Dennis Gabor: The father of seismic attribute analysis

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Introduction

- If you have been attending CREWES meetings for any length of time, you know the influence of the Gabor transform on the work of Gary Margrave.
- But you may not know that the Gabor transform was only one of three remarkable inventions in Gabor's landmark 1946 paper: "Theory of Communication", the others being time-frequency analysis and the complex signal.
- After a brief discussion of both time-frequency analysis and the Gabor transform, I will focus on the complex signal and the development of seismic attribute analysis.
- I will try to show the link among all seismic attributes.
- But first, let me tell you a little bit more about Dennis Gabor, the man.

Dennis Gabor

- Dennis Gabor was a Hungarian/British physicist who invented holography, for which he received the 1971 Nobel prize.
- In 1946, he also wrote one of the most influential papers of the 20th century, entitled: “Theory of communication”.
- In that paper, he invented the complex signal, the Gabor transform and time-frequency analysis, each of which has had a major impact on our profession.
- Today, I will discuss on how his complex signal lead to the development of seismic attribute analysis.

Dennis Gabor

Born	5 June 1900 Budapest, Kingdom of Hungary
Died	8 February 1979 (aged 78) London, England

The Fourier transform

- The standard approach to signal analysis is the Fourier transform, where a time signal $s(t)$ is transformed to a frequency signal $S(\omega)$:

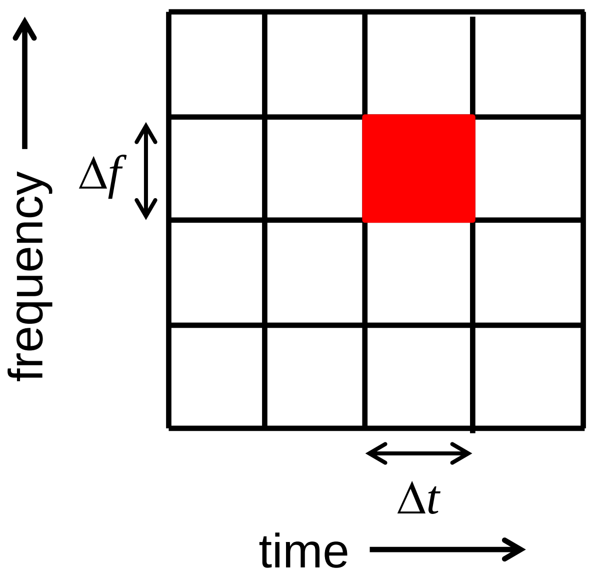
$$s(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\omega) e^{i\omega t} d\omega \Leftrightarrow S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt, \text{where}$$

$$\omega = 2\pi f, \; f = \text{frequency, and } S(\omega) = \text{the Fourier transform of } s(t).$$

- Note that we can also perform the inverse transform from frequency back to time.
- Gabor observed two problems with this:
  - (1) For perfect reconstruction of the forward and inverse transforms, an infinite signal length is required.
  - (2) The time signal is real but the frequency signal is complex.

Time-Frequency analysis

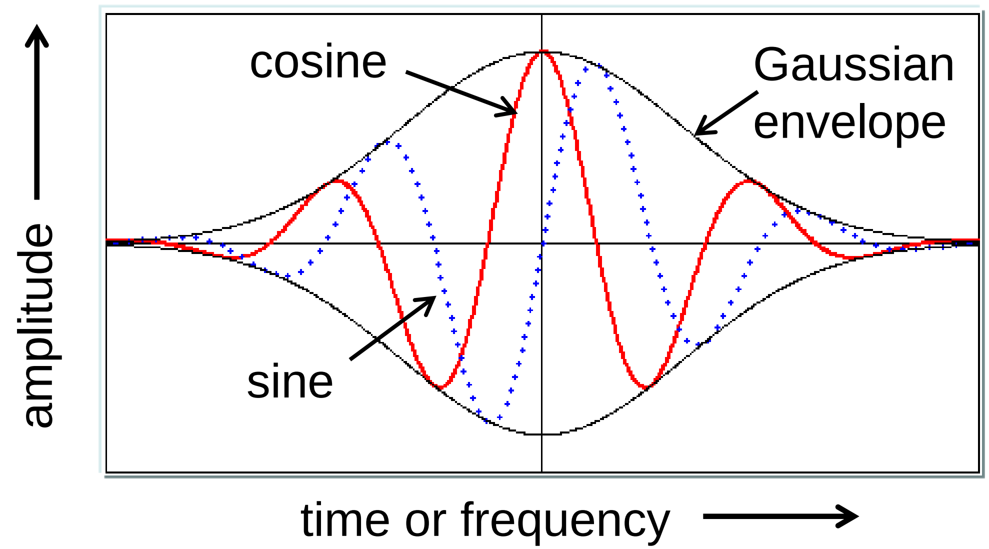
- To solve the first problem, and motivated by Heisenberg's uncertainty principle, Gabor proposed a time-frequency "quantum" of information which satisfied $\Delta t \Delta f \approx 1.0$.
- That is, he proposed creating a grid of squares in time-frequency space:



- The basic "quantum" of information is shown here by the red square of dimension Δt by Δf .
- For his "elementary signal" within each square, Gabor chose a sinusoidal function modulated by a Gaussian envelope.

The Gabor wavelet/transform

- Here is a pictorial representation of Gabor's "elementary signal", where the cosine is the real part of the signal and the sine is the imaginary part:



- This wavelet pair is now called the complex Gabor wavelet in wavelet transform theory.
- The use of Gaussian modulation lead to the Gabor transform.

The complex time signal

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- Gabor next went on to the second problem: how do we create a complex time signal, of which the observed signal is the real component?
- Since the Fourier transform of a real signal has a symmetric shape on both the positive and negative frequency side, Gabor proposed that we suppress the amplitudes belonging to negative frequencies, and multiply the amplitudes of the positive frequencies by two, which gives:

$$z(t) = s(t) + ih(t), \text{ where } i = \sqrt{-1},$$

$$z(t) = \text{the complex signal, and}$$

$$h(t) = \text{the Hilbert transform of } s(t).$$

- The Hilbert transform is a filter which applies a 90^{o} phase shift to every sinusoidal component of a signal.

Instantaneous attributes

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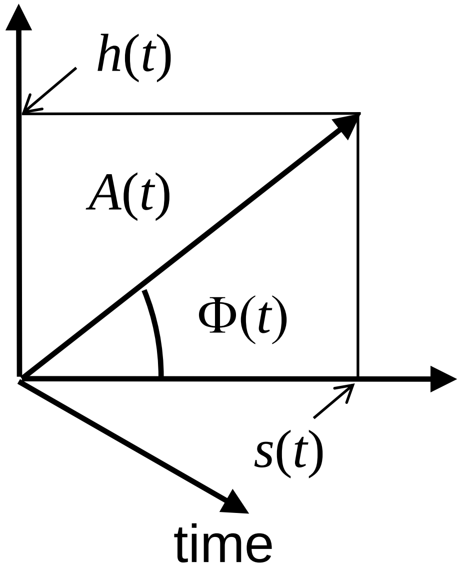
- Notice that the complex trace can be transformed from rectangular to polar coordinates, as shown below, to give the instantaneous amplitude $A(t)$ and instantaneous phase $\Phi(t)$:



$$z(t) = s(t) + ih(t) = A(t)e^{i\Phi(t)}$$
$$= A(t)\cos\Phi(t) + iA(t)\sin\Phi(t),$$
$$\text{where}: A(t) = \sqrt{s(t)^2 + h(t)^2},$$
$$\text{and}: \quad \Phi(t) = \tan^{-1}\left[\frac{h(t)}{s(t)}\right].$$

- The complex trace was introduced into geophysics by Taner et al. (1979), who also discussed its implementation.

