

# Estimation of phase and group velocities for the MASW survey using the 'slant stack generalized S-transform based' and 'phase shift' methods

Roohollah Askari

Robert J. Ferguson

Kristof DeMeersman

# Outline

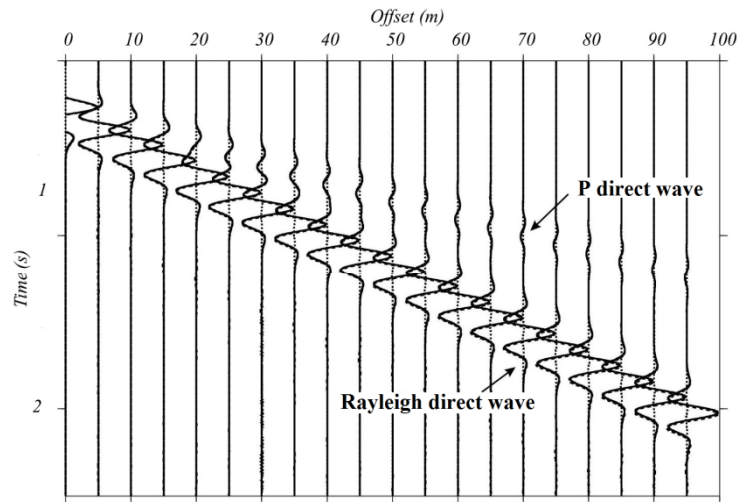
- **Objective of this study**
- **MASW method**
- **The S transform and its generalized version**
- **Slant stack generalized S transform based**
- **Phase-Shift method**
- **Conclusion**

# Objective of this study

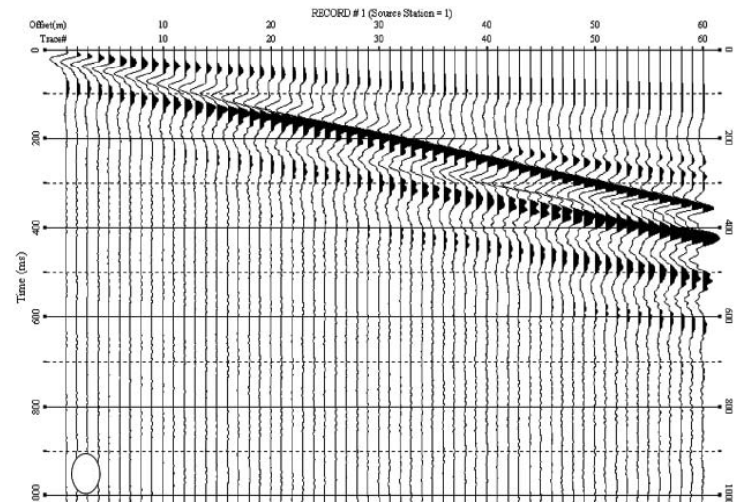
The Objective of this study is to expand a mathematical model to estimate group velocity and phase velocities of multi-modal ground-rolls for MASW survey.

# MASW Method

Multi-channel Analysis of Surface Waves (MASW) is a method for estimating shear wave velocity based on dispersed Rayleigh waves.



(a)

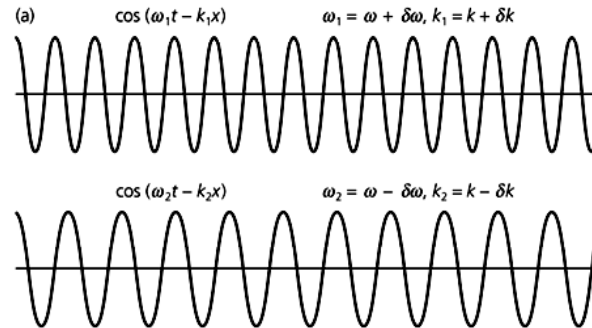


(b)

# Phase and Group Velocities

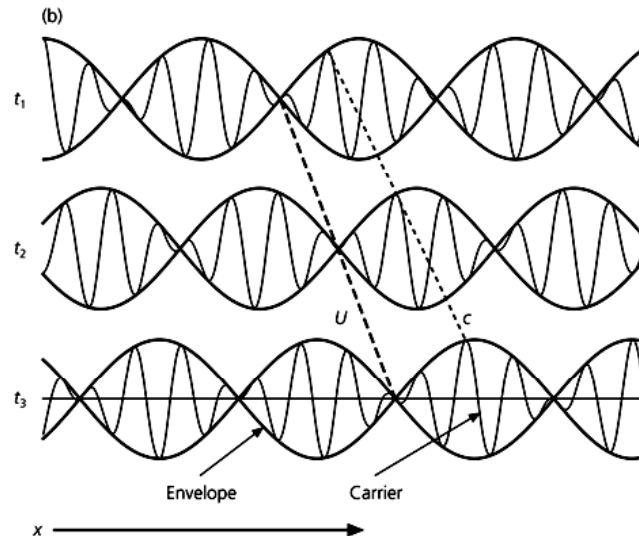
Phase velocity is the velocity of each frequency component of dispersed surface wave which is defined as

