Dispersion and the dissipative characteristics of surface waves in the generalized *S* transform domain

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Outline

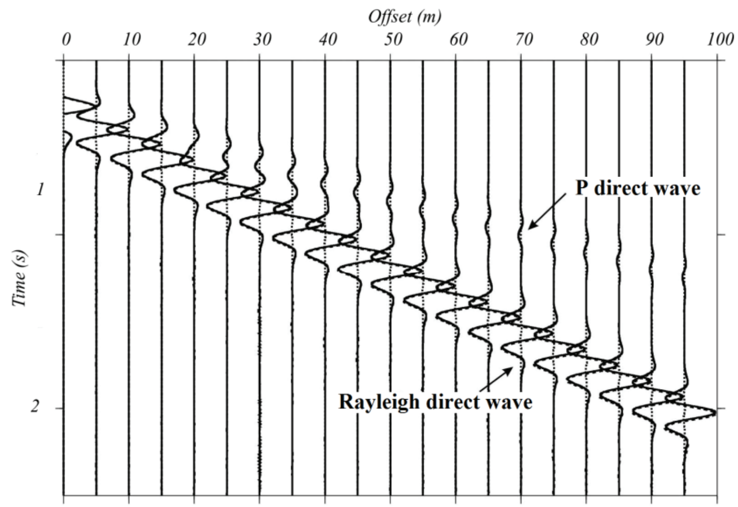
- **Objective of this study**
- **MASW method**
- **Time-Frequency Analysis**
  - **S transform**
  - **Generalized S transform**
- **Synthetic and Real Data Examples**
- **Conclusion**

Objective of this study

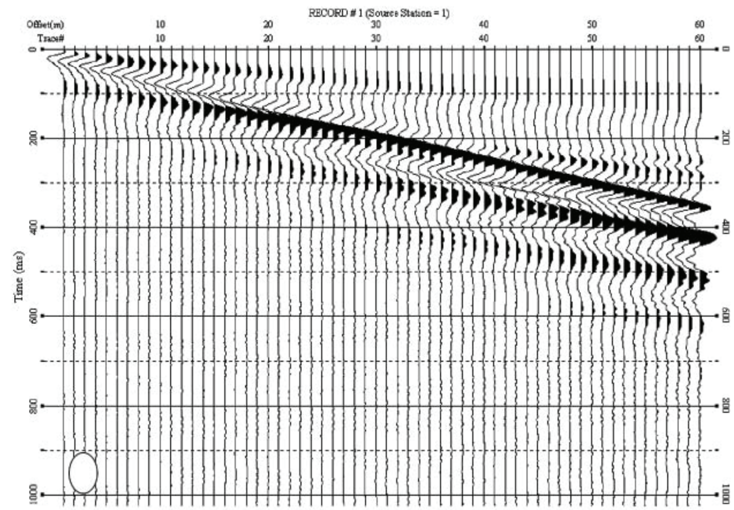
The objective of this study is to develop a mathematical model to estimate group velocity, phase velocity and also a frequency-dependent attenuation function for MASW survey.

MASW Method

Multi-channel Analysis of Surface Waves (MASW) is a method for estimating shear wave velocity based on dispersed Rayleigh waves.



