Frequency dependent AVO analysis

of P-, S-, and C-wave elastic and anelastic reflection data

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Outline

1. Geophysical setting: elastic & anelastic targets
2. Formulation
3. Results: expansions truncated at 2^{nd} order
4. Predictions & observations:

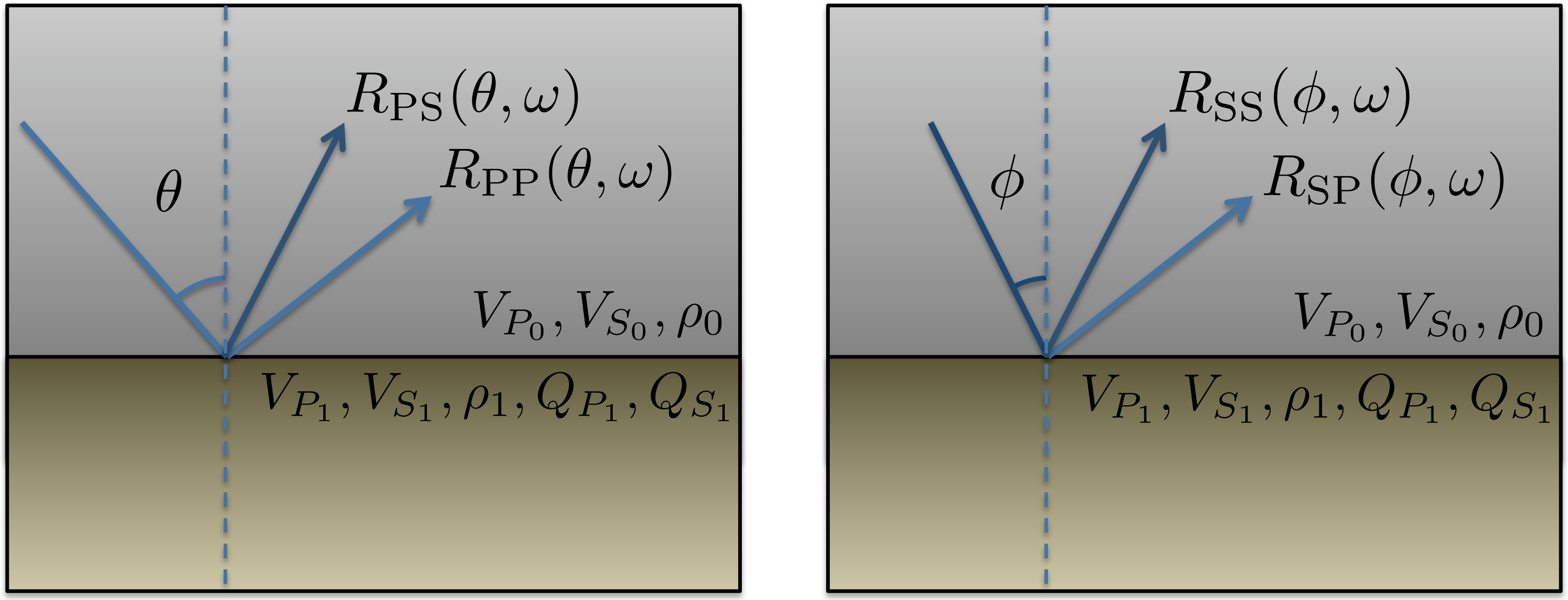
   I: elastic: R_{PS} and target V_P; importance of $V_P/V_S = 2$

   II: anelastic: mode conversions from Q_S / Q_P contrasts

   III: anelastic: R_{PS} and R_{SS} inversion; $Q^{-1} \approx \partial R/\partial\omega$

5. Conclusions

Geophysical setting



Elastic overburden, anelastic target

θ : P-wave angle of incidence

ϕ : S-wave angle of incidence

Theoretical tools

1. Knott-Zoeppritz: exact equations for R_{PP}, R_{PS}, etc.

   ✓ precise amplitudes regardless of angle (for plane waves)

   ✗ inaccessible: geophysical interpretation difficult

2. Aki-Richards approximation for R_{PP}, R_{PS}, etc.

   ✓ highly accessible to geophysical interpretation

   ✗ inaccurate RE. large contrasts, influence of parameters

Example: R_{PS} and target V_P

1. Knott-Zoeppritz: exact equations for R_{PP}, R_{PS}

$$A = \rho_1/\rho_0 \quad B = V_{S_0}/V_{P_0} \quad C = V_{P_1}/V_{P_0} \quad D = B(V_{S_1}/V_{S_0})$$

$$X = \sin\theta \quad S^Y = (1 - 2Y^2X^2) \quad S_Y = \sqrt{1 - Y^2X^2}$$

$$\begin{bmatrix} -X & -S_B & CX & -S_D \\ S_1 & -BX & S_C & DX \\ 2B^2XS_1 & BS^B & 2AD^2XS_C & -ADS^D \\ -S^B & 2B^2XS_B & ACS^D & 2AD^2XS_D \end{bmatrix} \begin{bmatrix} R_P \\ R_S \\ T_P \\ T_S \end{bmatrix} = \begin{bmatrix} X \\ S_1 \\ 2BS_1X \\ S^B \end{bmatrix}$$

Question: what influence does a ΔV_P have on R_{PS}?