$$V_p = f/k.$$

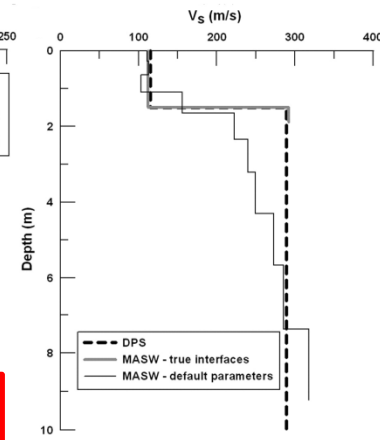
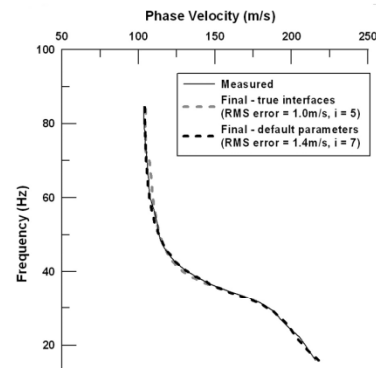
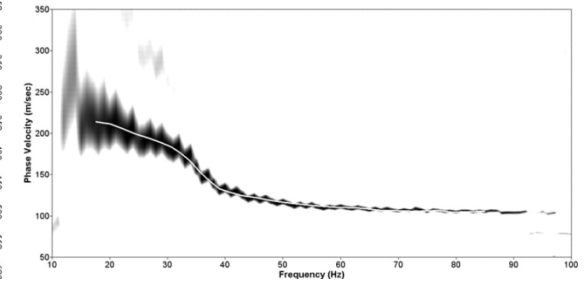
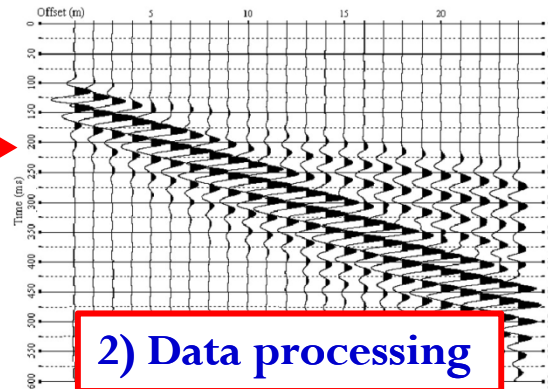


Group velocity is the velocity of a wave packet (envelope) of surface wave around frequency  $f$  which is defined as

$$V_g = df/dk$$



# Three steps in MASW survey



## 3) Inversion

[http://www.geovision.com/PDF/M\\_SASW.PDF](http://www.geovision.com/PDF/M_SASW.PDF)

<http://jeeg.geoscienceworld.org/content/vol13/issue2/images/large/eego>

# Time-Frequency Analysis

*The S transform (Stockwell et al, 1996)*

$$S[h(\tau)](t, f) = \int_{-\infty}^{+\infty} h(\tau) \left[ \frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(\tau-t)^2}{2}} \right] e^{-j2\pi f\tau} d\tau.$$

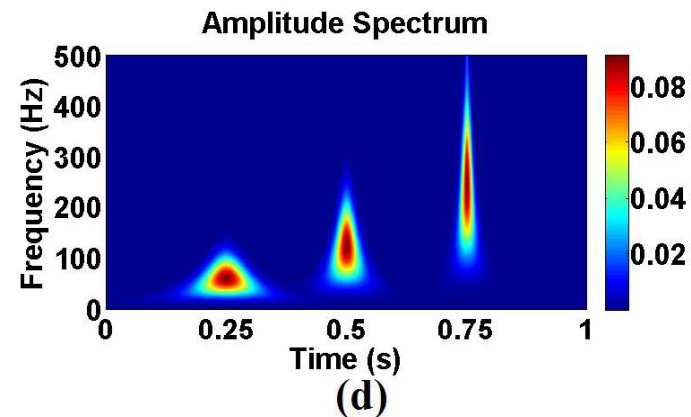
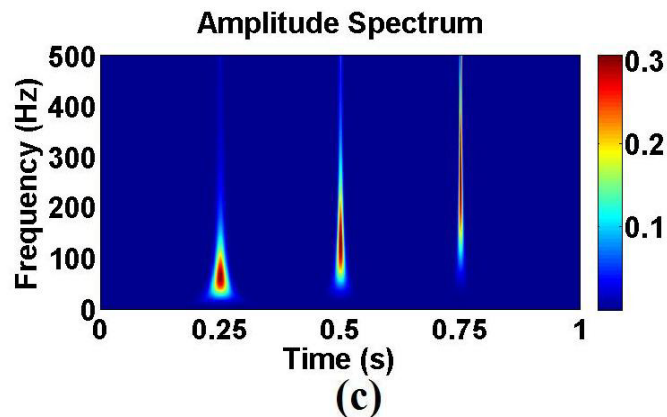
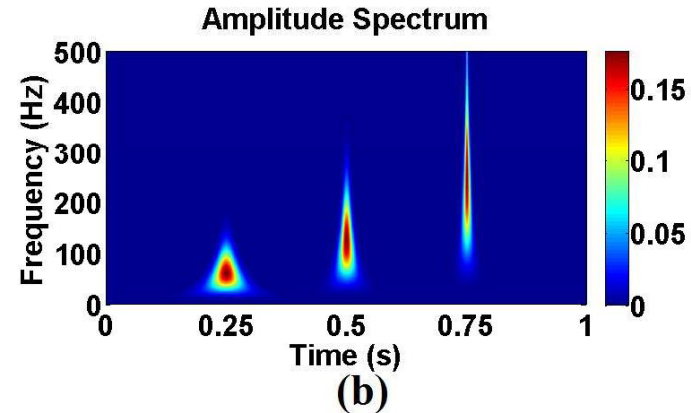
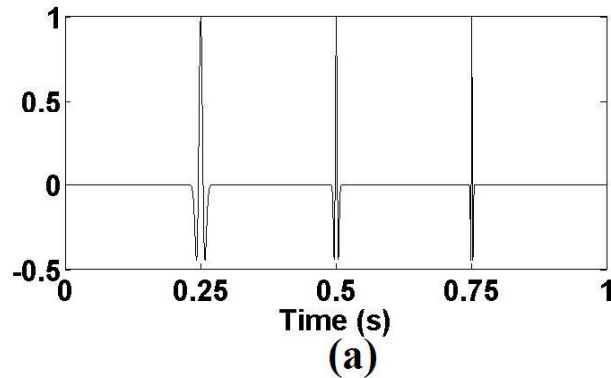
*The generalized S transform (Pinnegar and Mansinha, 2003)*

$$S_g[h(\tau)](t, f, \mathbf{p}) = \int_{-\infty}^{+\infty} h(\tau) w(\tau - t, f, \mathbf{p}) e^{-j2\pi f\tau} d\tau$$

*A version of the generalized S transform is defined using a scaling factor  $\sigma$ ,*

$$S_g[h(\tau)](t, f, \sigma) = \int_{-\infty}^{+\infty} h(\tau) \frac{|f|}{\sqrt{2\pi\sigma}} e^{-\frac{f^2(\tau-t)^2}{2\sigma^2}} e^{-j2\pi f\tau} d\tau$$

# The Generalized S Transform



(a) a signal. (b), (c) and (d) the generalized S transforms of (a) for  $\sigma=1, 0.25$  and  $4$  respectively.



