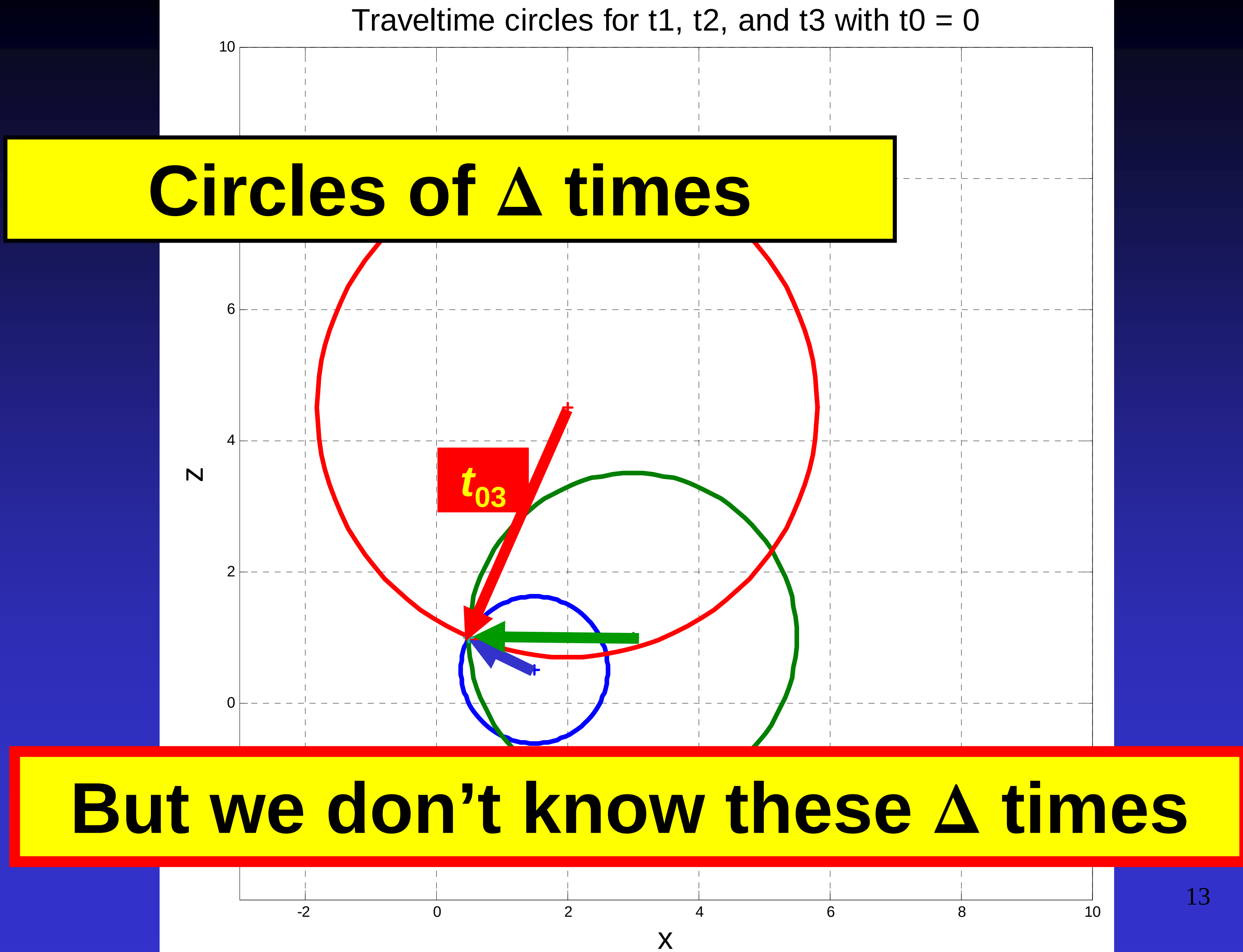
Clock times t_0, t_1, t_2, and t_3



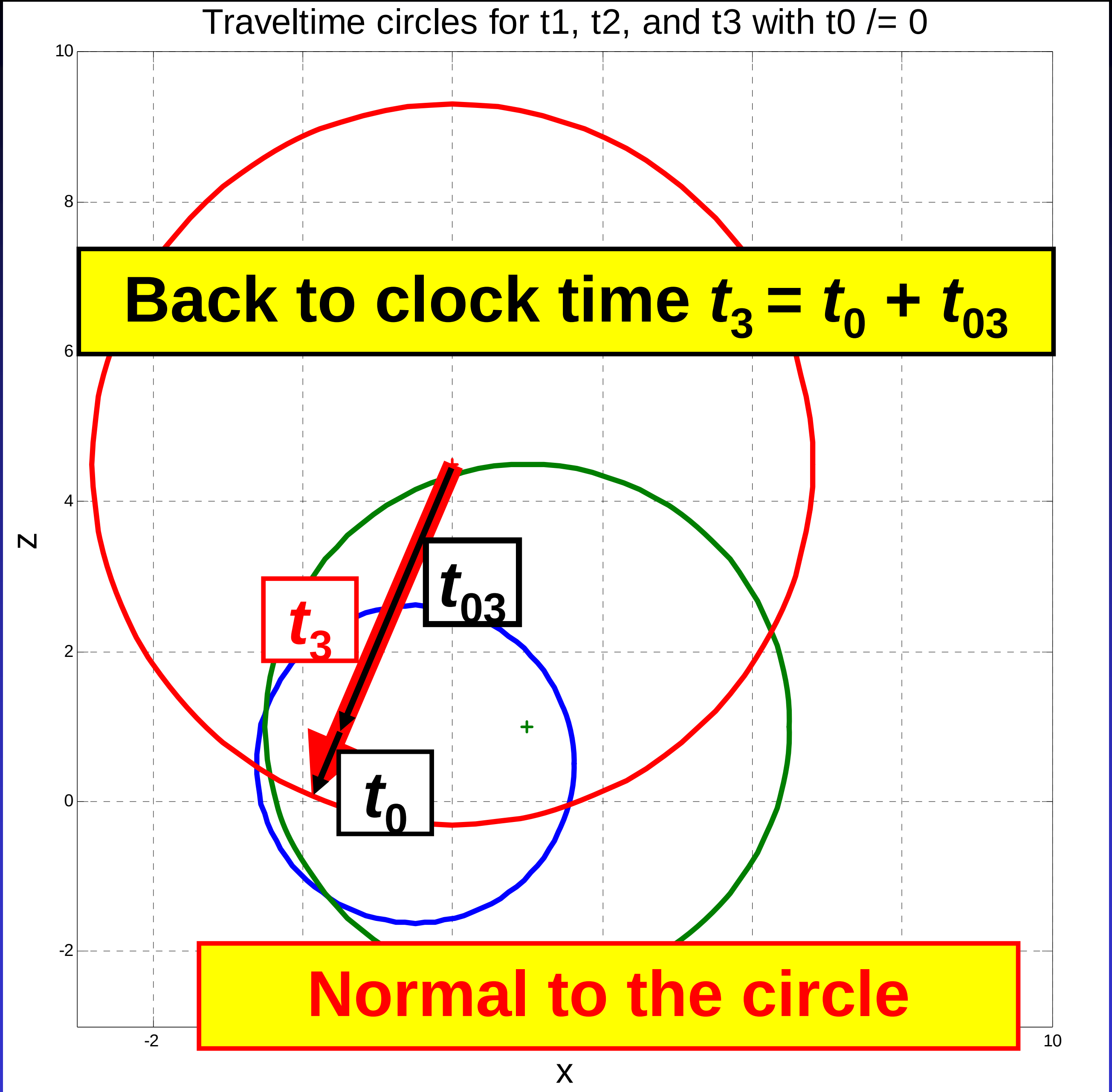


Radius of clock times t_1, t_2, and t_3

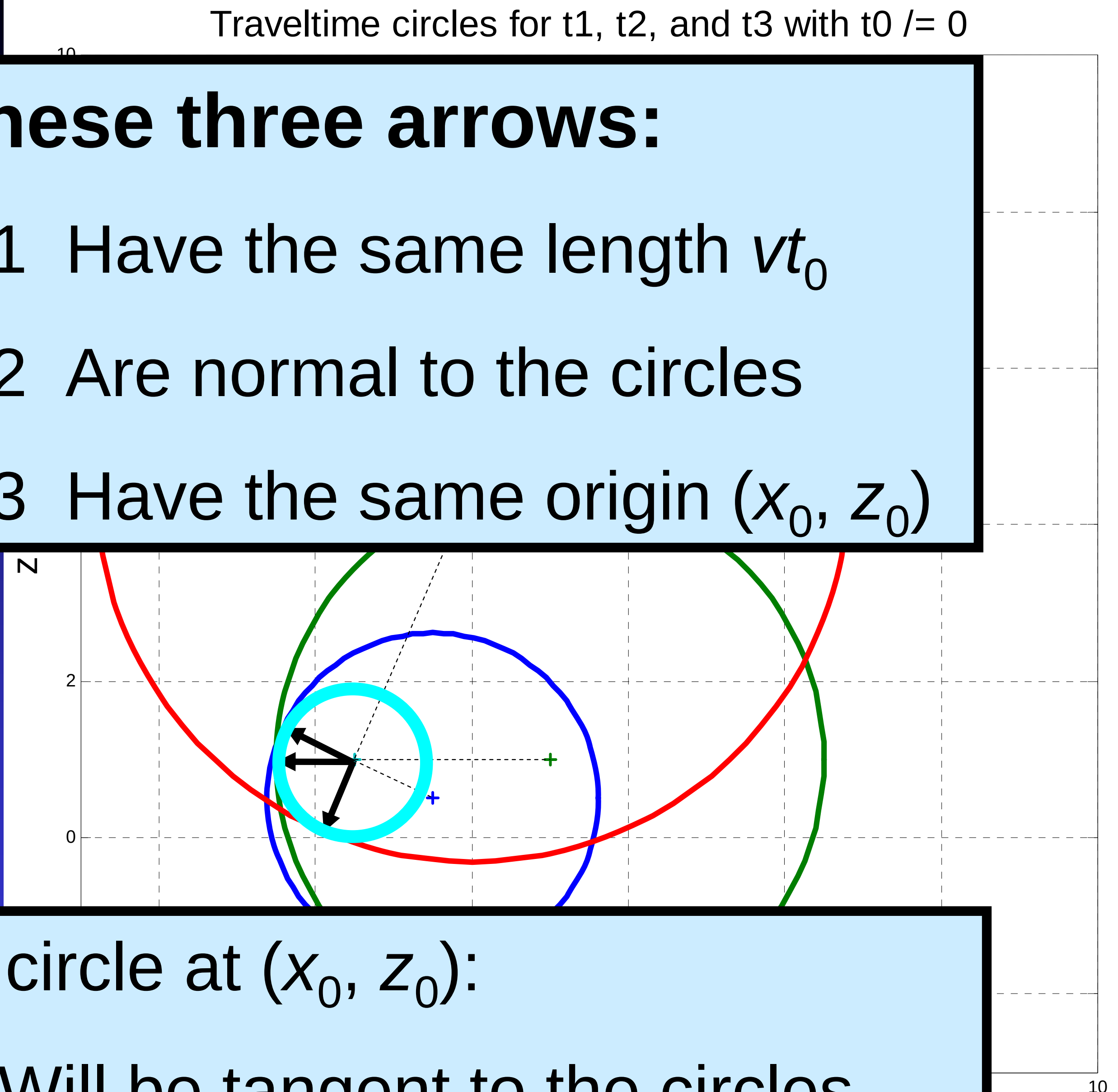
They don’t help find the source location, yet



Traveltime circles for t1, t2, and t3 with t0 = 0
Circles of Δ times
t_{03}
But we don't know these Δ times
10
6
4
2
0
-2
0
2
4
6
8
10
z
x



Traveltime circles for t1, t2, and t3 with t0 /= 0
10
8
6
4
2
0
-2
z
-2
10
x
Back to clock time $t_3 = t_0 + t_{03}$
t_3
t_{03}
t_0
Normal to the circle



These three arrows:

1 Have the same length vt_0

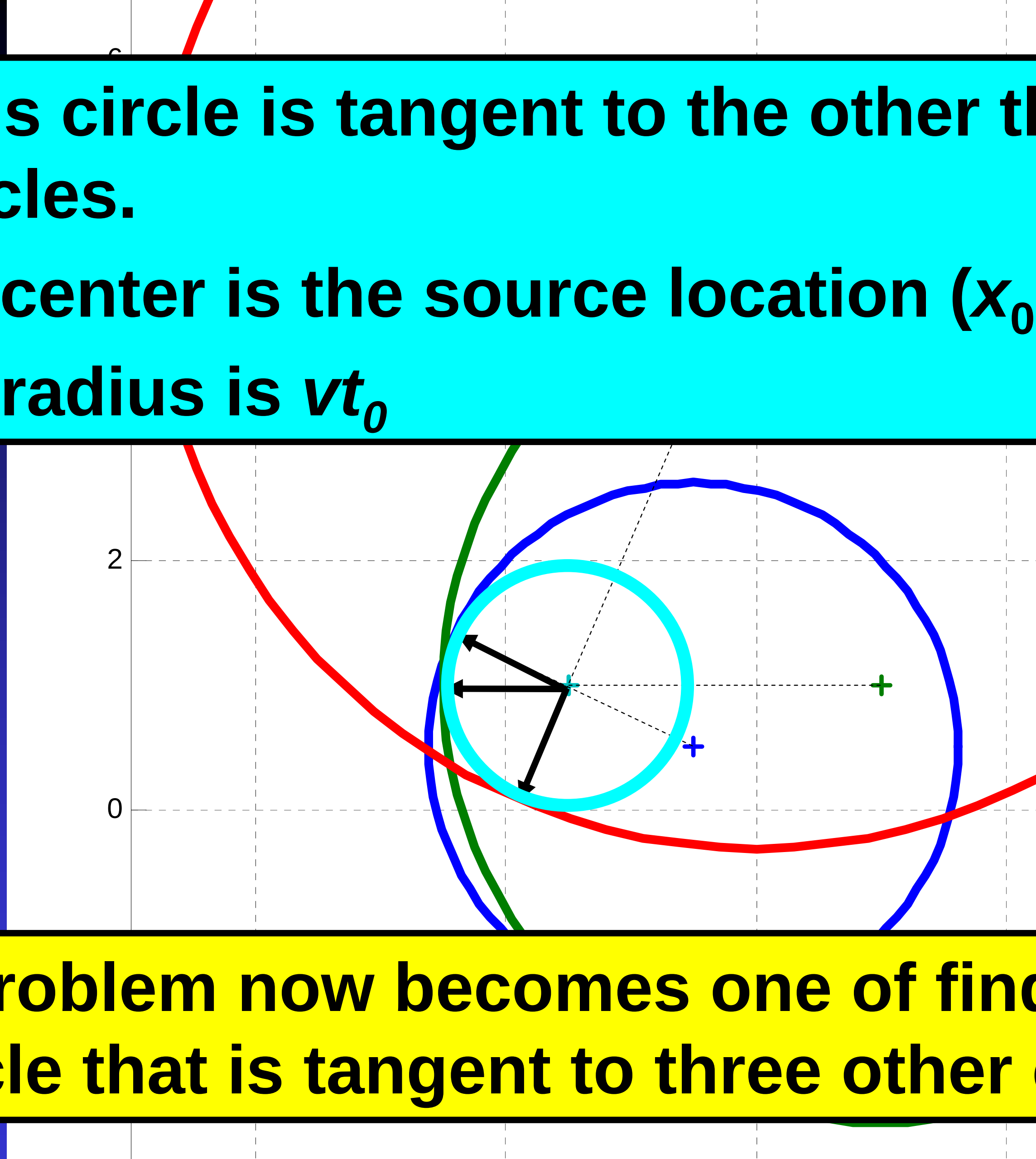
2 Are normal to the circles

3 Have the same origin (x_0, z_0)

A circle at (x_0, z_0):

Will be tangent to the circles

1. This circle is tangent to the other three circles.
2. Its center is the source location (x_0, z_0).
3. Its radius is vt_0