Answer: I haven't the foggiest idea.

✗ *Inaccessible to analysis*

Example: R_{PS} and target V_P

2. Aki-Richards: approximate form for R_{PS}:

$$R_{\mathrm{PS}} \approx -\frac{\gamma \tan\phi}{2}\left[(1+\delta)\,\frac{\Delta\rho}{\rho} + 2\delta\frac{\Delta V_S}{V_S}\right]$$

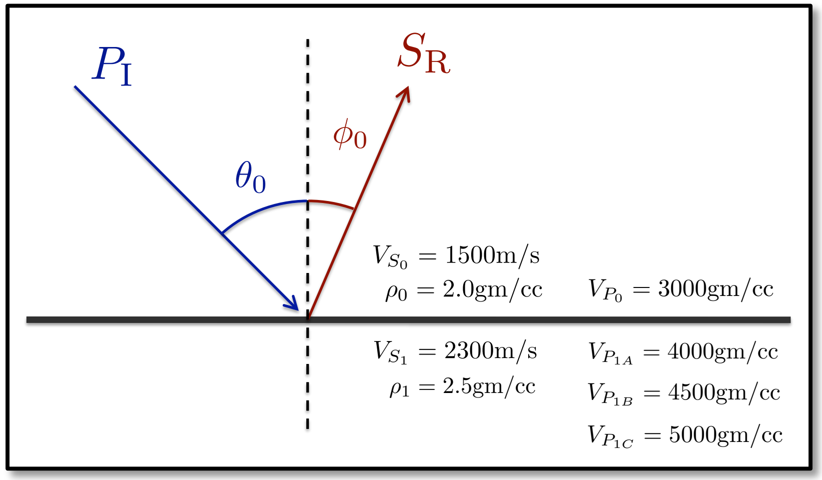
$$\text{where} \quad \delta = -2\frac{\sin^2\theta}{\gamma^2} + 2\frac{\cos\theta\cos\phi}{\gamma}$$

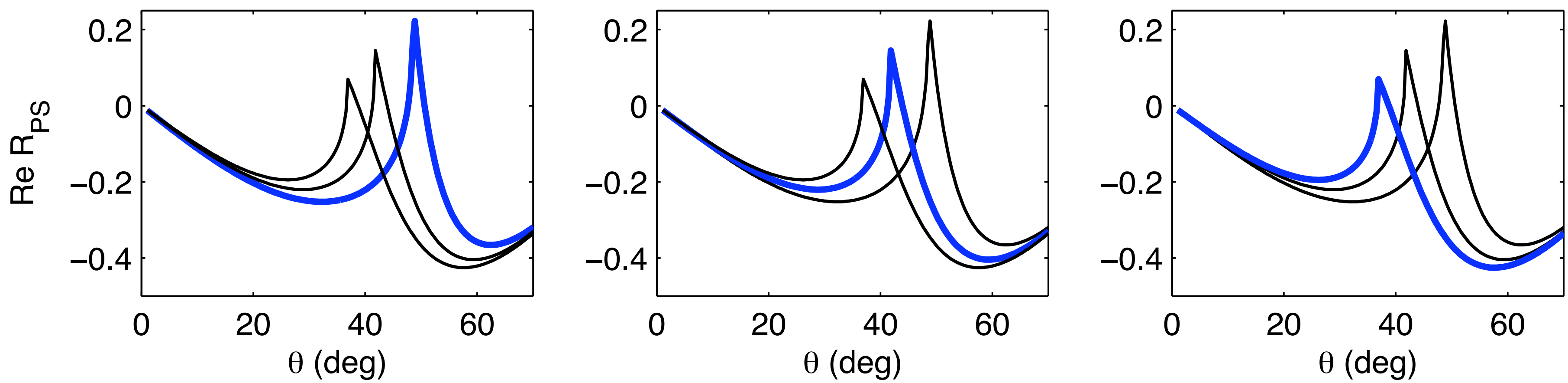
Question: what influence does a ΔV_P have on R_{PS}?

Answer: easy – no effect.

✓ *Highly accessible to analysis*

Example: R_{PS} and target V_P





✓ *Highly accessible to analysis*

✗ *Wrong – for certain angles/contrasts*

Formulation

1. Matrix forms for Knott-Zoeppritz
2. Adjust for finite Q_P, Q_S in target medium
3. All contrasts $\rightarrow$ 5 dimensionless perturbations:

$$a_{VP} = 1 - \frac{V_{P_0}^2}{V_{P_1}^2} \qquad a_{VS} = 1 - \frac{V_{S_0}^2}{V_{S_1}^2}$$

$$a_\rho = 1 - \frac{\rho_0}{\rho_1} \qquad a_{QP} = \frac{1}{Q_{P_1}} \qquad a_{QS} = \frac{1}{Q_{S_1}}$$