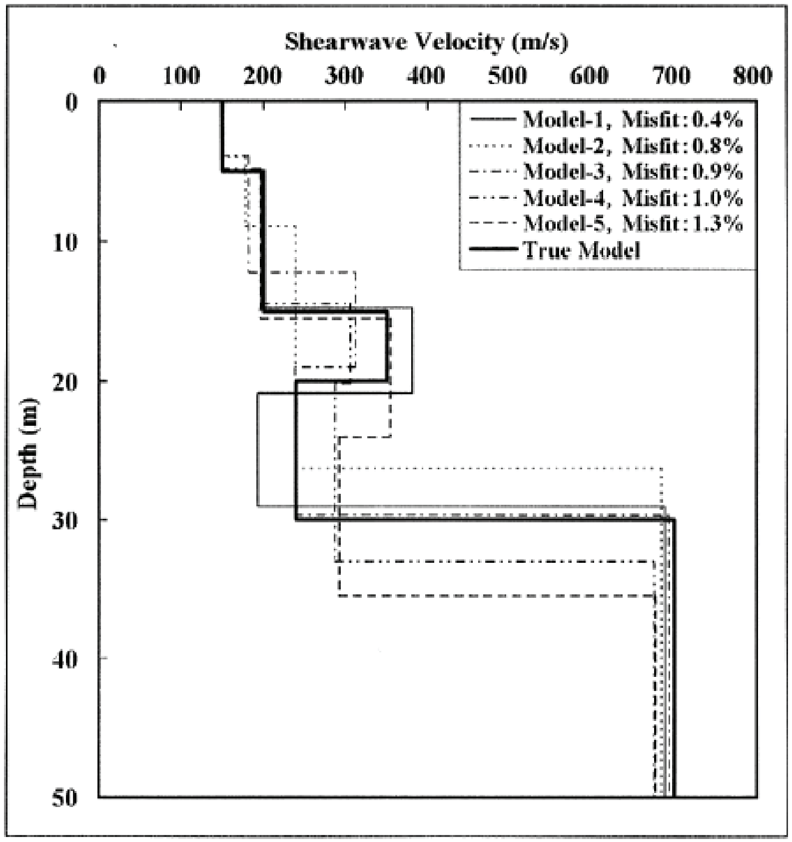
A



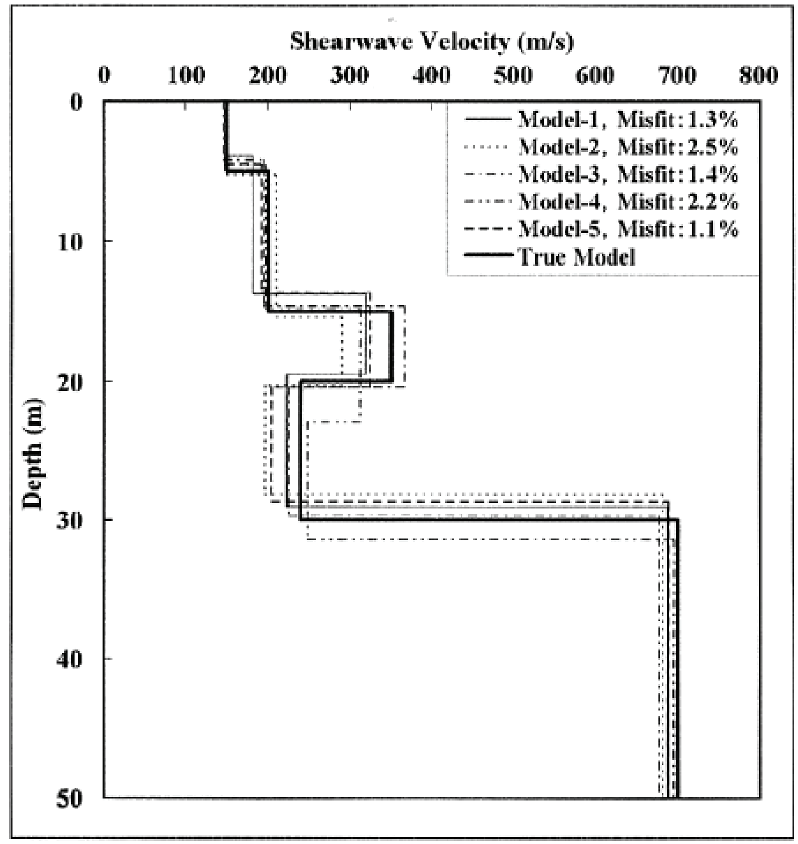
B

Luo et al., 2008
Gelis et al. 2005

The usefulness of MASW lies in the fact that it is able to detect hidden layers.



Results of synthetic inversions using the fundamental mode only, from 1–40 Hz.



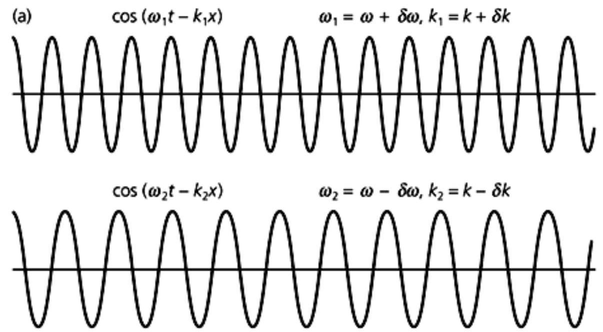
Results of synthetic inversions using the fundamental and first higher modes over 1–40 Hz.

Feng et al. 2005.

Phase and Group Velocities

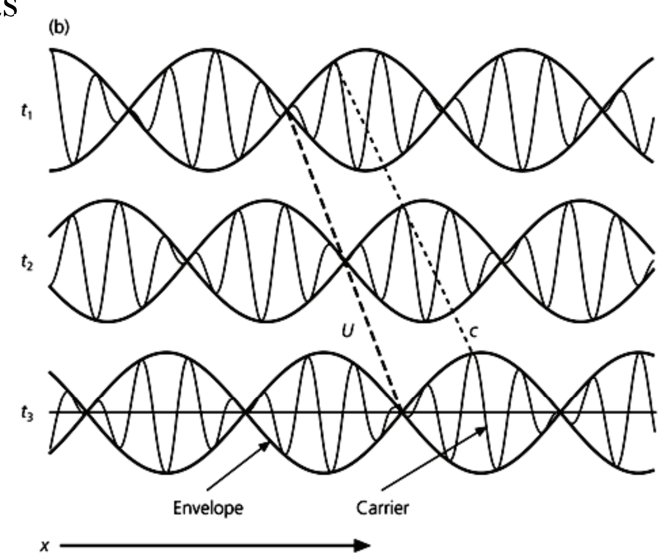
Phase velocity is the velocity of each frequency component of dispersed surface wave which is defined as

Vp= *f/k.*



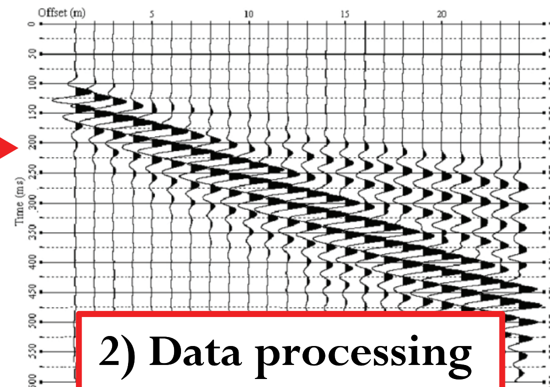
Group velocity *is* the velocity of a wave pocket (envelope) of surface wave around frequency f which is defined as

