Plane-wave reflection coefficients for anisotropic media *et al*.

Chuck Ursenbach

CREWES Sponsors Meeting

November 29, 2007

WARNING

This presentation contains slides of mathematical expressions which may be offensive to some.

Viewer discretion is advised.

Outline

- Programming plane-wave reflection coefficients for anisotropic media
- Zoeppritz Explorer updates
- Fluid Properties calculator
- Hodogram Explorer
- Papers:
  - Ursenbach & Haase: “Plane-wave reflection coefficients for anisotropic media: Practical implementation”
  - Ursenbach & Lawton: “Seismic modeling of acid-gas injection in a deep saline reservoir”
  - Ursenbach: “New and updated web applet Explorers”

From DATA to PLAY

- DATA
- DATE
- DARE
- DIRE
- DIRT
- DIET
- DUET
- SUET
- SUED
- SLED
- PLED
- PLOD
- PLOY
- PLAY

From $(\theta_{\text{inc}},\phi_{\text{inc}})$ to R_{PP}

- From to $(\theta_{\text{inc}},\phi_{\text{inc}})$ to V_{inc} (incident wave speed)
- From V_{inc} to s_1,s_2 (horizontal slowness)
- From s_1,s_2 to s_3 (vertical slowness)
- From $\underline{s} = (s_1,s_2,s_3)$ to $\underline{P}$ (polarization vector)
- From $\underline{s}$, $\underline{P}$ to R_{PP} (reflection coefficient)

Schoenberg &
Protázio (1992)

Christoffel equations

$$\Gamma_{ik} = \sum_{jl} c_{ijkl} s_j s_l$$

general Christoffel matrix

Orthorhombic case

$$\Gamma_{11} = c_{11}s_1^2 + c_{66}s_2^2 + c_{55}s_3^2$$

$$\Gamma_{22} = c_{66}s_1^2 + c_{22}s_2^2 + c_{44}s_3^2$$

$$\Gamma_{33} = c_{55}s_1^2 + c_{44}s_2^2 + c_{33}s_3^2$$

$$\Gamma_{12} = \Gamma_{21} = \left(c_{12} + c_{66}\right) s_1 s_2$$

$$\Gamma_{13} = \Gamma_{31} = \left(c_{13} + c_{55}\right) s_1 s_3$$

$$\Gamma_{23} = \Gamma_{32} = \left(c_{23} + c_{44}\right) s_2 s_3$$

3 X 3 X 3 X 3
→ 6 X 6

Christoffel equation

$$\left(\underline{\underline{\Gamma}} - \rho \underline{\underline{I}}\right) \underline{P} = \underline{0}$$

From to (θ_{inc},ϕ_{inc}) to V_{inc}

$$\left(\underline{\underline{\Gamma}} - \rho \underline{\underline{I}}\right)\underline{P} = \underline{0} \quad \xrightarrow{\times\ V^2/\rho} \quad \left(\underline{\underline{\Lambda}} - V^2 \underline{\underline{I}}\right)\underline{P} = \underline{0}$$

$$\Lambda_{ik} = \sum_{jl} \frac{c_{ijkl}}{\rho}\left(Vs_j\right)\left(Vs_l\right) = \sum_{jl} a_{ijkl} n_j n_l$$

$$\underline{n} = \left(\cos\phi_{\text{inc}} \sin\theta_{\text{inc}},\ \sin\phi_{\text{inc}} \sin\theta_{\text{inc}},\ \cos\theta_{\text{inc}}\right)$$

$$\left|\underline{\underline{\Lambda}} - V^2 \underline{\underline{I}}\right| = 0 \quad \longleftarrow$$

In forward modeling, everything is known in this except for V

$$\Big\downarrow$$

$$AV^6 + BV^4 + CV^2 + D = 0 \quad \longrightarrow \quad V_{qP}, V_{qSV}, V_{qSH}$$

(Everything in this slide pertains to the upper layer)

From V_{inc} to s_1, s_2

For an incident P-wave:

$$V_{\text{inc}} = V_{qP}$$

$$\underline{s}_{\text{inc}} = \left\{ \boxed{\frac{n_1}{V_{qP}}, \frac{n_2}{V_{qP}}}, \frac{n_3}{V_{qP}} \right\}$$

By Snell’s Law, horizontal slowness components s_1 and s_2 are the same for all reflected and transmitted waves as for the incident wave.