4. Expand R_{PP}, R_{PS}, R_{SP}, R_{SS} in orders of each & $\sin\theta$
5. Truncate at 1^{st} or 2^{nd} order

1^{st} order ~ Aki-Richards approximation with incidence angle

2^{nd} order ~ low-moderate angle, low-large contrast

3^{rd} order and higher ~ negligible

Results (anelastic P-P example)

$$R_{\mathrm{PP}}(\theta_0,\omega) = R_{\mathrm{PP}}^{(1)}(\theta_0,\omega) + R_{\mathrm{PP}}^{(2)}(\theta_0,\omega) + ...,$$

The output is the set of Δs for the expansion:

$$R_{\mathrm{PP}}^{(1)}(\theta_0,\omega) = \Delta_p a_{VP} + \Delta_s a_{VS} + \Delta_\rho a_\rho + \Delta_{Q_p} a_{QP} + \Delta_{Q_s} a_{QS}$$

$$\begin{aligned} R_{\mathrm{PP}}^{(2)}(\theta_0,\omega) = {} & \Delta_{pp} a_{VP}^2 + \Delta_{ps} a_{VP} a_{VS} + \Delta_{p\rho} a_{VP} a_\rho + \Delta_{pQp} a_{VP} a_{QP} \\ & + \Delta_{pQs} a_{VP} a_{QS} + \Delta_{ss} a_{VS}^2 + \Delta_{s\rho} a_{VS} a_\rho + \Delta_{sQp} a_{VS} a_{QP} \\ & + \Delta_{sQs} a_{VS} a_{QS} + \Delta_{\rho\rho} a_\rho^2 + \Delta_{\rho Qp} a_\rho a_{QP} + \Delta_{\rho Qs} a_\rho a_{QS} \\ & + \Delta_{QpQp} a_{QP}^2 + \Delta_{QsQp} a_{QP} a_{QS} + \Delta_{QsQs} a_{QS}^2 \end{aligned}$$

Results (anelastic P-P example)

Interpretability & ease of use.

No target anelasticity?

Q_P contrast only?

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Results (anelastic P-P example)

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$$R^{(1)}_{\mathrm{PP}}(\theta_0,\omega) = \Delta_p a_{VP} + \Delta_s a_{VS} + \Delta_\rho a_\rho + \cancel{\Delta_{Q_p} a_{QP}} + \cancel{\Delta_{Q_s} a_{QS}}$$

$$\begin{aligned} R^{(2)}_{\mathrm{PP}}(\theta_0,\omega) = {} & \Delta_{pp} a^2_{VP} + \Delta_{ps} a_{VP} a_{VS} + \Delta_{p\rho} a_{VP} a_\rho + \cancel{\Delta_{pQp} a_{VP} a_{QP}} \\ & + \cancel{\Delta_{pQs} a_{VP} a_{QS}} + \Delta_{ss} a^2_{VS} + \Delta_{s\rho} a_{VS} a_\rho + \cancel{\Delta_{sQp} a_{VS} a_{QP}} \\ & + \cancel{\Delta_{sQs} a_{VS} a_{QS}} + \Delta_{\rho\rho} a^2_\rho + \cancel{\Delta_{\rho Qp} a_\rho a_{QP}} + \cancel{\Delta_{\rho Qs} a_\rho a_{QS}} \\ & + \cancel{\Delta_{QpQp} a^2_{QP}} + \cancel{\Delta_{QsQp} a_{QP} a_{QS}} + \cancel{\Delta_{QsQs} a^2_{QS}} \end{aligned}$$

Results (anelastic P-P example)

Interpretability & ease of use.

No target anelasticity?

Q_P contrast only? ✓

$\neq 0!$ (Lines et al., 2011)

$$R^{(1)}_{\mathrm{PP}}(\theta_0,\omega) = \cancel{\Delta_p a_{VP}} + \cancel{\Delta_s a_{VS}} + \cancel{\Delta_\rho a_\rho} + \Delta_{Q_p} a_{QP} + \cancel{\Delta_{Q_s} a_{QS}}$$