Vg= *df/dk*

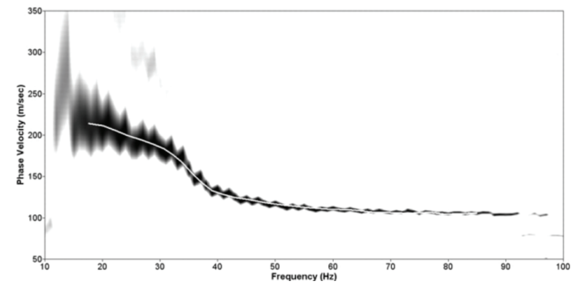


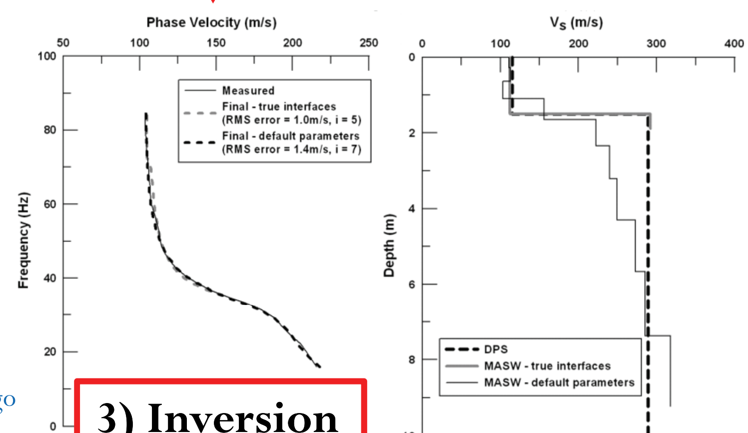
Three steps in MASW survey

1) Data Acquisition



2) Data processing





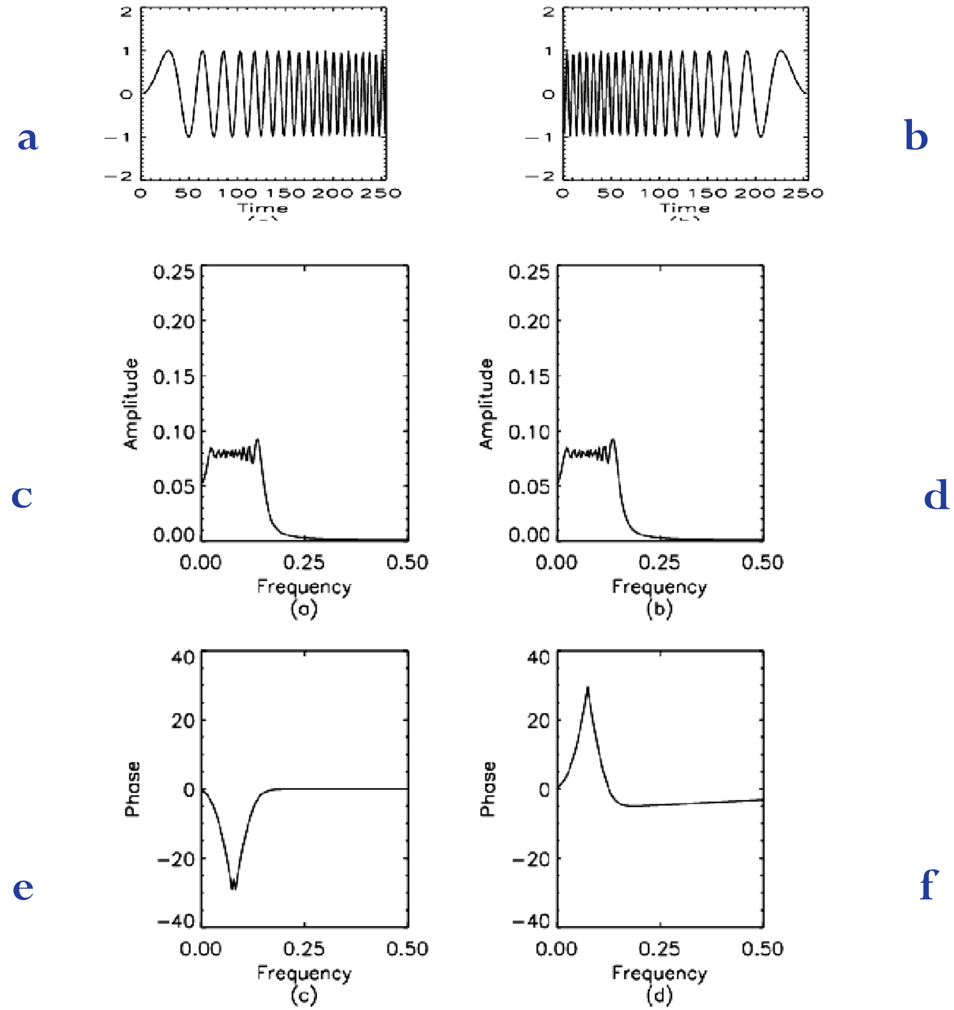
3) Inversion

http://www.geovision.com/PDF/M_SASW.PDF

http://jeeg.geoscienceworld.org/content/vol13/issue2/images/large/eego

The Fourier Transform

We use the Fourier transform to know about frequency content of a signal. The Fourier transform does provide us any straightforward information about time distribution of frequency components.



Time-Frequency Analysis

The S transform (Stockwell et al, 2003)

$$S[h(\tau)](t,f) = \int_{-\infty}^{+\infty} h(\tau)\left[\frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(\tau-t)^2}{2}}\right] e^{-j2\pi f\tau} d\tau.$$

The generalized S transform (Pinnegar and Mansinha, 2003)

$$S_g[h(\tau)](t,f,\boldsymbol{p}) = \int_{-\infty}^{+\infty} h(\tau) w(\tau - t, f, \boldsymbol{p}) e^{-j2\pi f\tau} d\tau$$

A version of the generalized S transform is defined using a scaling factor γ,

$$S_g[h(\tau)](t,f,\gamma) = \int_{-\infty}^{+\infty} h(\tau) \frac{|f|}{\sqrt{2\pi}\gamma} e^{-\frac{f^2(\tau-t)^2}{2\gamma^2}} e^{-j2\pi f\tau} d\tau$$

Estimation of phase, group velocities and attenuation

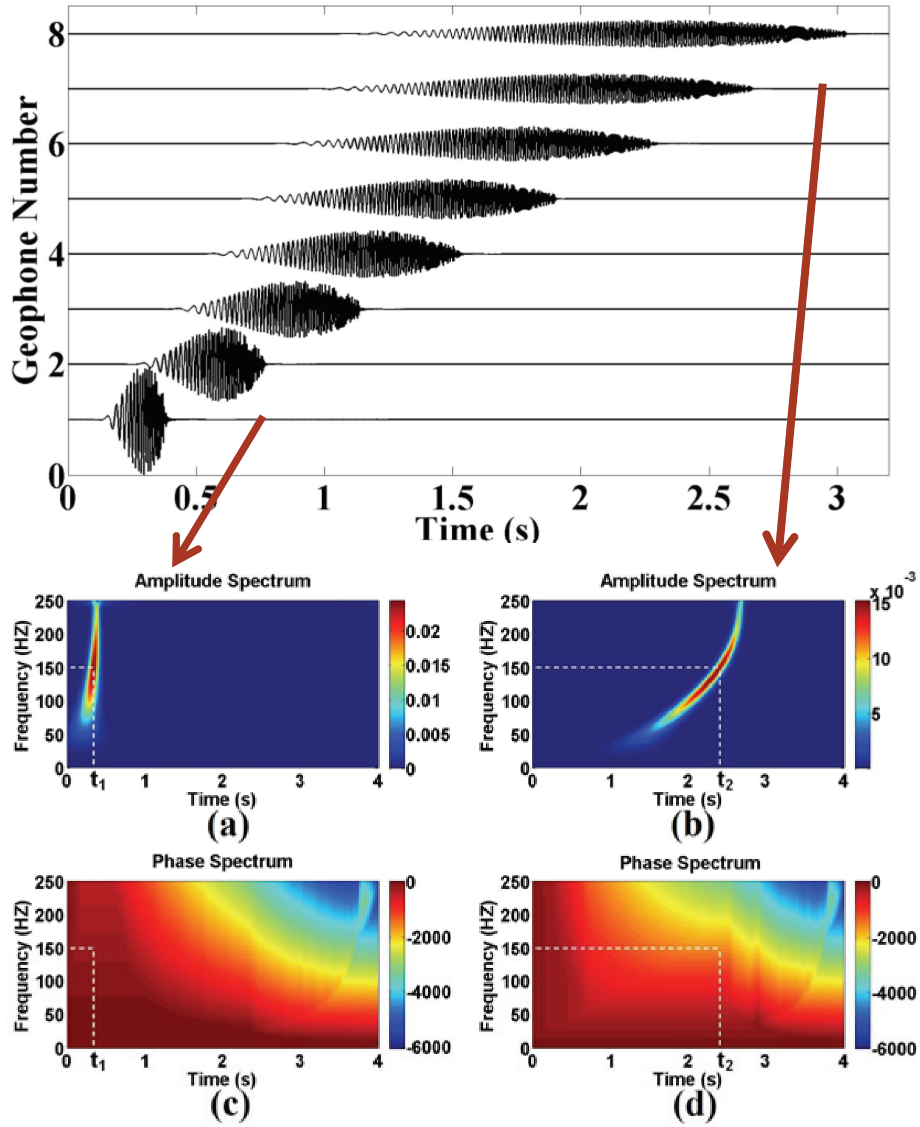
Assuming geometrical spreading correction has been applied on surface wave data, if $h_1(\tau)$ is the wavelet at station 1, the wavelet $h_2(\tau)$ recorded at station 2 can be expressed