Instantaneous frequency

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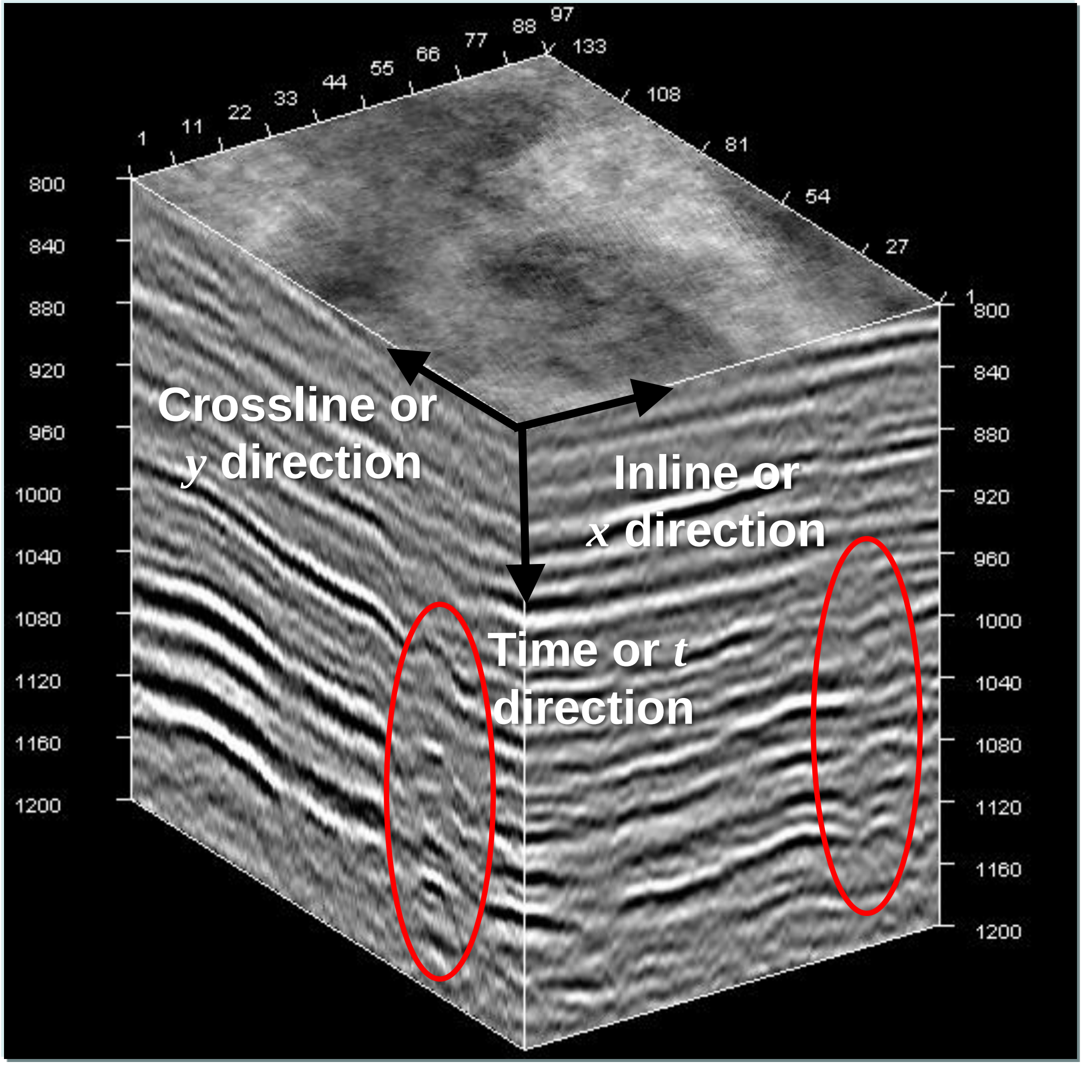
- Taner et al. (1979) also introduced the instantaneous frequency of the complex seismic trace, which was initially derived by J. Ville in a 1948 paper entitled: "*Théorie et applications de la notion de signal analytique*".
- The instantaneous frequency is the time derivative of the instantaneous phase:

$$\omega(t) = \frac{d\Phi(t)}{dt} = \frac{d\tan^{-1}\left(h(t)/s(t)\right)}{dt} = \frac{s(t)\dfrac{dh(t)}{dt} - h(t)\dfrac{ds(t)}{dt}}{A(t)^2}$$

- Note that to compute $\omega(t)$ we need to differentiate both the seismic trace and its Hilbert transform.
- Like the Hilbert transform, the derivative applies a 90^{o} phase shift, but it also applies a high frequency ramp.

A seismic volume

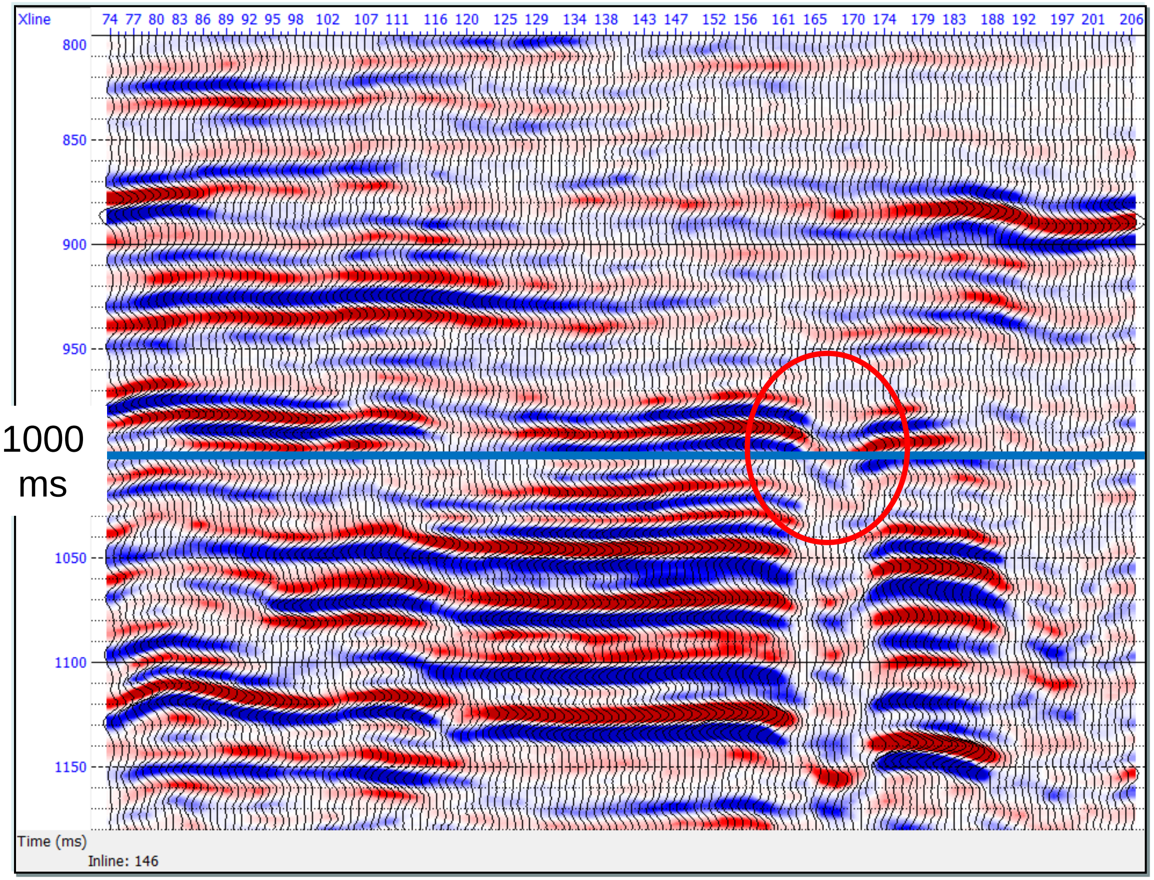
- Here is a 3D seismic volume that was recorded over a karsted terrain (Hardage et al., 1996)
- It consists of 97 inlines and 133 crosslines, each with 200 samples (800–1200 ms).
- The karst features are illustrated by the red ellipses.



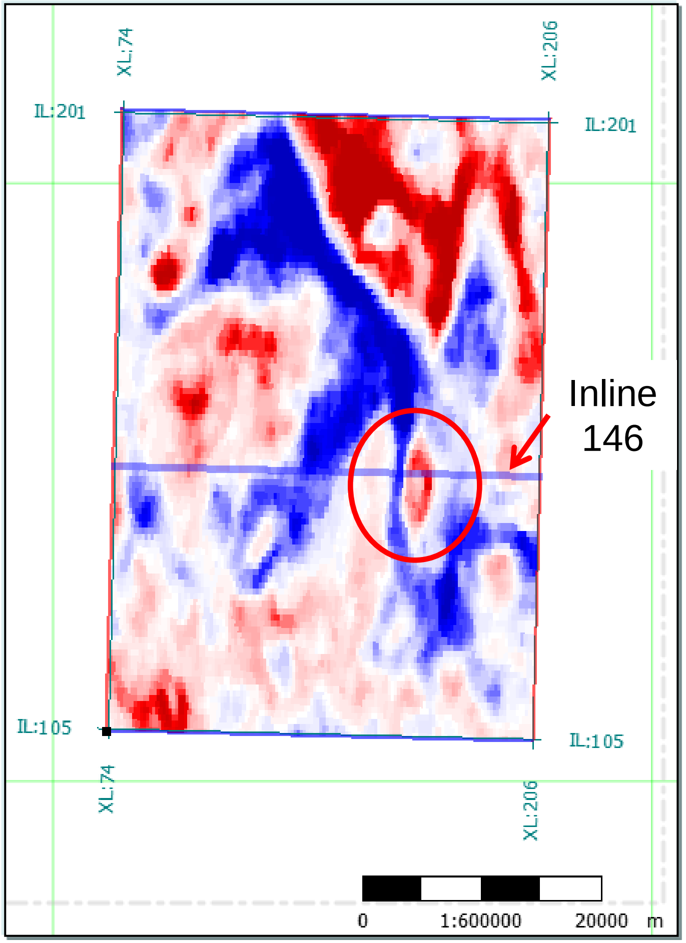
Slices through the volume

Here are vertical and horizontal slices through the seismic volume:

1000 ms



Inline seismic section 146



Time slice at 1000 ms

Note the karst feature at the east end of the line.

