Elastodynamic FWI in 2D with partial stacking

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Outline

- Partial stacking
- Elastodynamics
- FWI
- Numerical experiments
- Web-interface

Motivation

Objectives

- Friendly FWI:
  - simple
  - fast
  - stable
- FWI:
  - density FWI
  - bulk modulus FWI
  - FWI stacking

Approximation scheme

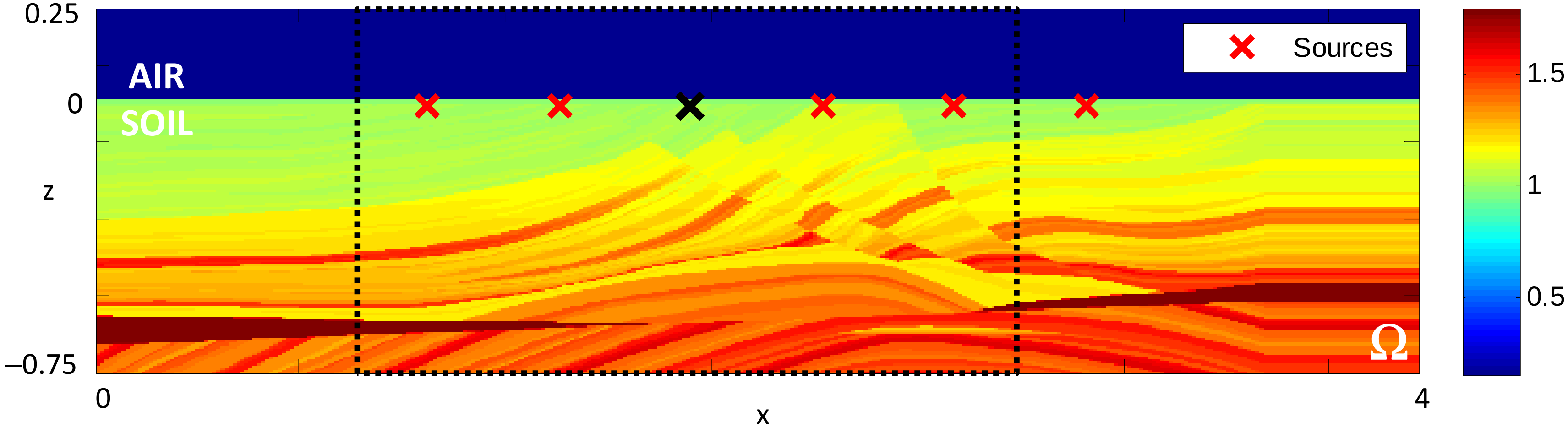
Accuracy

Stability

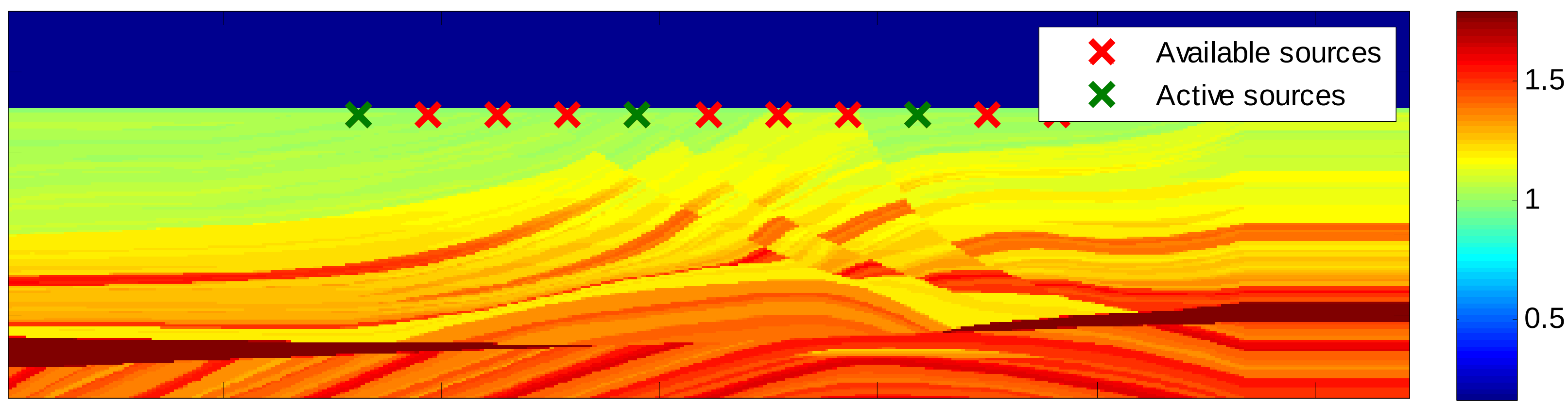
Computational difficulty

Partial stacking

Full stack



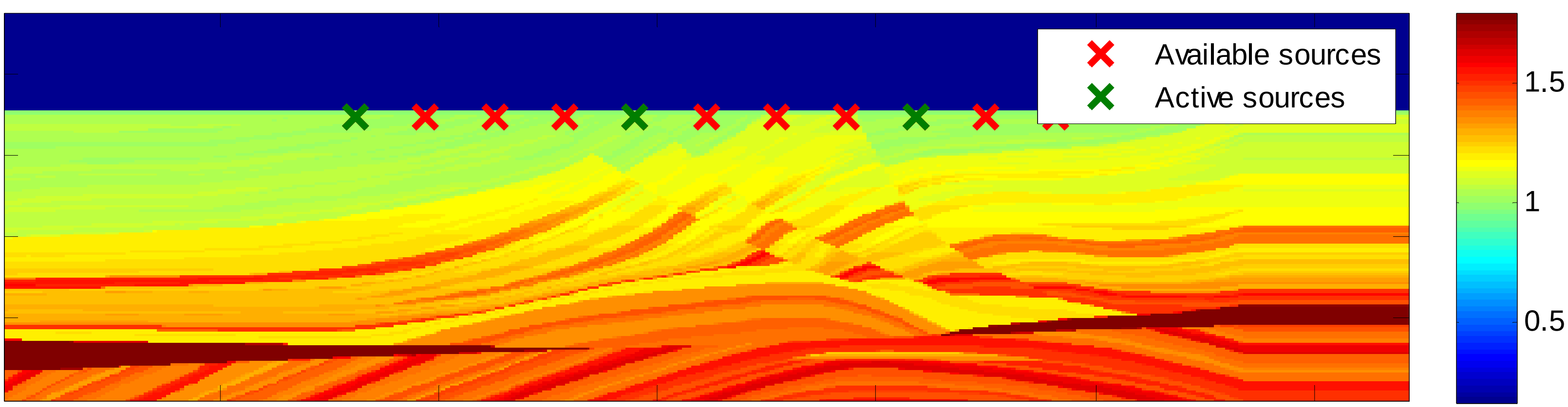
Partial stacking



Source switching

FWI iteration	Sources										
	1	2	3	4	5	6	7	8	9	10	11
1	■				■				■		
2		■				■				■	
3			■				■				■
4				■				■			
5	■				■				■		
6				■				■			
7			■				■				■
8		■				■				■	
9	■				■				■		