$$H_2(f) = e^{-\lambda(f)d} e^{-j2\pi k(f)d} H_1(f)$$

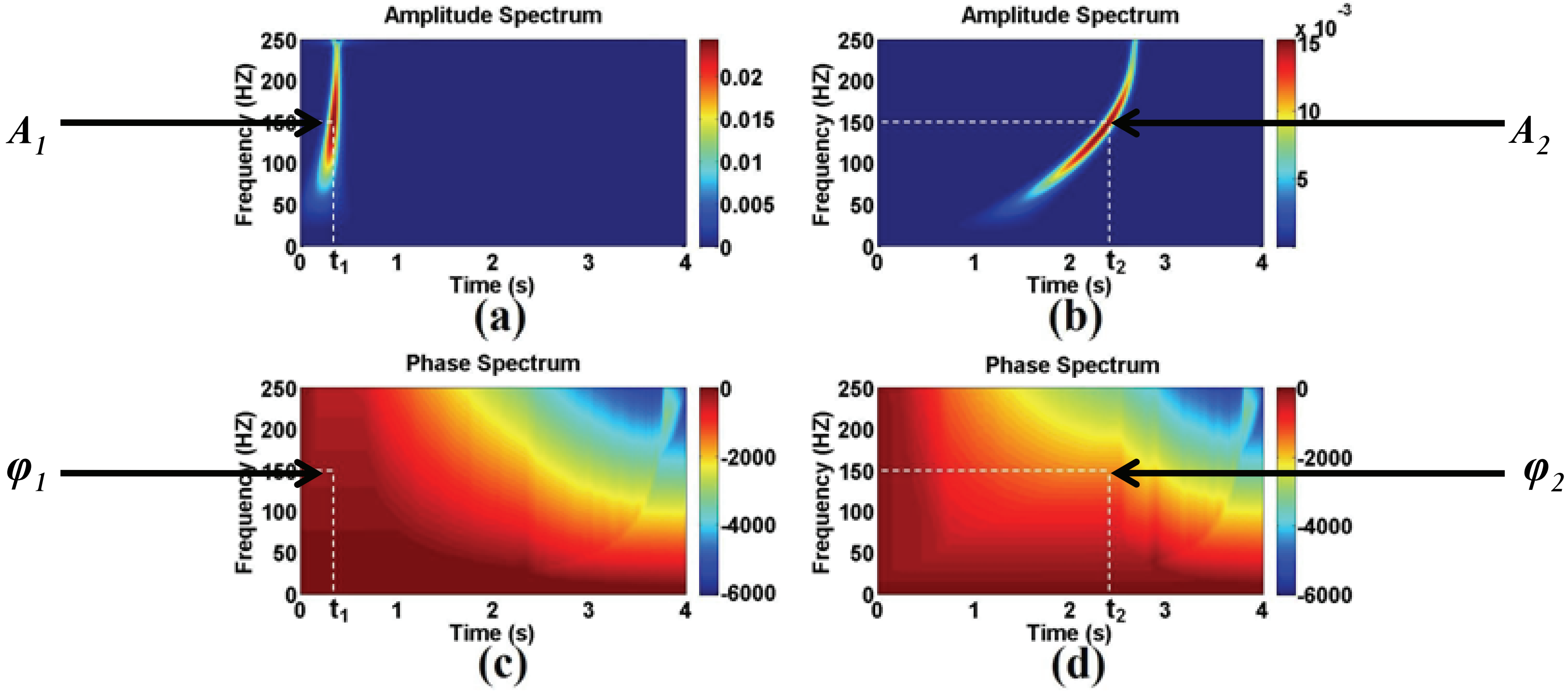
Assuming

$$\lambda(f+\alpha) = \lambda(f) + O(\alpha),$$

$$k(f+\alpha) = k(f) + \alpha k'(f) + O(\alpha^2),$$



(a) and (c) the amplitude spectrum; (b) and (d) the phase spectrum for traces 1 and 7 respectively