From s_1, s_2 to s_3

$$\Gamma_{11} = c_{11} s_1^2 + c_{66} s_2^2 + c_{55} \boxed{s_3^2}$$

$$\Gamma_{22} = c_{66} s_1^2 + c_{22} s_2^2 + c_{44} \boxed{s_3^2}$$

$$\Gamma_{33} = c_{55} s_1^2 + c_{44} s_2^2 + c_{33} \boxed{s_3^2}$$

$$\Gamma_{12} = \Gamma_{21} = \left(c_{12} + c_{66}\right) s_1 s_2$$

$$\Gamma_{13} = \Gamma_{31} = \left(c_{13} + c_{55}\right) s_1 \boxed{s_3}$$

$$\Gamma_{23} = \Gamma_{32} = \left(c_{23} + c_{44}\right) s_2 \boxed{s_3}$$

$$\left(\underline{\underline{\Gamma}} - \rho \underline{\underline{I}}\right) \underline{P} = \underline{0}$$

$$\left| \underline{\underline{\Gamma}}^U - \rho^U \underline{\underline{I}} \right| = 0 \longleftarrow \text{Everything known except } s_3 \longrightarrow \left| \underline{\underline{\Gamma}}^L - \rho^L \underline{\underline{I}} \right| = 0$$

$$A^U s_3^6 + B^U s_3^4 + C^U s_3^2 + D^U = 0 \qquad A^L s_3^6 + B^L s_3^4 + C^L s_3^2 + D^L = 0$$

$$s_3^{U,qP},\ s_3^{U,qSV},\ s_3^{U,qSH} \qquad s_3^{L,qP},\ s_3^{L,qSV},\ s_3^{L,qSH}$$

From $\underline{s} = (s_1, s_2, s_3)$ to $\underline{P}$

Everything known except $\underline{P}$

$$\left(\underline{\underline{\Gamma}}^{U} - \rho^{\mathrm{U}}\,\underline{\underline{I}}\right)\underline{P} = \underline{0}$$

$$\left(\underline{\underline{\Gamma}}^{L} - \rho^{L}\,\underline{\underline{I}}\right)\underline{P} = \underline{0}$$

- Only the direction of $\underline{P}$ can be determined uniquely
- The magnitude is arbitrary – determined by normalization
- But use $1 = P_1^2 + P_2^2 + P_3^2$, not $1 = \left|P_1\right|^2 + \left|P_2\right|^2 + \left|P_3\right|^2$

From $\underline{s}$, $\underline{P}$ to R_{PP}

Schoenberg and Protázio (1992) define two 3 X 3 matrices for each layer:

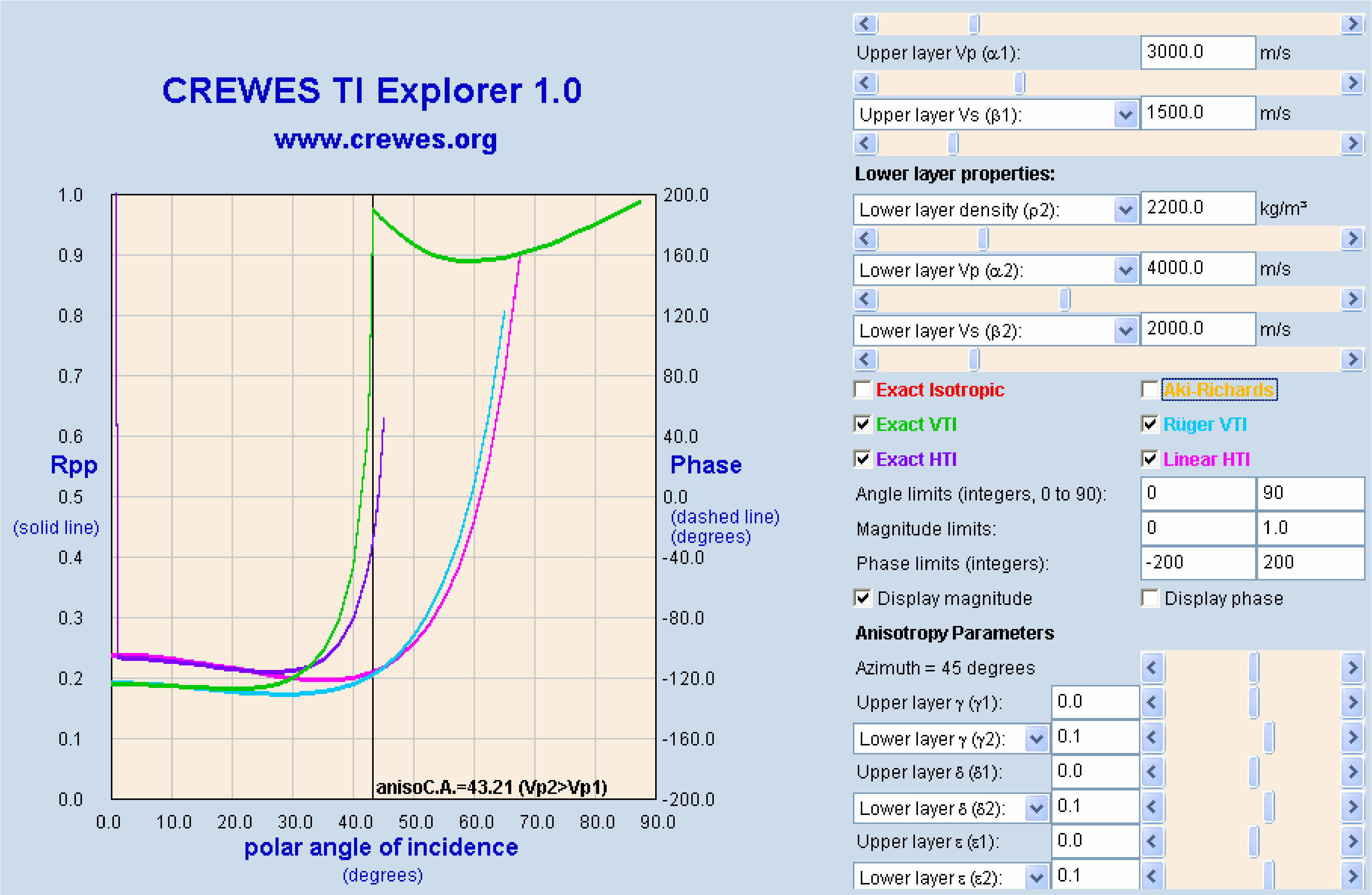
$$\mathbf{X}^U = \mathbf{X}^U\left(\underline{\underline{c}}^U, \underline{s}^U, \underline{P}^U\right) \qquad \mathbf{X}^L = \mathbf{X}^L\left(\underline{\underline{c}}^L, \underline{s}^L, \underline{P}^L\right)$$