Instantaneous attributes

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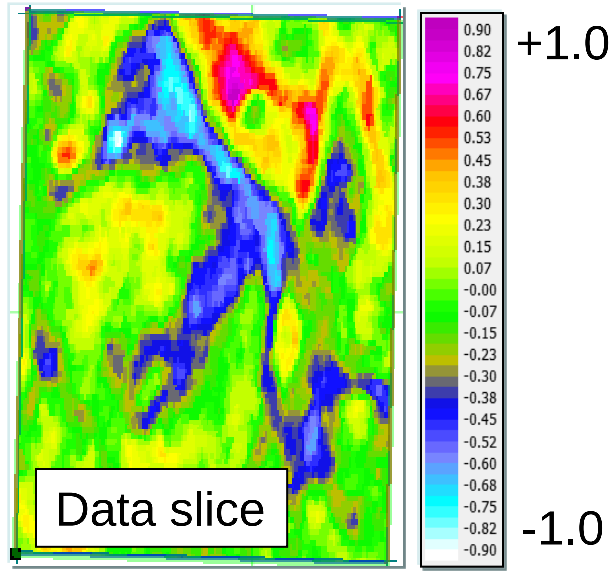
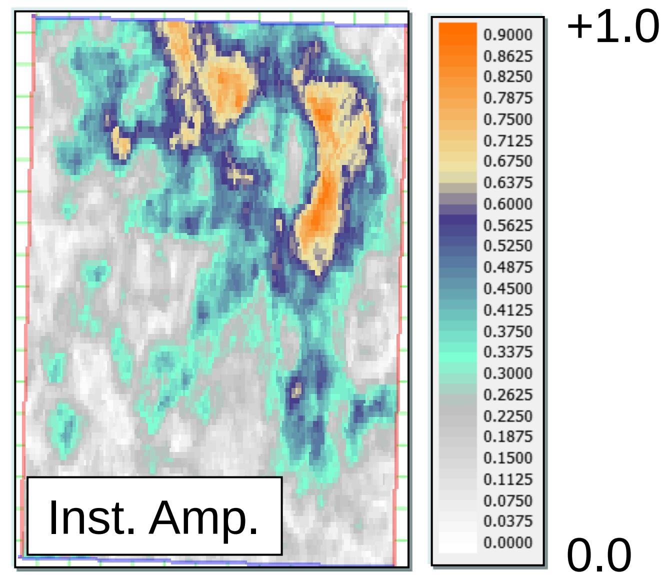
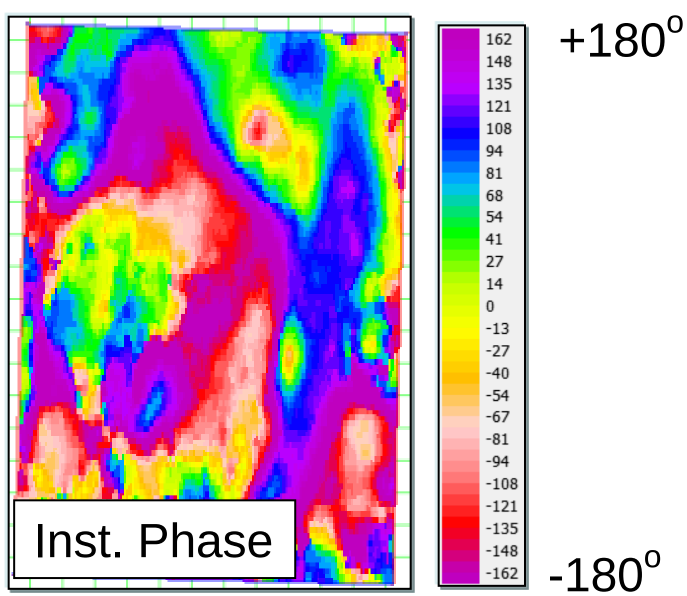
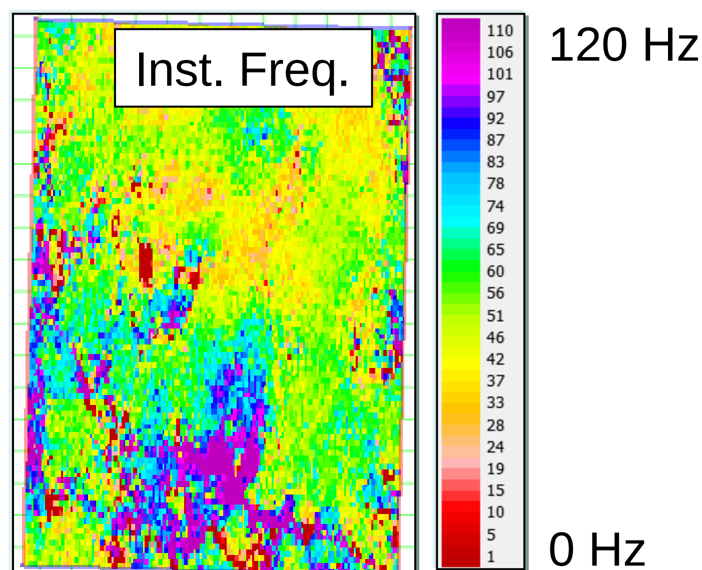
This figure shows the instantaneous attributes associated with the seismic amplitude slice at 1000 ms.









New attributes

- Attribute analysis until the mid-90s was thus based on the three basic attributes introduced by Gabor and Ville in the 1940s: instantaneous phase, frequency, and amplitude.
- Then, in the space of two years, papers on two new approaches to attribute analysis appeared:
  - The coherency method (Bahorich and Farmer, 1995)
  - 2-D (and 3-D) complex trace analysis (Barnes, 1996)
- The coherency method was a new approach which relied on cross-correlations between traces.
- It was hard to see how instantaneous attributes and coherency were related.
- However, the link was provided in the paper by Barnes on 2-D and 3-D complex trace analysis.

The coherency method

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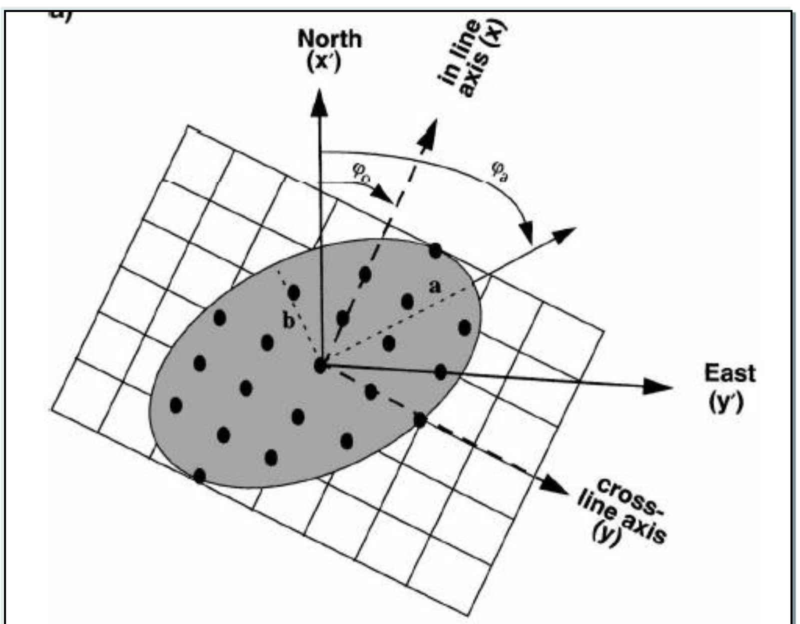
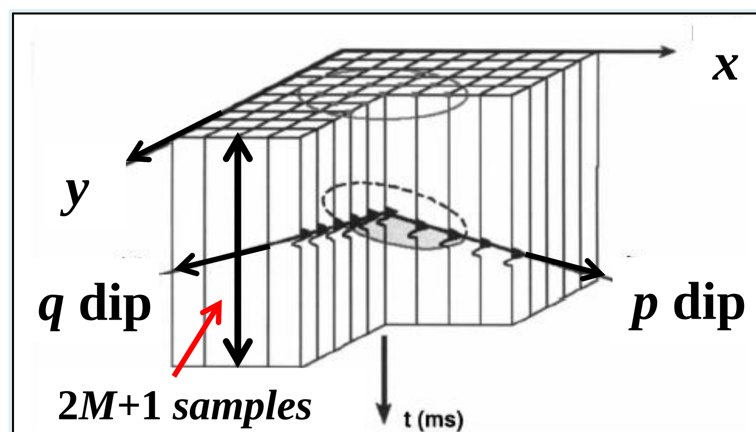
- The first coherency method involved finding the maximum correlation coefficients between adjacent traces in the x and y directions, and taking their harmonic average.
- Marfurt et al. (1998) extended this by computing the semblance of all combinations of J traces in a window.