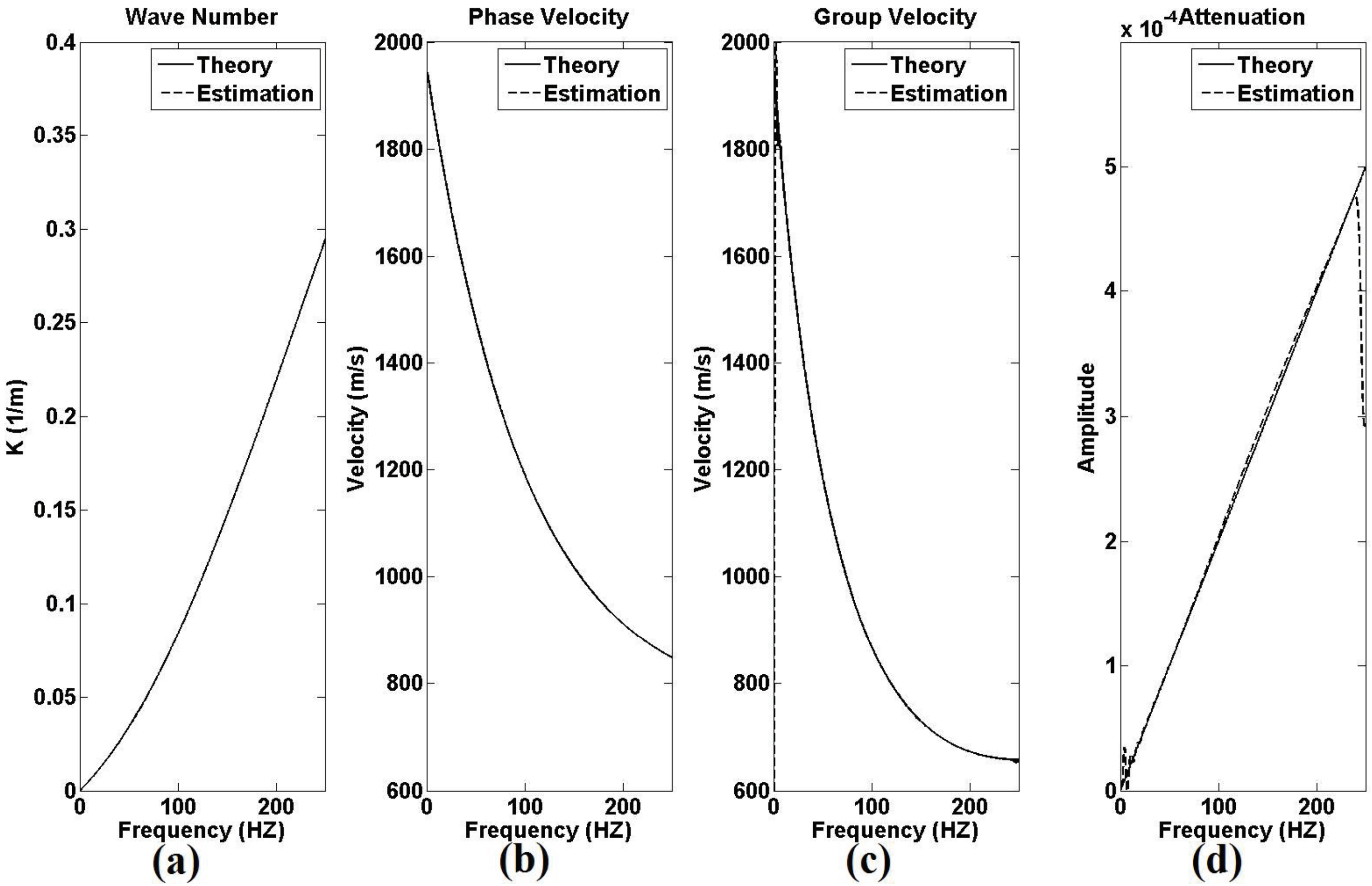
$$\lambda(f) = \frac{[\log(A_1(f)/A_2(f))]}{d}$$

$$v_g = \frac{d}{\Delta t}$$

$$k(f) = \frac{\Delta\varphi}{2\pi d}$$

$$v_p(f) = f/k(f)$$

The estimated propagation model parameters

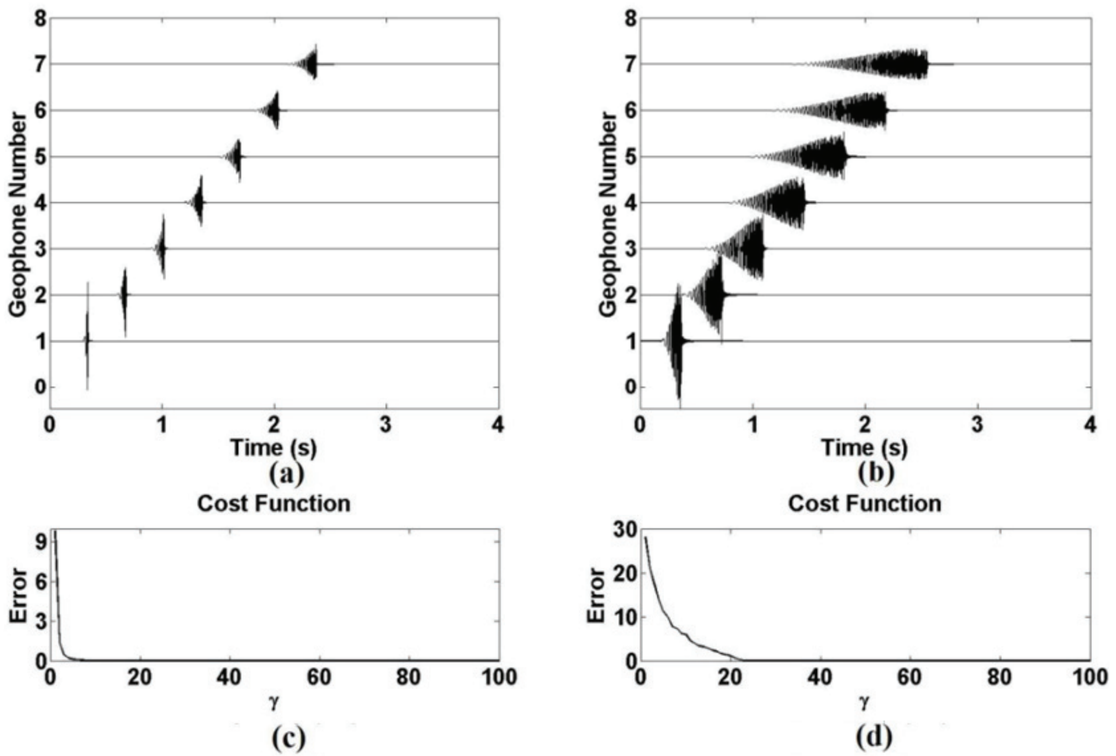


(a) The wave number. (b) The phase velocity. (c) The group velocity. (d) The attenuation function.