Exact solution (Density)



Elastodynamics

$$\begin{cases} \frac{\partial}{\partial t}\rho\frac{\partial u_x}{\partial t} = \frac{\partial}{\partial x}(\lambda + 2\mu)\frac{\partial u_x}{\partial x} + \frac{\partial}{\partial x}\lambda\frac{\partial u_z}{\partial z} + \frac{\partial}{\partial z}\mu\frac{\partial u_z}{\partial x} + \frac{\partial}{\partial z}\mu\frac{\partial u_x}{\partial z} \\ \frac{\partial}{\partial t}\rho\frac{\partial u_z}{\partial t} = \frac{\partial}{\partial x}\mu\frac{\partial u_z}{\partial x} + \frac{\partial}{\partial x}\mu\frac{\partial u_x}{\partial z} + \frac{\partial}{\partial z}\lambda\frac{\partial u_x}{\partial x} + \frac{\partial}{\partial z}(\lambda + 2\mu)\frac{\partial u_z}{\partial z} \end{cases}$$

(u_x, u_z) - deformation fields

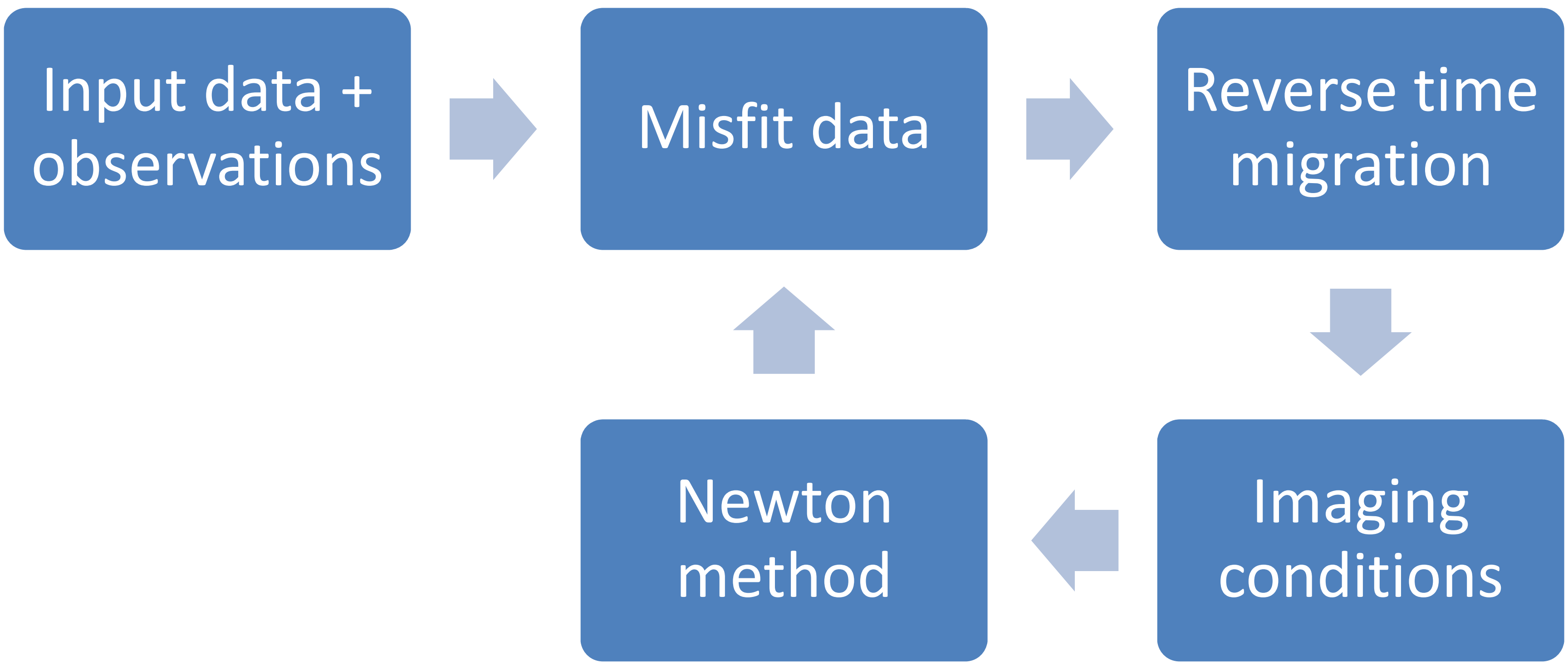
Unknowns:

ρ – density field

λ – bulk modulus

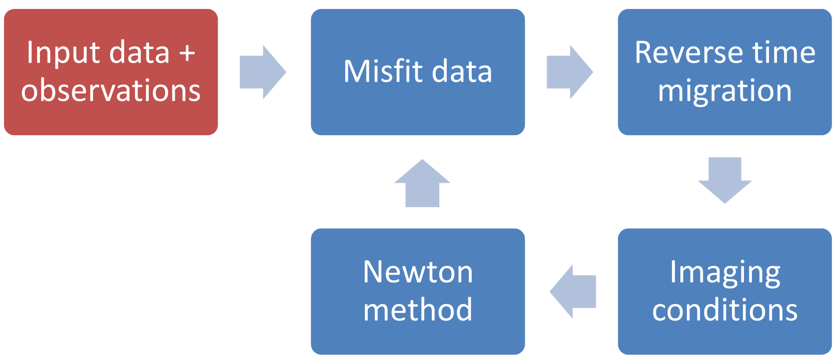
Virieux J. 1986, P-SV wave propagation in heterogeneous media: Velocity-stress finite-difference method. Geophysics 51, No 4, 889-901

FWI


Input data + observations
Misfit data
Reverse time migration
Imaging conditions
Newton method

Input data

FWI

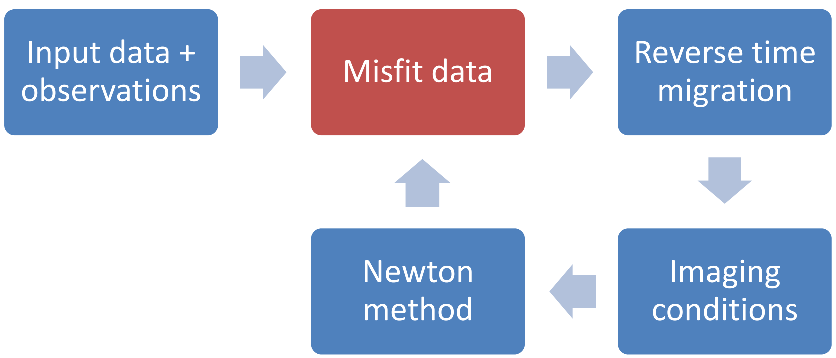


Modelling parameters

- Temporal and spatial scales
- Surface relief
- Under surface density
- Synthetic seismograms
- Good density field initial guess
- Source wavelet