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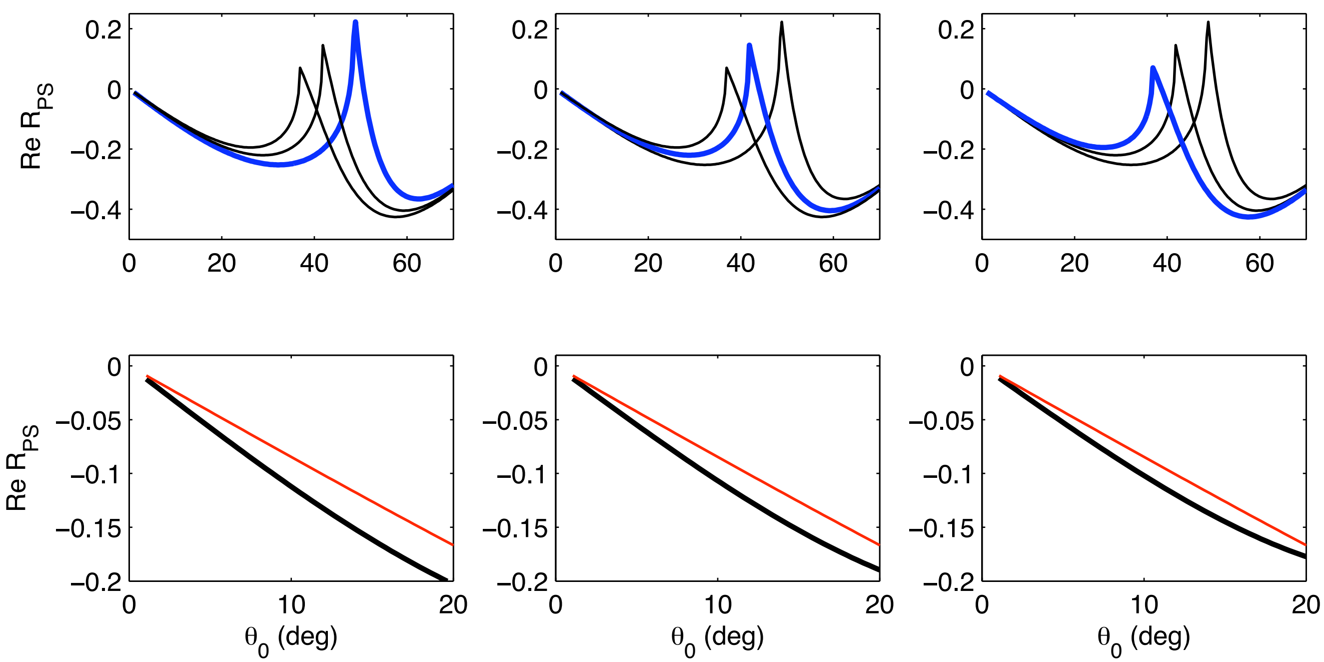
Observation I: R_{PS} and target V_P

$$R_{\mathrm{PS}}(\theta_0) = \underbrace{\Lambda_s a_{VS} + \Lambda_\rho a_\rho}_{\text{Linear: no influence of } V_P}$$

$$+\underbrace{\Lambda_{ps} a_{VP} a_{VS} + \Lambda_{p\rho} a_{VP} a_\rho}_{\text{Nonlinear: coupling of } V_S,\ \rho \text{ with } V_P} + \Lambda_{ss} a_{VS}^2 + \Lambda_{s\rho} a_{VS} a_\rho + \Lambda_{\rho\rho} a_\rho^2 + \ldots$$

Linear



Observation I: R_{PS} and target V_P

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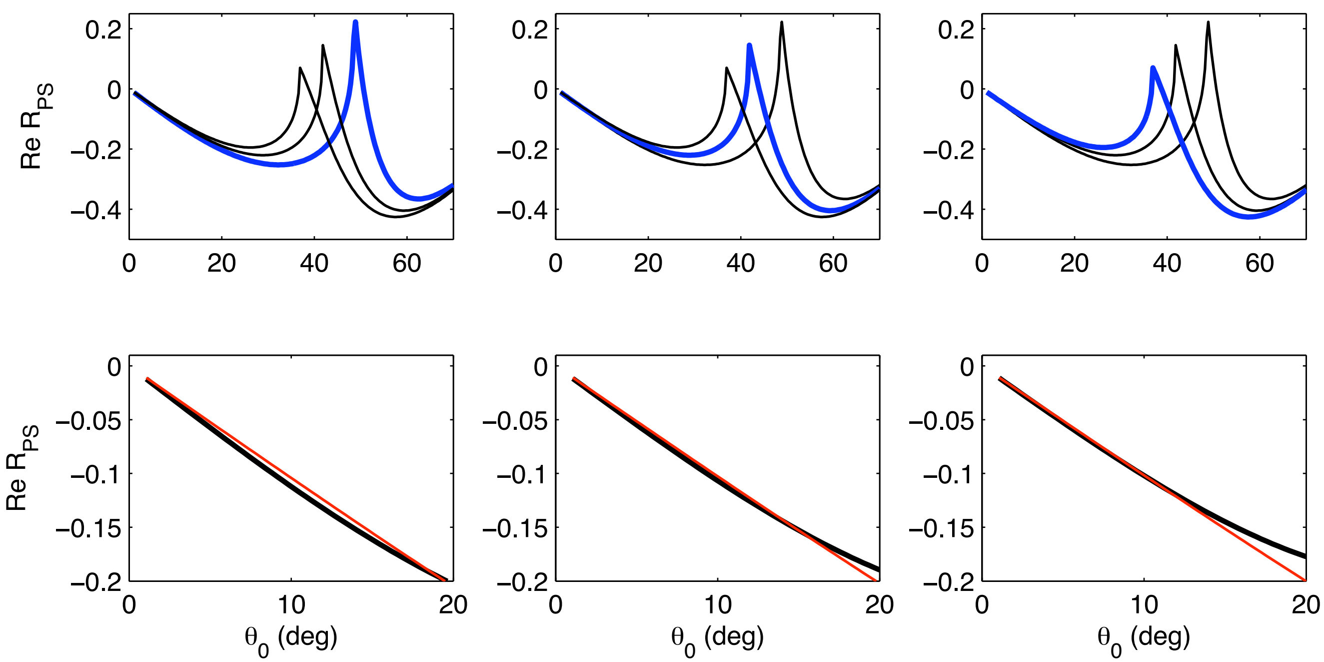
Nonlinear