- This involves searching over all x dips p and y dips q, over a $2M + 1$ sample window:



Marfurt et al. (1998)

Coherency slice

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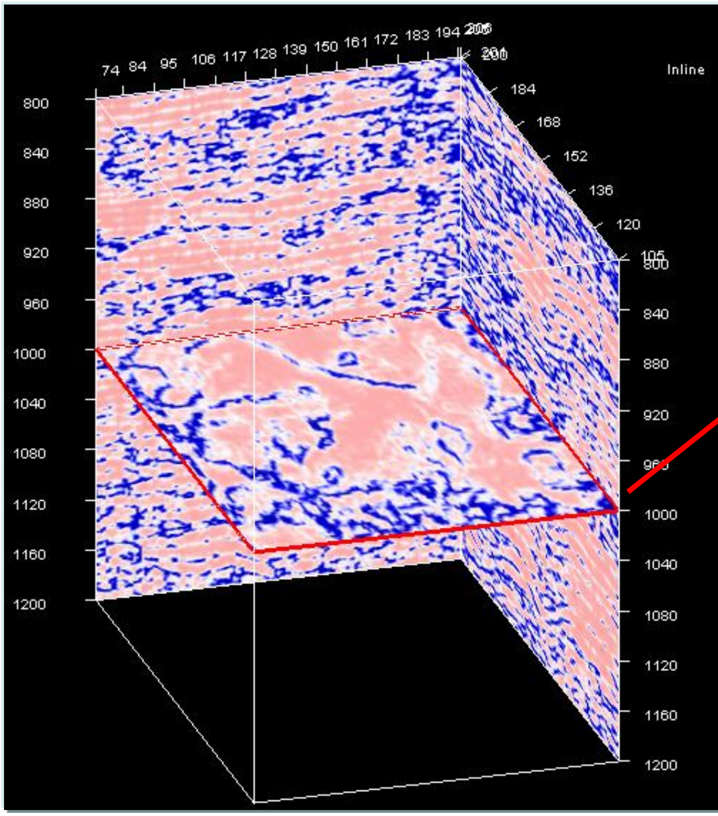
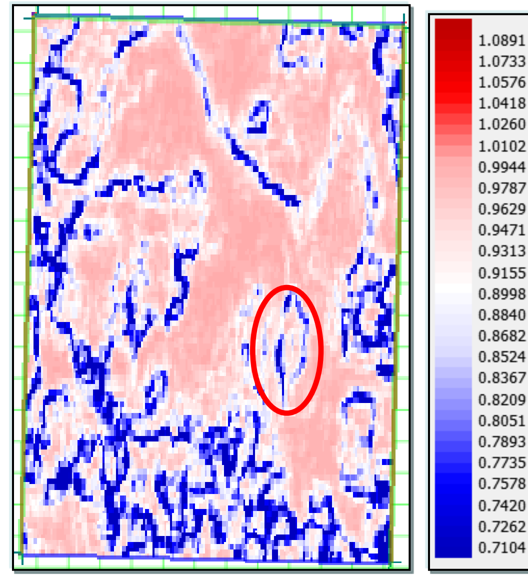
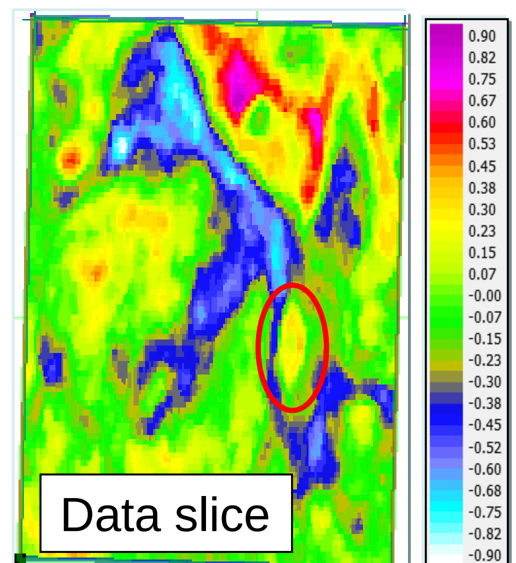




1.1

0.7



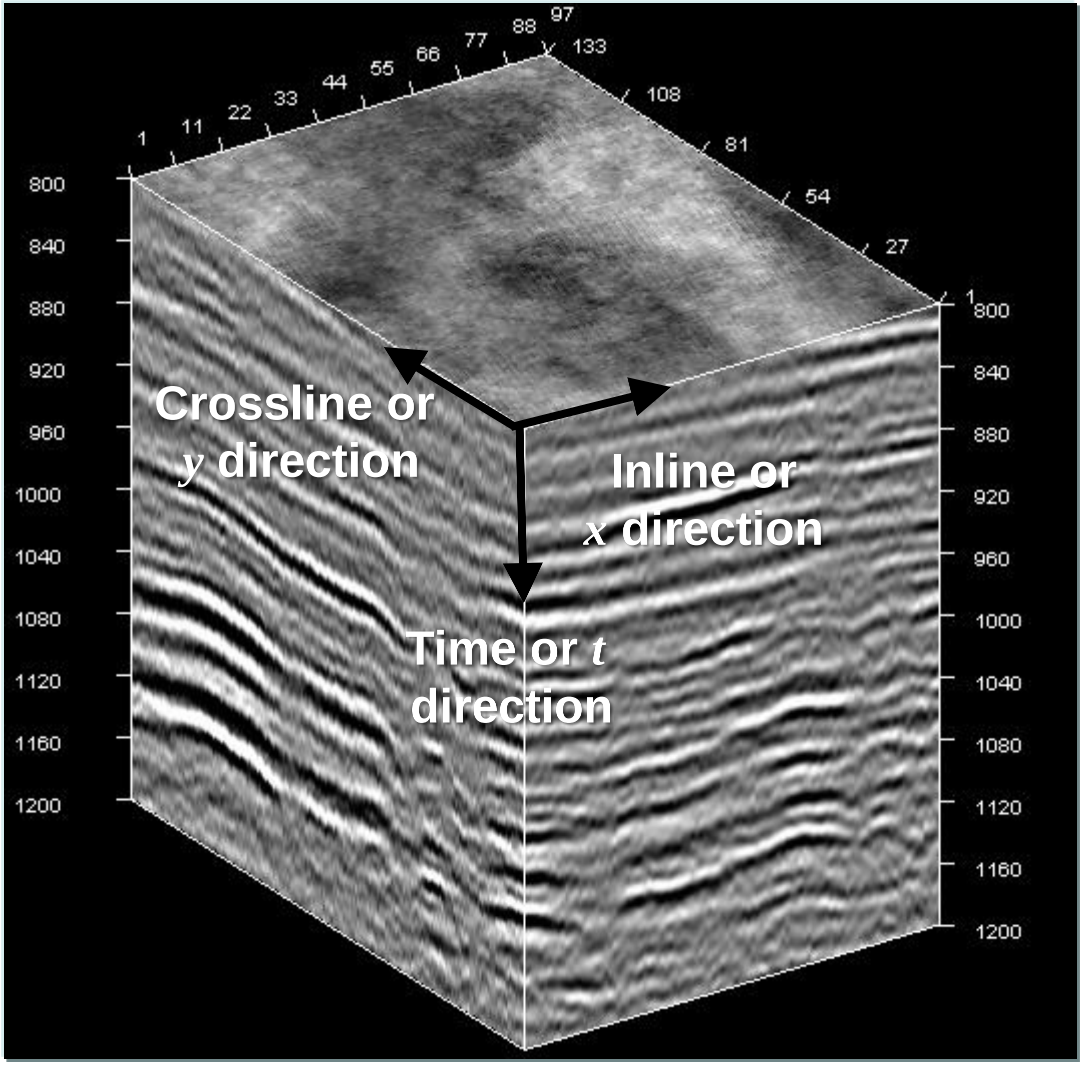
+1.0

-1.0

The x, y and z slices through a coherency volume with the 1000 ms data and coherency slices on the right. Note the low amplitude discontinuities and the highlighted event.

