Numerical Experiments in Diffraction Theory: Edge Diffraction

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CREWES

Theory

Theoretical development is done within the context of zero order asymptotic ray theory (ART) – geometrical optics solution.

Point source:

3D spreading:

2.5D solution:

Geometrical Arrival:

$$\mathbf{U}_G \;\; \mathbf{r},\omega \;\; = \frac{F \;\; \omega \;\; \mathbf{\Pi}}{L_G \;\; \mathbf{r}} \exp\left[i\omega\tau_G \;\; \mathbf{r} \;\; \right]\mathbf{e}$$

Diffracted Arrival:

$$\mathbf{U}_{\mathcal{D}} \;\; \mathbf{r},\omega = \frac{F \;\; \omega \, W \;\; \omega,w \; \mathbf{\Pi}}{L_{\mathcal{D}} \;\; \mathbf{r}} \exp\left[i\omega\tau_{\mathcal{D}} \;\; \mathbf{r} \;\; \right]\mathbf{e}$$

$L \;\; \mathbf{r}$ – 3D geometrical spreading.

$\mathbf{\Pi}$ – Reflection & Transmission Coefficients.

$\tau \;\; \mathbf{r}$ – travel time.

$\mathbf{e}$ – vector decomposition of displacement - $\mathbf{U} \;\; \mathbf{r},\omega$.

$W \;\; \omega,w$ – diffraction coefficient.

$F \;\; \omega$ – Fourier time transform of source wavelet.

Klem-Musatov, K.D., 1995, Theory of edge waves and their use in seismology, SEG Publications, F. Hron and L.R. Lines, Editors.

Diffraction Coefficient

- Obtained from the saddle point approximation to a standard diffraction type integral.

- May be expressed in terms of the incomplete gamma function:

$$W(\omega,w)=\pm\frac{\exp\left[-i\pi w^2/2\right]}{2\sqrt{\pi}}\,\Gamma\left(1/2,-i\pi w^2/2\right)$$

"+" in shadow zone - "-" in illuminated zone

- Argument w is defined by: $w^2 \propto \tau_D(\mathbf{r})-\tau_G(\mathbf{r})$

- For large argument: $W(\omega,w)\propto 1/\sqrt{\omega}$

Numerical Results

- Two simple wedge models are considered.
- Only a single geometrical arrival plus the relevant diffracted arrival are shown.
- Synthetics for a VSP and two AVO situations are graphically presented – both V and H components of particle displacement.
- In the final figure the two AVO are added together as this is one of more useful aspects of ART.
- The velocity and density parameters for the two models are given in the following figure.

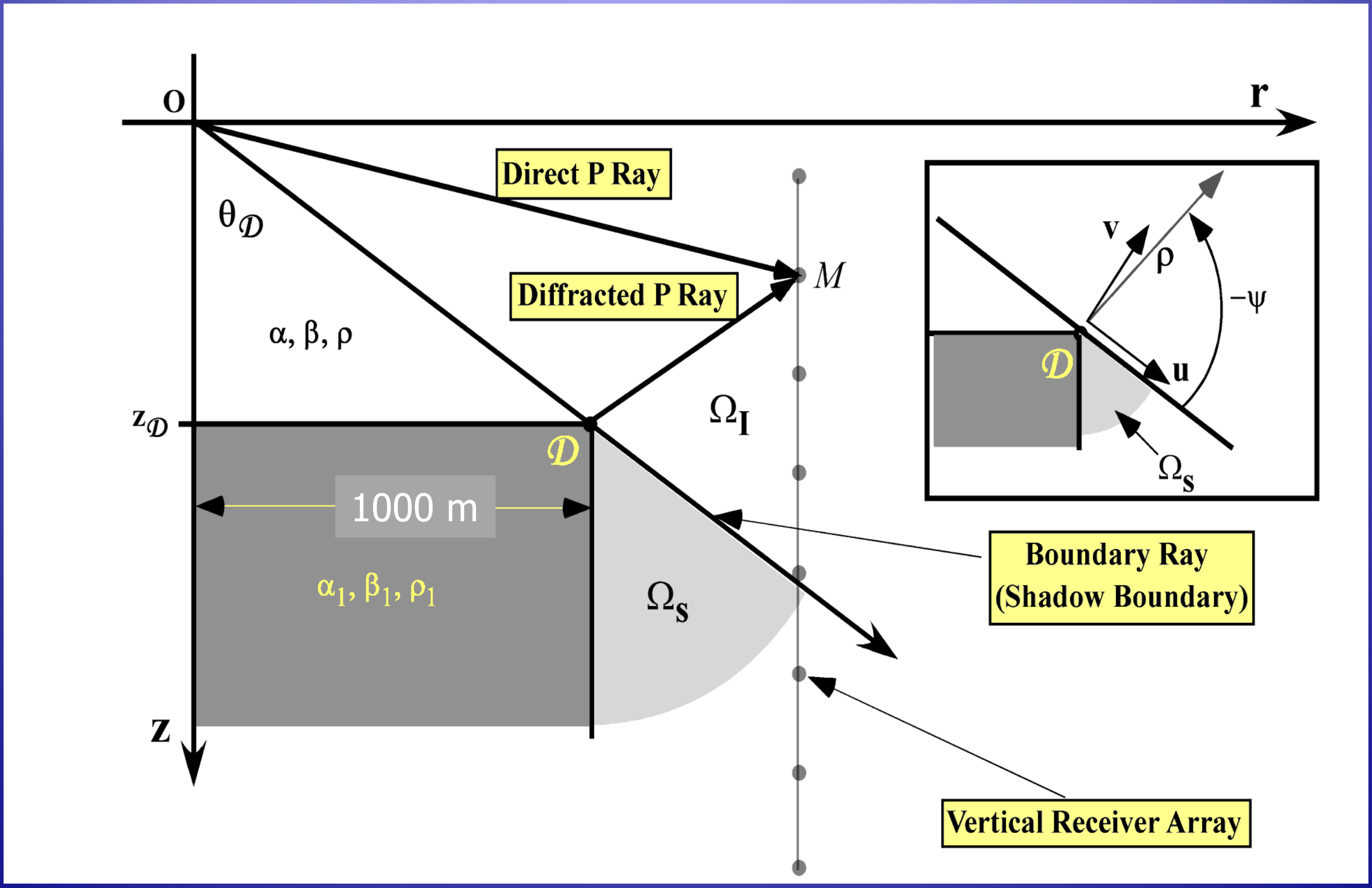
Model Parameters

	P Velocity (km/s)	S Velocity (km/s)	Density (gm/cm^3)
Wedge I	2.50	1.44	2.20
Halfspace	2.00	1.15	1.80