Observation II: $V_P/V_S = 2$

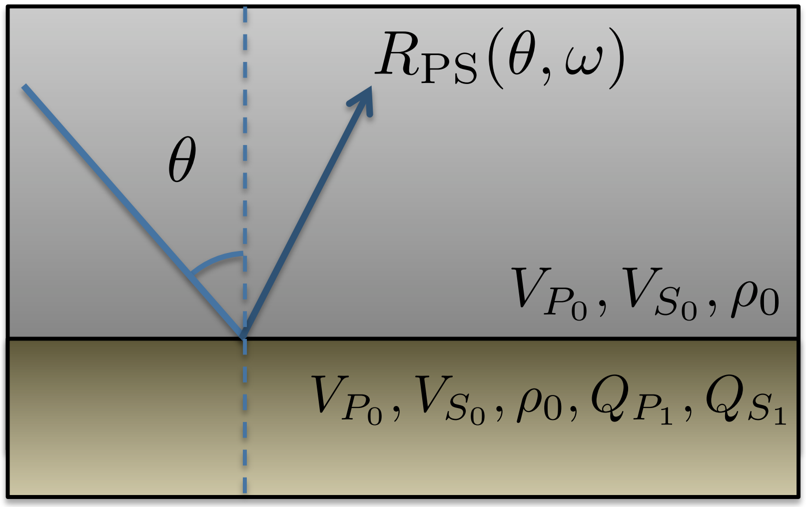
ALL 2nd order coupling involving ρ

have coefficients Δ proportional to

$$\Delta_{\rho[\cdot]} \propto 1 - 2\left(\frac{V_{S_0}}{V_{P_0}}\right)$$

A V_P-V_S ratio of 2 has an inherent mathematical importance for the Zoeppritz equations, originating from elastodynamics + BCs alone.

Prediction: mode conversions from Q_S contrasts



Lines, Wong, Sondergeld, Treitel & others: P-P reflections from contrasts in Q only.

Mode conversions: R_{PS}, R_{SP}?

$$\Lambda_{Q_p}(\theta_0, \omega) = 0 \qquad \Lambda_{QpQp}(\theta_0, \omega) = 0 \qquad \Lambda_{QsQs}(\theta_0, \omega) = 0$$

$$\boxed{R_{\mathrm{PS}}(\theta_0, \omega) = \Lambda_{Q_p} a_{QP} + \Lambda_{Q_s} a_{QS} + \Lambda_{QpQp} a_{QP}^2 + \Lambda_{QpQs} a_{QP} a_{QS} + \Lambda_{QsQs} a_{QS}^2}$$

$$\Lambda_{Qs}(\theta_0, \omega) = 2F_S(\omega) \left(\frac{V_{S_0}}{V_{P_0}} \right) \sin \theta_0$$

$$\Lambda_{QpQs}(\theta_0, \omega) = \left(\frac{V_{S_0}}{V_{P_0}} \right) F_P(\omega) F_S(\omega) \sin \theta_0$$

Prediction: mode conversions from Q_S contrasts

- A Q_S contrast alone generates a mode conversion
- The amplitude of the Q_S mode conversion is proportional to

$$\log\left(\frac{\omega}{\omega_s}\right) \times \left(\frac{V_{S_0}}{V_{P_0}}\right) \times \sin\theta_0$$

- A Q_P contrast alone generates no mode conversion
- However, Q_P contrasts will influence, to 2^{nd} order, the amplitudes of mode conversions due to Q_S contrasts, with a strength proportional to

$$\log\left(\frac{\omega}{\omega_p}\right) \times \log\left(\frac{\omega}{\omega_s}\right) \times \left(\frac{V_{S_0}}{V_{P_0}}\right) \times \sin\theta_0$$