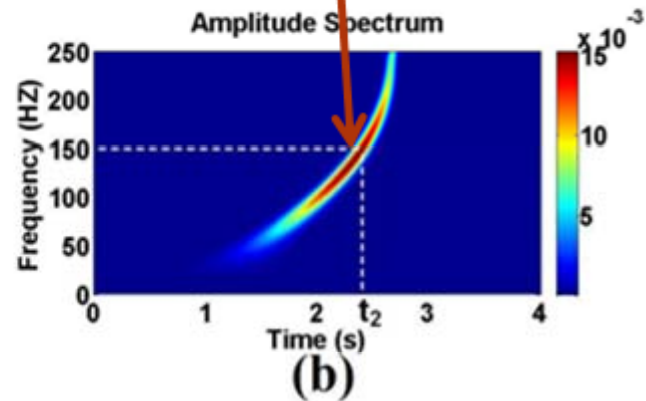
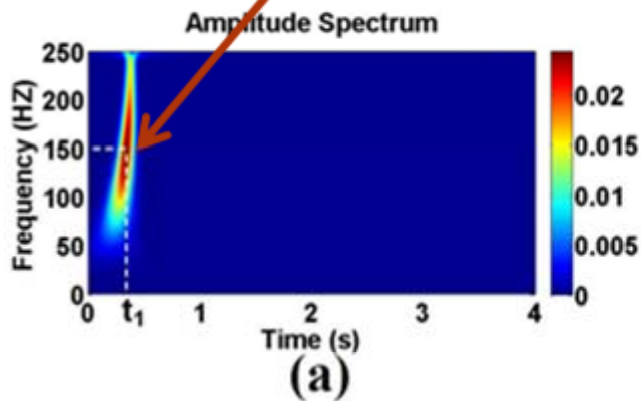
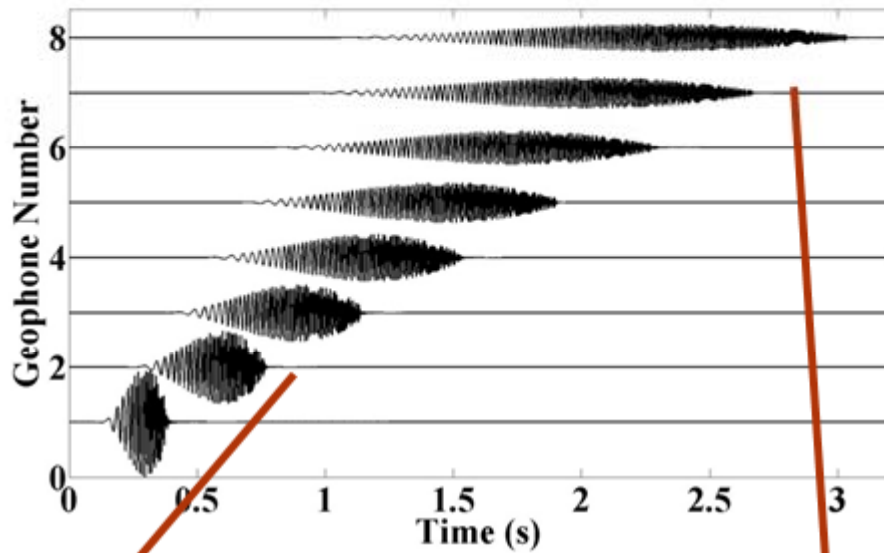
# Estimation of group velocities

Assuming geometrical spreading correction has been applied on surface wave data, if  $h_1(\tau)$  is the wavelet at station 1, the wavelet  $h_2(\tau)$  recorded at station 2 can be expressed

$$H_2(f) = e^{-\lambda(f)d} e^{-j2\pi k(f)d} H_1(f)$$

Assuming

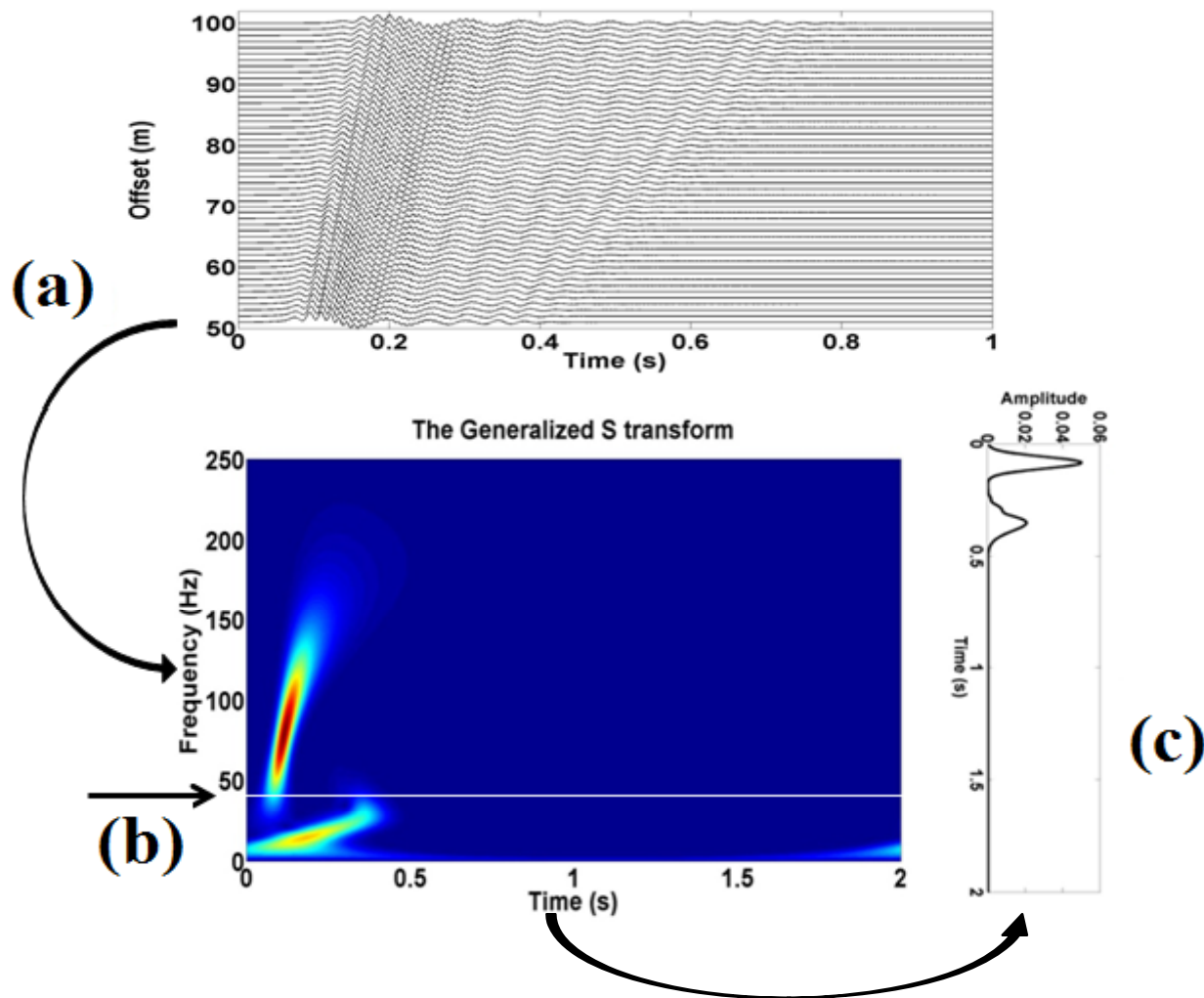
$$k(f + \alpha) = k(f) + \alpha k'(f) + O(\alpha^2),$$