First Cost Function

An optimum value for γ is obtained using a cost function

$$E(\gamma) = \left\| S_{g_i}(t,f) - S_{g_r}\left(t - D_i/v_g(f), f\right) e^{-\lambda_{\gamma(f)} D_i} e^{-i2\pi k_\gamma(f) D_i} \right\|^2$$

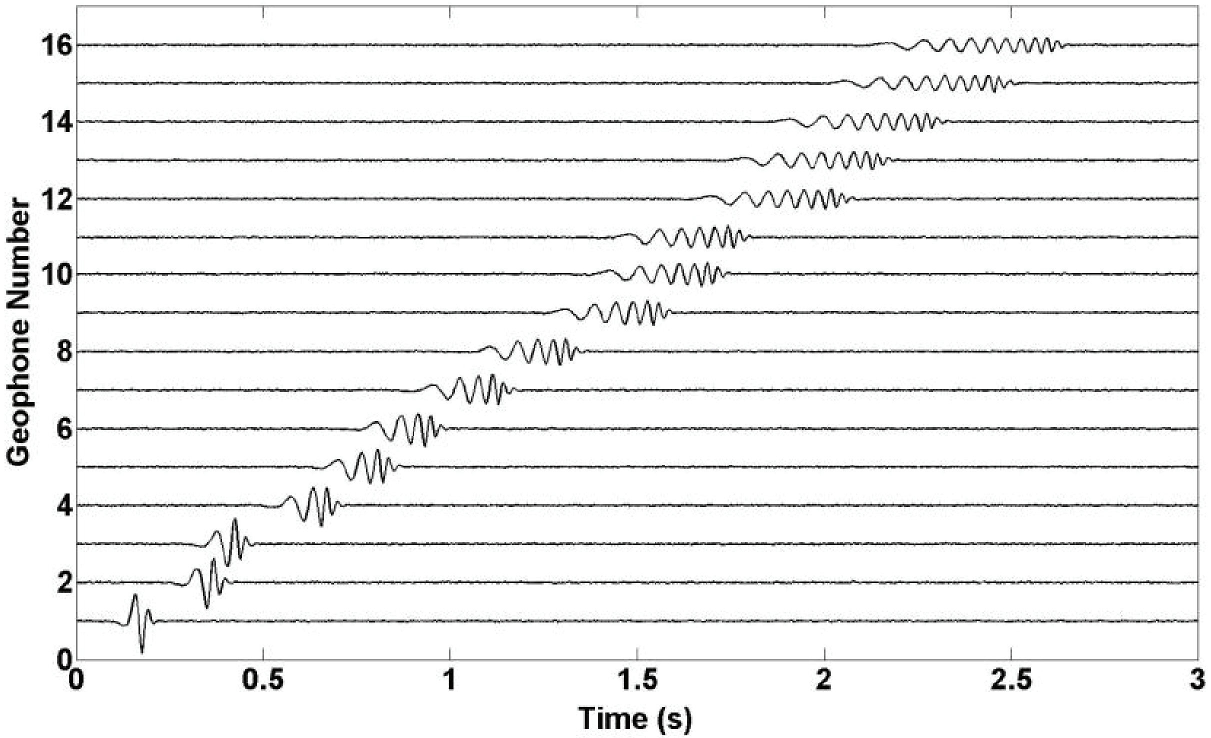


(a) and (b) two different dispersed data sets. (c) and (d) the cost functions for (a) and (b) respectively.