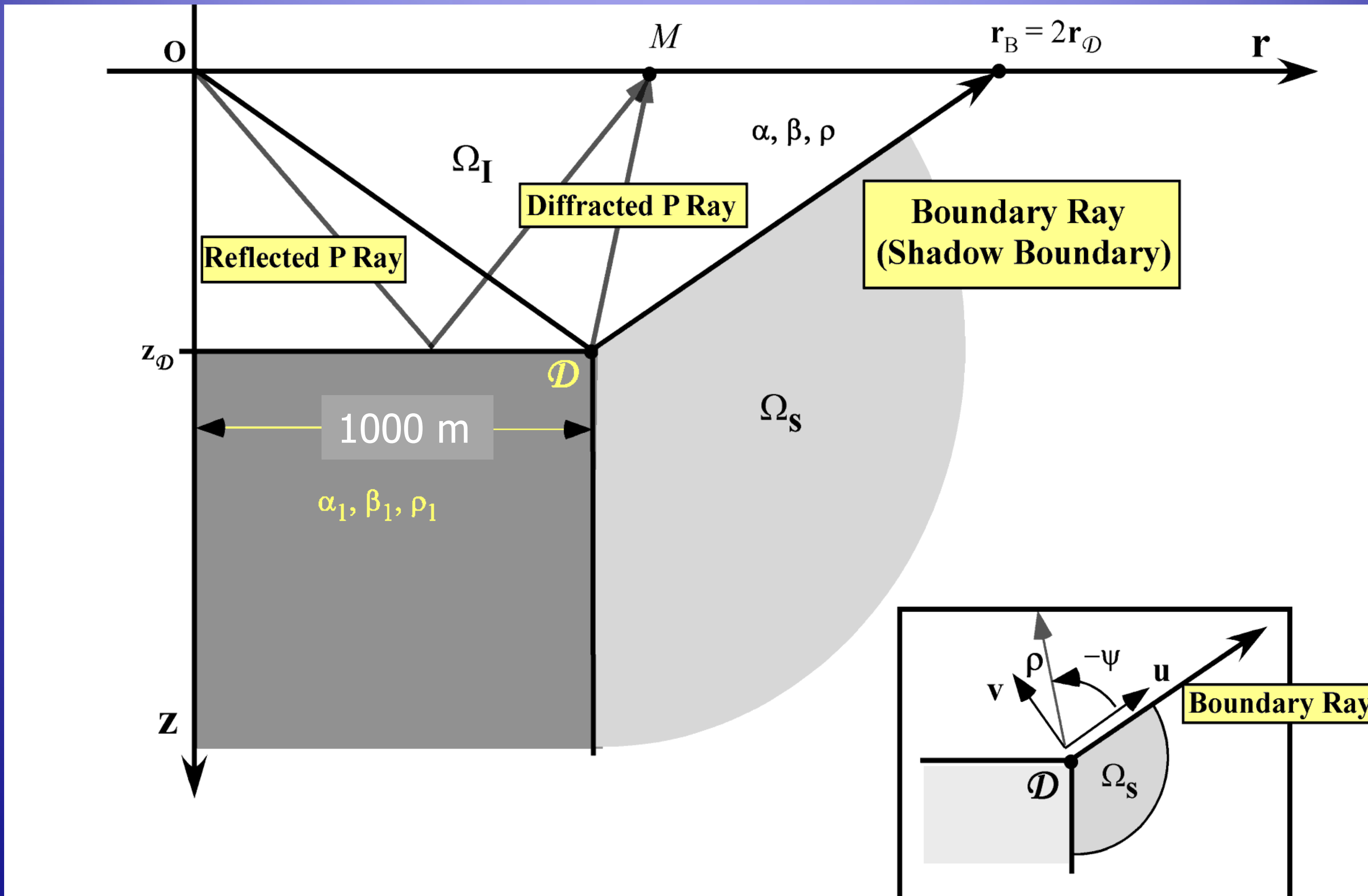
Model I

Model II

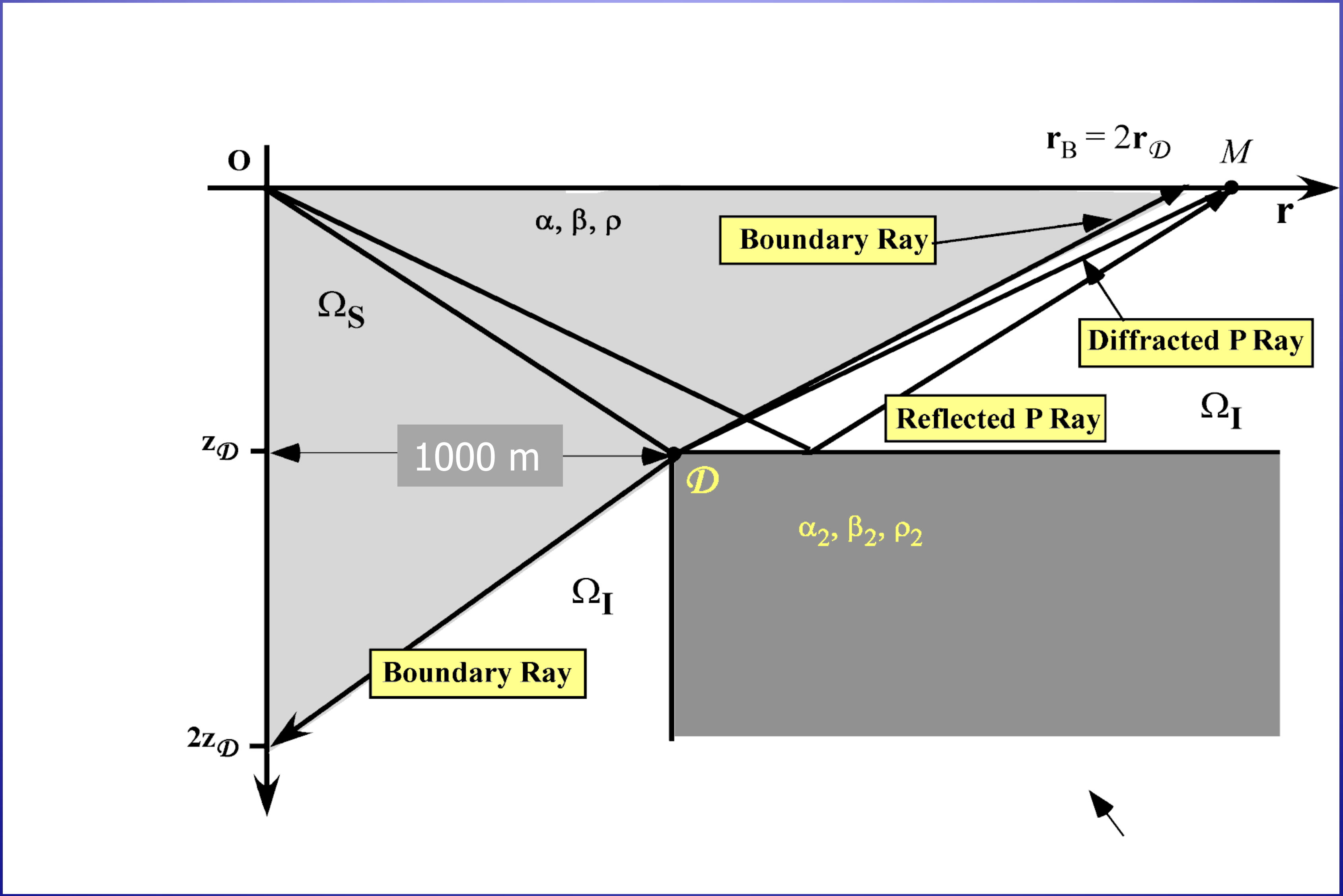
	P Velocity (km/s)	S Velocity (km/s)	Density (gm/cm^3)
Wedge II	1.60	0.92	1.50
Halfspace	2.00	1.15	1.80


O
r
Direct P Ray
θ𝒟
Diffracted P Ray
M
α, β, ρ
z𝒟
𝒟
Ω_I
1000 m
α1, β1, ρ1
Ω_S
z
Boundary Ray
(Shadow Boundary)
Vertical Receiver Array
v
ρ
−ψ
u
Ω_S

Model I: VSP

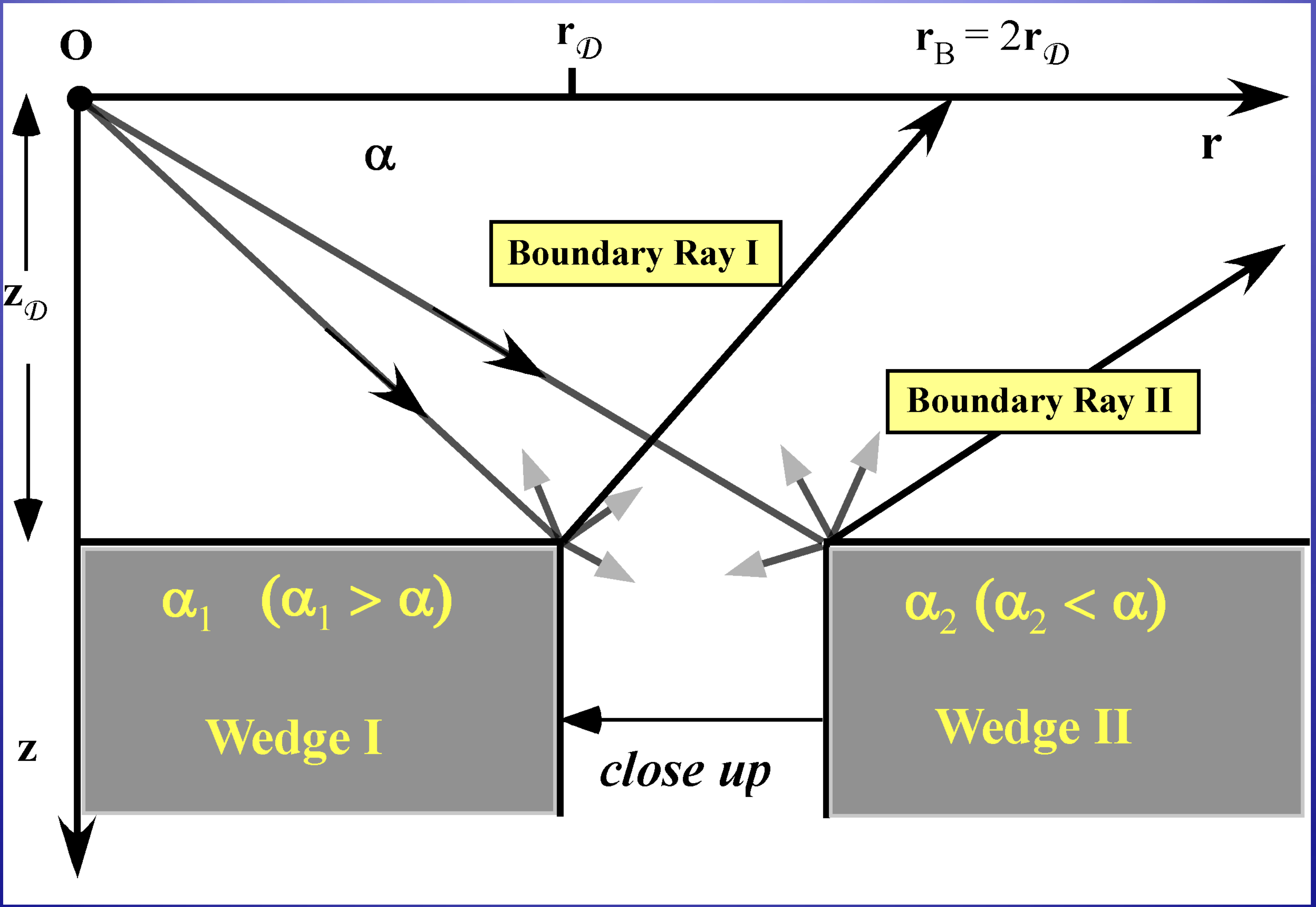

O
M
$\mathbf{r}_B = 2\mathbf{r}_{\mathcal{D}}$
r
α, β, ρ
Ω_I
Diffracted P Ray
Boundary Ray
(Shadow Boundary)
Reflected P Ray
$z_{\mathcal{D}}$
$\mathcal{D}$
1000 m
Ω_S
$\alpha_1, \beta_1, \rho_1$
z
ρ
$-\psi$
u
v
Boundary Ray
$\mathcal{D}$
Ω_S

Model I: AVO



O
$\mathbf{r}_B = 2\mathbf{r}_{\mathcal{D}}$
M
r
α, β, ρ
Boundary Ray
Ω_S
Diffracted P Ray
Reflected P Ray
Ω_I
$z_{\mathcal{D}}$
1000 m
$\mathcal{D}$
$\alpha_2, \beta_2, \rho_2$
Ω_I
Boundary Ray
$2z_{\mathcal{D}}$

