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Applied Mathematics Perspectives Workshops

“Medical and Seismic Imaging”

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Gabor multipliers for pseudodifferential operators and wavefield propagators

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Outline

Gabor theory - time-frequency display

Gabor multipliers are built on a modification of the Fourier transform

$$\mathcal{F}f(\omega) = \int_{-\infty}^{\infty} f(s)e^{-2\pi i s\omega}\, ds,$$

where we insert a shifted “window function” $g(s-t)$ to get a time-frequency display of the data:

$$\mathcal{G}f(t,\omega) = \int_{-\infty}^{\infty} f(s)g(s-t)e^{-2\pi i s\omega}\, ds.$$

This $\mathcal{G}f(t,\omega)$ is called the Gabor transform of signal $f(t)$.

Example: Vibroseis sweep, Fourier

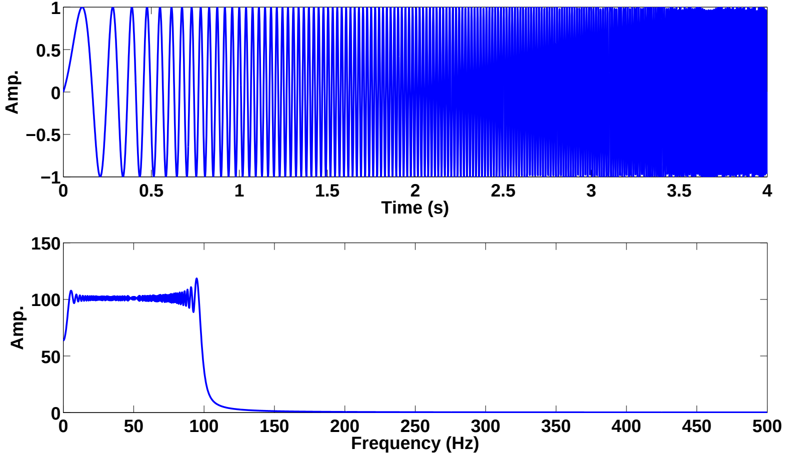

Figure: A synthetic Vibroseis sweep, 2 to 100 Hz, and FFT.

Example: Vibroseis sweep, Gabor

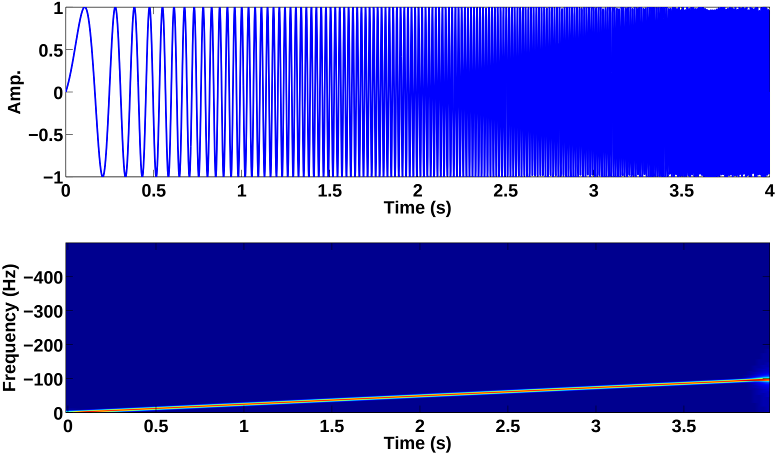

Figure: A synthetic Vibroseis sweep, 2 to 100 Hz, and Gabor transform.

Example: Vibroseis + harmonics, Fourier

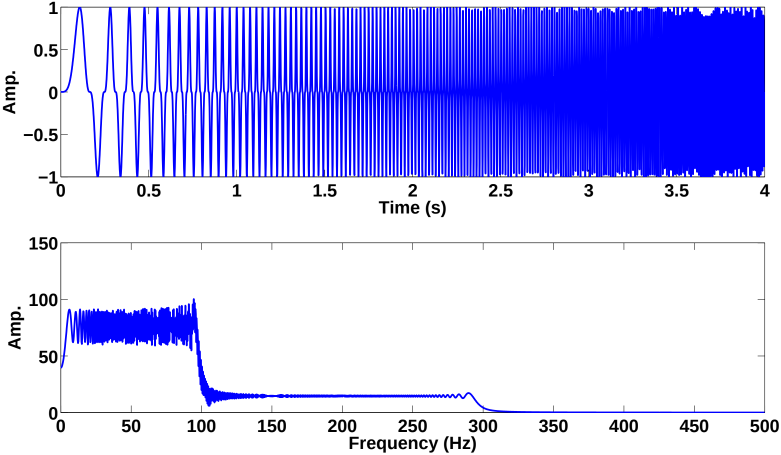

Figure: A synthetic Vibroseis sweep, with harmonics apparent in FFT.