(a) and (b) the amplitude spectrum for traces 1 and 7 respectively

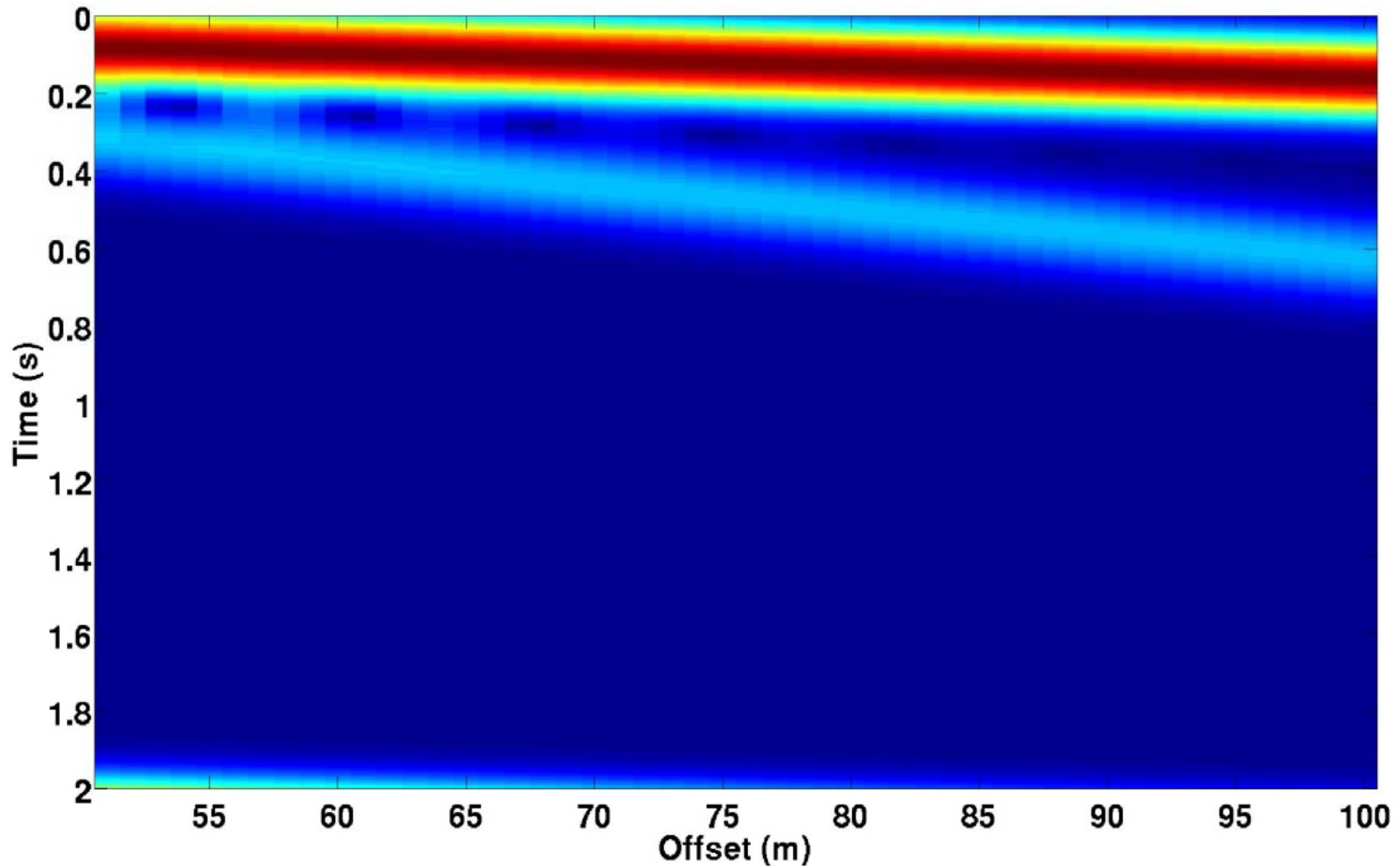
$$v_g = \frac{d}{\Delta t}$$

# Slant Stack Generalized S transform Based



(a) A synthetic data containing a two modal ground roll. (b) The generalized S transform of the first trace in Figure 1. (c) Time representation of the generalized S transform at the single frequency 40Hz.