Misfit data

FWI

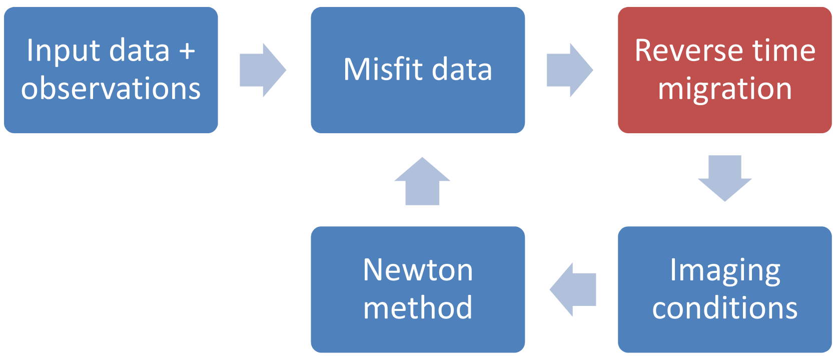


Forward propagation problem

- Elastodynamic eq.
- Source in stress tensor
- Courant condition
- Reflective boundary conditions
- Misfit = observed – estimated seismogram

Reverse time migration

FWI

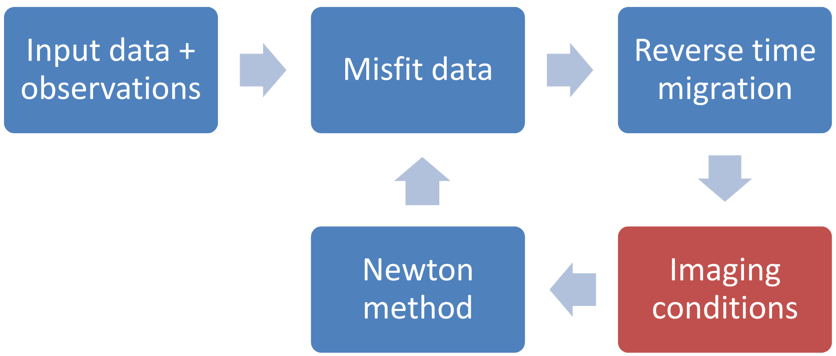


Adjoint problem solution

- Elastodynamic is self-adjoint
- Homogenous initial conditions at $t = T$
- Misfit is a source for corresponding velocity
- Time direction switching at any moment

Imaging conditions

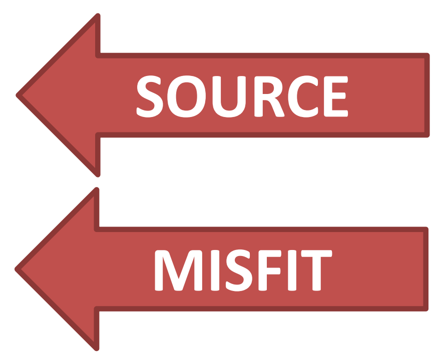
FWI



Misfit correlated with source

1. Source FRW propagated

2. Source RTM $(t = -t)$
   + Misfit RTM (adjoint)



Imaging conditions

- Density field ρ:

$$\int_{\Omega}\int_0^T \Delta\rho\left(\frac{\partial\Phi_x}{\partial t}\frac{\partial u_x}{\partial t}+\frac{\partial\Phi_z}{\partial t}\frac{\partial u_z}{\partial t}\right)dtdxdz=\int_{\partial\Omega}\int_0^T\left[(\dots)\Delta u+(\dots)\frac{\partial\Delta u}{\partial x}+(\dots)\frac{\partial\Delta u}{\partial z}\right]dtdS$$

- Bulk modulus λ:

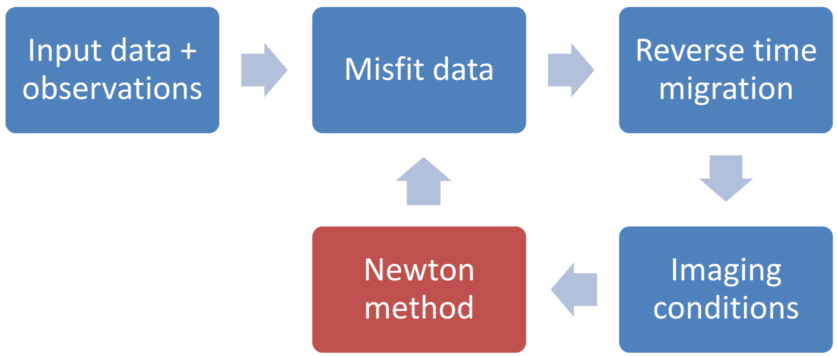
$$\int_{\Omega}\int_0^T \Delta\lambda\left(\frac{\partial\Phi_x}{\partial x}+\frac{\partial\Phi_z}{\partial z}\right)\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right)dtdxdz=\int_{\partial\Omega}\int_0^T\left[(\dots)\Delta u+(\dots)\frac{\partial\Delta u}{\partial x}+(\dots)\frac{\partial\Delta u}{\partial z}\right]dtdS$$

Tarantola A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, 74, No. 8, 1259-1266

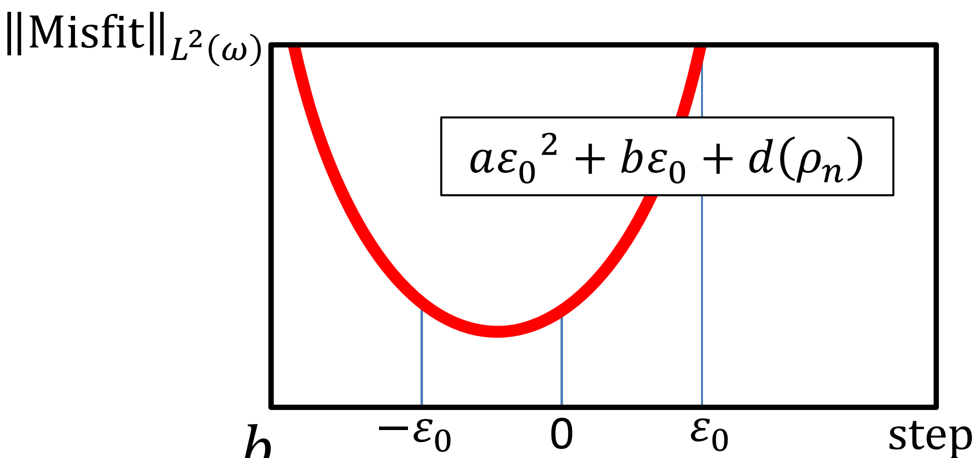
Hasanov A., Pektas B. and Erdem A. 2011, Comparative analysis of inverse coefficient problems for parabolic equations. Part I: adjoint problem approach, Inverse Problems in Science and Engineering, 19:5, 599-615