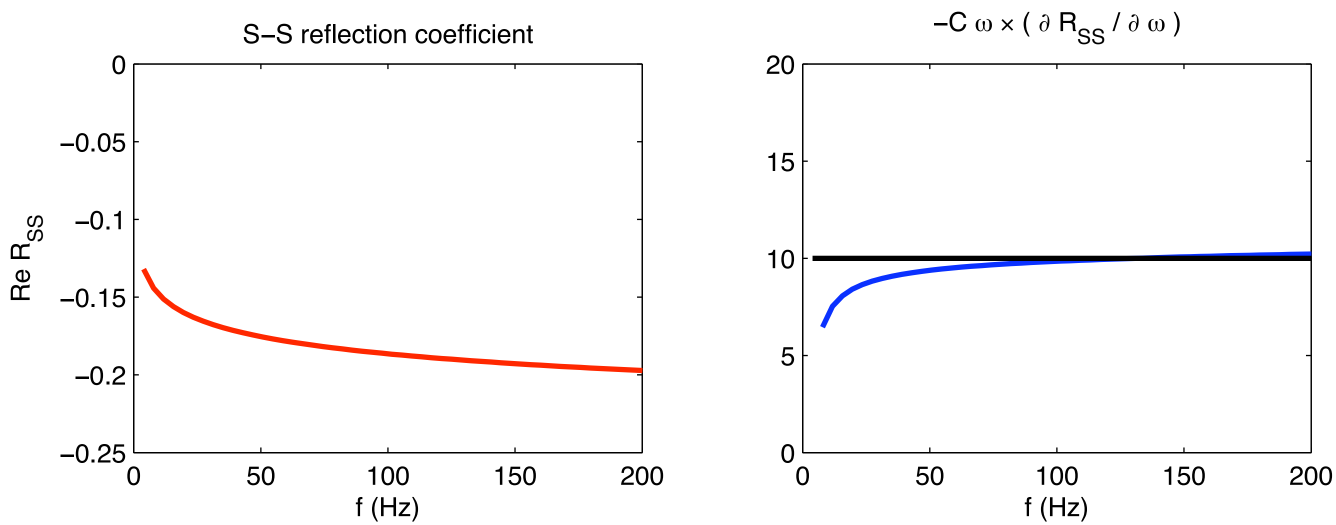
Linear anelastic inversion

Frequency rate of change of R_{PP}, R_{PS}, R_{SS}

$$Q_S^{-1} \approx -\left(\frac{\pi}{2} \times \frac{V_{P_0}}{V_{S_0}} \times \frac{\omega}{\sin\theta}\right)\frac{\partial}{\partial\omega}R_{\mathrm{PS}}(\theta,\omega)$$

$$Q_P^{-1} \approx (2\pi \times \omega)\frac{\partial}{\partial\omega}R_{\mathrm{PP}}(0,\omega)$$

$$Q_S^{-1} \approx -[2\pi \times \omega \times (1 - 7\sin^2\phi)^{-1}]\frac{\partial}{\partial\omega}R_{\mathrm{SS}}(\phi,\omega)$$



Conclusions

1. Accuracy v. interpretability – scalable analysis
2. 2^{nd} order influences for low angles, large contrasts
3. Predictions & observations:

   I: elastic: R_{PS} and target V_P; importance of $V_P/V_S = 2$

   II: anelastic: mode conversions from Q_S / Q_P contrasts

   III: anelastic: R_{PS} and R_{SS} inversion; $Q \approx \partial R/\partial\omega$

4. **Practical** frequency dependent AVO and/or AVF analysis? Next talk.