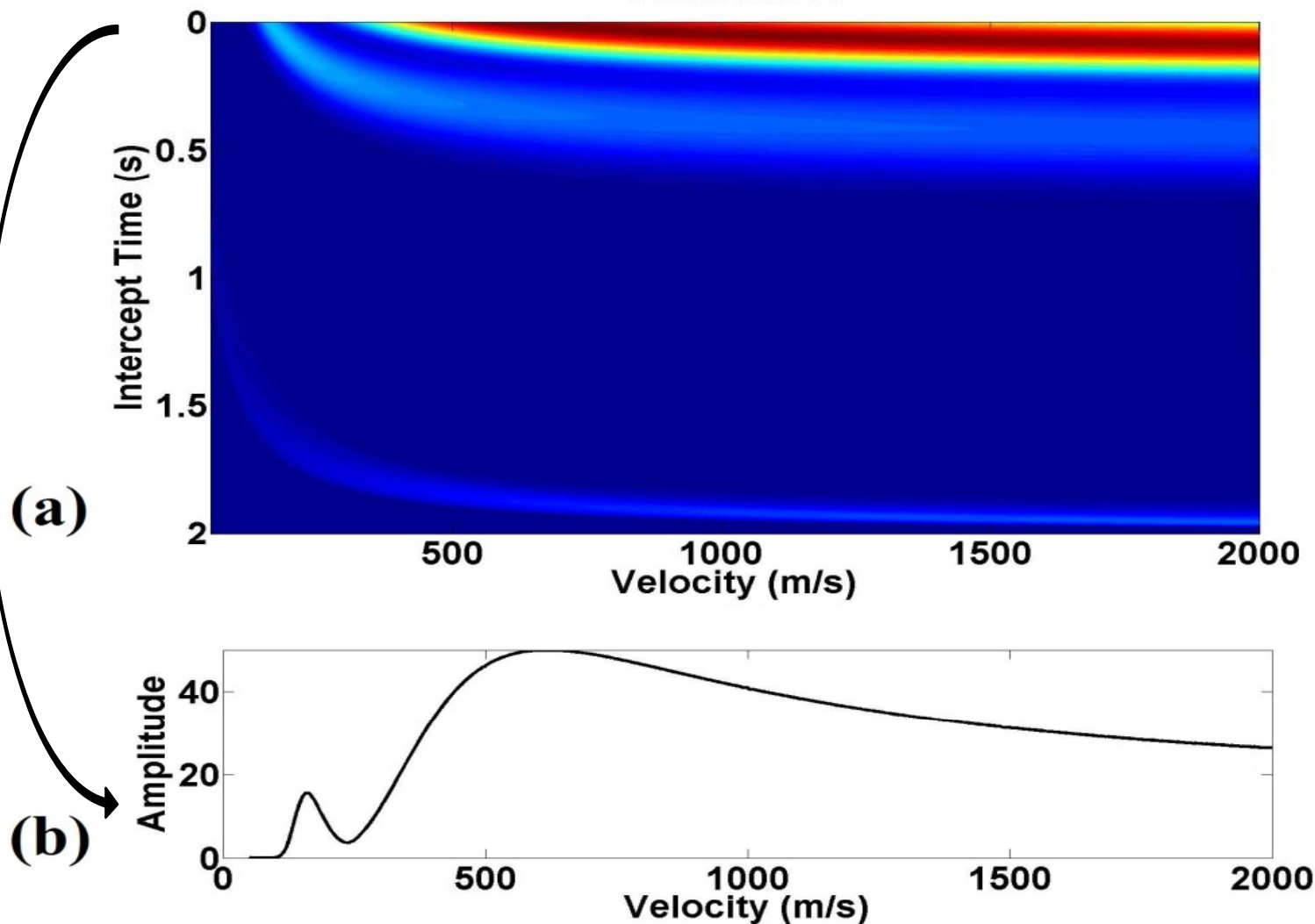
# Slant Stack Generalized S transform Based



A pseudo-seismic record based of the generalized S transform of the traces of the record in Figure 1 at Frequency=40Hz.

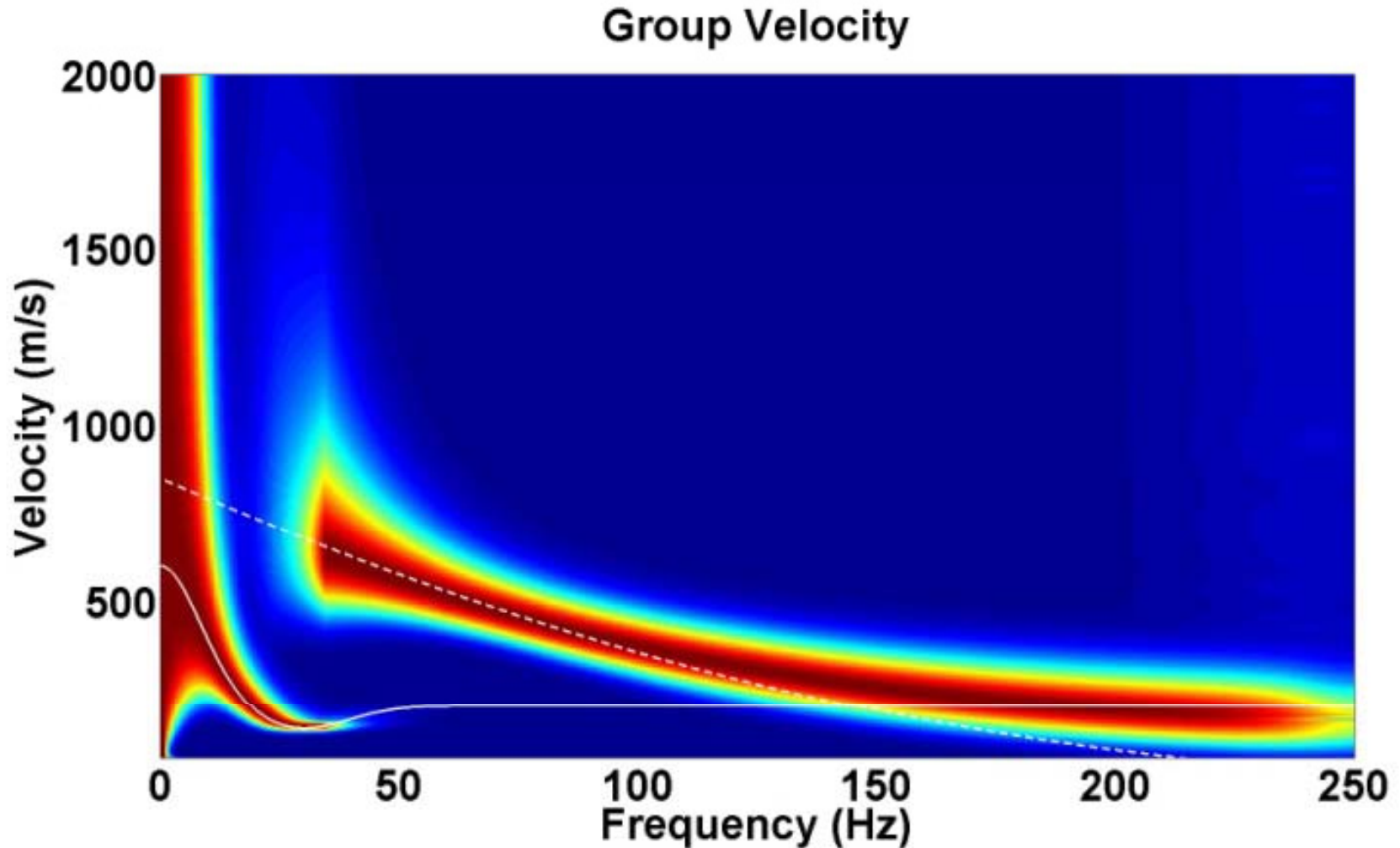
# Slant Stack Generalized S transform Based