Model II: AVO



Model I and II: AVO

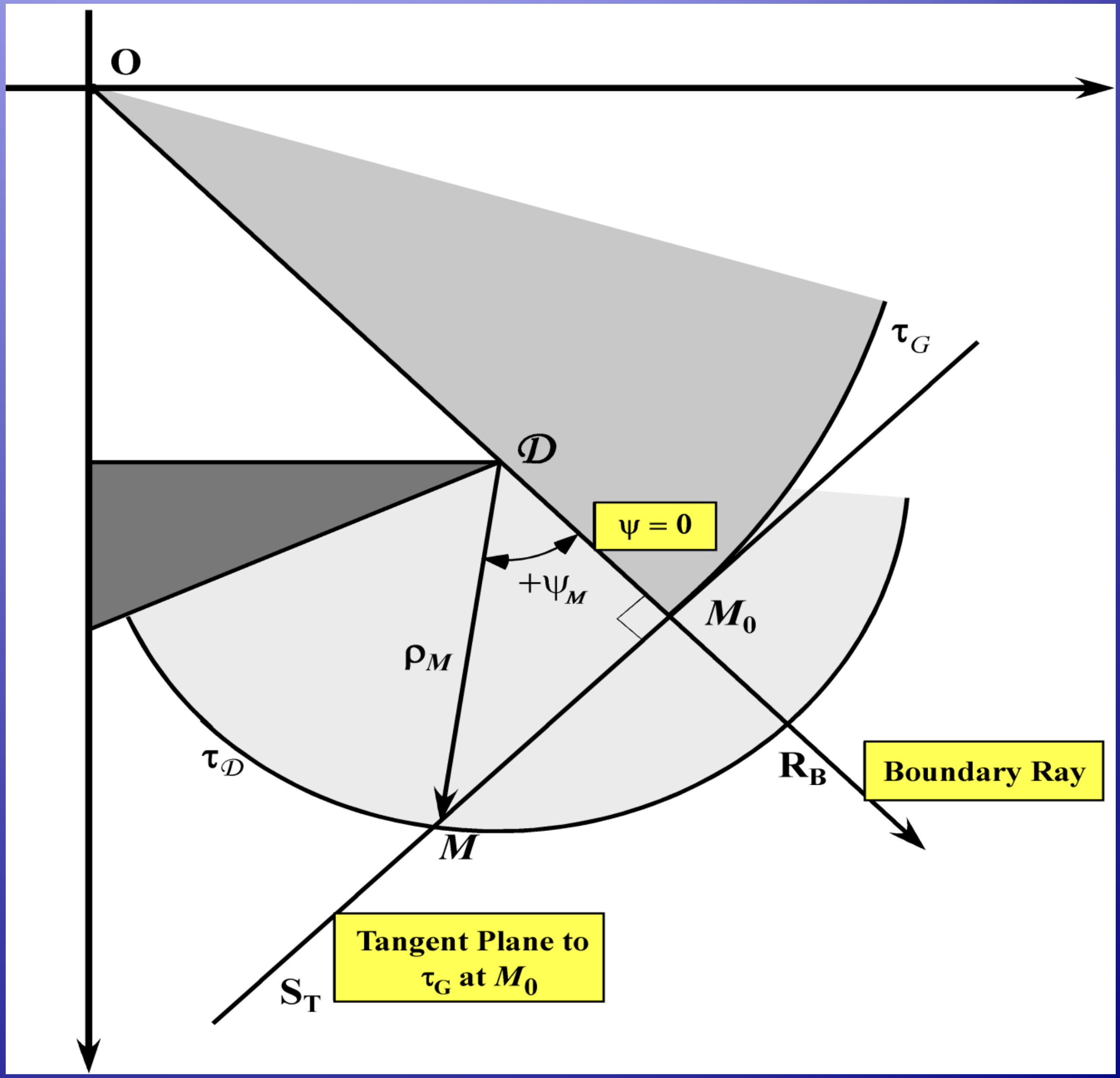
Analytic Continuation of $\tau_G(\mathbf{r})$ From Illuminated to Shadow Zone

Illuminated Region:

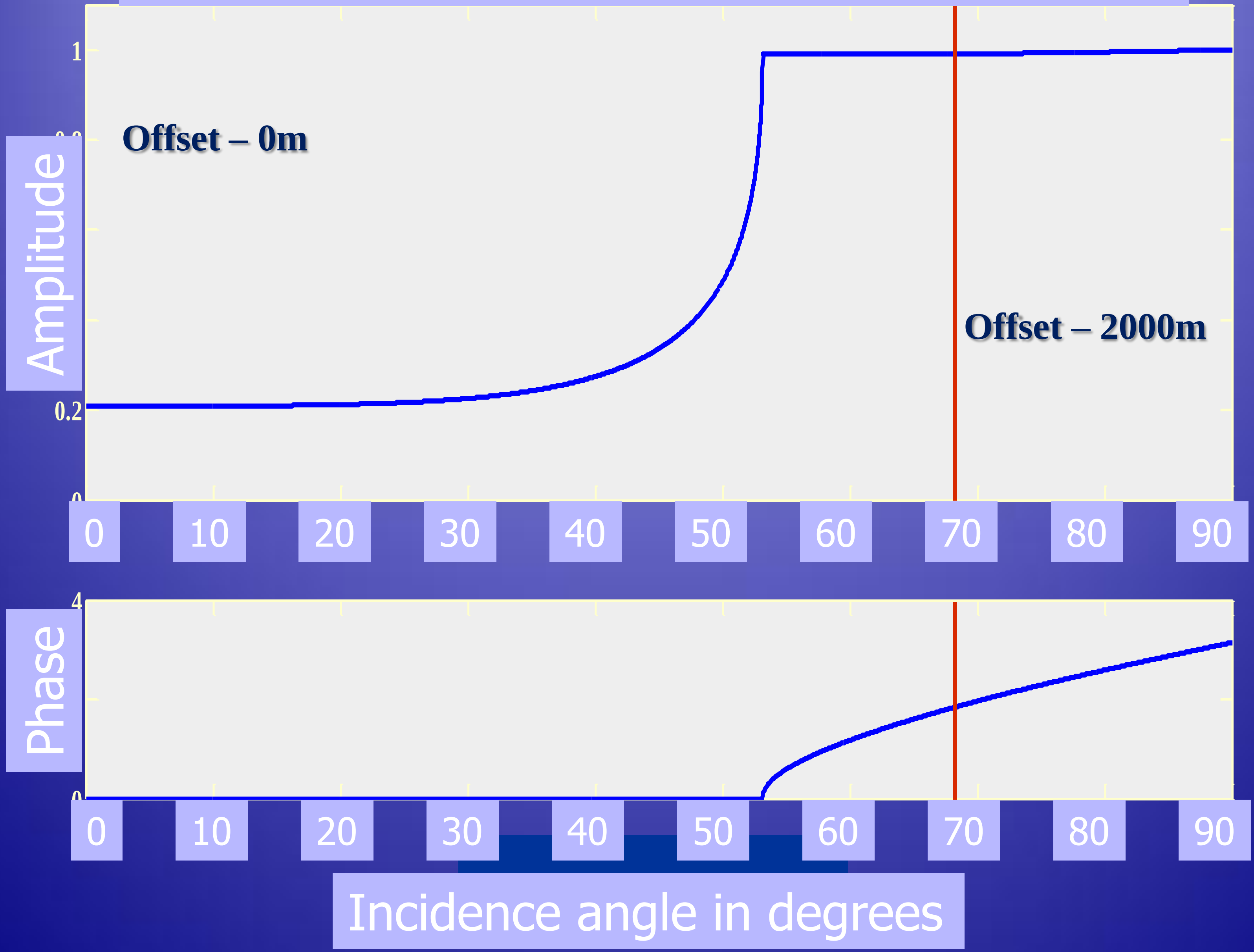
$$w^2 = \left[\frac{2\omega}{\pi}\left(\tau_D(M) - \tau_G(M)\right)\right]$$

Shadow Region:

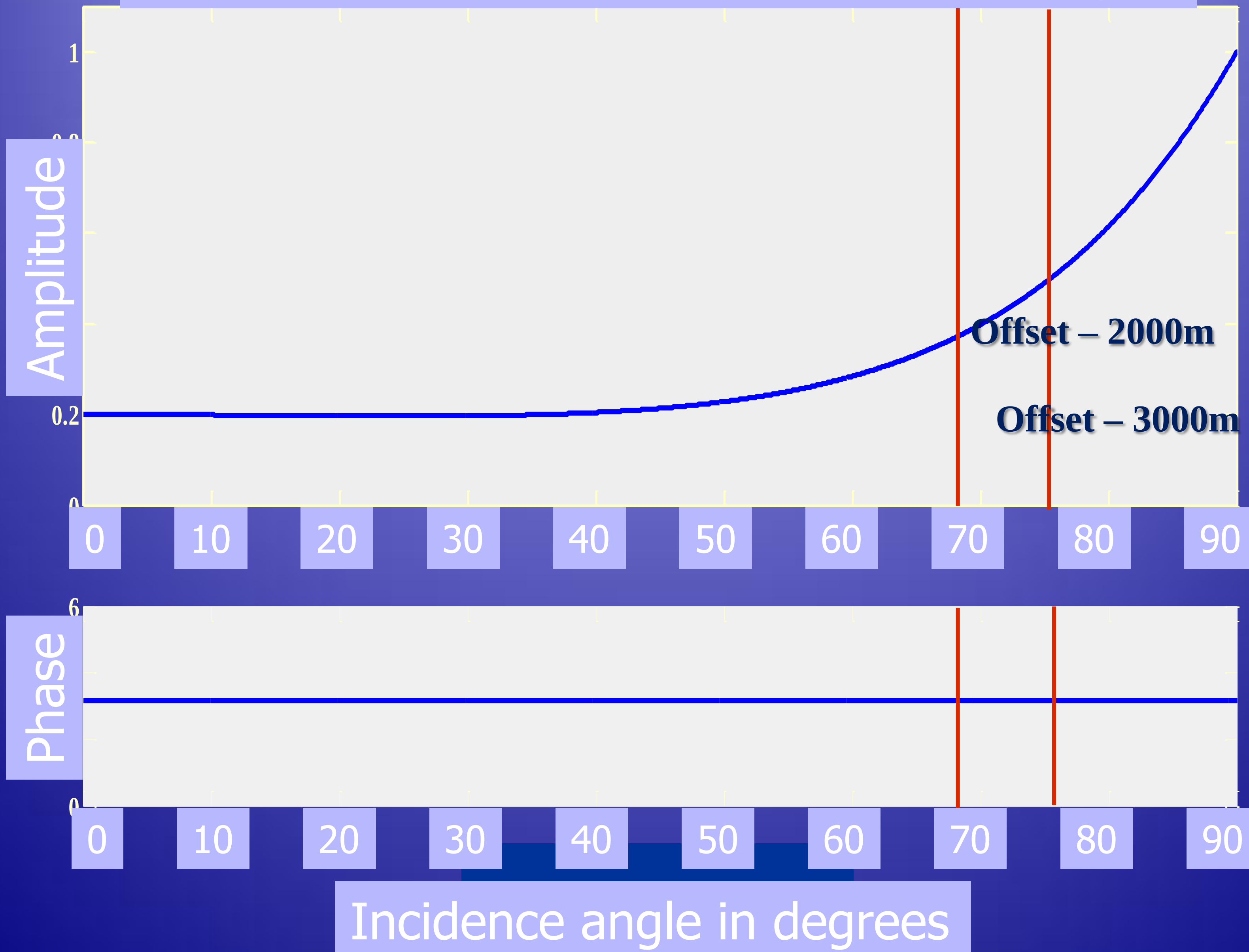
$$w^2 = \left[\frac{2\omega}{\pi}\left(\frac{\rho_M}{\alpha_1}\left(1 - \cos\psi_M\right)\right)\right]$$

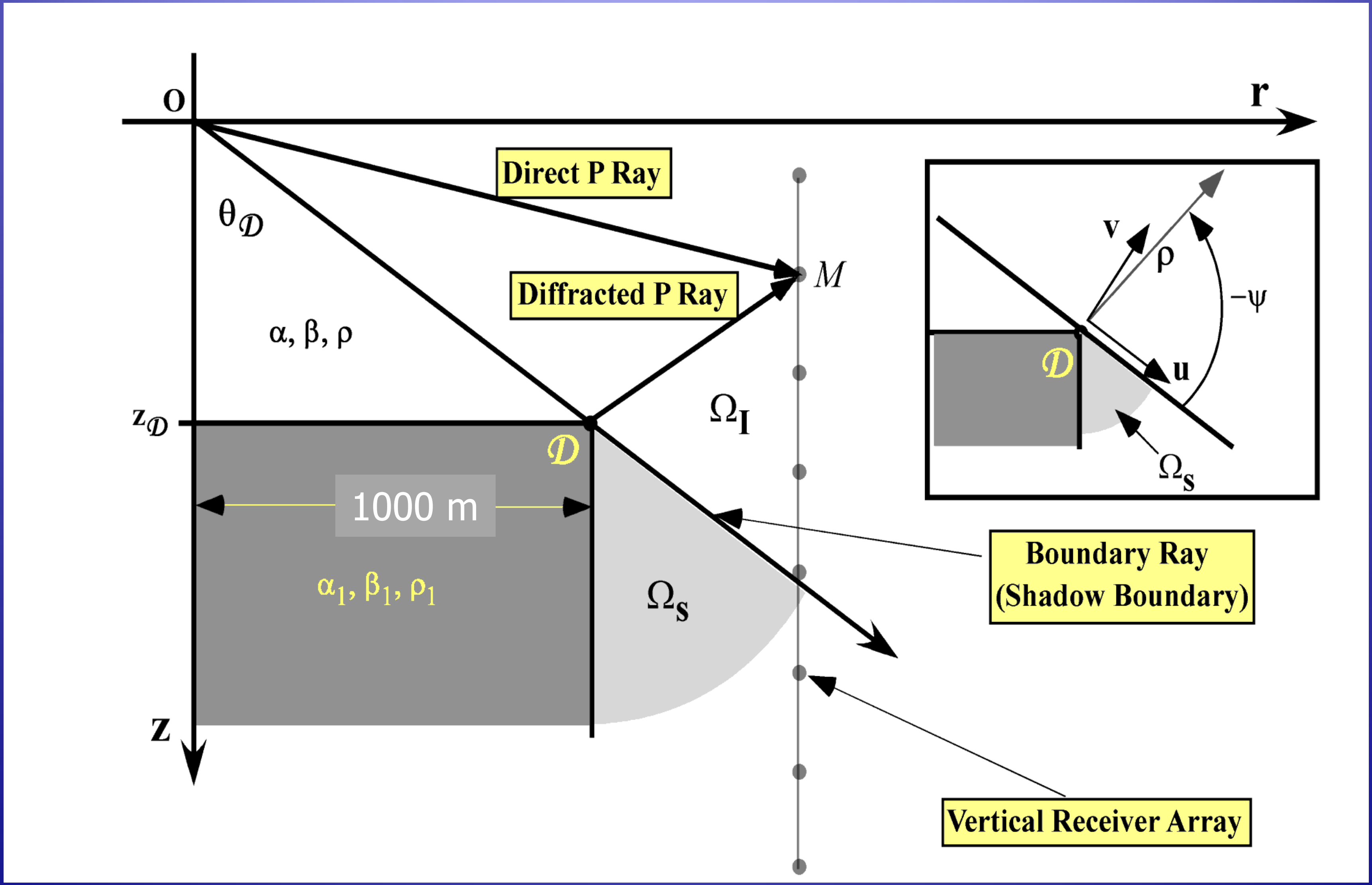


P1P1 Reflection coefficient Halfspace/WedgeI



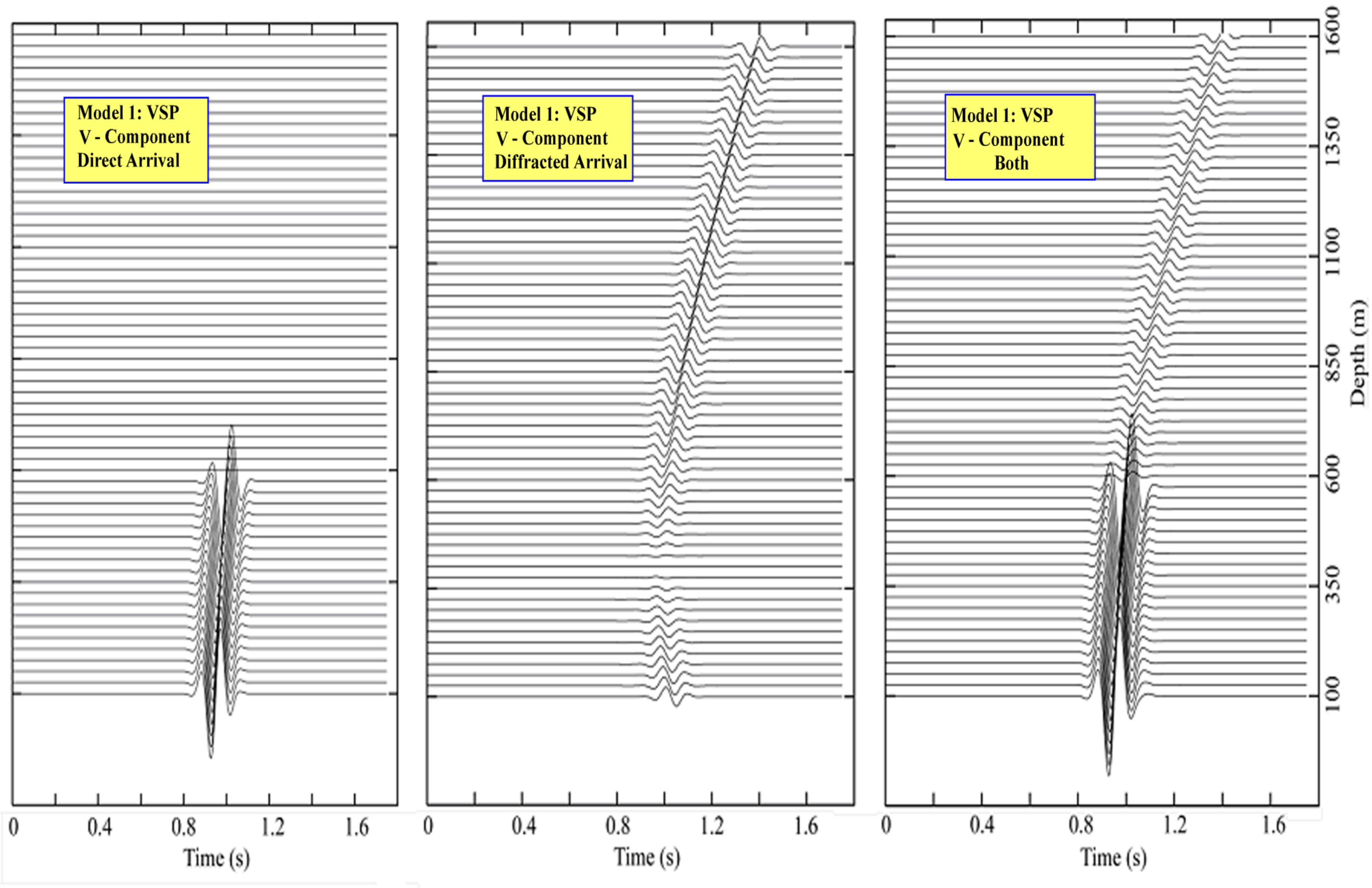
P1P1 Reflection coefficient Halfspace/WedgeII