Newton method

FWI



Scalar parabolic approximation



$$\Delta\rho = \frac{b}{2a} \cdot I,$$

$$\rho_{n+1} = \rho_n + \sum_{\substack{\text{partial}\\ \text{stack}}} \sigma_i \Delta\rho_i$$

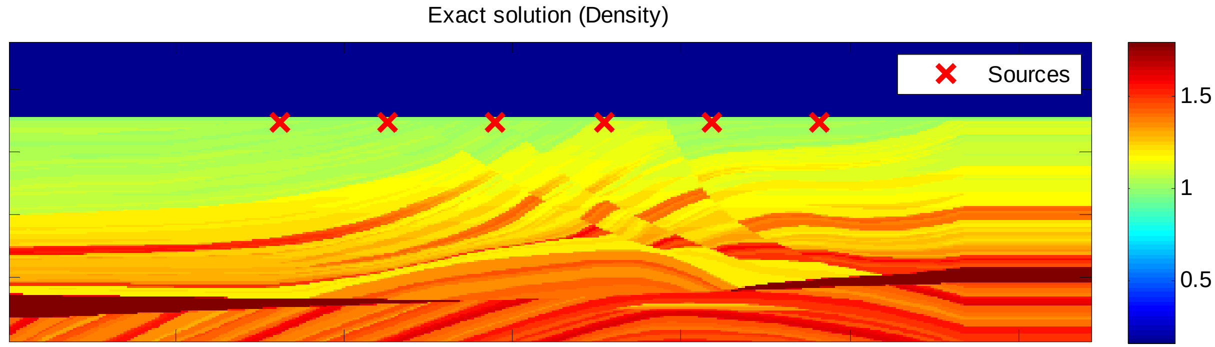
Numerical experiments

- Partial stacking vs. full stack
- Noise resistance study
- Density FWI vs. bulk modulus FWI