$$\mathbf{Y}^U = \mathbf{Y}^U\left(\underline{\underline{c}}^U, \underline{s}^U, \underline{P}^U\right) \qquad \mathbf{Y}^L = \mathbf{Y}^L\left(\underline{\underline{c}}^L, \underline{s}^L, \underline{P}^L\right)$$

Boundary conditions are encoded in the following expressions:

$$\mathbf{D} = \left(\mathbf{X}^U\right)^{-1}\mathbf{X}^L + \left(\mathbf{Y}^U\right)^{-1}\mathbf{Y}^L$$

$$\mathbf{R} = \begin{bmatrix} \enclose{circle}{R_{PP}} & R_{SP} & R_{TP} \\ R_{PS} & R_{SS} & R_{TS} \\ R_{PT} & R_{ST} & R_{TT} \end{bmatrix} = \left[\left(\mathbf{X}^U\right)^{-1}\mathbf{X}^L - \left(\mathbf{Y}^U\right)^{-1}\mathbf{Y}^L\right]\mathbf{D}^{-1} \qquad \mathbf{T} = \begin{bmatrix} T_{PP} & T_{SP} & T_{TP} \\ T_{PS} & T_{SS} & T_{TS} \\ T_{PT} & T_{ST} & T_{TT} \end{bmatrix} = 2\mathbf{D}^{-1}$$



CREWES TI Explorer 1.0

www.crewes.org

Upper layer Vp (α1): 3000.0 m/s

Upper layer Vs (β1): 1500.0 m/s

Lower layer properties:

Lower layer density (ρ2): 2200.0 kg/m³

Lower layer Vp (α2): 4000.0 m/s

Lower layer Vs (β2): 2000.0 m/s

☐ Exact Isotropic
☐ Aki-Richards
☑ Exact VTI
☑ Rüger VTI
☑ Exact HTI
☑ Linear HTI