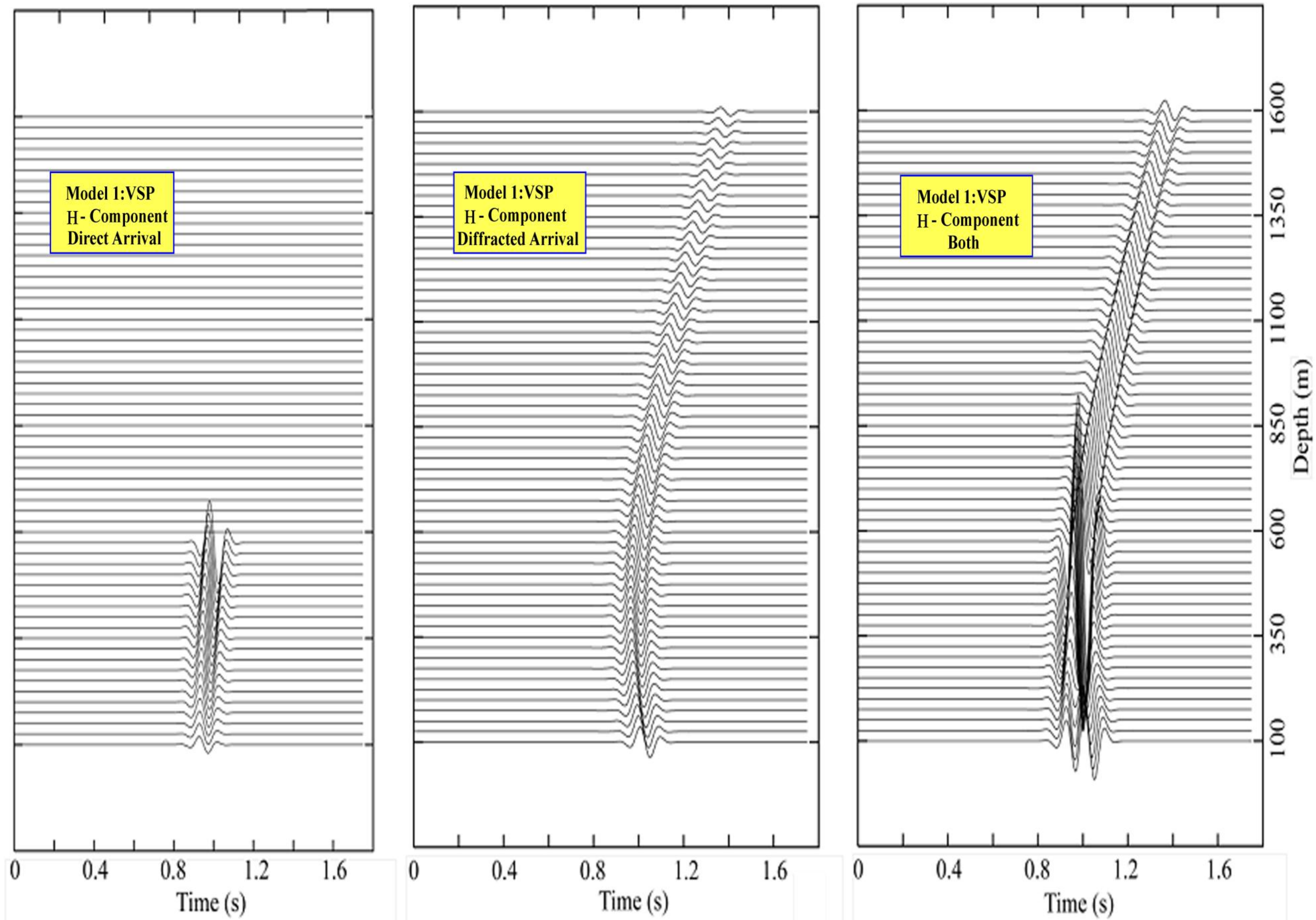




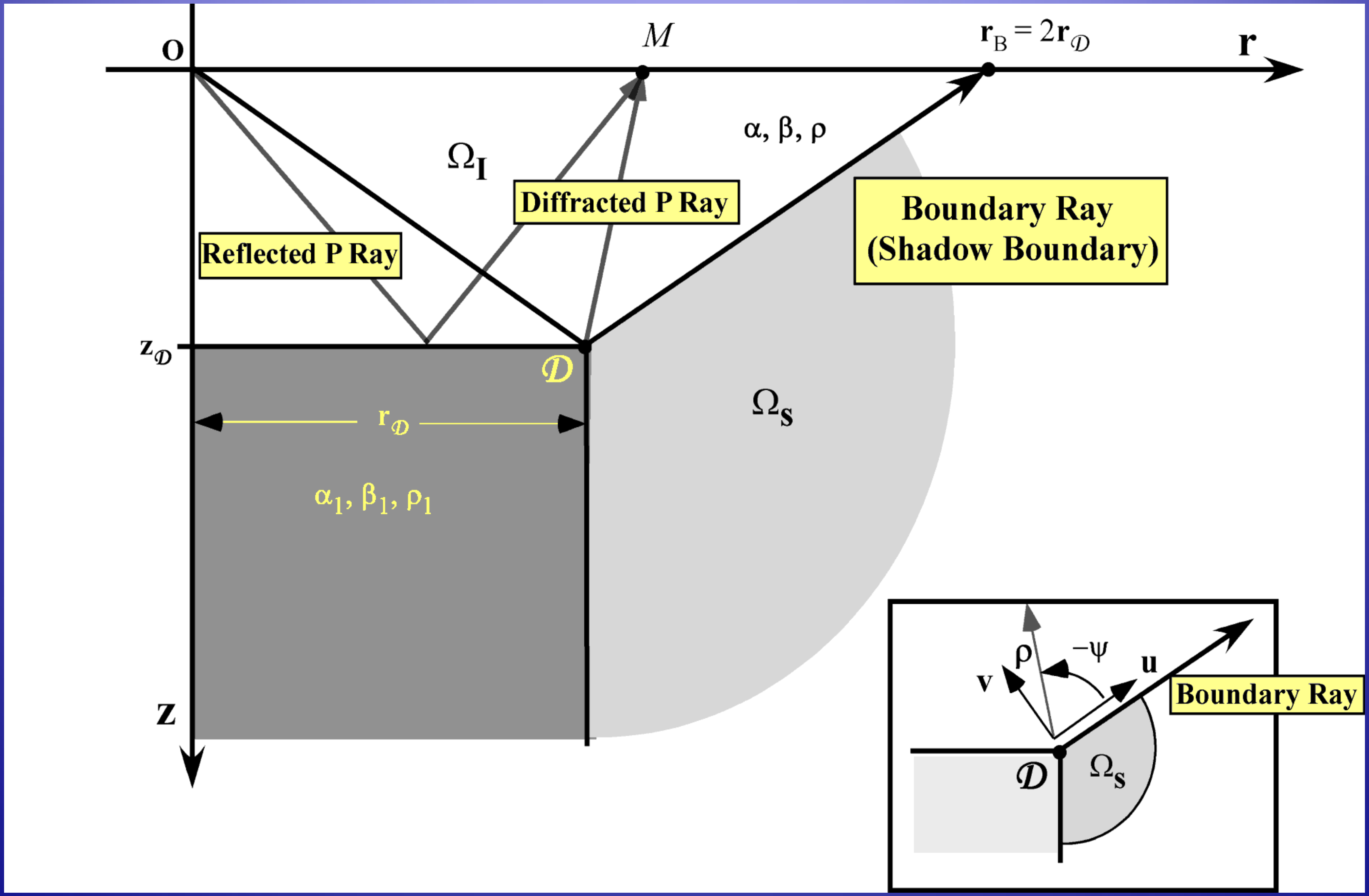
O
r
z
Direct P Ray
Diffracted P Ray
M
θ𝒟
α, β, ρ
z𝒟
𝒟
Ω_I
1000 m
α1, β1, ρ1
Ω_S
Boundary Ray
(Shadow Boundary)
Vertical Receiver Array
v
ρ
−ψ
u
Ω_S

Model I: VSP


Model 1: VSP
V - Component
Direct Arrival
Model 1: VSP
V - Component
Diffracted Arrival
Model 1: VSP
V - Component
Both
0
0.4
0.8
1.2
1.6
Time (s)
Depth (m)
1600
1350
1100
850
600
350
100