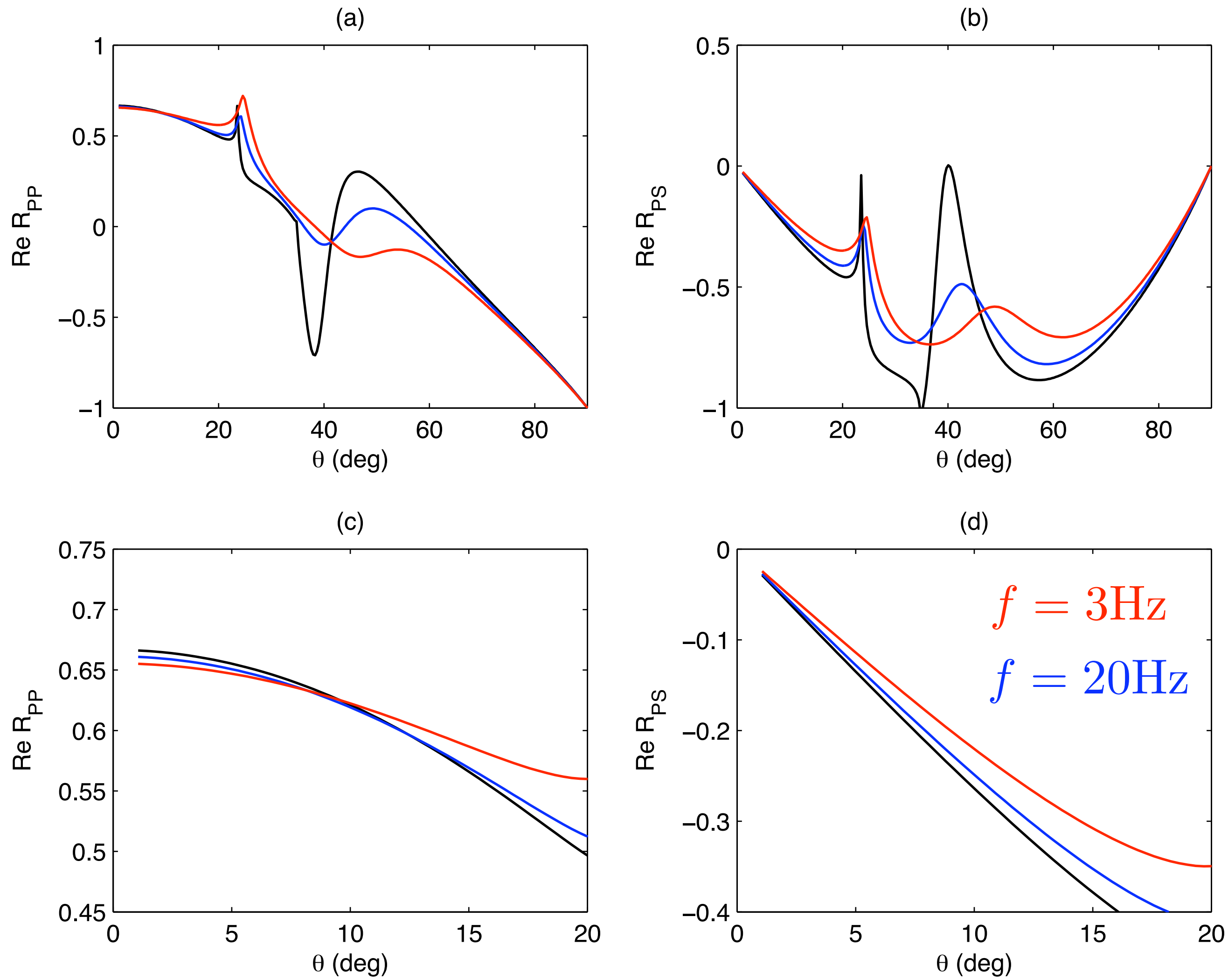
Acknowledgments

CREWES project sponsors & researchers

NSERC

Linear anelastic inversion

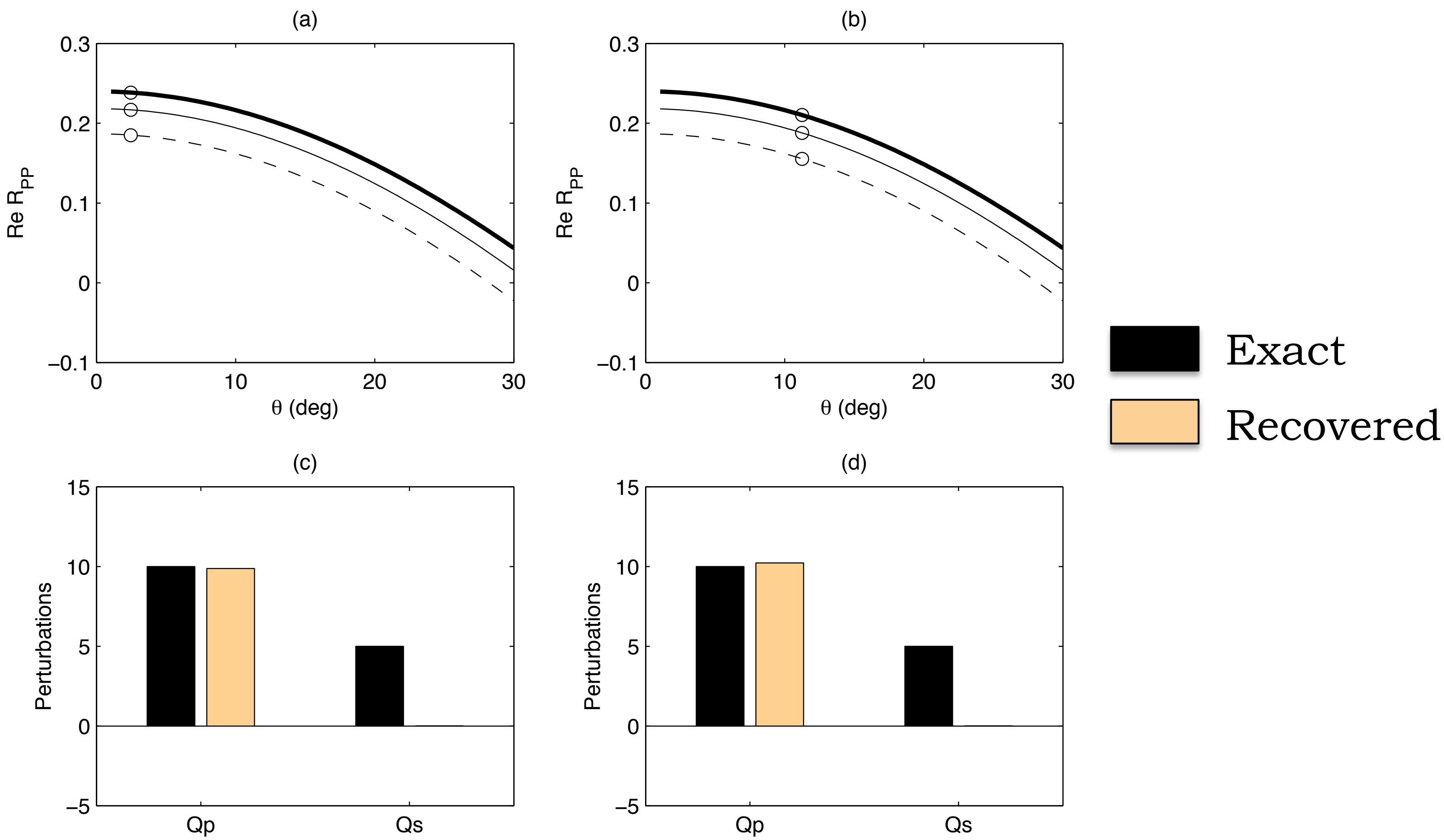
$R_{PP}(\theta)$, $R_{PS}(\theta)$ in the elastic limit (-) and for 2 frequencies (-,-)



$$V_{P_0} = 2000\mathrm{m/s}$$
$$V_{P_1} = 5000\mathrm{m/s}$$
$$V_{S_0} = 1500\mathrm{m/s}$$
$$V_{S_1} = 3500\mathrm{m/s}$$
$$\rho_0 = 2.0\mathrm{gm/cc}$$
$$\rho_1 = 4.0\mathrm{gm/cc}$$
$$Q_{P_1} = 30$$
$$Q_{S_1} = 5$$