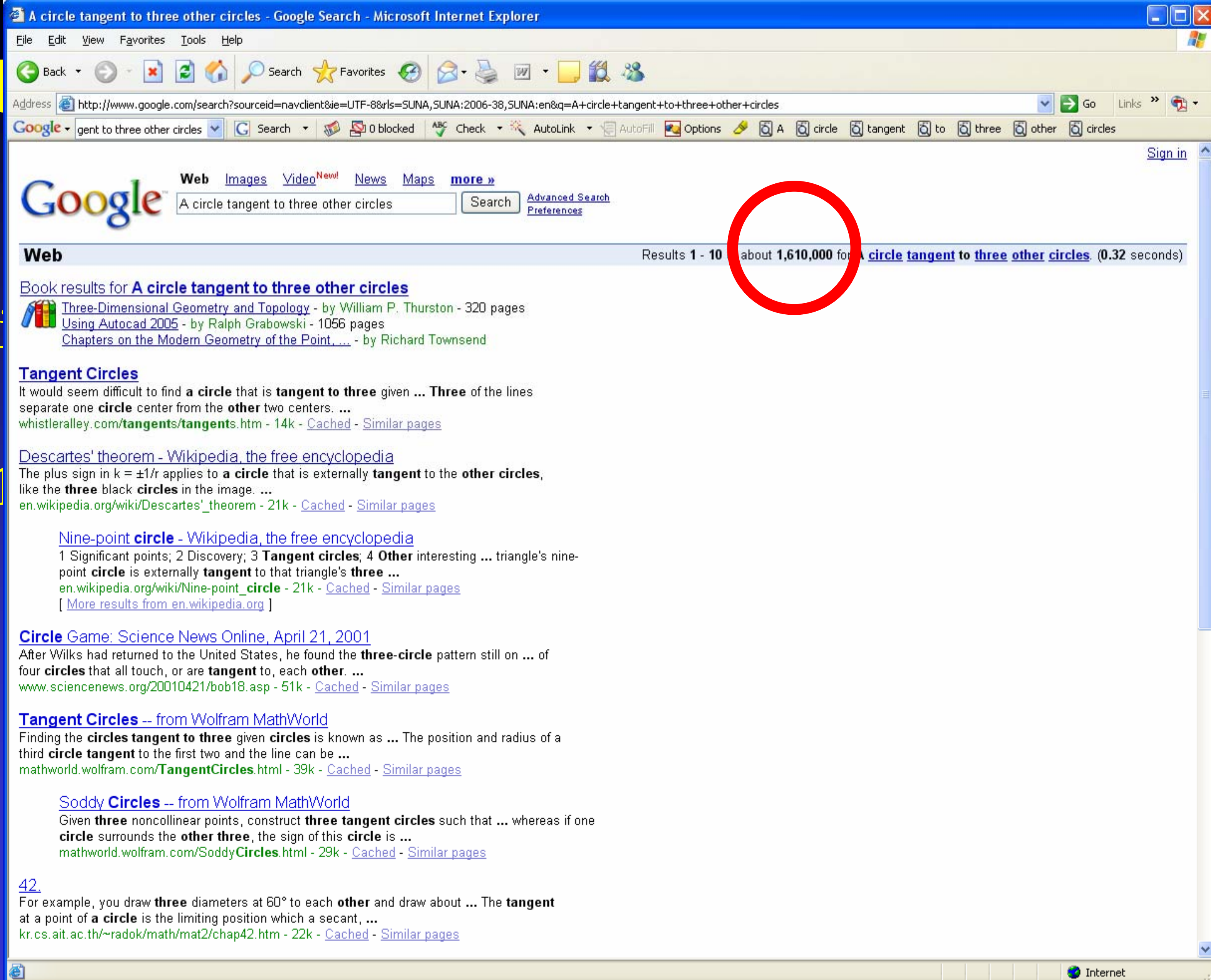
Our problem now becomes one of finding this circle that is tangent to three other circles.