Full stack density field FWI

Exact solution

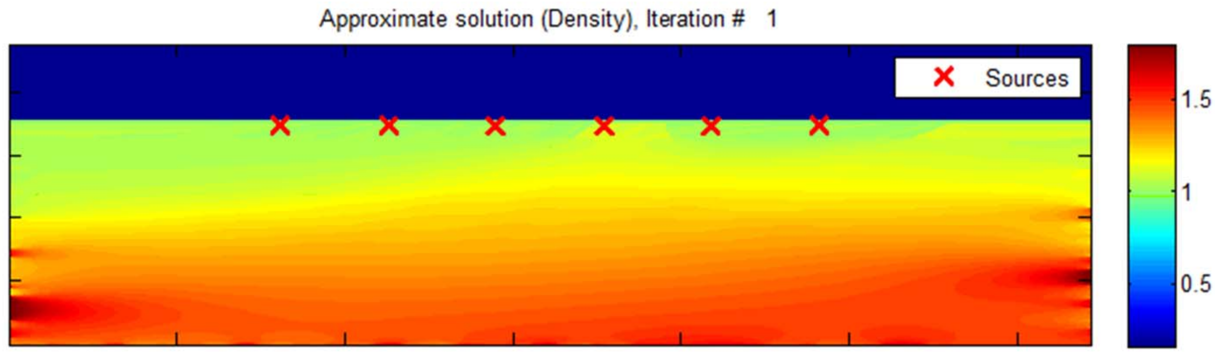
grid: 321×475



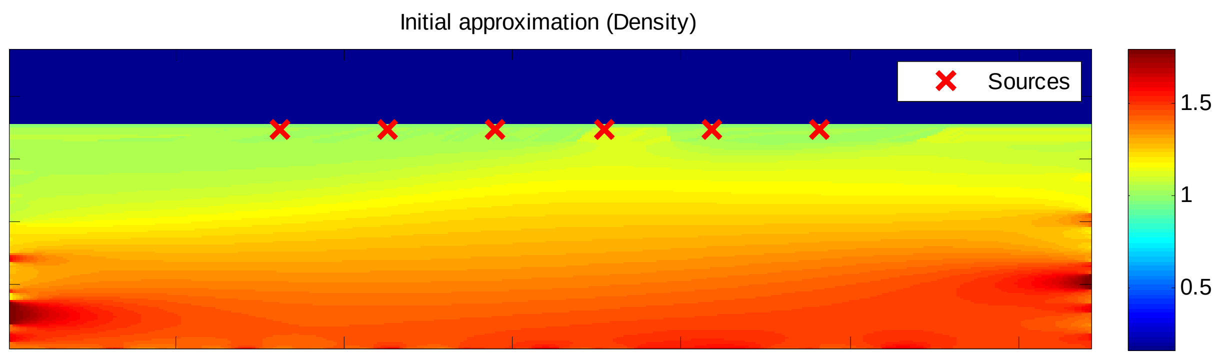
Estimated solution

16 filters ×

8 iterations

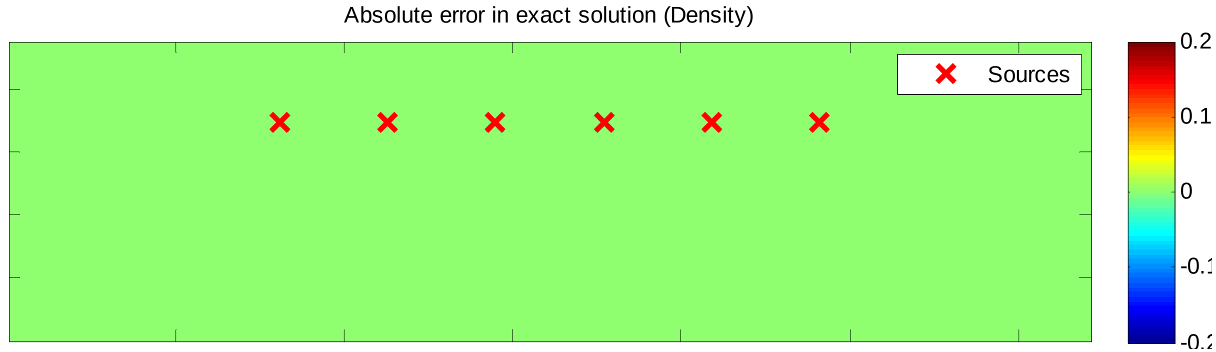


Initial guess

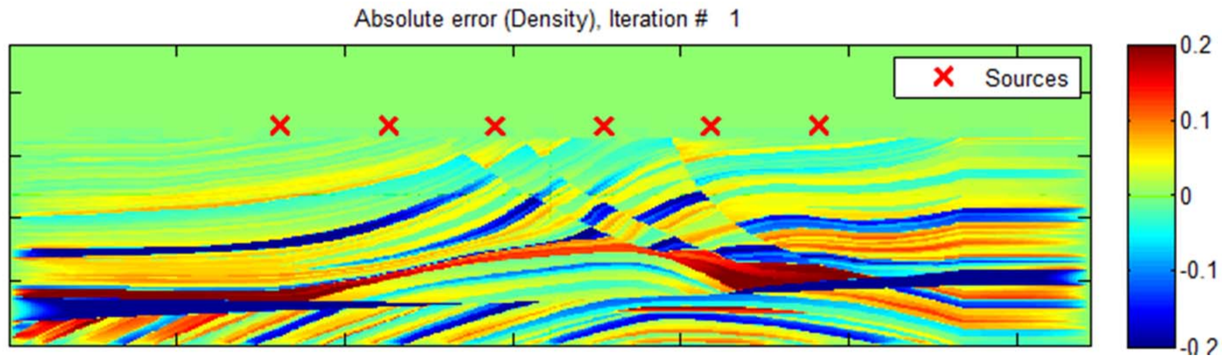


Full stack density field FWI

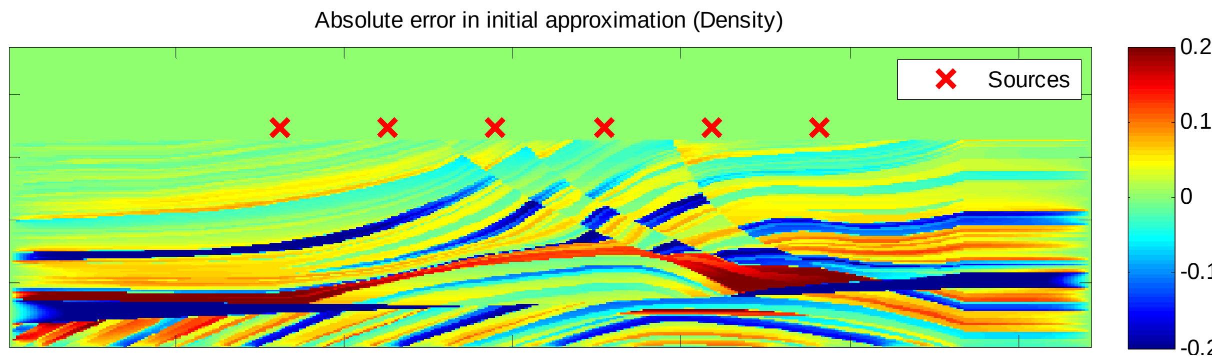
Absolute error
In exact solution



Absolute error
in estimated solution