Angle limits (integers, 0 to 90): 0 90

Magnitude limits: 0 1.0

Phase limits (integers): -200 200

☑ Display magnitude
☐ Display phase

Anisotropy Parameters

Azimuth = 45 degrees

Upper layer γ (γ1): 0.0

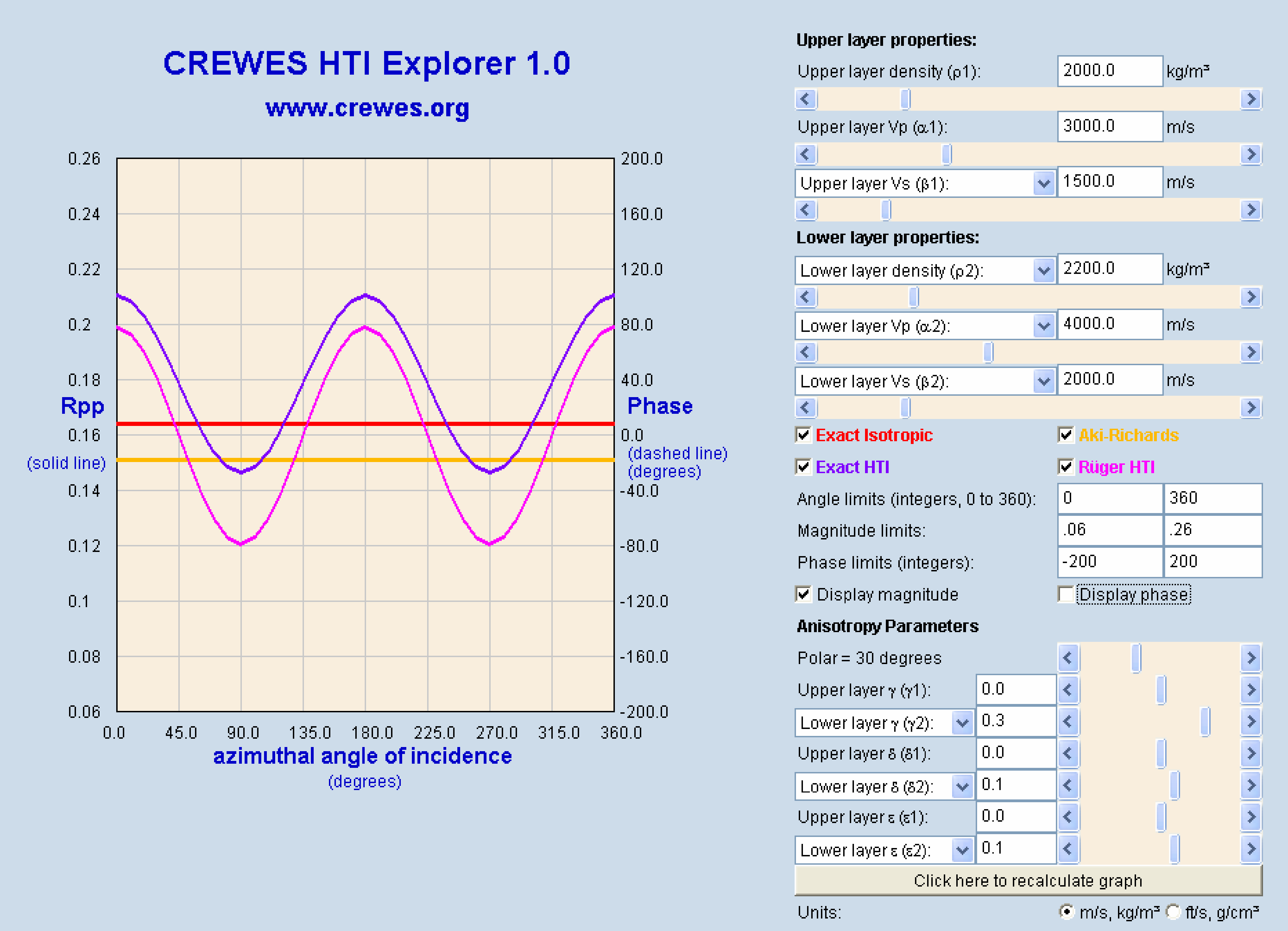
Lower layer γ (γ2): 0.1

Upper layer δ (δ1): 0.0

Lower layer δ (δ2): 0.1

Upper layer ε (ε1): 0.0

Lower layer ε (ε2): 0.1



CREWES HTI Explorer 1.0

www.crewes.org

Upper layer properties:

Upper layer density (ρ1): 2000.0 kg/m³

Upper layer Vp (α1): 3000.0 m/s

Upper layer Vs (β1): 1500.0 m/s

Lower layer properties:

Lower layer density (ρ2): 2200.0 kg/m³

Lower layer Vp (α2): 4000.0 m/s

Lower layer Vs (β2): 2000.0 m/s

☑ Exact Isotropic ☑ Aki-Richards

☑ Exact HTI ☑ Rüger HTI

Angle limits (integers, 0 to 360): 0 360

Magnitude limits: .06 .26

Phase limits (integers): -200 200

☑ Display magnitude ☐ Display phase

Anisotropy Parameters

Polar = 30 degrees

Upper layer γ (γ1): 0.0

Lower layer γ (γ2): 0.3

Upper layer δ (δ1): 0.0

Lower layer δ (δ2): 0.1

Upper layer ε (ε1): 0.0

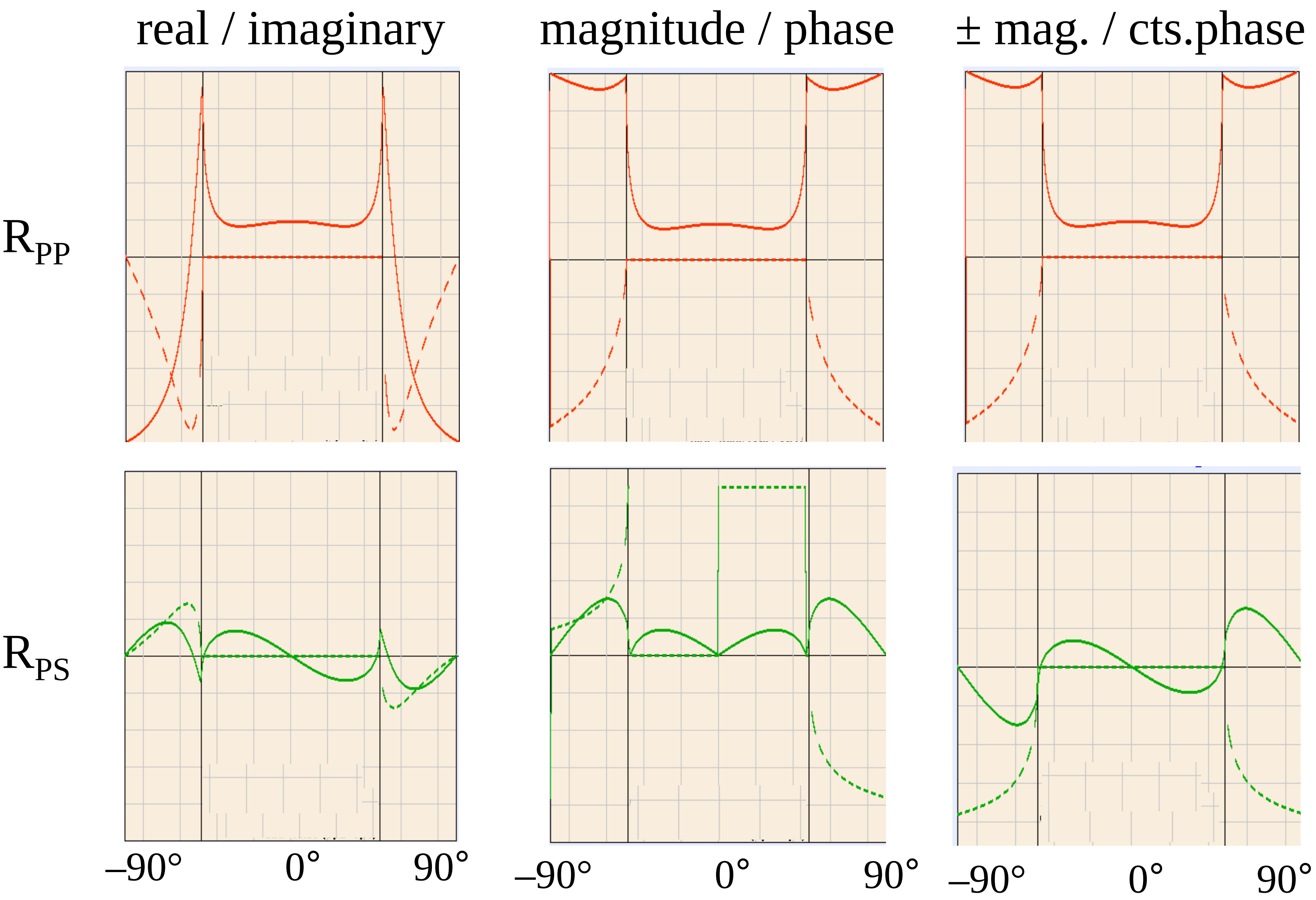
Lower layer ε (ε2): 0.1

Units: ● m/s, kg/m³ ○ ft/s, g/cm³

Future work

- General improvements (critical angles, post-critical curves, fix instabilities)
- Mixed symmetries (e.g. VTI over HTI)
- Non-aligned HTI media
- Orthorhombic symmetry Explorer
- Studies of linearization
- Reflectivity of point-source waves