Go to the internet and look up…

- A circle tangent to three other circles
- 1,610,000 responses

Go t

- A ci
- 1,61



A circle tangent to three other circles - Google Search - Microsoft Internet Explorer
File Edit View Favorites Tools Help
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Address http://www.google.com/search?sourceid=navclient&ie=UTF-8&rls=SUNA,SUNA:2006-38,SUNA:en&q=A+circle+tangent+to+three+other+circles
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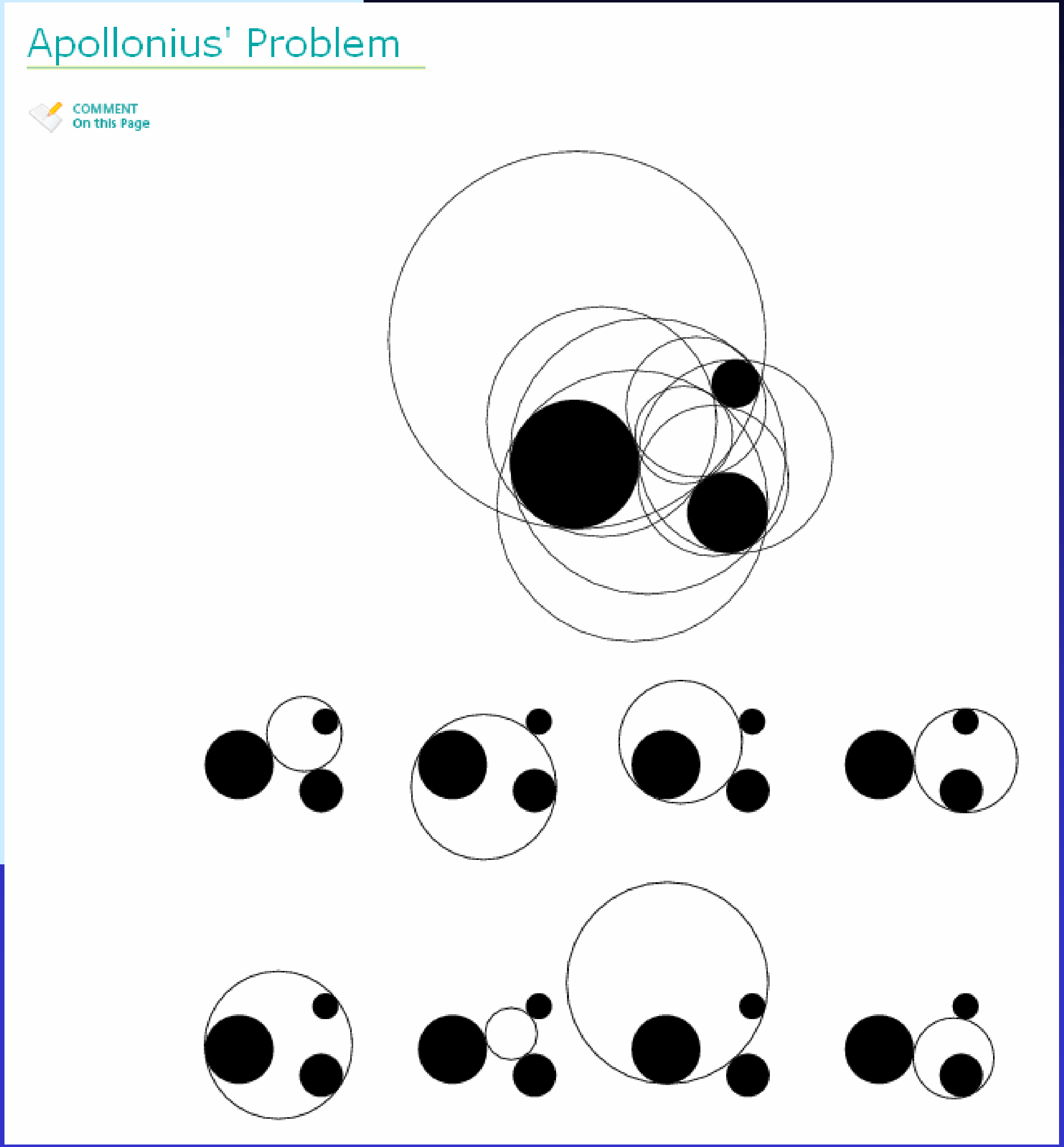
Search
Advanced Search
Preferences
Web
Results 1 - 10 of about 1,610,000 for A circle tangent to three other circles. (0.32 seconds)
Book results for A circle tangent to three other circles
Three-Dimensional Geometry and Topology - by William P. Thurston - 320 pages
Using Autocad 2005 - by Ralph Grabowski - 1056 pages
Chapters on the Modern Geometry of the Point, ... - by Richard Townsend
Tangent Circles
It would seem difficult to find a circle that is tangent to three given ... Three of the lines separate one circle center from the other two centers. ...
whistleralley.com/tangents/tangents.htm - 14k - Cached - Similar pages
Descartes' theorem - Wikipedia, the free encyclopedia
The plus sign in k = ±1/r applies to a circle that is externally tangent to the other circles, like the three black circles in the image. ...
en.wikipedia.org/wiki/Descartes'_theorem - 21k - Cached - Similar pages
Nine-point circle - Wikipedia, the free encyclopedia
1 Significant points; 2 Discovery; 3 Tangent circles; 4 Other interesting ... triangle's nine-point circle is externally tangent to that triangle's three ...
en.wikipedia.org/wiki/Nine-point_circle - 21k - Cached - Similar pages
[More results from en.wikipedia.org]
Circle Game: Science News Online, April 21, 2001
After Wilks had returned to the United States, he found the three-circle pattern still on ... of four circles that all touch, or are tangent to, each other. ...
www.sciencenews.org/20010421/bob18.asp - 51k - Cached - Similar pages
Tangent Circles -- from Wolfram MathWorld
Finding the circles tangent to three given circles is known as ... The position and radius of a third circle tangent to the first two and the line can be ...
mathworld.wolfram.com/TangentCircles.html - 39k - Cached - Similar pages
Soddy Circles -- from Wolfram MathWorld
Given three noncollinear points, construct three tangent circles such that ... whereas if one circle surrounds the other three, the sign of this circle is ...
mathworld.wolfram.com/SoddyCircles.html - 29k - Cached - Similar pages
42.
For example, you draw three diameters at 60° to each other and draw about ... The tangent at a point of a circle is the limiting position which a secant, ...
kr.cs.ait.ac.th/~radok/math/mat2/chap42.htm - 22k - Cached - Similar pages
Internet