Slant Stack



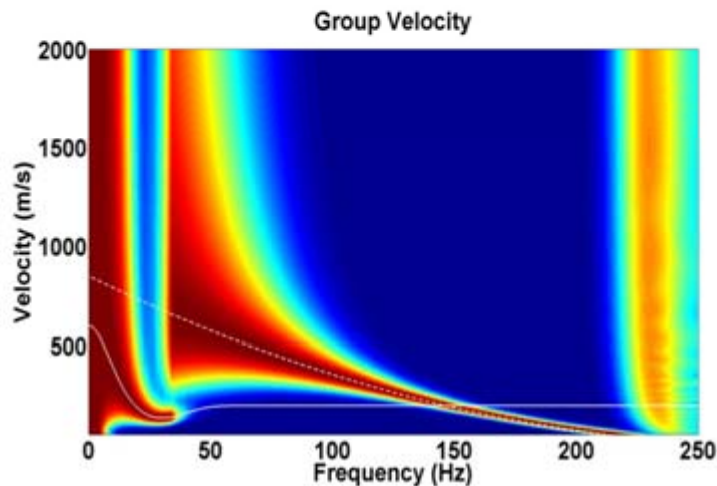
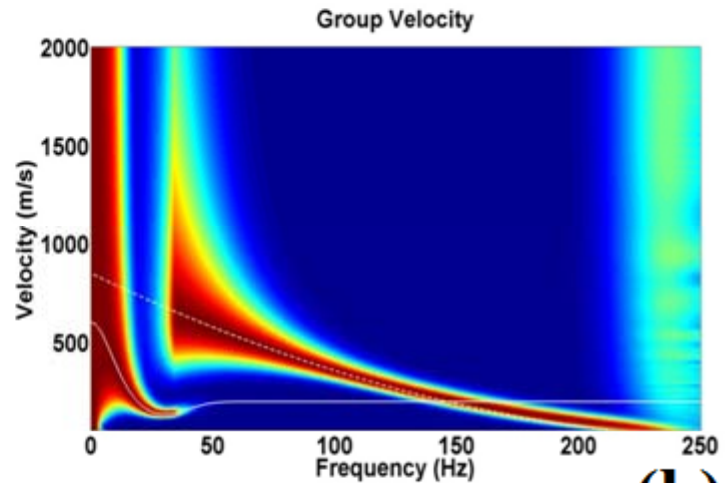
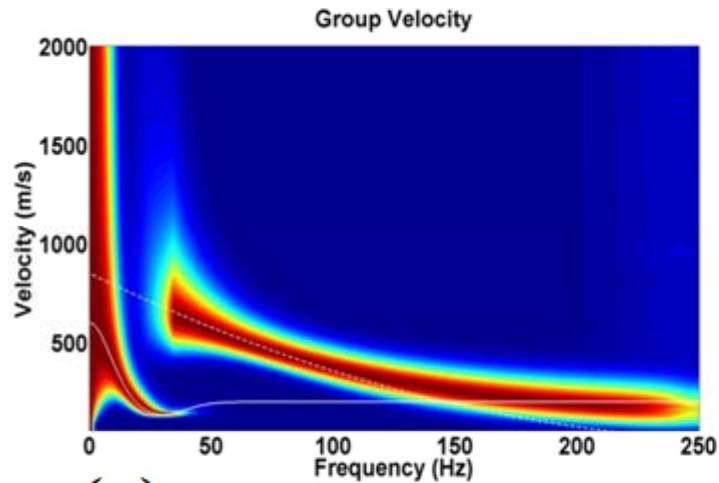
(a) The slant stack of the pseudo-record in Figure 7. (b) Single representation of the slant stack at intercept time  $T=0$ .