Zoeppritz Explorer – update

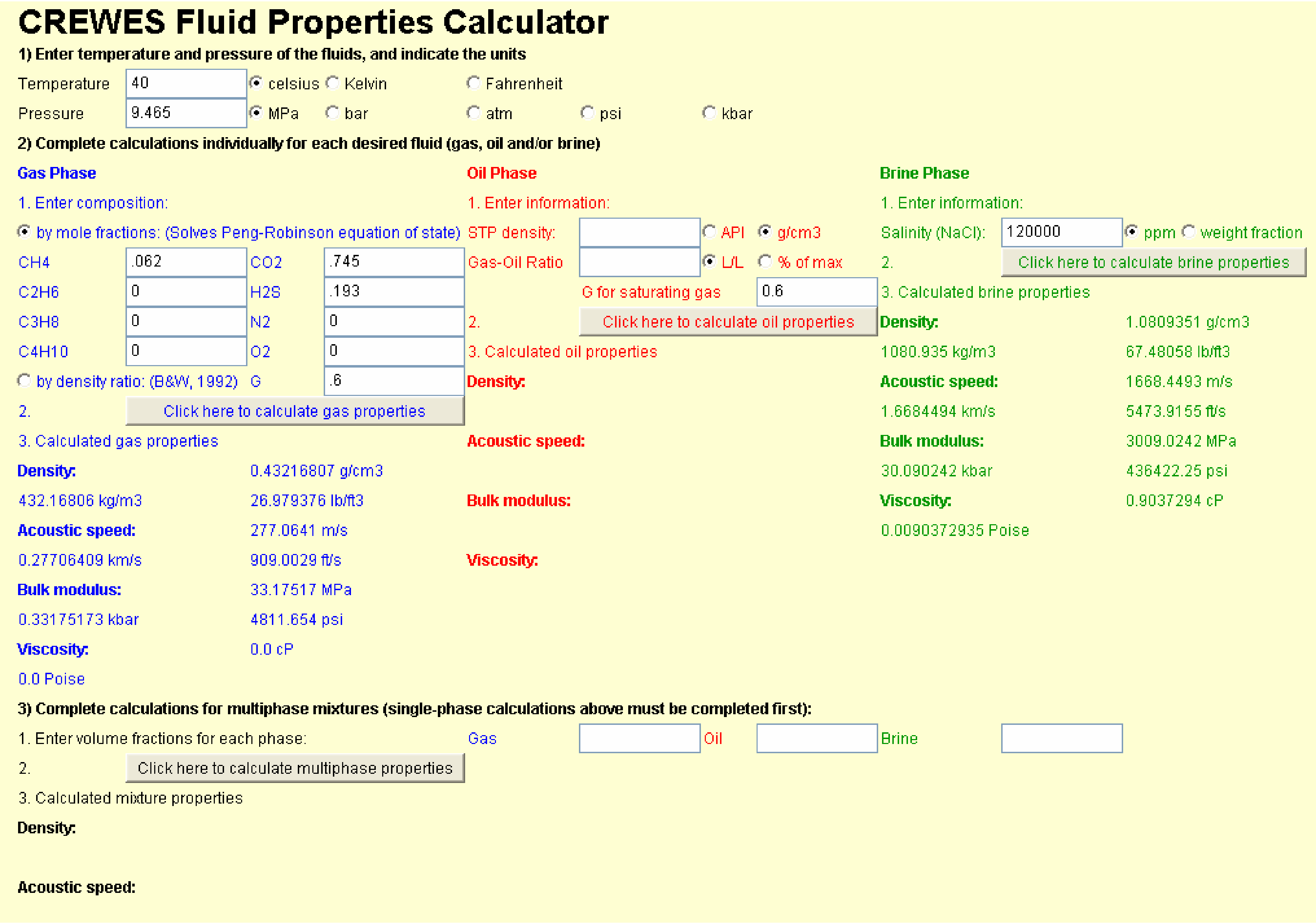


Acid-gas injection

- Scenario: injection of H_2S and CO_2 in deep saline reservoirs
- Can seismic track progress?
- For fluid substitution, require acoustic properties of acid gas (ρ, V_P)
- Peng-Robinson EOS – non-aqueous

EOS ⟶ Acoustic Properties

$$\begin{tikzcd}
\boxed{P = P(T, v, x_i)} \arrow[dd] & & \\
& \boxed{\beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T} \arrow[dr] & \\
\boxed{v = v(T, P, x_i)} \arrow[ur] \arrow[r] \arrow[dr] \arrow[dd] & \boxed{\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P} \arrow[r] & \boxed{\beta_S = \beta_T - \frac{Tv\alpha^2}{C_P}} \arrow[dd] \\
& \boxed{C_P - C_P^{\mathrm{id}} = -T\int_0^{\infty}\left(\frac{\partial^2 v}{\partial T^2}\right)_v dv} \arrow[ur] & \\
\boxed{\rho \propto \frac{1}{v}} & & \boxed{K = \frac{1}{\beta_S}}
\end{tikzcd}$$



CREWES Fluid Properties Calculator

1) Enter temperature and pressure of the fluids, and indicate the units

Temperature: 40 ● celsius ○ Kelvin ○ Fahrenheit

Pressure: 9.465 ● MPa ○ bar ○ atm ○ psi ○ kbar

2) Complete calculations individually for each desired fluid (gas, oil and/or brine)

Gas Phase

1. Enter composition:

● by mole fractions: (Solves Peng-Robinson equation of state)

CH4: .062
CO2: .745
C2H6: 0
H2S: .193
C3H8: 0
N2: 0
C4H10: 0
O2: 0

○ by density ratio: (B&W, 1992) G: .6

2. Click here to calculate gas properties

3. Calculated gas properties

Density: 0.43216807 g/cm3
432.16806 kg/m3
26.979376 lb/ft3

Acoustic speed: 277.0641 m/s
0.27706409 km/s
909.0029 ft/s

Bulk modulus: 33.17517 MPa
0.33175173 kbar
4811.654 psi

Viscosity: 0.0 cP
0.0 Poise

Oil Phase

1. Enter information:

STP density: ○ API ● g/cm3

Gas-Oil Ratio: ● L/L ○ % of max

G for saturating gas: 0.6

2. Click here to calculate oil properties

3. Calculated oil properties

Density:

Acoustic speed:

Bulk modulus:

Viscosity:

Brine Phase

1. Enter information:

Salinity (NaCl): 120000 ● ppm ○ weight fraction

2. Click here to calculate brine properties

3. Calculated brine properties

Density: 1.0809351 g/cm3
1080.935 kg/m3
67.48058 lb/ft3

Acoustic speed: 1668.4493 m/s
1.6684494 km/s
5473.9155 ft/s

Bulk modulus: 3009.0242 MPa
30.090242 kbar
436422.25 psi

Viscosity: 0.9037294 cP
0.0090372935 Poise

3) Complete calculations for multiphase mixtures (single-phase calculations above must be completed first):

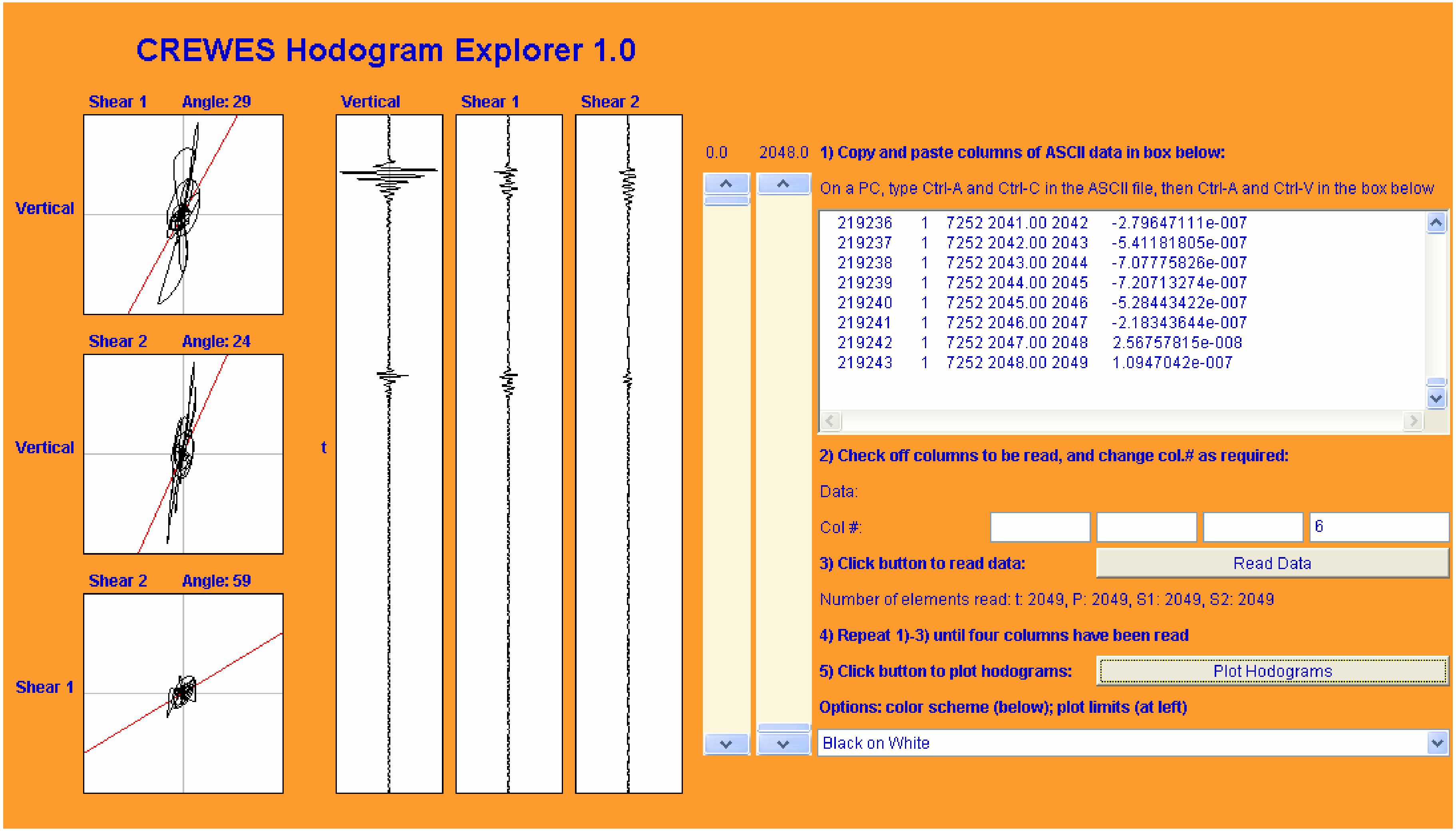
1. Enter volume fractions for each phase: Gas Oil Brine

2. Click here to calculate multiphase properties

3. Calculated mixture properties

Density:

Acoustic speed:



CREWES Hodogram Explorer 1.0

1) Copy and paste columns of ASCII data in box below:

On a PC, type Ctrl-A and Ctrl-C in the ASCII file, then Ctrl-A and Ctrl-V in the box below

```
219236    1   7252 2041.00 2042   -2.79647111e-007
219237    1   7252 2042.00 2043   -5.41181805e-007
219238    1   7252 2043.00 2044   -7.07775826e-007
219239    1   7252 2044.00 2045   -7.20713274e-007
219240    1   7252 2045.00 2046   -5.28443422e-007
219241    1   7252 2046.00 2047   -2.18343644e-007
219242    1   7252 2047.00 2048    2.56757815e-008
219243    1   7252 2048.00 2049    1.0947042e-007
```

2) Check off columns to be read, and change col.# as required:

Data:

Col #: 6

3) Click button to read data:

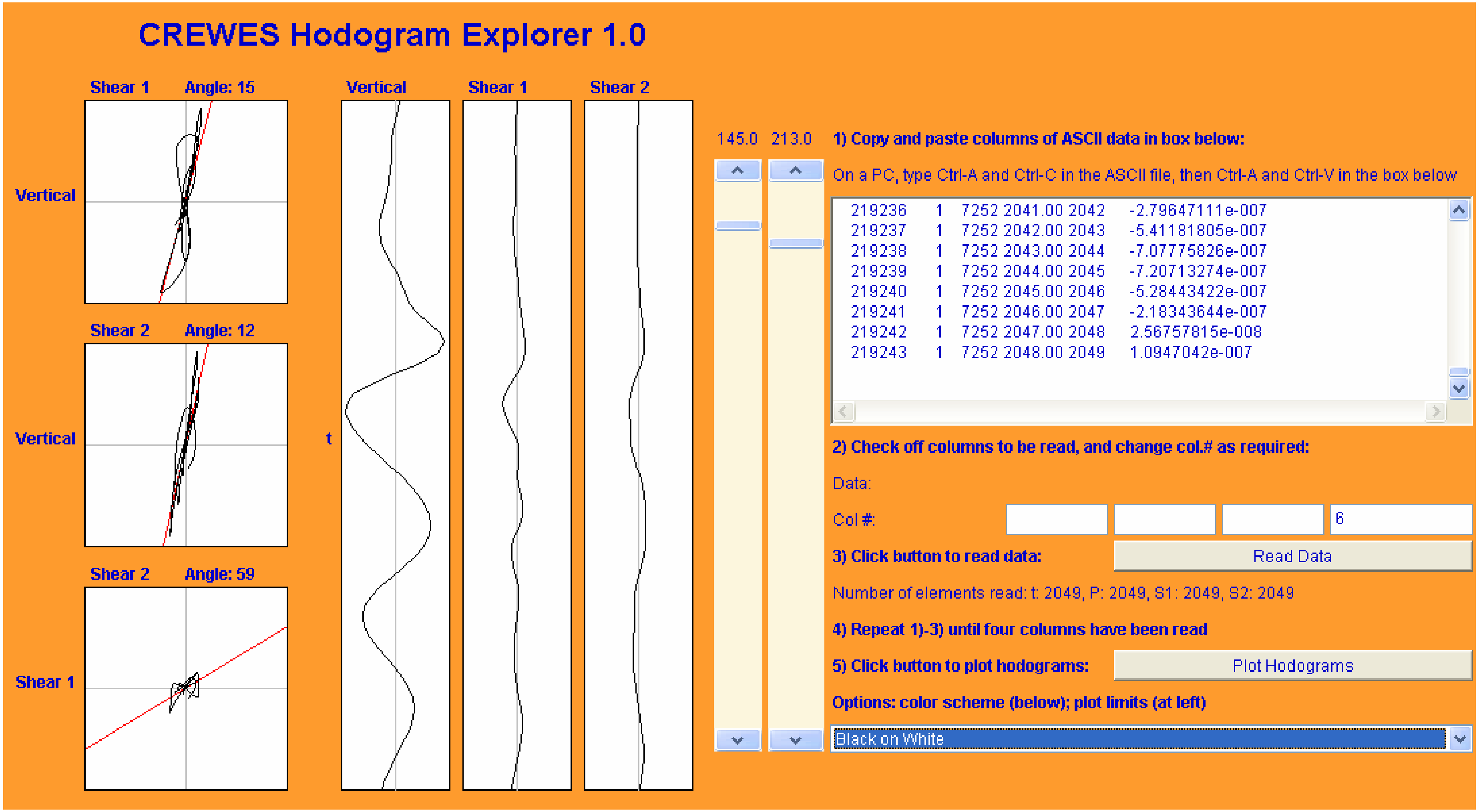
Number of elements read: t: 2049, P: 2049, S1: 2049, S2: 2049

4) Repeat 1)-3) until four columns have been read

5) Click button to plot hodograms:

Options: color scheme (below); plot limits (at left)

Black on White



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219241    1   7252 2046.00 2047   -2.18343644e-007
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