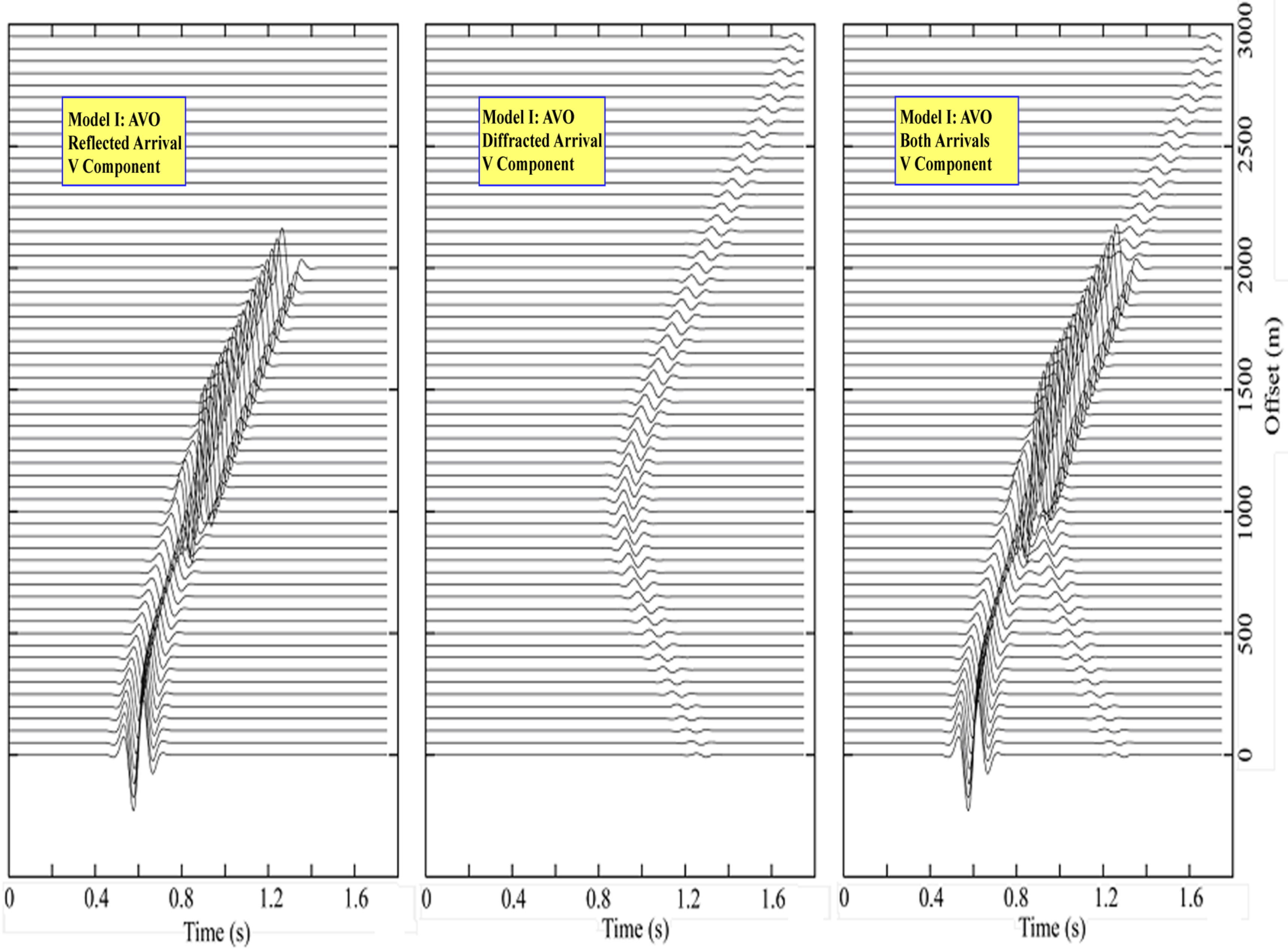


Model 1:VSP
H - Component
Direct Arrival
Model 1:VSP
H - Component
Diffracted Arrival
Model 1:VSP
H - Component
Both
0
0.4
0.8
1.2
1.6
Time (s)
100
350
600
850
1100
1350
1600
Depth (m)

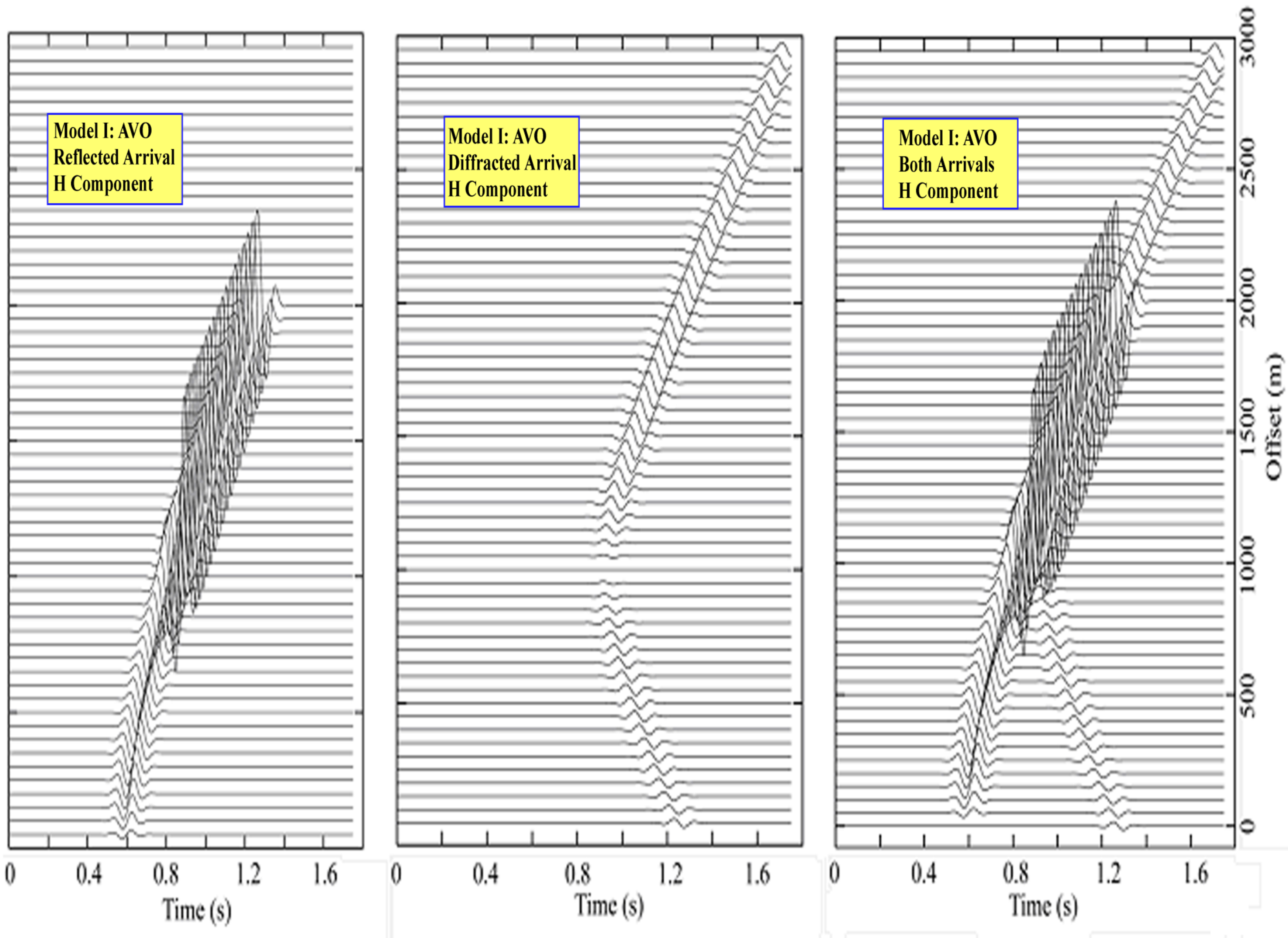


O
M
$r_B = 2r_{\mathcal{D}}$
r
α, β, ρ
Ω_I
Diffracted P Ray
Boundary Ray
(Shadow Boundary)
Reflected P Ray
$z_{\mathcal{D}}$
$\mathcal{D}$
Ω_s
$r_{\mathcal{D}}$
$\alpha_1, \beta_1, \rho_1$
z
ρ
$-\psi$
v
u
Boundary Ray
$\mathcal{D}$
Ω_s