Example: Vibroseis + harmonic, Gabor

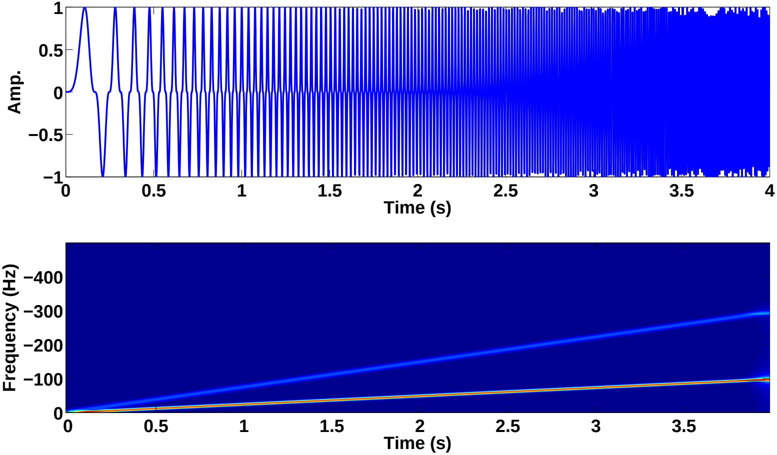

Figure: A synthetic Vibroseis sweep, with 3rd harmonic apparent in Gabor.

Gabor multiplier: filtering a range of time–freq values

The Gabor transform $\mathcal{G}f(t,\omega)$ is modified by function $\alpha(t,\omega)$.
Result is the product

$$\alpha(t,\omega)\mathcal{G}f(t,\omega),$$

where $\alpha(t,\omega)$ is set to one to pass energy at certain t,ω, set to zero to block energy at other t,ω.

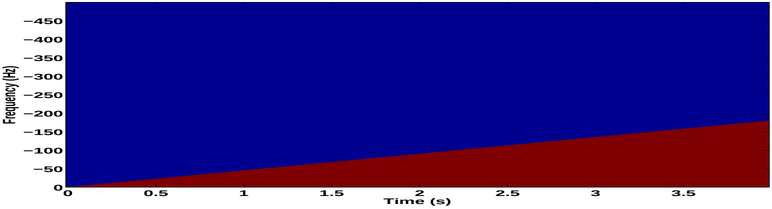

Figure: The permissible region of frequencies to pass, in t-ω domain.

Gabor multiplier: the filtered result

The inverse Gabor transform is applied to the product $\alpha(t,\omega)\mathcal{G}f(t,\omega)$

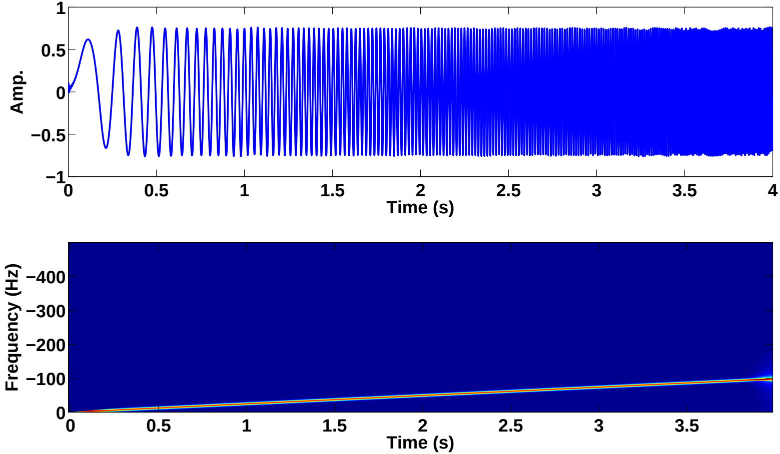

Figure: The result of non-stationary filtering: harmonic is removed.