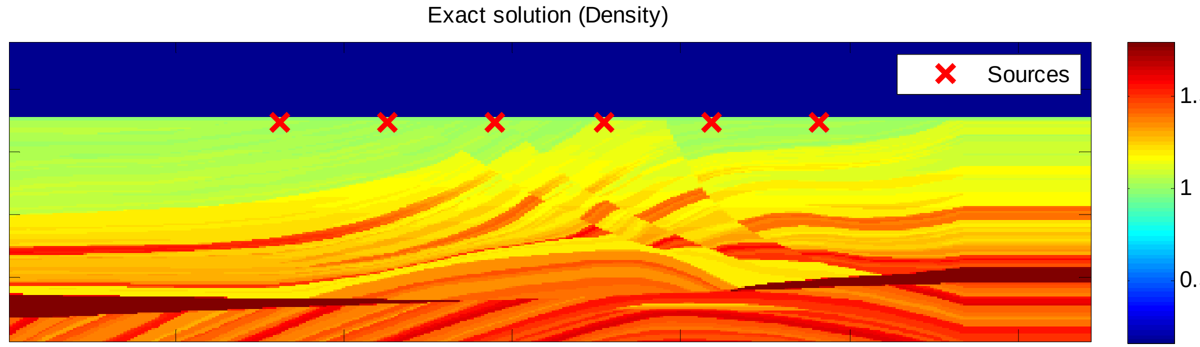


Absolute error
in initial guess

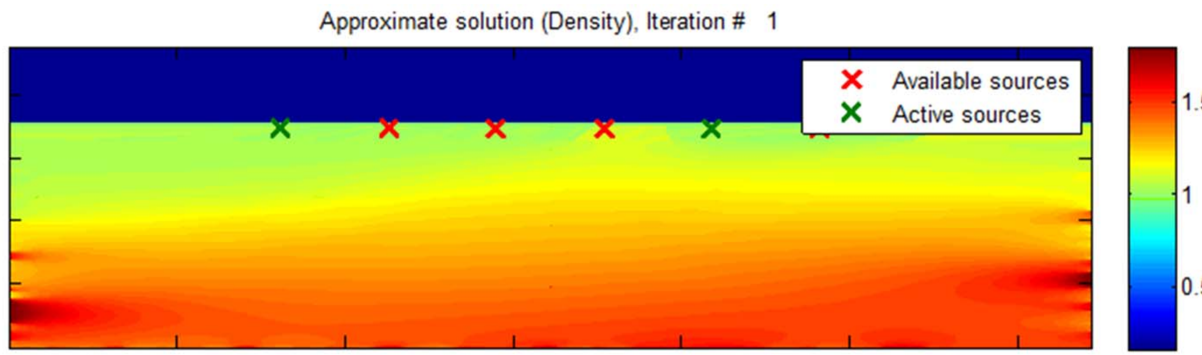


Partial stacking vs. full stack

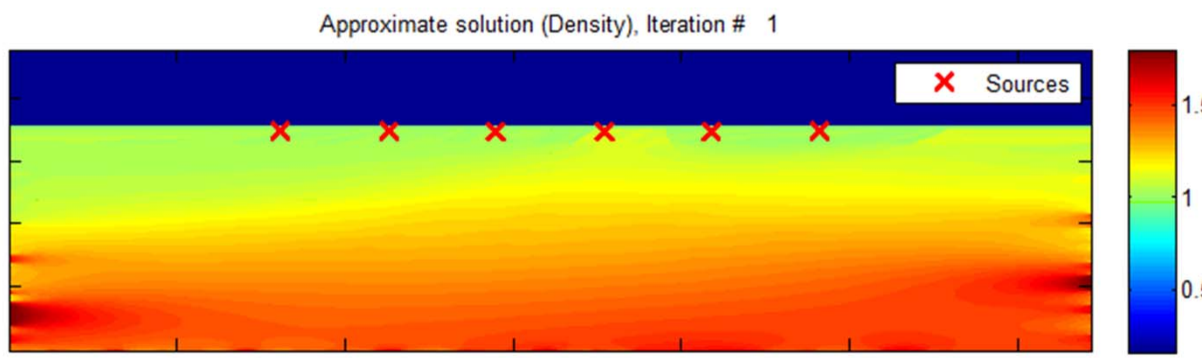
Exact solution
grid: 321×475



Partial stacking
25% shots

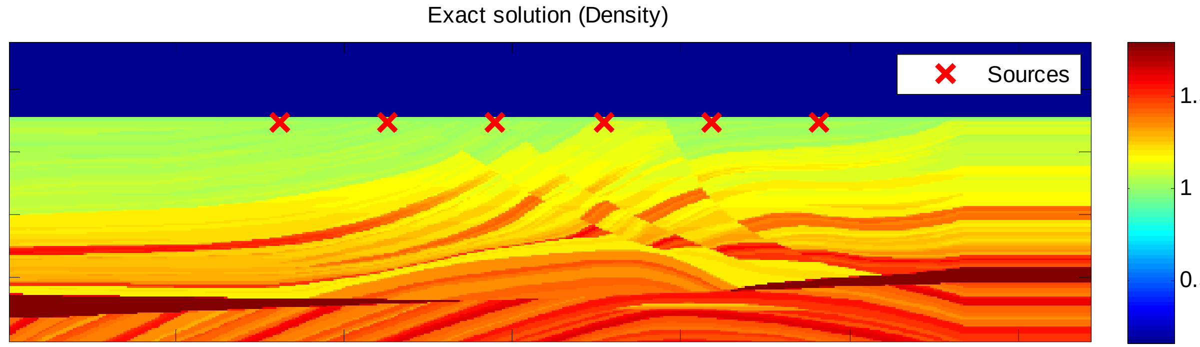


Full stack

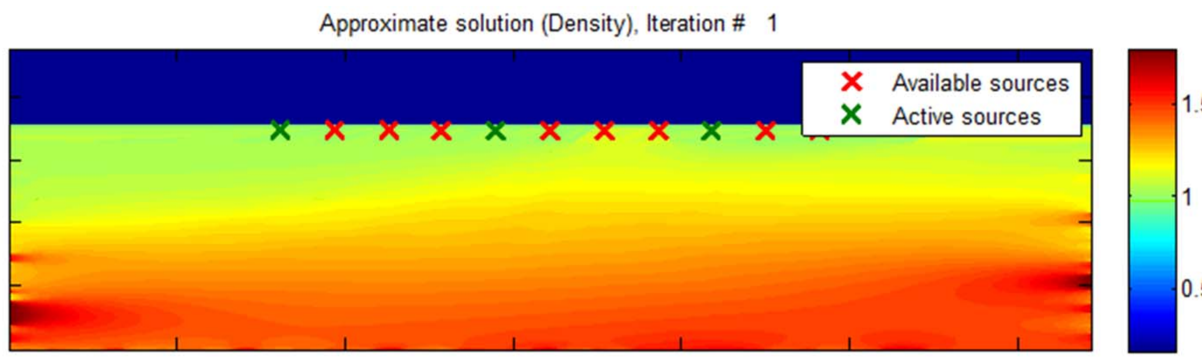


More data in partial stacking

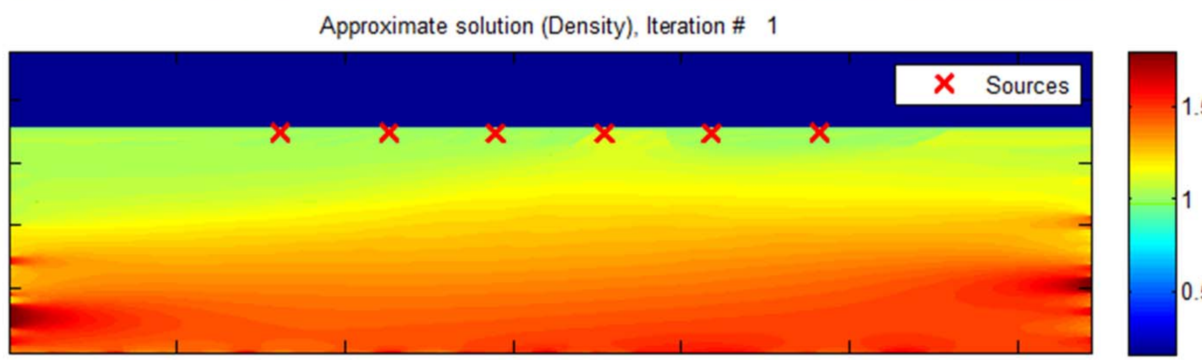