Apollonius 262 BC to 190 BC

The Great Geometer

Introduced terms:

- parabola
- ellipse
- hyperbola



Wolfram MathWorld

the web's most extensive mathematics resource

The three-circle problem was solved by Viète (Boyer 1968), and the solutions are called Apollonius circles. There are eight total solutions. The simplest solution is obtained by solving the three simultaneous quadratic equations

$$(x-x_1)^2+(y-y_1)^2-(r\pm r_1)^2=0 \tag{1}$$
$$(x-x_2)^2+(y-y_2)^2-(r\pm r_2)^2=0 \tag{2}$$
$$(x-x_3)^2+(y-y_3)^2-(r\pm r_3)^2=0 \tag{3}$$

in the three unknowns x, y, r for the eight triplets of signs (Courant and Robbins 1996). Expanding the equations gives

$$(x^2+y^2-r^2)-2\,x\,x_i-2\,y\,y_i\mp 2\,r\,r_i+(x_i^2+y_i^2-r_i^2)=0 \tag{4}$$

for $i=1,2,3$. Since the first term is the same for each equation, taking $(2)-(1)$ and $(3)-(1)$ gives

$$a\,x+b\,y+c\,r=d \tag{5}$$
$$a'\,x+b'\,y+c'\,r=d', \tag{6}$$

where

$$a = 2\,(x_1-x_2) \tag{7}$$
$$b = 2\,(y_1-y_2) \tag{8}$$
$$c = \pm 2\,(r_1-r_2) \tag{9}$$
$$d = (x_1^2+y_1^2-r_1^2)-(x_2^2+y_2^2-r_2^2) \tag{10}$$

and similarly for a', b', c' and d' (where the 2 subscripts are replaced by 3s). Solving these two simultaneous linear equations gives

$$x = \frac{b'\,d-b\,d'-b'\,c\,r+b\,c'\,r}{a\,b'-b\,a'} \tag{11}$$
$$y = \frac{-a'\,d+a\,d'+a'\,c\,r-a\,c'\,r}{a\,b'-a'\,b}, \tag{12}$$

which can then be plugged back into the quadratic equation (1) and solved using the quadratic formula.

It turns out…

- In general there are 8 possible solutions
- But our case is limited by the problem to only 2 solutions.