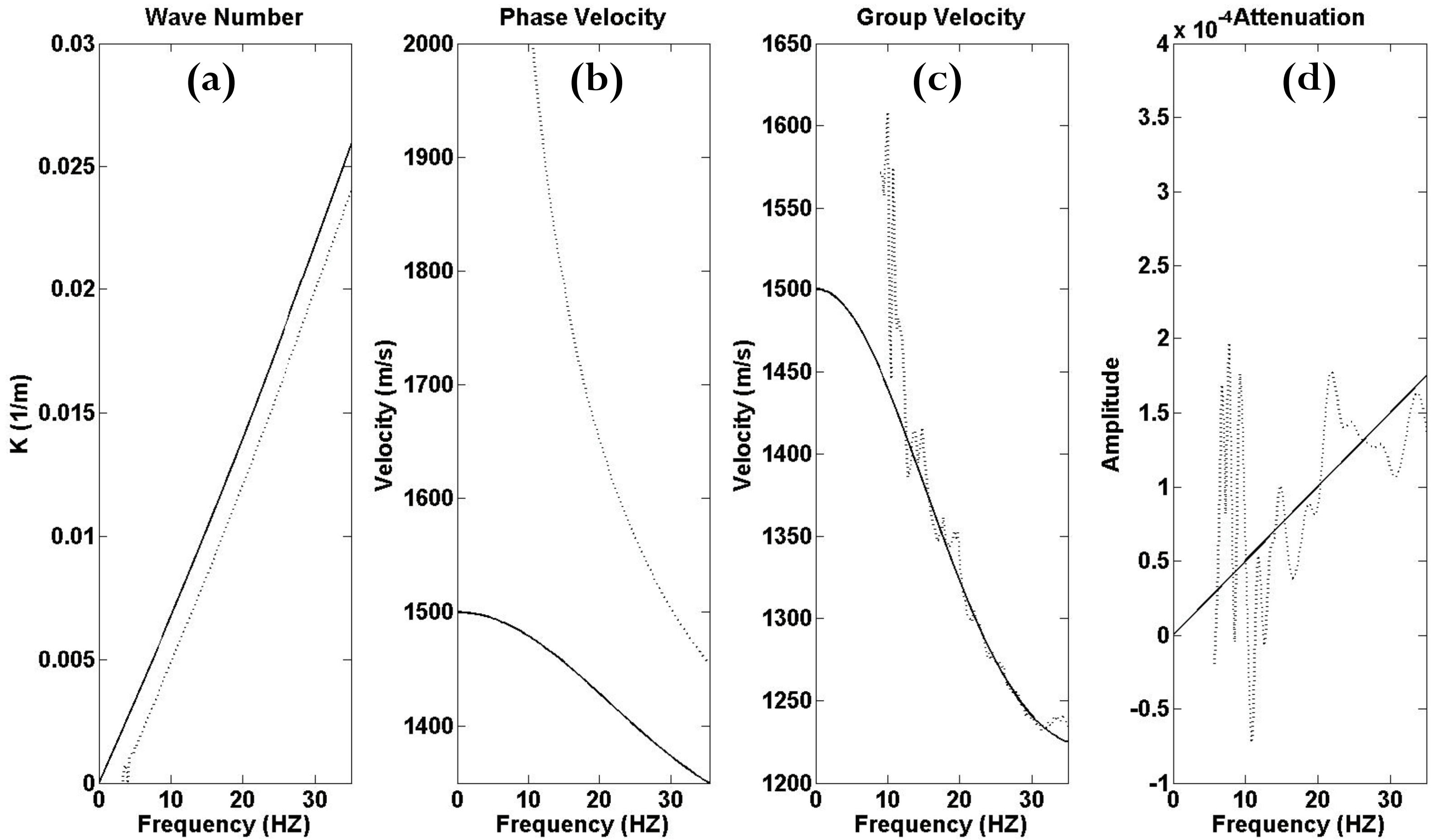
Second Cost Function

When signal to noise ratio is small at low frequencies, absolute phase is perturbated and therefore wave number is perturbated as well.

$$k_m \approx k_e + \varepsilon.$$



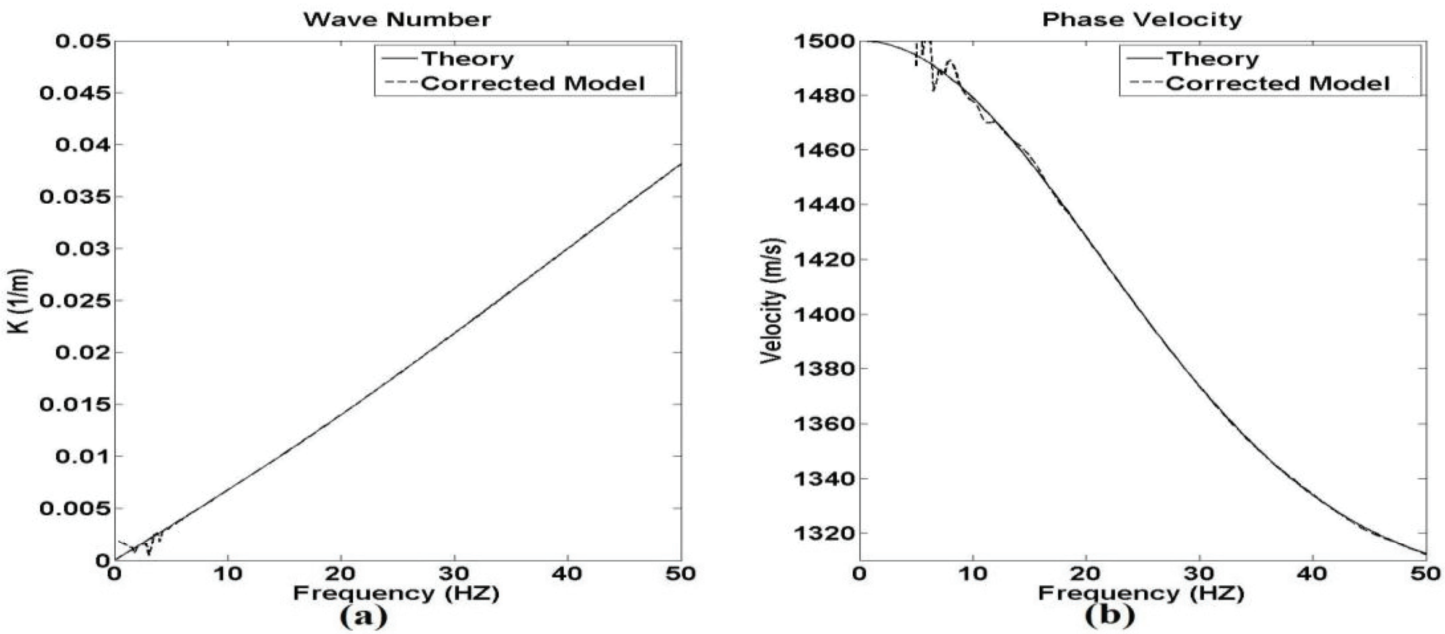
Estimated propagation model parameters.



(a) The wave number. (b) The phase velocity. (c) The group velocity. (d) The attenuation function.

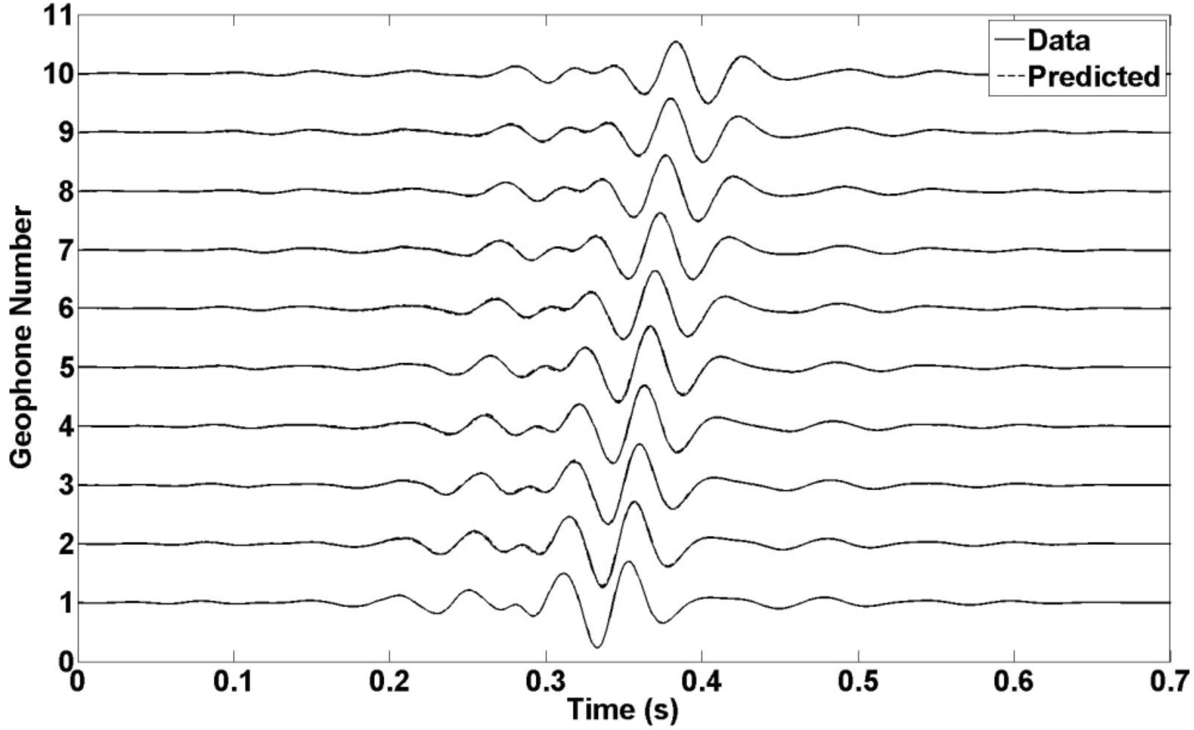
The second cost function to estimate wave number perturbation is defined as