Exact solution
grid: 321×475



Partial stacking
25% of 11 shots

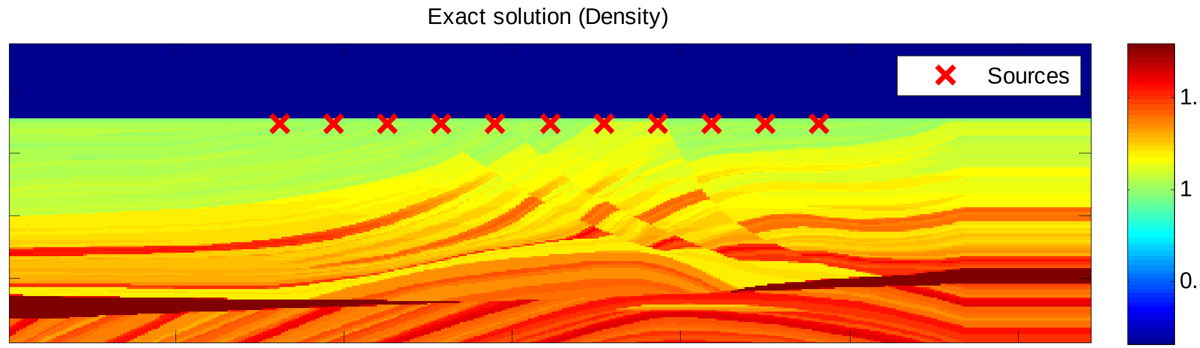


Full stack
6 shots

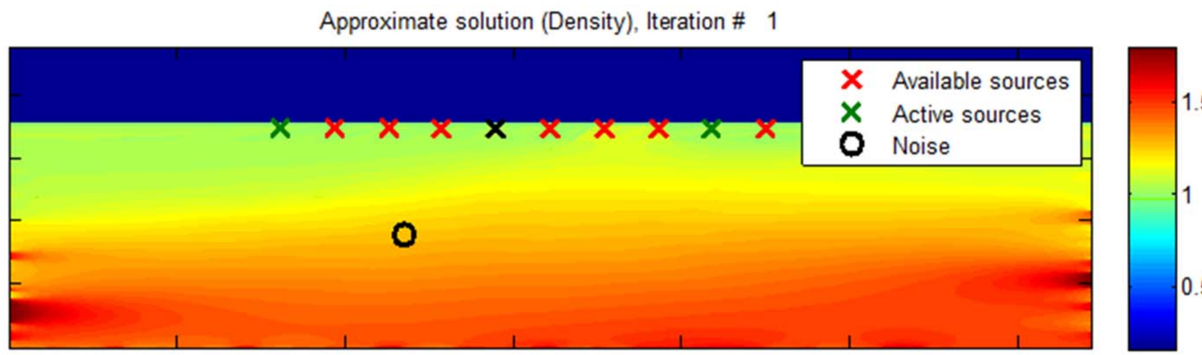


Periodical noise impact

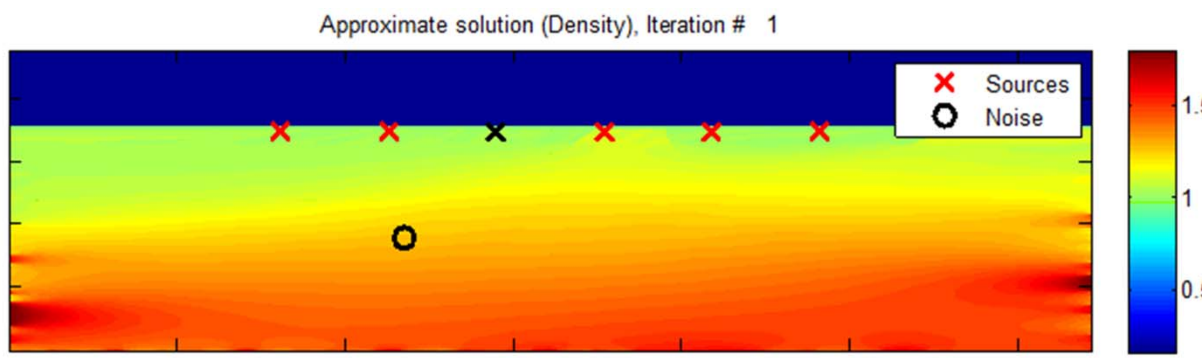
Exact solution
grid: 321×475



Partial stacking
25% of 11 shots



Full stack
6 shots

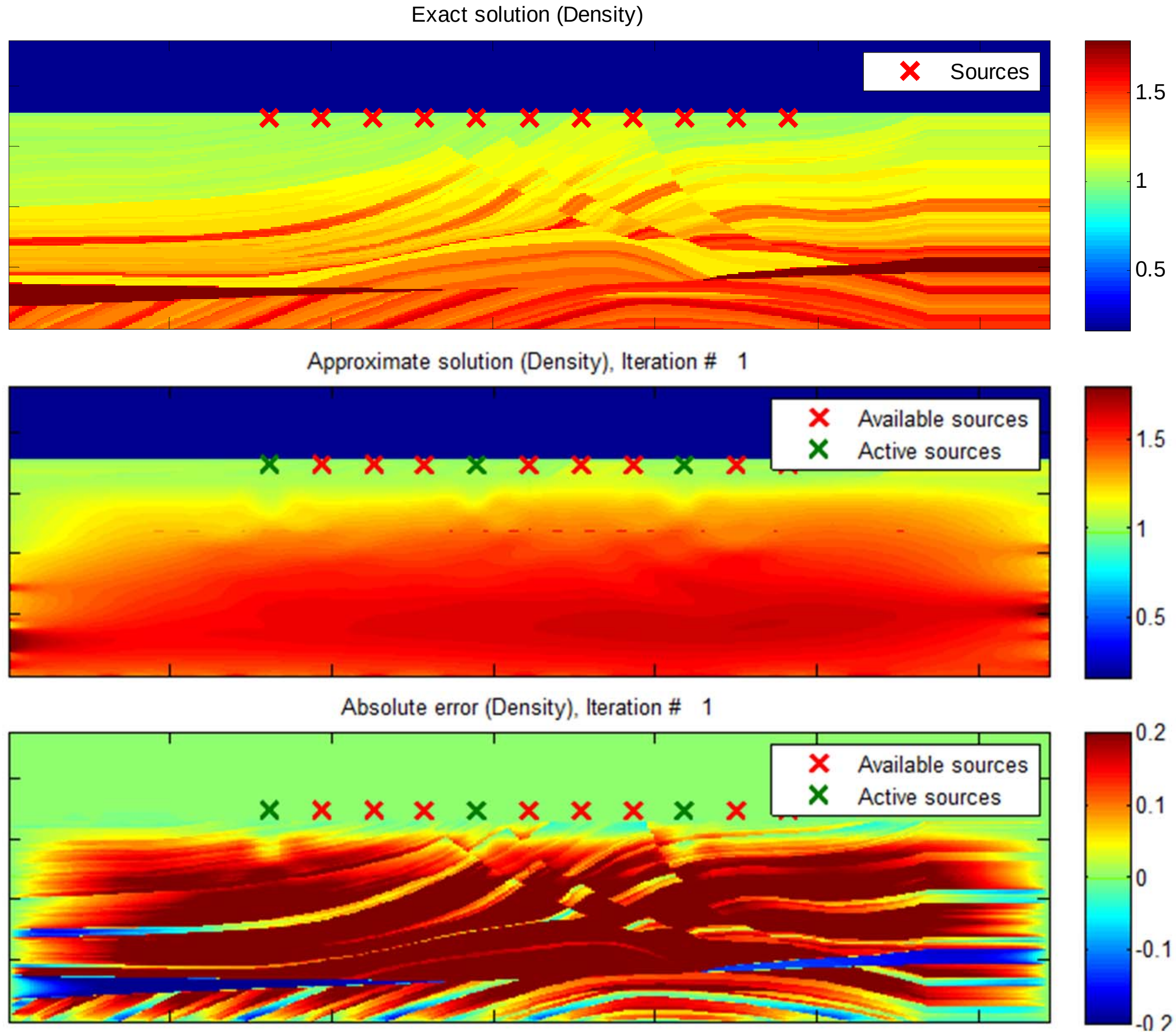


Poor initial guess

Exact solution
grid: 321×475