Gabor multiplier: mathematical form

Forward transform, multiply by α, apply adjoint (inverse):

$$f \mapsto G_\alpha f = \mathcal{G}^* M_\alpha \mathcal{G} f.$$

Theorem

There is an approximate functional calculus for Gabor multipliers G_α.

Meaning: we can represent a series of physical operations (transmission, reflection, attenuation of signals) with a series of multipliers.

Theorem

With a good choice of windows, and slowly changing symbol α, the Gabor multiplier approximates pseudodifferential operators.

$$G_\alpha \approx K_\alpha \text{ or } A_\alpha.$$

Differential operators

The wave equation gives an example of a pseudodifferential operator:

$$f \mapsto \nabla^2 f - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} f$$

Phase shift operators, attenuation operators are also pseudodifferential operators, given in the form

$$K_\alpha f(t) = \int_{-\infty}^{\infty} \alpha(t,\omega)\widehat{f}(\omega)e^{2\pi i\omega t}\, d\omega.$$

Our work shows these operators (and their adjoints) can be approximated by Gabor multipliers G_α, which are fast to compute.

Phase shift operator: $\alpha(t, \omega) = \exp(-2\pi i t_o \omega)$

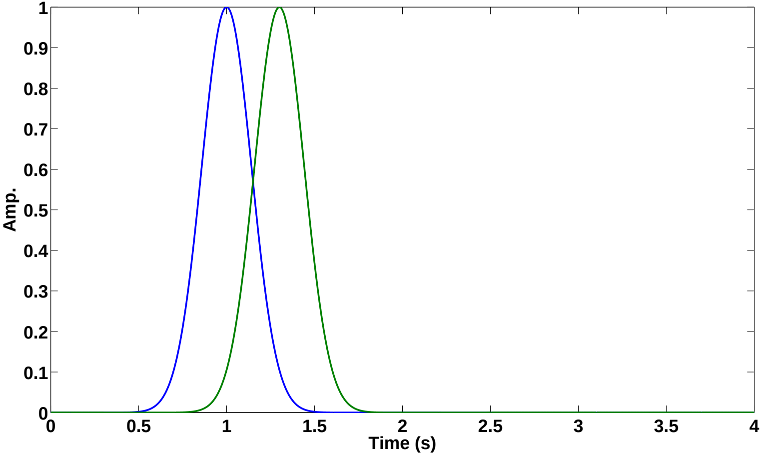

Figure: One pulse, and the phase shifted translate.

Time-variant attenuation: $\alpha(t,\omega) = \exp(-\pi t\omega/Q)$

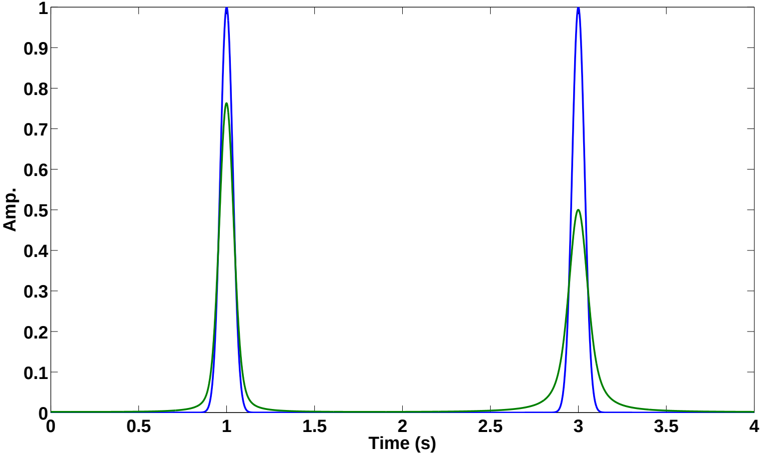

Figure: Two pulses, and their Q-attenuated versions.