Linear anelastic inversion

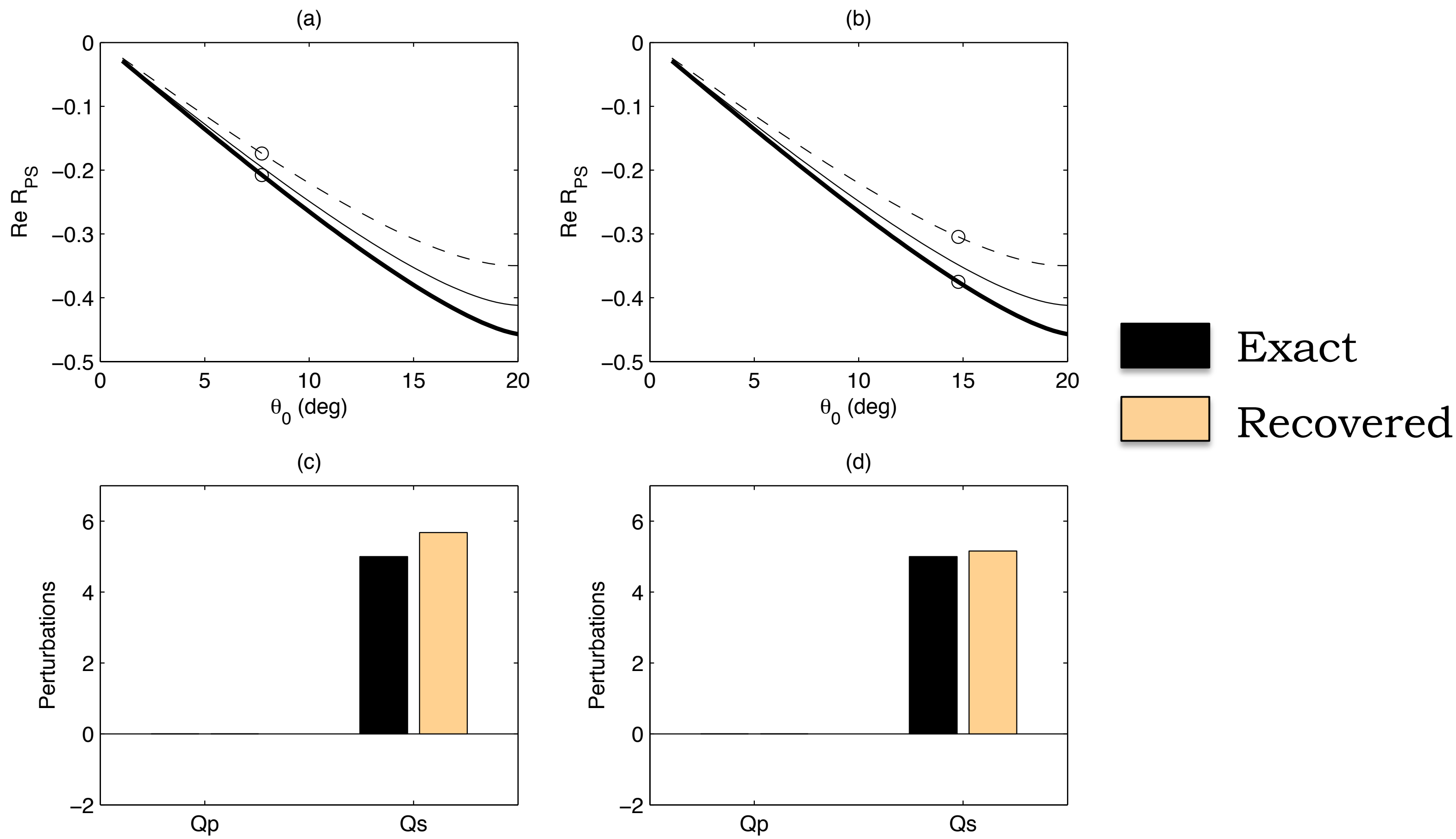
$$\text{DATA} = [\ R_{PP}(\theta_{fixed},\omega_1),\ R_{PP}(\theta_{fixed},\omega_2),\ R_{PP}(\theta_{fixed},\omega_3)\ ]$$



From R_{PP} AVF: target Q_P recovered, Q_S not

Linear anelastic inversion

$$\text{DATA} = [\ R_{PS}(\theta_{fixed},\omega_1),\ R_{PS}(\theta_{fixed},\omega_3)\ ]$$



From R_{PS} AVF: target Q_S recovered, Q_P not