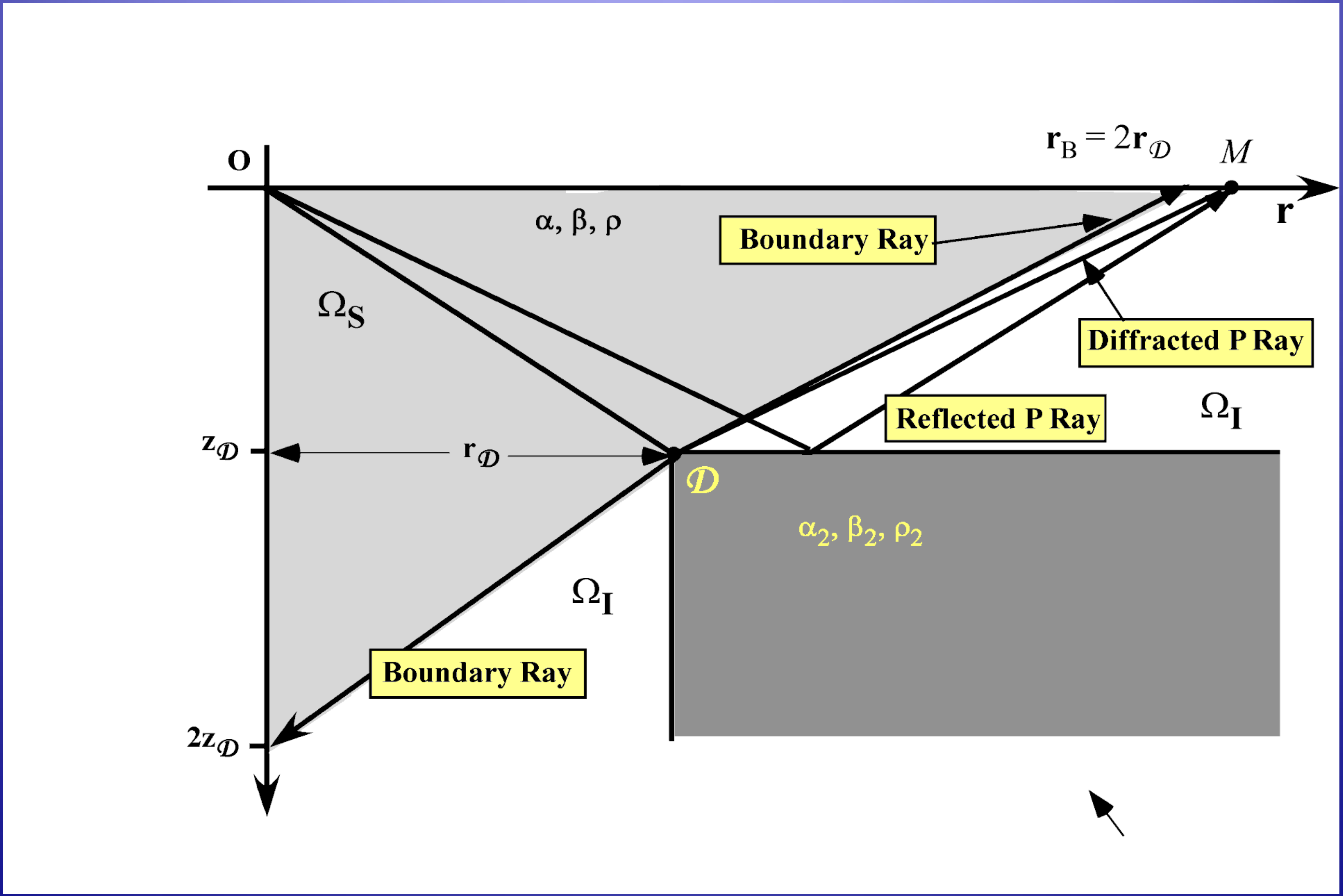
Model I: AVO



Model I: AVO
Reflected Arrival
V Component
Model I: AVO
Diffracted Arrival
V Component
Model I: AVO
Both Arrivals
V Component
0
0.4
0.8
1.2
1.6
Time (s)
Offset (m)
0
500
1000
1500
2000
2500
3000

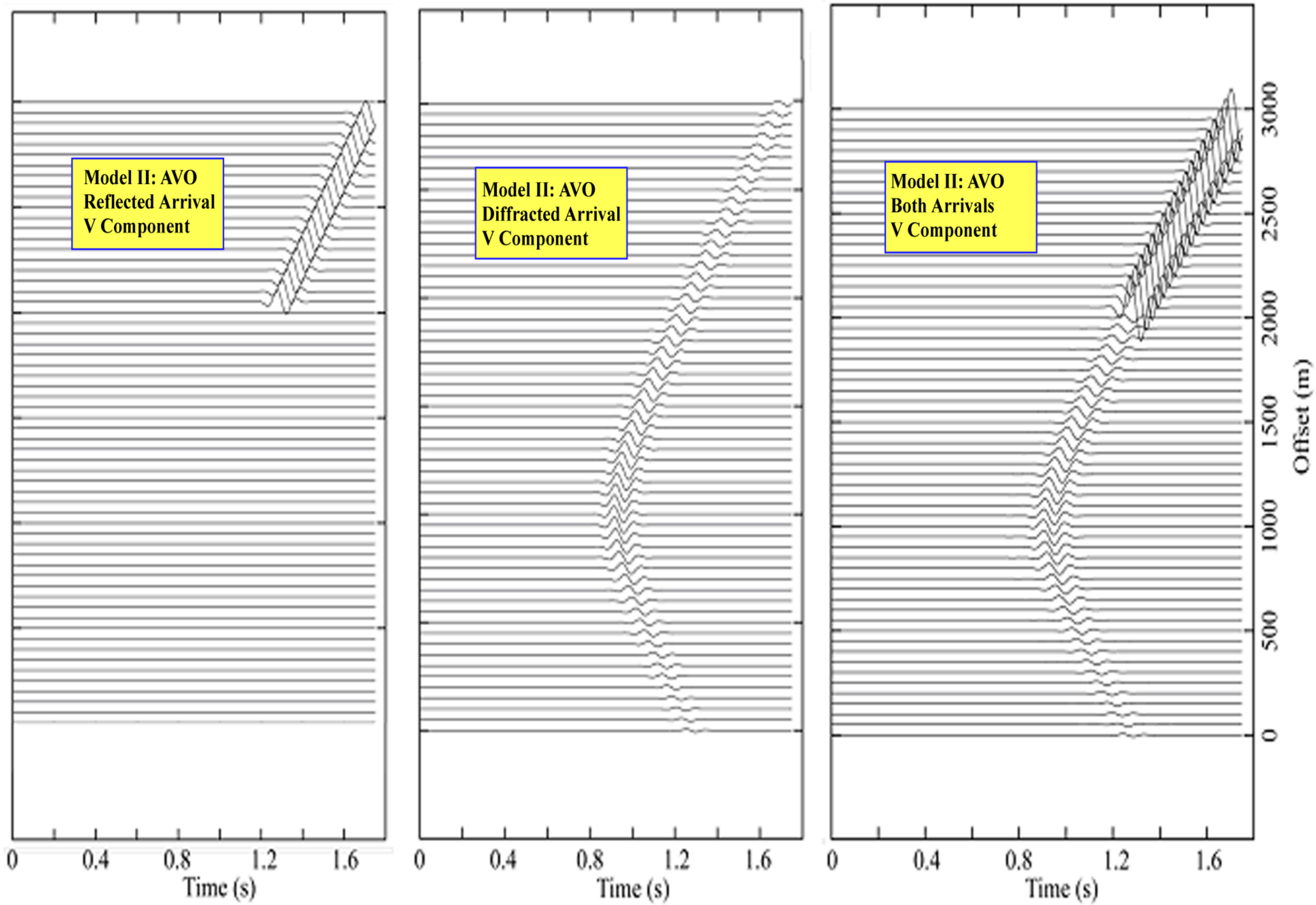


Model I: AVO
Reflected Arrival
H Component
Model I: AVO
Diffracted Arrival
H Component
Model I: AVO
Both Arrivals
H Component
0
0.4
0.8
1.2
1.6
Time (s)
0
500
1000
1500
2000
2500
3000
Offset (m)

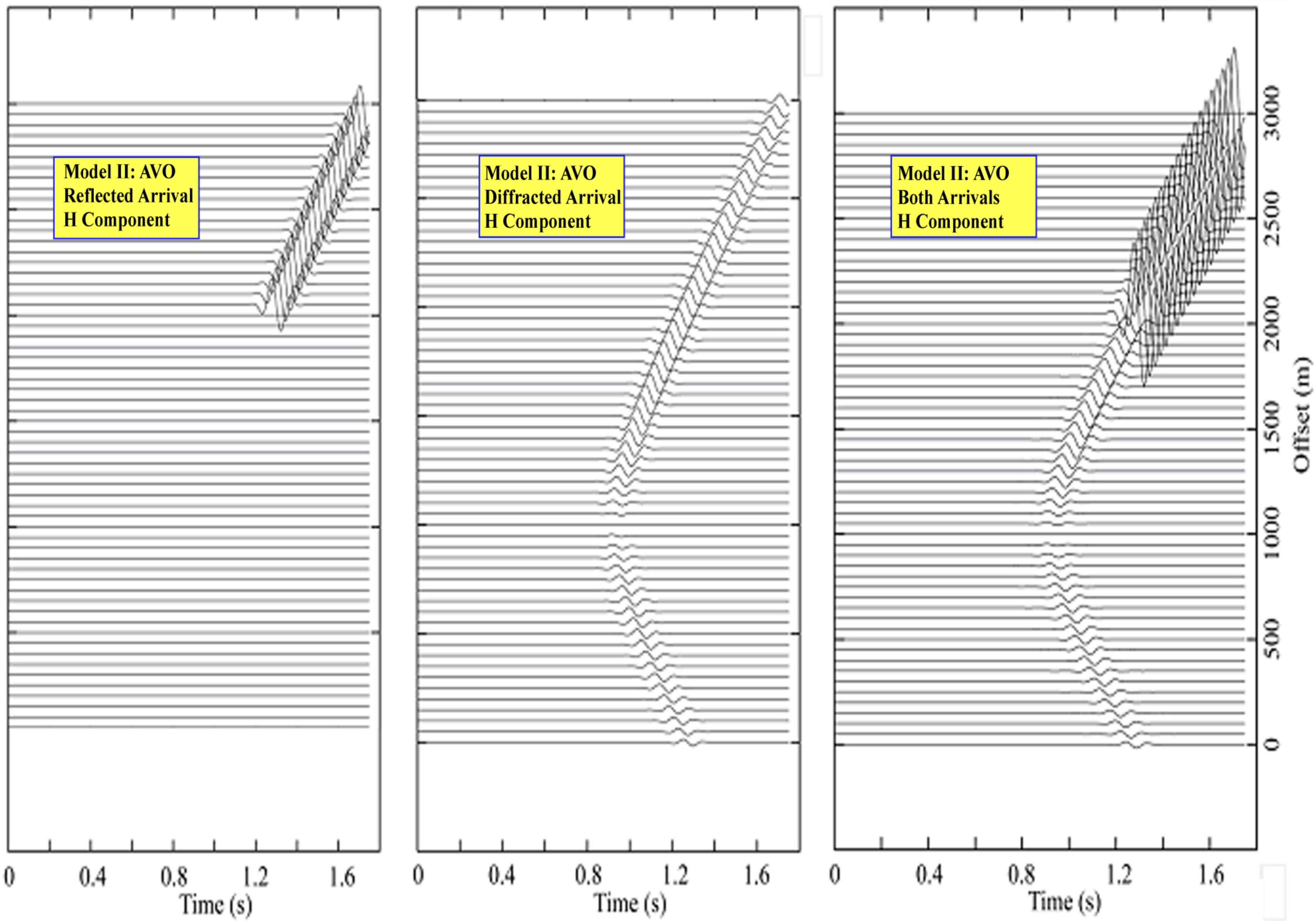


O
$r_B = 2r_{\mathcal{D}}$
M
r
α, β, ρ
Boundary Ray
Ω_S
Diffracted P Ray
Reflected P Ray
Ω_I
$z_{\mathcal{D}}$
$r_{\mathcal{D}}$
$\mathcal{D}$
$\alpha_2, \beta_2, \rho_2$
Ω_I
Boundary Ray
$2z_{\mathcal{D}}$