Time-variant attenuation: $\alpha(t,\omega) = \exp(-\pi t\omega^2/Q)$

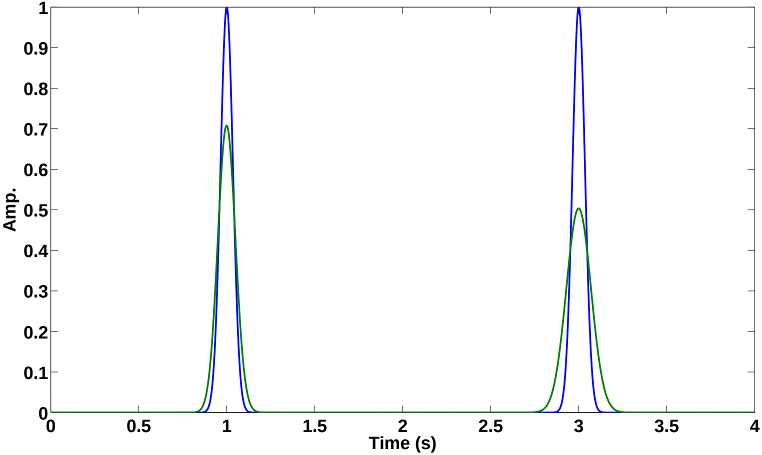

Figure: Two pulses, modified Q-attenuated versions.

Time-variant attenuation: $\alpha = \exp(-\pi t\omega/Q)$ + min phase

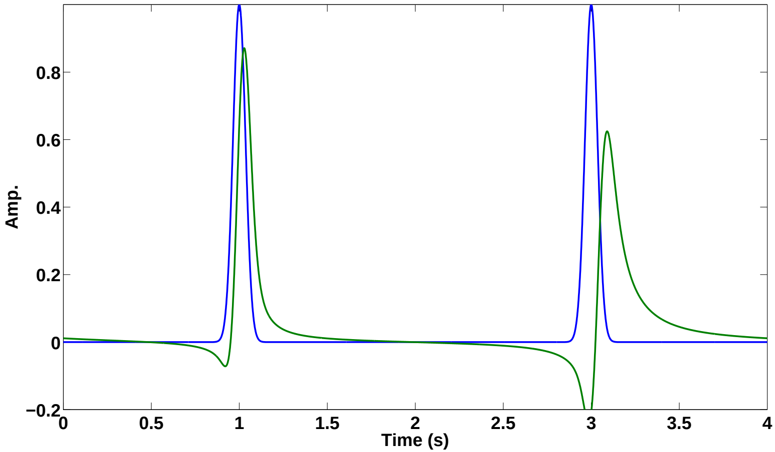

Figure: Two pulses, min phase Q-attenuated versions.

Gabor decon – nonstationary deconvolution

Gabor deconvolution is based on an approximate factorization of signals in the Gabor transform domain.

With source $s(t)$, reflectivity $r(t)$ and recorded seismic data $d(t)$, we have

$$\mathcal{G}d(t,\omega) \approx \widehat{s}(\omega)\alpha(t,\omega)\mathcal{G}r(t,\omega),$$

where
- $\mathcal{G}d(t,\omega)$ is the Gabor transform of the data,
- $\widehat{s}$ is the Fourier transform of the source,
- $\alpha(t,\omega)$ is the nonstationary attenuation,
- $\mathcal{G}r(t,\omega)$ is the Gabor transform of the reflectivity.

What's new? A mathematical derivation of this result.

Gabor decon – nonstationary deconvolution

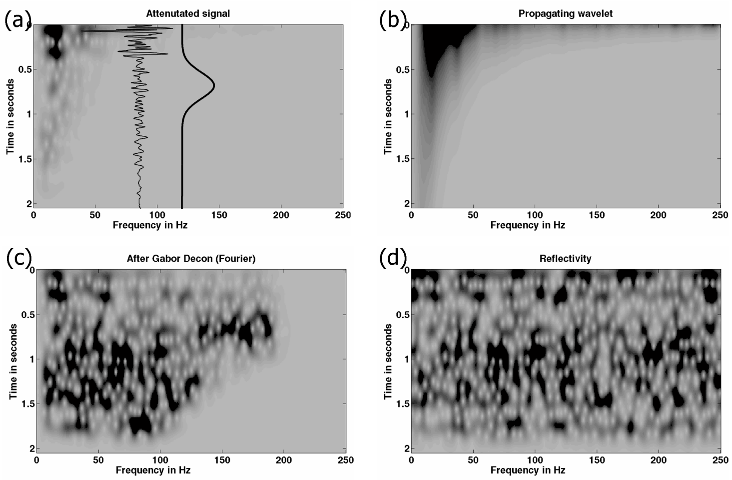

Figure: Time-frequency plots of a seismic signal, source, reflectivity.