# Group Velocity of the Record Based on Scaling factor $\sigma=1$ .



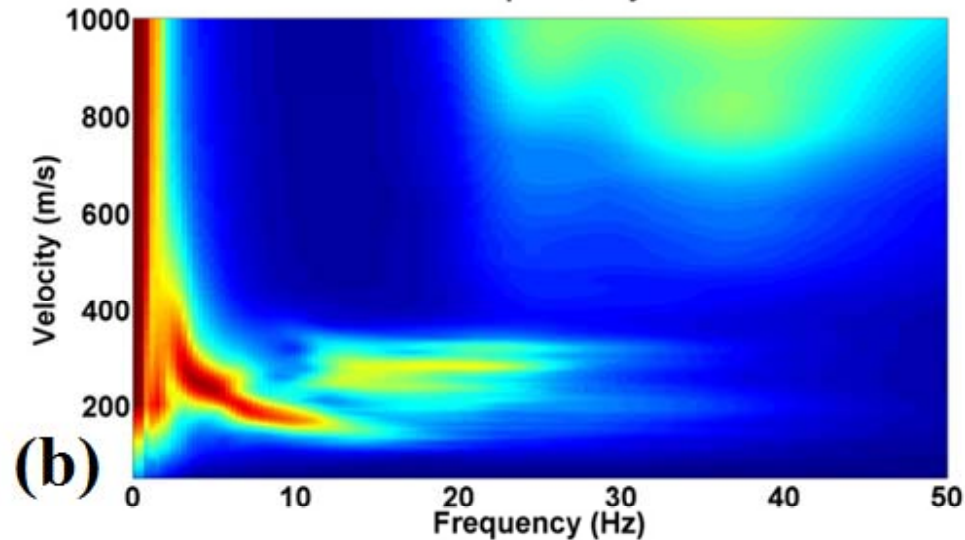
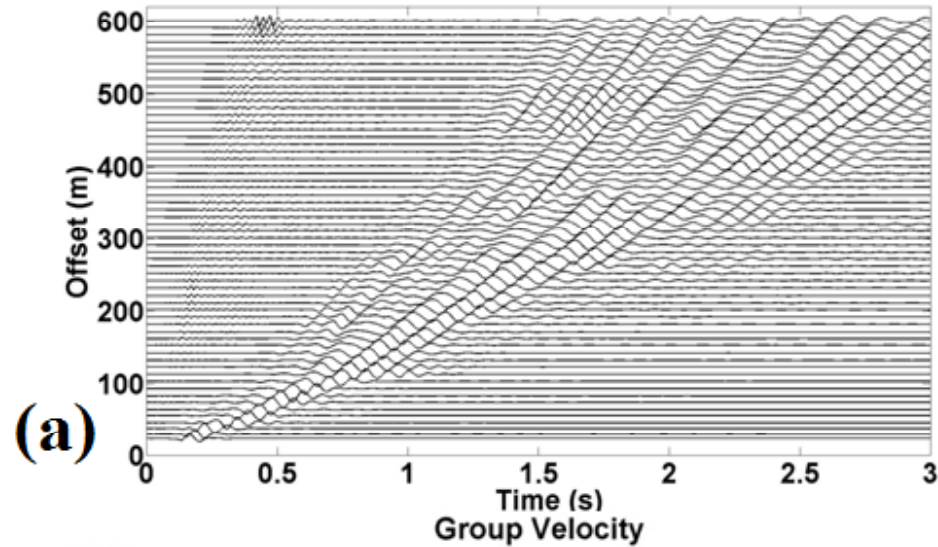
The estimated group velocity for the record in Figure 1 based on the scaling factor  $\sigma=1$ . The solid and dashed lines correspond to the theoretical values.

## Group Velocity of the Record



(a), (b), and (c) the estimated group velocity for the record in Figure 1 based on the scaling factor  $\sigma=1$ ,  $\sigma=5$ , and  $\sigma=20$  respectively.

# A Real Data Example



(a) A real data. (b) The estimated group velocity for the record based on the scaling factor  $\sigma=1$ .



# Phase Shift (Park et al (1998))

Considering a seismic record in the offset-time domain  $u(t,x)$  contacting ground rolls, the Fourier transform for each trace is expressed

$$U(x, \omega) = \int u(x, t) e^{-i\omega t} dt.$$

The equation above could be rewritten as

$$U(x, \omega) = P(x, \omega) A(x, \omega)$$

Phase spectrum



Amplitude spectrum

# Phase Shift

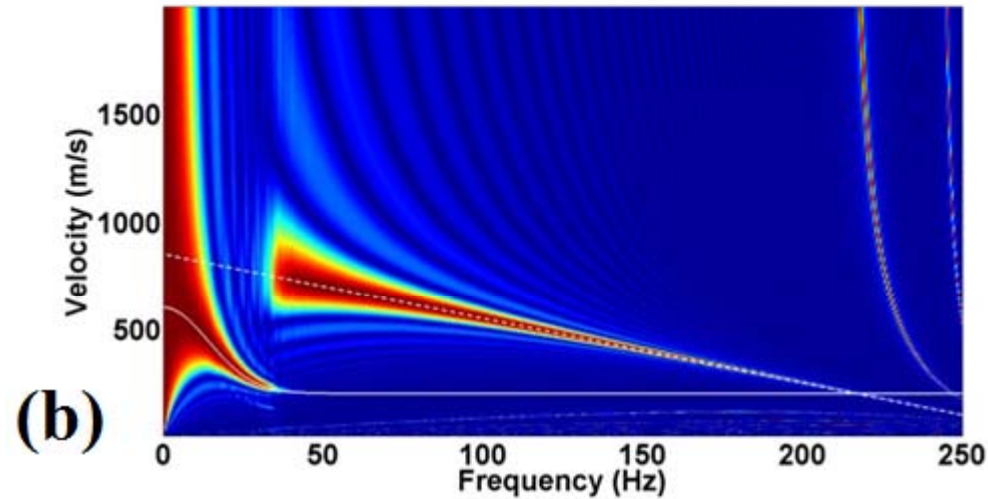
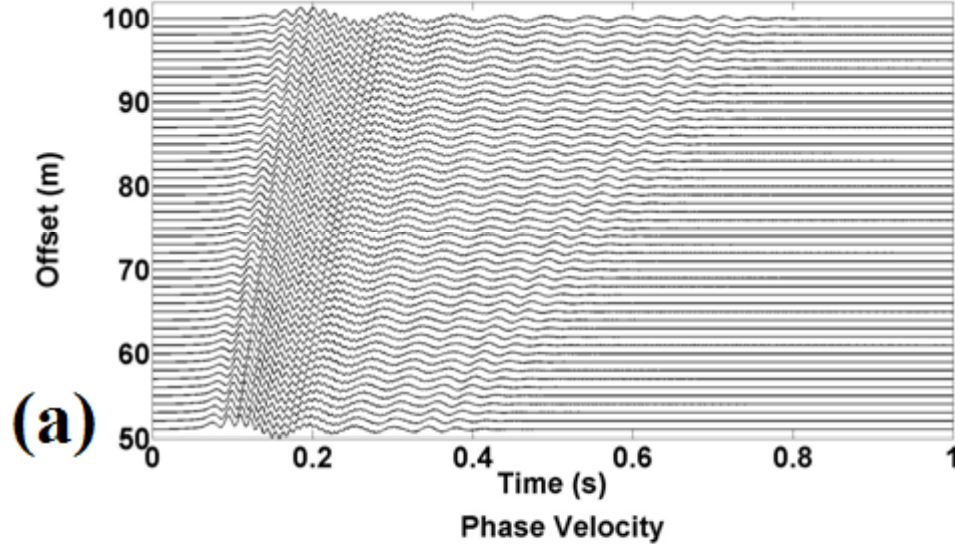
Applying the operator  $e^{i2\pi fx/v_p}$  where  $v_p$  is a phase velocity to the following integral