Instantaneous *x* and *y* freq.

- Re-visiting our earlier 3D dataset, notice that we only computed the frequency attribute in the time direction
- Since the Hilbert transform is actually a function of three coordinates (i.e. $\Phi(t,x,y)$) we can also compute frequencies in the inline (x) and crossline (y) directions.



Instantaneous wavenumber

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- Analogous to instantaneous frequency, Barnes (1996) defined the instantaneous wavenumbers k_x and k_y:

$$k_x = \frac{\partial \Phi(t, x, y)}{\partial x} \quad \text{and} \quad k_y = \frac{\partial \Phi(t, x, y)}{\partial y}.$$

- The instantaneous time dips in the x and y direction, p and q, are given as:

$$p = \frac{k_x}{\omega} \ \text{and} \ q = \frac{k_y}{\omega}.$$

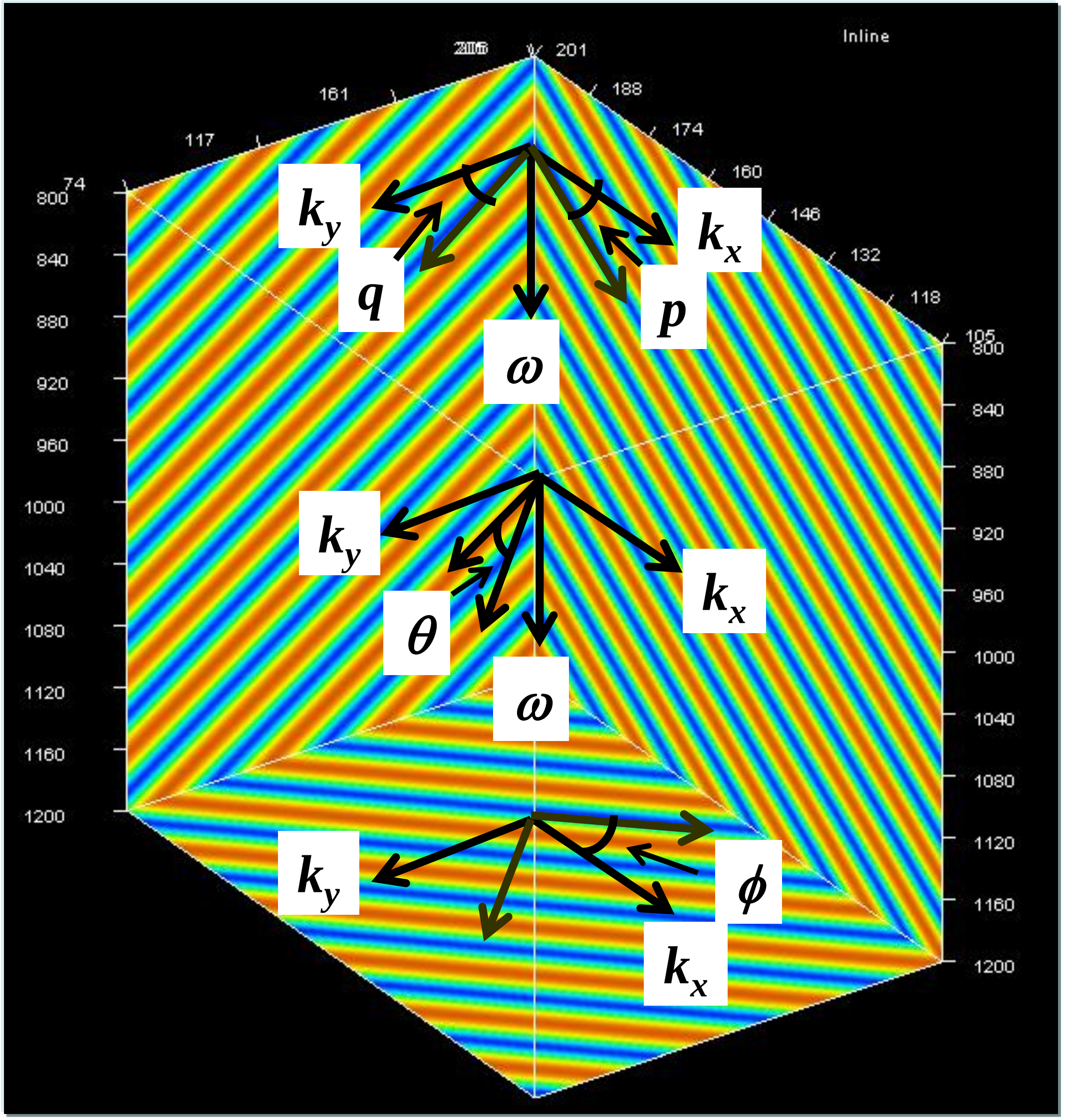
- Using p and q, the azimuth ϕ and true time dip θ are:

$$\phi = \tan^{-1}(p/q) = \tan^{-1}(k_x/k_y),$$

$$\text{and} \ \theta = \sqrt{p^2 + q^2}.$$

Dip and azimuth attributes

- To illustrate this, let's look at a dipping cosine wave in 3D.
- In this display, we have sliced it along the *x, y* and *t* axes.
- For this dipping event, it is clear how the dips and azimuths are related to the inst. frequencies.
- Next, we will look at azimuth on our karst example.