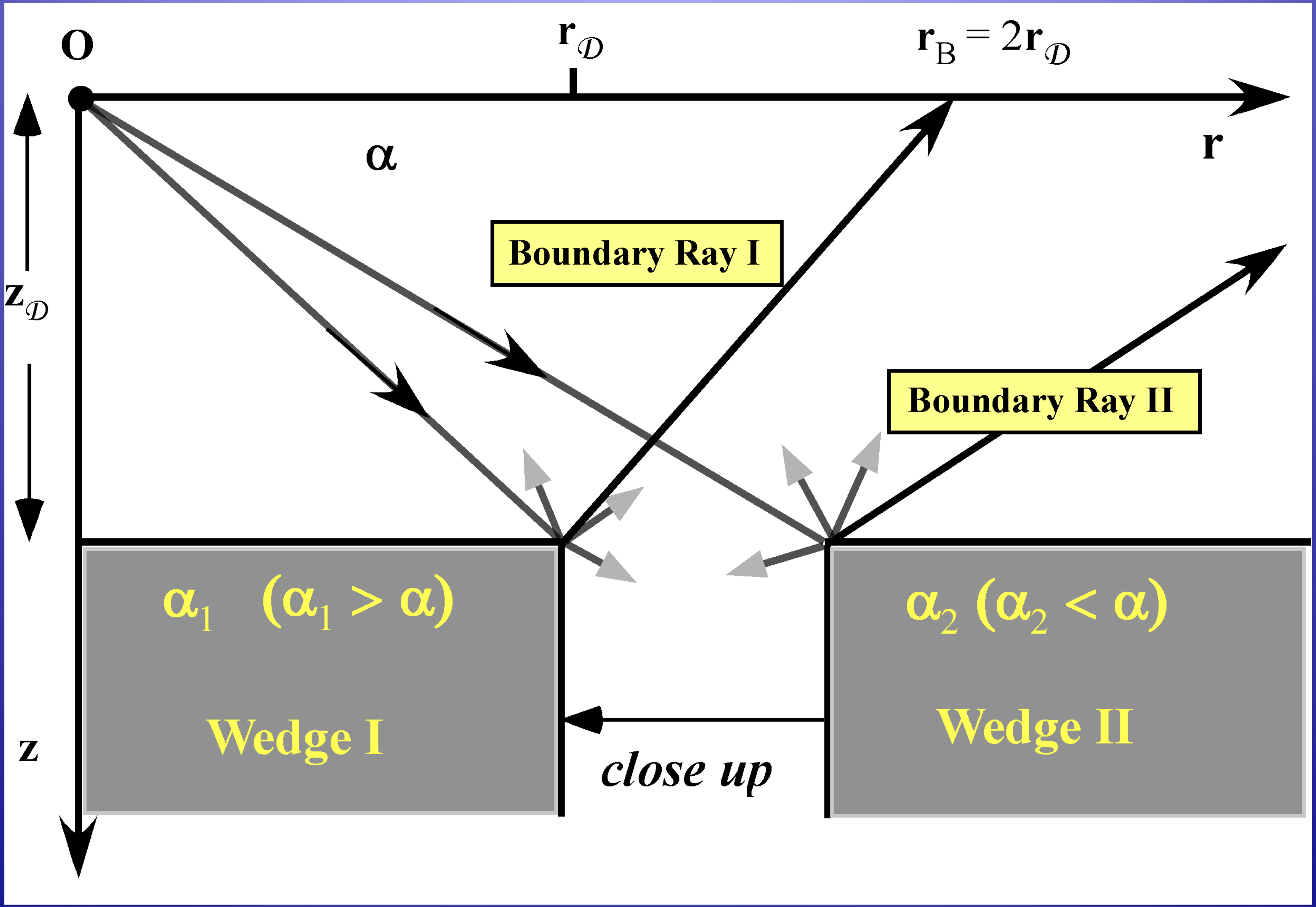
Model II: AVO



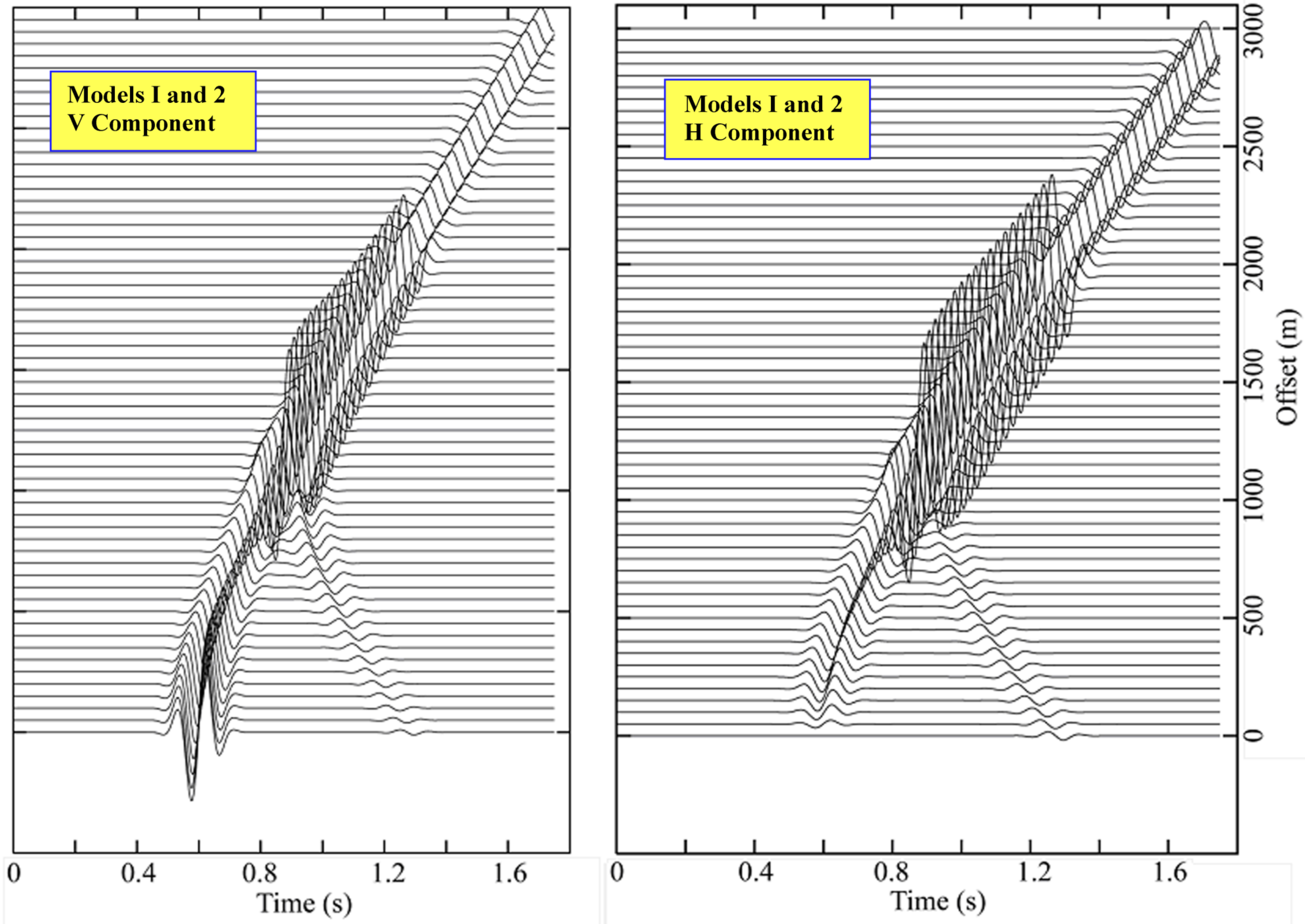
Model II: AVO
Reflected Arrival
V Component
Model II: AVO
Diffracted Arrival
V Component
Model II: AVO
Both Arrivals
V Component
0
0.4
0.8
1.2
1.6
Time (s)
Offset (m)
0
500
1000
1500
2000
2500
3000



Model II: AVO
Reflected Arrival
H Component
Model II: AVO
Diffracted Arrival
H Component
Model II: AVO
Both Arrivals
H Component
0
0.4
0.8
1.2
1.6
Time (s)
Offset (m)
0
500
1000
1500
2000
2500
3000



Model I and II: AVO


Models I and 2
V Component
0
0.4
0.8
1.2
1.6
Time (s)
Models I and 2
H Component
0
0.4
0.8
1.2
1.6
Time (s)
0
500
1000
1500
2000
2500
3000
Offset (m)

Conclusions/Discussion

- Diffraction theory based on zero order asymptotic ray theory (ART) has been presented.
- Two simple models and their union were investigated and synthetic seismograms were shown.
- Geometrical arrival and a diffracted arrival are, apart from the diffraction coefficient, quite similar.
- The theory presented here can be applied to converted waves.
- The diffraction coefficient may be expressed in terms of a number of standard functions for which source code is freely available.
- The extension to a true 3D structure does not require any significant modification of the theory, only the unglamorous task of model building.

Acknowledgements

- Thanks to CREWES Sponsors and NSERC.

- Thank you for listening.