$$E(\varepsilon) = \left\| S_{g_i}(t,f) - S_{g_r}\left(t - D_i/v_g(f), f\right) e^{-\lambda(f)D_i} e^{-i2\pi k_e(f)D_i} e^{-i2\pi\varepsilon D_i} \right\|^2$$



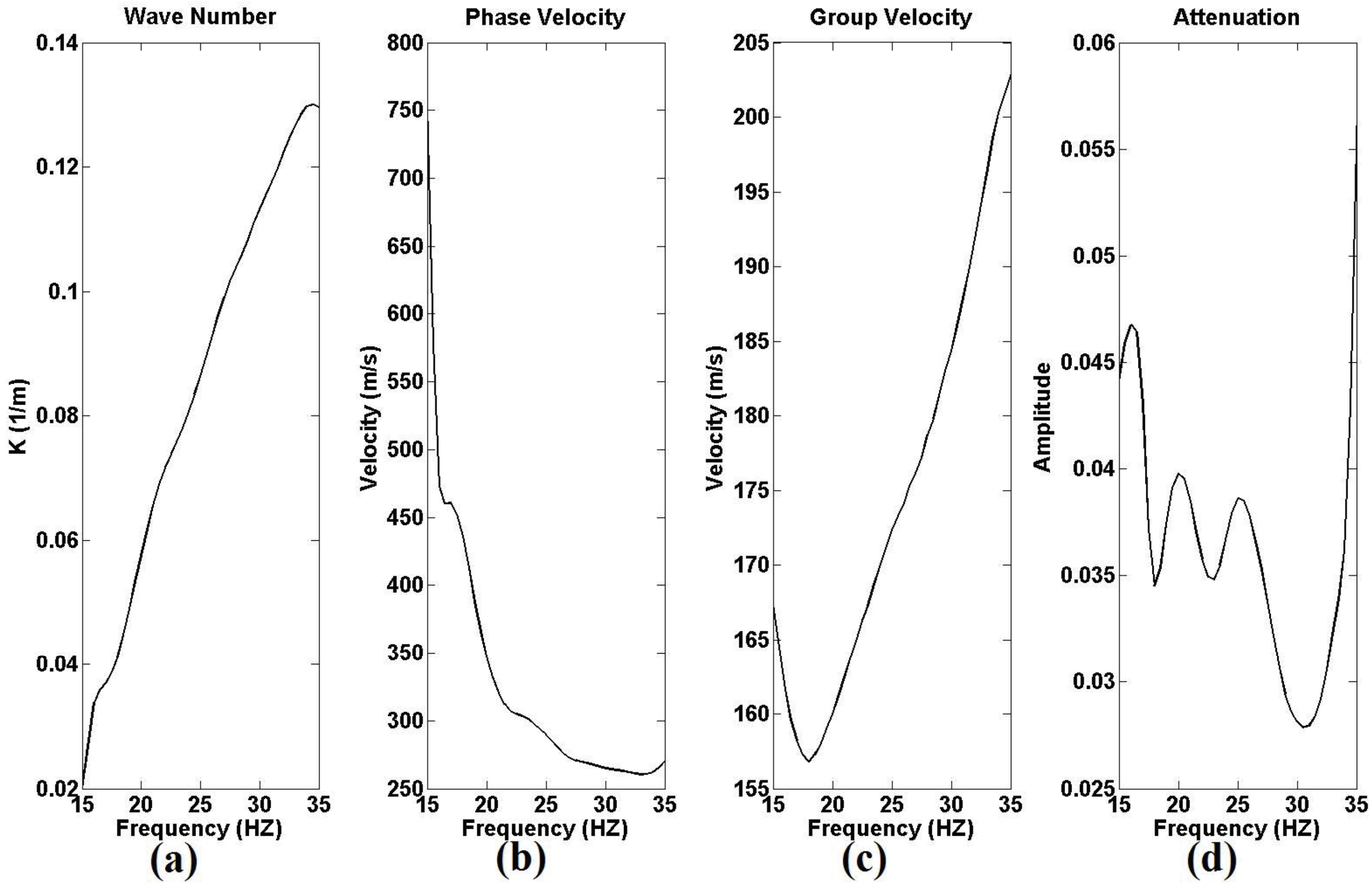
(a) The wave number. (b) The phase velocity corrected based on computed optimum value for epsilon.

Real Data



Data
Predicted
Geophone Number
Time (s)
0
1
2
3
4
5
6
7
8
9
10
11
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Estimated propagation model parameters.



(a) The wave number. (b) The phase velocity. (c) The group velocity. (d) The attenuation function.

Conclusion

The advantage of the *S* transform and its generalized versions lies in the fact that they provide frequency-dependent resolution while maintaining a direct relationship with the Fourier spectrum. Using this property, wave number and phase velocity, group velocity and dependent attenuation function are directly obtained with respect to the ridges of the transform.

Scaling factor in the generalized S transform expands the application of method in highly dispersive medium. When medium is highly dispersive, a larger value of the scaling factor should be chosen.

The cost function in the *S* domain generalizes the application of the method for noisy data, especially data with a low signal to noise ratio at low frequencies. The cost function is highly non-linear, so it could be minimized using global optimization methods such as Simulating Annealing.

Acknowledgements

CREWES team and its industrial sponsors.