Instantaneous azimuth

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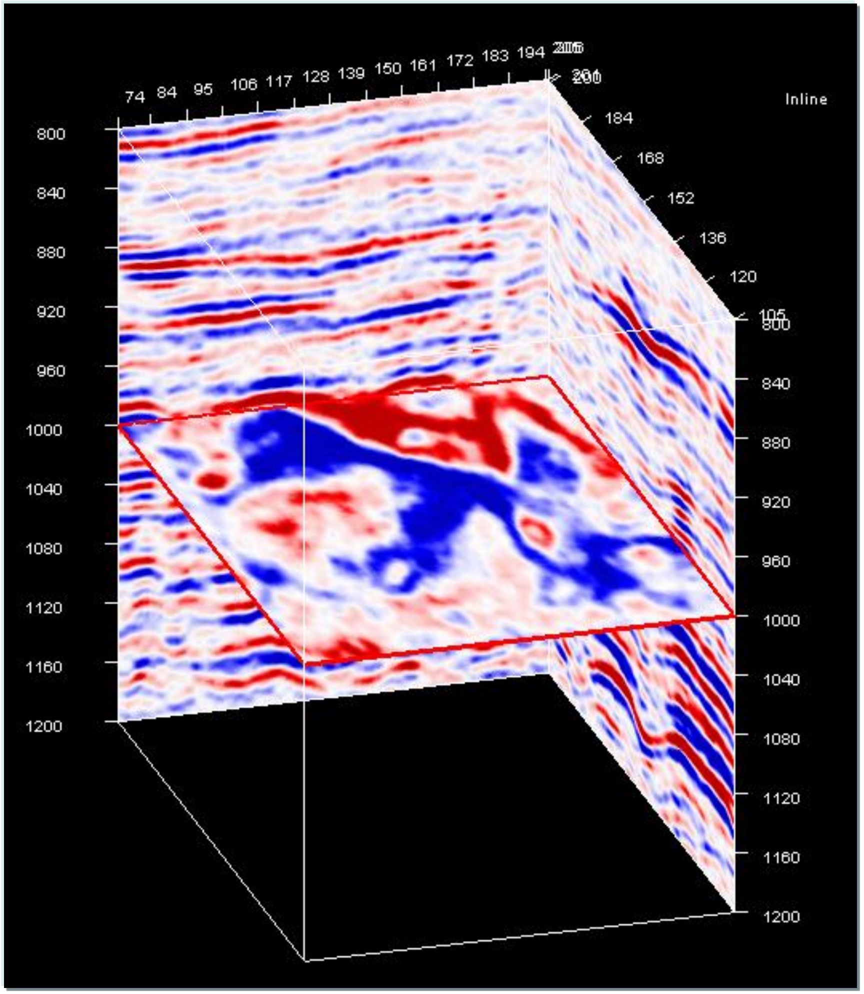
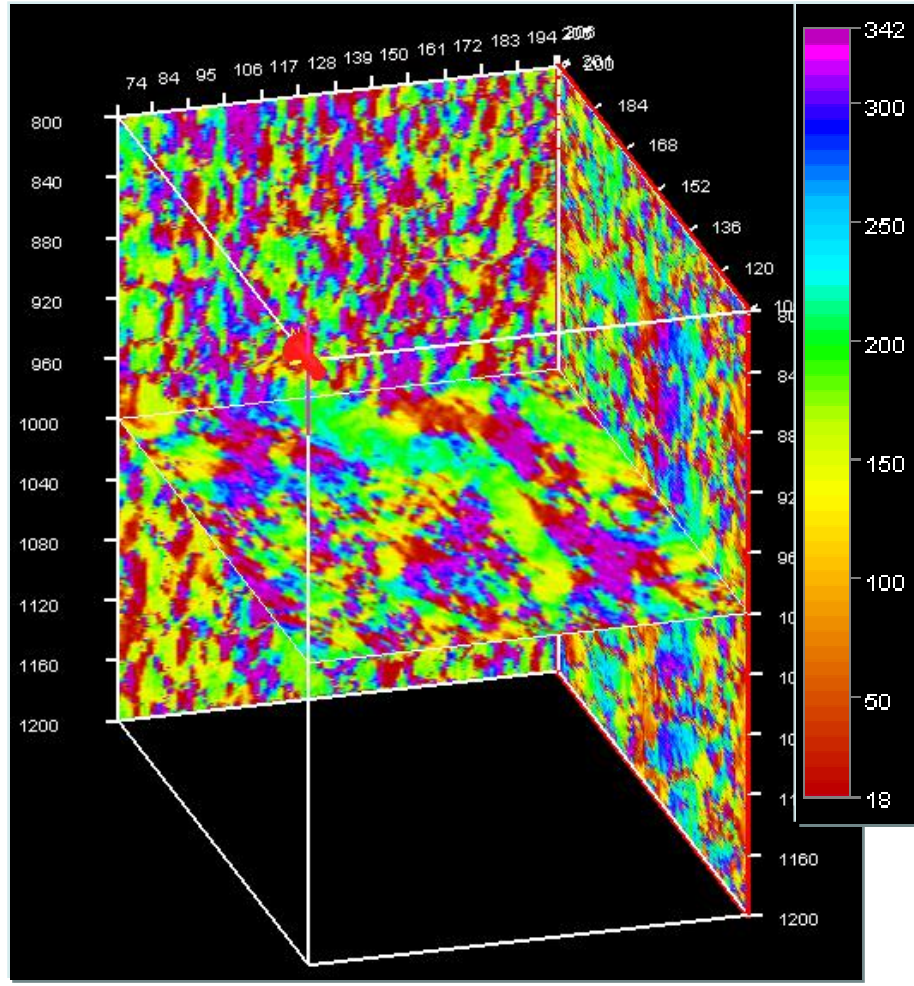


Seismic with 3 x 3 alpha-trim mean



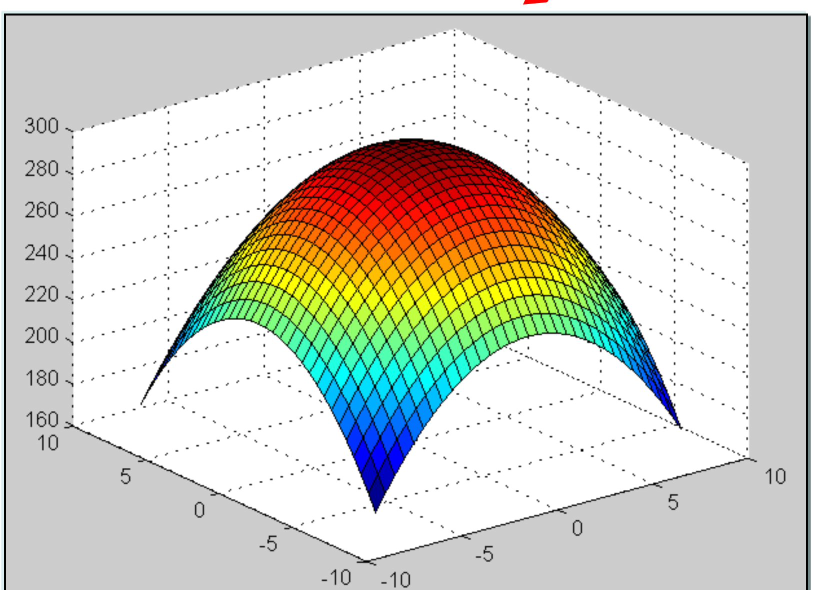
Instantaneous Azimuth

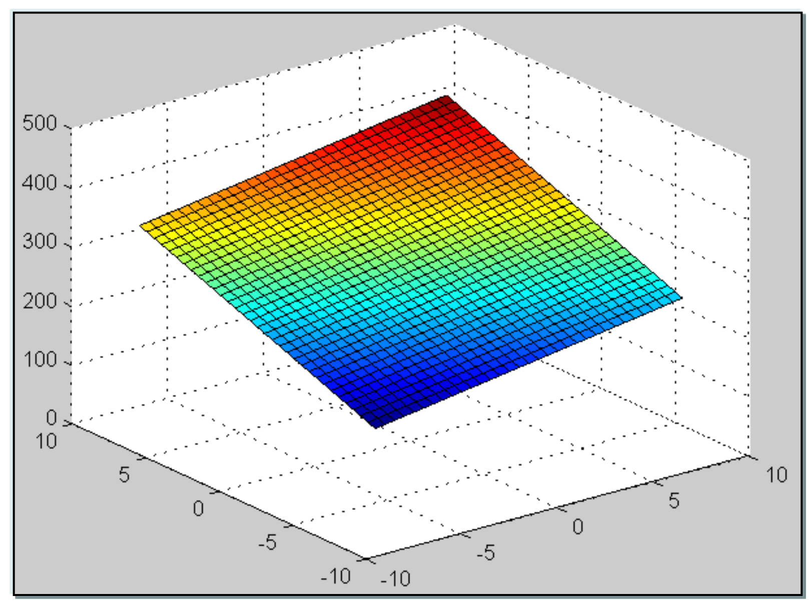
Curvature attributes

- Roberts (2001) shows that curvature can be estimated from a time structure map by fitting the local quadratic surface given by:

$$t(x,y)=\boxed{ax^2+by^2+cxy}+\boxed{dx+ey+f}$$

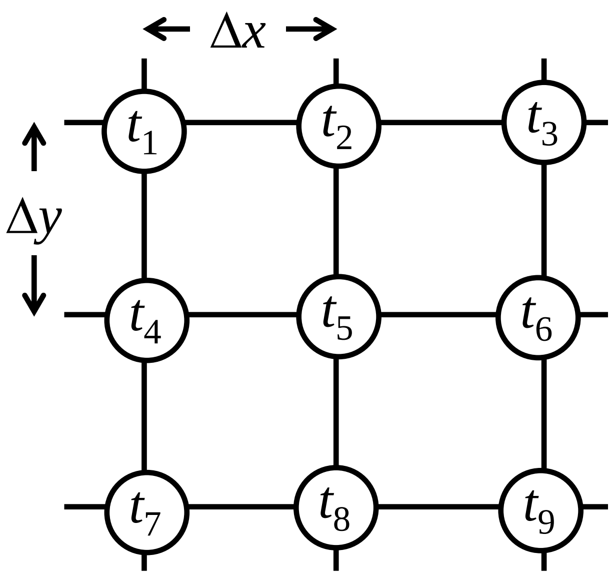
- This is a combination of an ellipsoid and a dipping plane.