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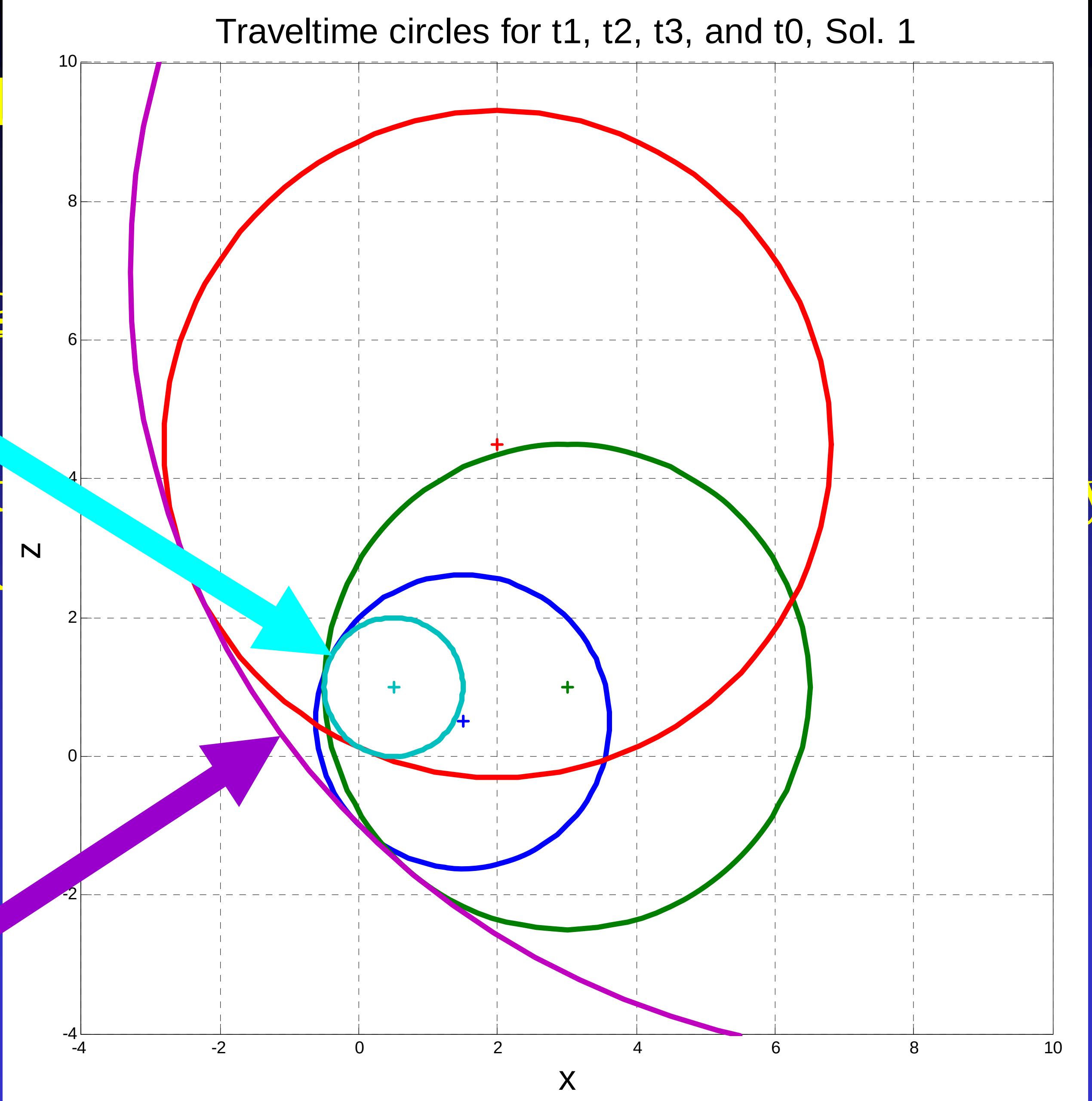

3D solution

- Use the identical form as for the 2D solution
- Requires four receivers and their clock times
- Find one sphere tangent to four spheres
- Only two solutions

Comments on the clock time

- The clock times can be very large relative to the Δ times. (Difference of large numbers)

- Subtract the minimum clock time from all other clock times:
  - simpler solutions
  - 2D: two circles and one point
  - 3D: three spheres and one point
  - t_0 will be negative

Final words

- Receivers at any location
- Circular or spherical wavefronts
- Source time and location is computed
- Assumes locally constant velocities
- Many types of solutions for Apollonius' problem
- Illustrated with 2D data, solved for 3D data
- Applications for:
  - Microseismic events, fraccing, CO_2 sequestration
  - Monitoring of hazardous geological sites (Frank slide), volcanoes, …
  - Computing gridded traveltimes
  - Converting ray traveltimes to gridded traveltimes


The End