$$V(f, v_p) = \int \frac{U(x, \omega)}{|U(x, \omega)|} e^{i2\pi fx/v_p} dx = \int P(x, \omega) e^{i2\pi fx/v_p} dx,$$

we obtain a two dimensional image of the phase velocities versus frequencies (Park et al., 1998).

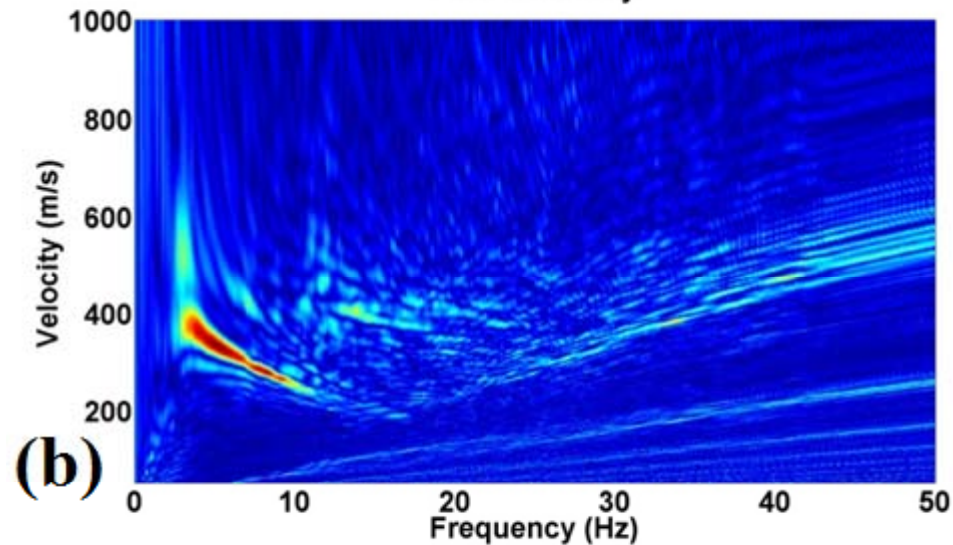
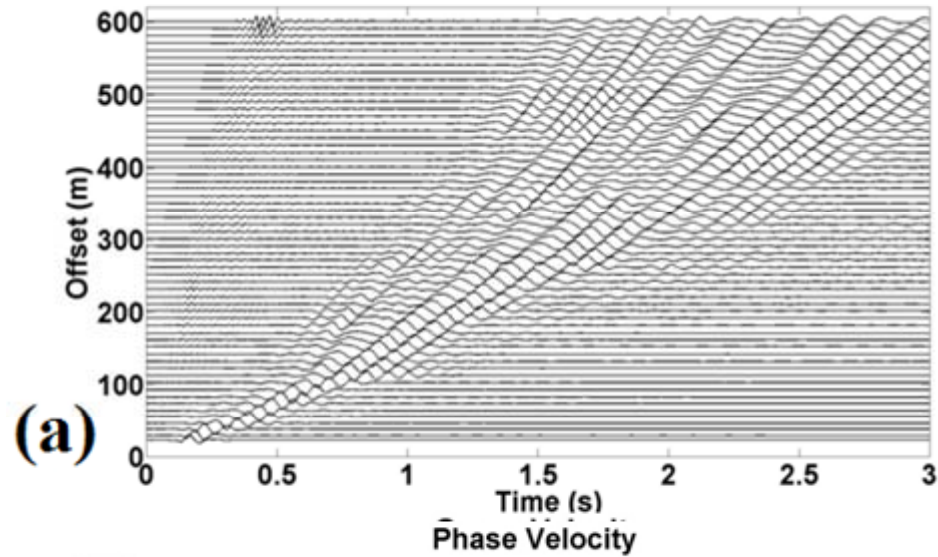
It is clear that at those frequencies where  $v_p$  is equal to the phase velocities of the signal, the integral will have a maximum.

# A Synthetic Data Example



(a) A synthetic data composed of two modal ground roll. (b) The estimated phase velocity.

# A Real Data Example



(a) A real data composed of two modal ground roll. (b) The estimated phase velocity.

# Conclusion

Slant Stack S-transform based is a flexible method to estimate group velocity of ground-roll.

The resolution of the group velocity is manipulated by a scaling factor. As smaller scaling factor should be chosen for the low frequency ground roll. However, for higher frequencies, a larger scaling factor should be chosen. Therefore, there is a trade of between the resolution of low and high frequency ground roll. Based on our imperial observations, for the ranges of the frequencies that we deal with in the MASW survey (3-100Hz), scaling factors from 1 to 5 are recommended.

Phase-shift method is a very fast and high resolution technique for estimation of phase velocity.

# Acknowledgements

Special thanks to

- ✓ *CGGVeritas* for providing me the opportunity and the facilities to do this study.



Thanks to

- ✓ *Petrobank* for providing us with the real data.
- ✓ Also thank to *CREWES* and its *industrial sponsors*.