Curvature attributes

- Roberts (2001) computes the curvature attributes by first picking a 3D surface on the seismic data and then finding the coefficients *a* through *f* from the map grid shown below:



- For example, the linear dips in the *x* and *y* directions (*d* and *e*), are:

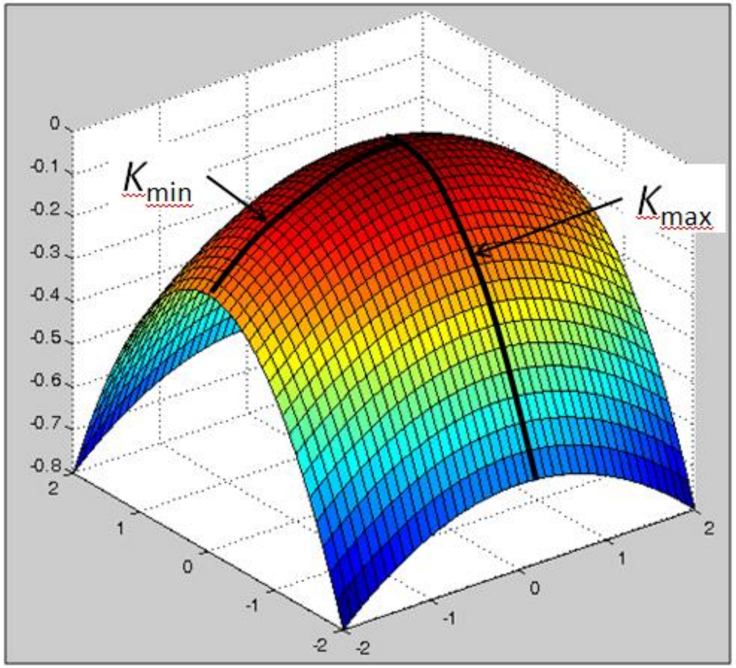
$$d = \frac{\partial t}{\partial x} = \frac{(t_3 + t_6 + t_9) - (t_1 + t_4 + t_7)}{2\Delta x}$$

$$e = \frac{\partial t}{\partial y} = \frac{(t_7 + t_8 + t_9) - (t_1 + t_2 + t_3)}{2\Delta y}$$

- Klein et al. (2008) show how to generalize this to each point on the seismic volume by finding the optimum time shift between pairs of traces using cross-correlation.

Curvature attributes

- The coefficients d and e are identical to dips p and q defined earlier, so when $a = b = c = 0$, we have a dipping plane and can also define the true dip and azimuth as before.
- For a curved surface, Roberts (2001) defines the following curvature attributes ($K_{\min}$ and $K_{\max}$ are shown on the surface):



$$K_{\max} = K_{mean} + \sqrt{K_{mean}^2 - K_{gauss}},$$

$$K_{\min} = K_{mean} - \sqrt{K_{mean}^2 - K_{gauss}},$$

$$\text{where}: \ K_{gauss} = \frac{4ab - c^2}{(1 + d^2 + e^2)^{1/2}}, \text{and}$$

$$K_{mean} = \frac{a(1+e^2) + b(1+d^2) - cde}{(1+d^2+e^2)^{3/2}}.$$

Curvature and instantaneous attributes

- Differentiating the Roberts quadratic, we find that at $x = y = 0$ we get the following relationships for the coefficients:

$$a = \frac{1}{2}\left(\frac{\partial p}{\partial x}\right), b = \frac{1}{2}\left(\frac{\partial q}{\partial y}\right), c = \frac{1}{2}\left(\frac{\partial p}{\partial y} + \frac{\partial q}{\partial x}\right), d = p \text{ and } e = q.$$

- This leads to the following quadratic relationship:

$$t(x, y) = \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)x^2 + \frac{1}{2}\left(\frac{\partial q}{\partial y}\right)y^2 + \frac{1}{2}\left(\frac{\partial p}{\partial y} + \frac{\partial q}{\partial x}\right)xy + px + qy + f$$

- Thus, all of the curvature attributes can be derived from the instantaneous dip attributes described earlier, using a second differentiation.
- The next figures shows a comparison between azimuth maximum curvature derived the two different ways.

Azimuth comparison

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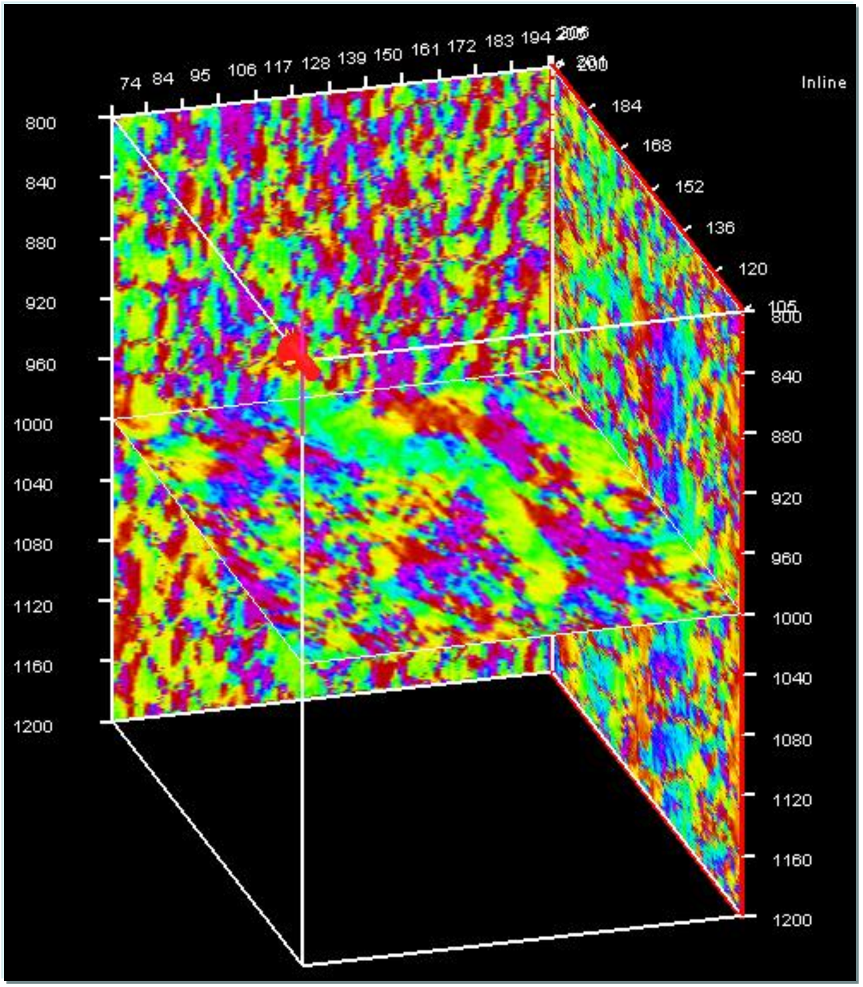
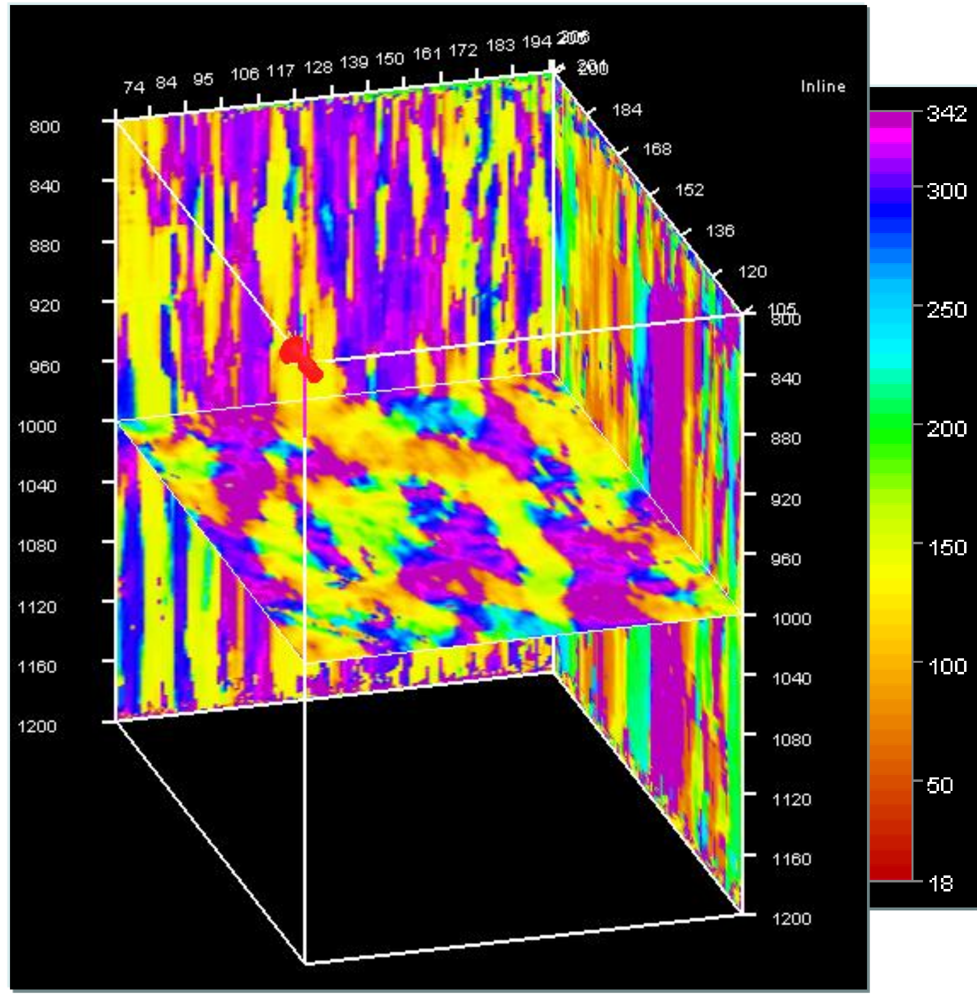
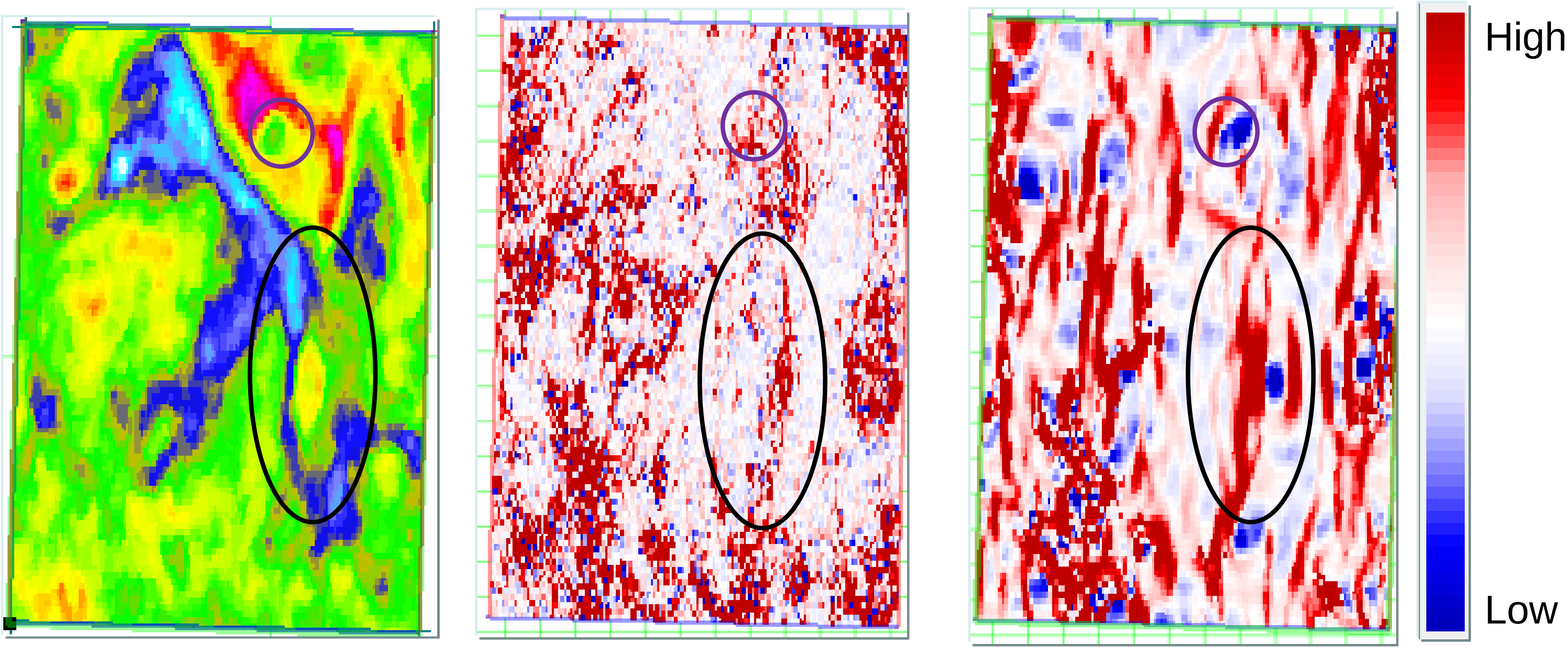