Generalized frames

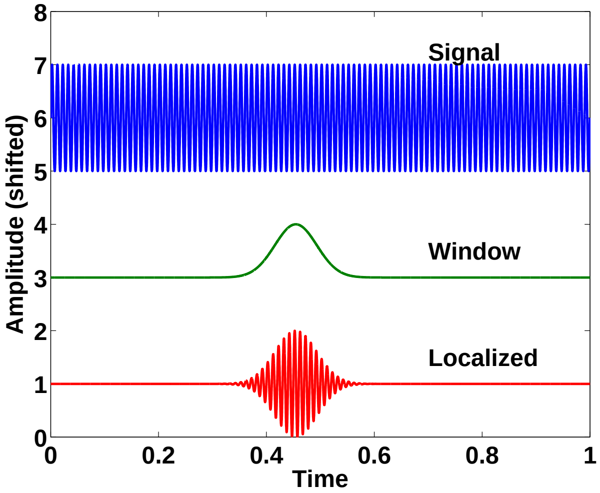

Figure: We localize a signal by multiplying with a window.

Many windows

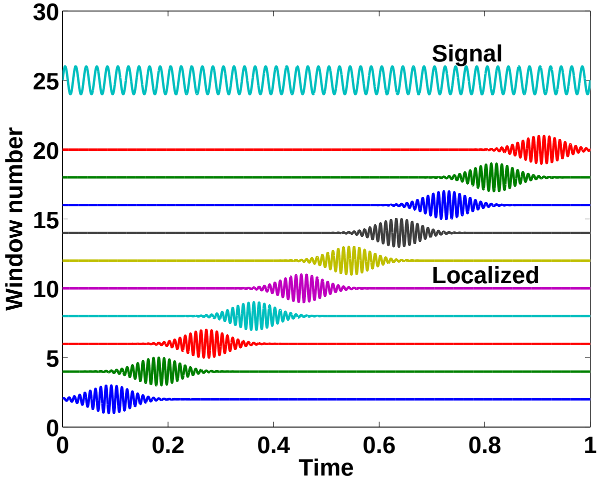

Figure: Many windows used. Apply FFT, algorithms to these localized versions.

Data flow

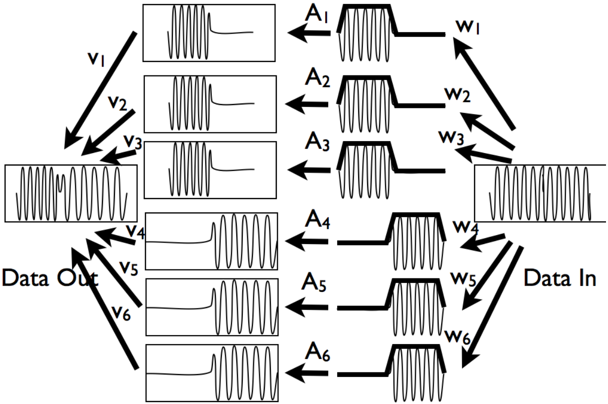

Figure: Many windows for data, to process, then recombine.

Generalized frame operators

A generalized frame operator is given by combining these windows and local operators:

$$f \mapsto \sum_k W_k^* A_k W_k f,$$

where the W_k are the windowing, the A_k the local operators.

Theorem

The partition of unity condition $\sum_k W_k^* W_k = 1$ gives boundedness, stability results for the generalized frame operators.

We can partition the signal by time, location, frequency band, or physical parameters such as local velocity.

Theorem

A Gabor multiplier is a special case of the generalized frame operators.

Example: use local operators to propagate waves

Apply local windows to follow the velocity contours. Propagate waves through each approx. velocity field individually.

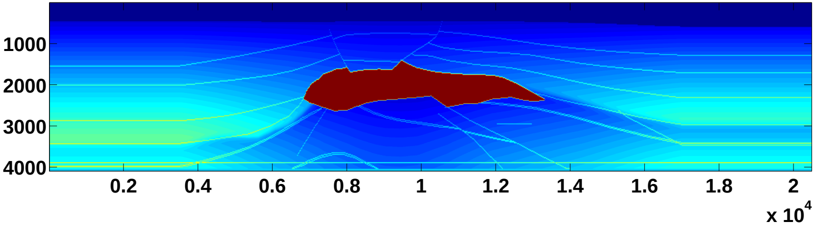

Figure: EAGE Salt model velocity field, 1500-4500 m/s, simulation.