Instantaneous Azimuth



Azimuth from curvature by correlation

Max Curvature Comparison



Data slice   Inst. maximum curvature   Corr. maximum curvature

- Notice that the curvature features are similar, but that instantaneous curvature shows higher frequency events.
- However, lower frequency events are present on the correlation approach.

Conclusions

- In this talk, I have shown how Dennis Gabor can be thought of as the "father" of modern seismic attribute analysis.
- In his 1947 paper, Gabor invented the concept of the complex signal, which allowed us to derive the instantaneous phase, amplitude and frequency.
- These attributes were introduced into geophysics by Taner et al. (1979) and were initially just computed in time.
- Correlation attributes involve cross-correlating pairs of traces, where coherency is based on the correlation coefficient and curvature on the correlation time-shift.
- By computing instantaneous frequency in x and y as well as time, we can derive dip, azimuth and curvature.
- Thus, we can see Gabor's initial work as being the pre-cursor of most seismic attribute methods.

Acknowledgements

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References

al-Dossary, S., and K. J. Marfurt, 2006, 3D volumetric multispectral estimates of reflector curvature and rotation: Geophysics, **71**, 41–51.

Bahorich, M. S., and S. L. Farmer, 1995, 3-D seismic discontinuity for faults and stratigraphic features, The coherence cube: The Leading Edge, **16**, 1053–1058.

Barnes, A. E., 1996, Theory of two-dimensional complex seismic trace analysis: Geophysics, **61**, 264–272.

Cohen, L., 1995, Time-Frequency Analysis: Prentice-Hall PTR.

Gabor, D., 1946, Theory of communication, part I: J. Int. Elect. Eng., v. 93, part III, p. 429-441.

Gersztenkorn, A., and K. J. Marfurt, 1999, Eigenstructure based coherence computations as an aid to 3D structural and stratigraphic mapping: Geophysics, **64**, 1468–1479.

Klein, P., L. Richard and H. James, 2008, 3D curvature attributes: a new approach for seismic interpretation: First Break, 26, 105–111.

References

Marfurt, K. J., 2006, Robust estimates of 3D reflector dip: Geophysics, 71, 29-40

Marfurt, K. J., and R. L. Kirlin, 2000, 3D broadband estimates of reflector dip and amplitude: Geophysics, **65**, 304–320.

Marfurt, K. J., R. L. Kirlin, S. H. Farmer, and M. S. Bahorich, 1998, 3D seismic attributes using a running window semblance-based algorithm: Geophysics, **63**, 1150–1165.

Marfurt, K. J., V. Sudhakar, A. Gersztenkorn, K. D. Crawford, and S. E. Nissen, 1999, Coherency calculations in the presence of structural dip: Geophysics **64**, 104–111.

Roberts, A., 2001, Curvature attributes and their application to 3D interpreted horizons: First Break, 19, 85–100.

Taner, M. T., F. Koehler, and R. E. Sheriff, 1979,Complex seismic trace analysis: Geophysics, **44**, 1041–1063.

Appendix: The coherency method

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- Defining the covariance matrix between locations i and j as:

$$C(p,q) = \begin{bmatrix} c_{11} & \cdots & c_{1J} \\ \vdots & \ddots & \vdots \\ c_{J1} & \cdots & c_{JJ} \end{bmatrix}, \quad c_{ij} = \sum_{t=-M\Delta t}^{M\Delta t} s(t - px_i - qy_i)\, s(t - px_j - qy_j),$$

the two coherency measures are as follows:

$$coh_1 = \max\left[\left(\frac{c_{12}}{(c_{11}c_{22})^{1/2}} \frac{c_{13}}{(c_{11}c_{33})^{1/2}}\right)^{1/2}\right] \text{ and } coh_2 = \max\left[\frac{a^T C(p,q) a}{Tr\left[C(p,q)\right]}\right],$$

where $a^T = \left[1, \quad \ldots, \quad 1\right]$ and $Tr\left[C(p,q)\right]$ = sum of main diagonal of $C(p,q)$.

- A third measure (Gursztenkorn and Marfurt, 1999) is:

$$coh_3 = \max\left[\frac{\lambda_1}{Tr\left[C(p,q)\right]}\right], \quad \lambda_1 = \text{max eigenvalue of } C(p,q).$$