Example: snapshot 21

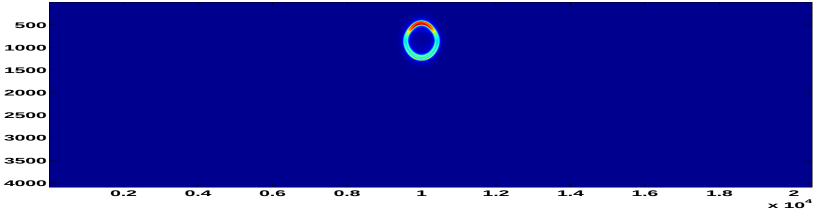

Figure: EAGE Salt model velocity field, 1500-4500 m/s, simulation.

Example: snapshot 41

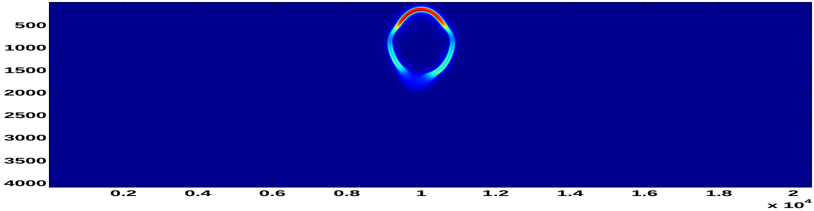

Figure: EAGE Salt model velocity field, 1500-4500 m/s, simulation.

Example: snapshot 61

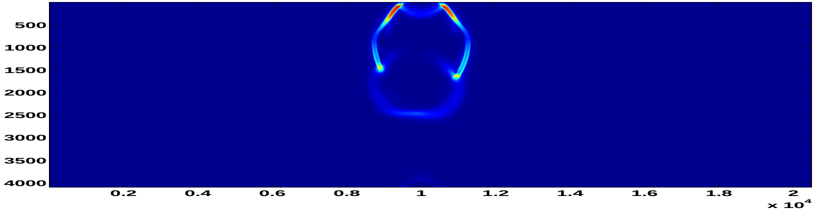

Figure: EAGE Salt model velocity field, 1500-4500 m/s, simulation.

Example: snapshot 81

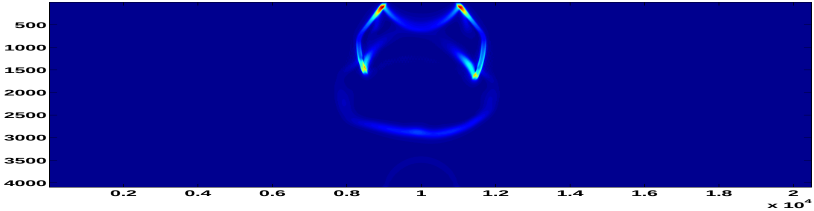

Figure: EAGE Salt model velocity field, 1500-4500 m/s, simulation.

Example: snapshot 100

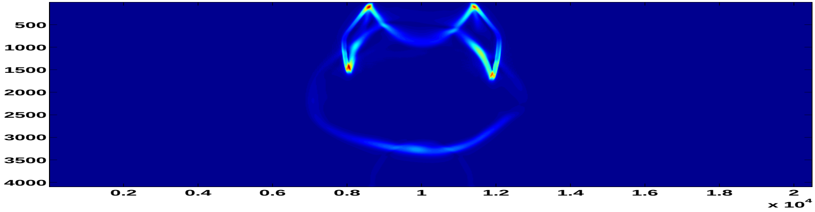

Figure: EAGE Salt model velocity field, 1500-4500 m/s, simulation.

Summary

– the Gabor transform is a suite of localized Fourier transforms, gives an informative time-frequency display.

– the Gabor multiplier is a suite of localized Fourier multipliers.

– Gabor multipliers model differential operators, including reflection, attenuation, propagation.

– Gabor multipliers are a special case of generalized frame operators, which partition signals in time, frequency, spatial slices.

– Mathematical results developed to analyze, improve numerical methods.

Acknowledgements

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