Partial stacking
25% of 11 shots

Absolute error

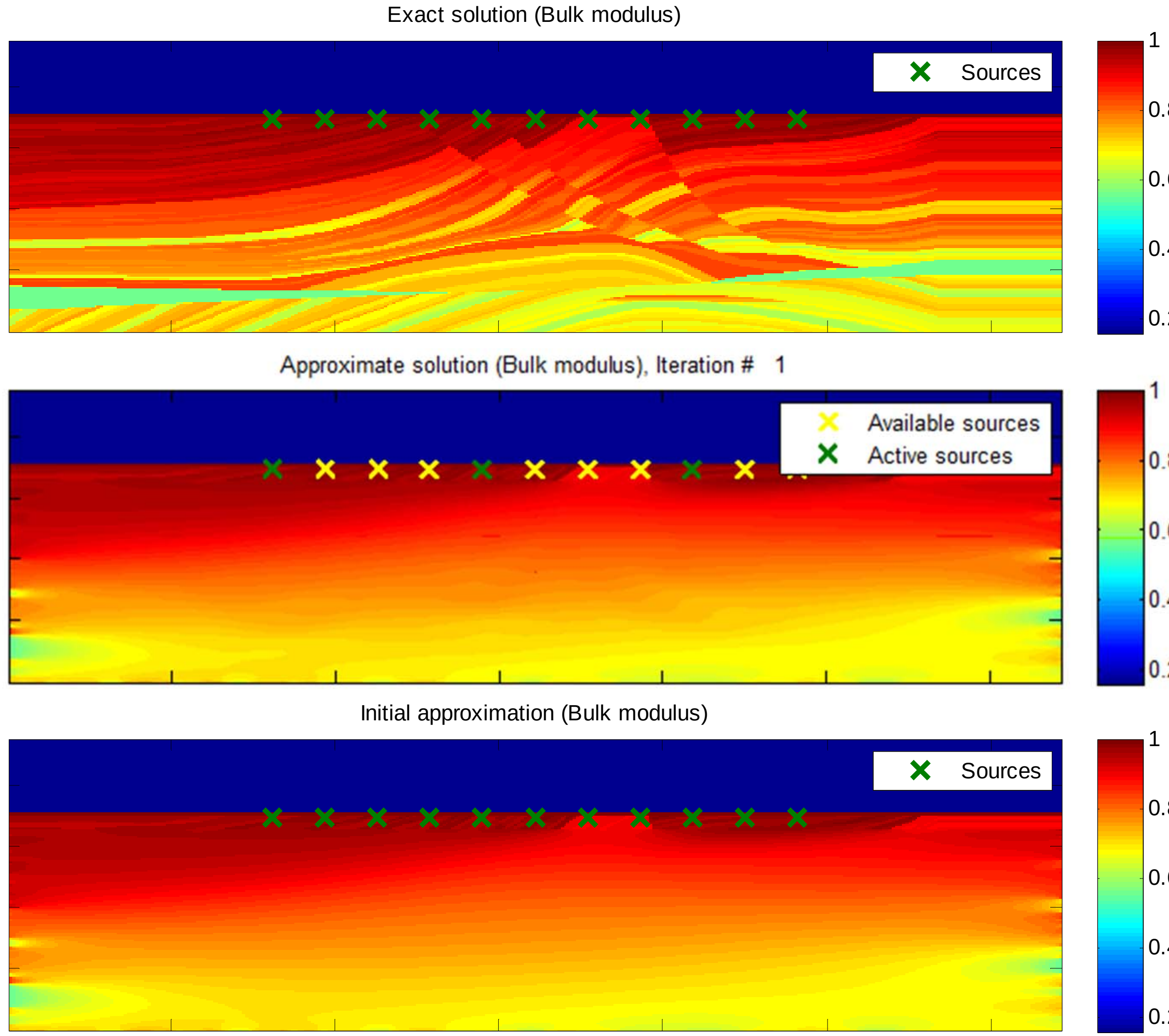


Bulk modulus FWI

Exact solution
grid: 321×475

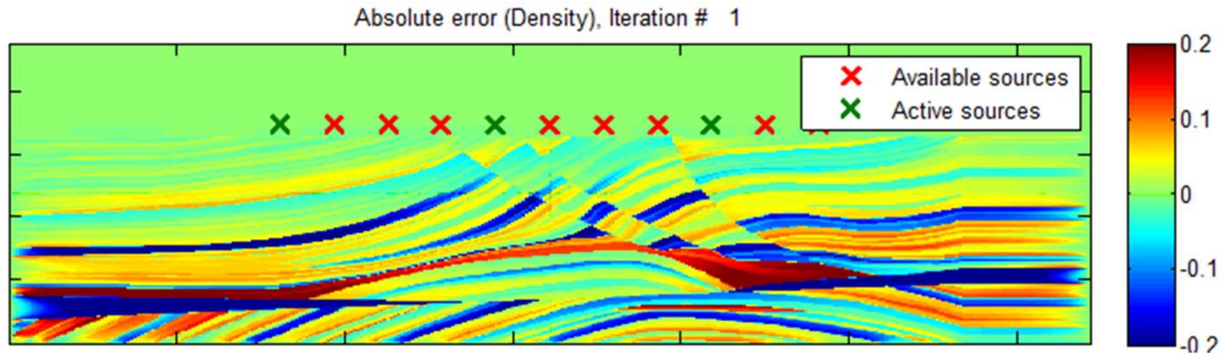
Estimated solution
16 filters ×
8 iterations

Initial guess

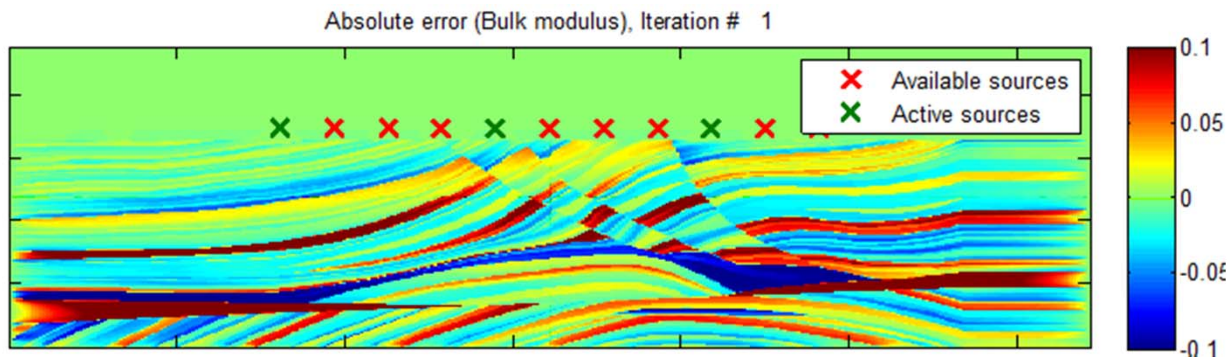


Density FWI vs. bulk modulus FWI

Absolute error
Density FWI



Absolute error
Bulk modulus FWI



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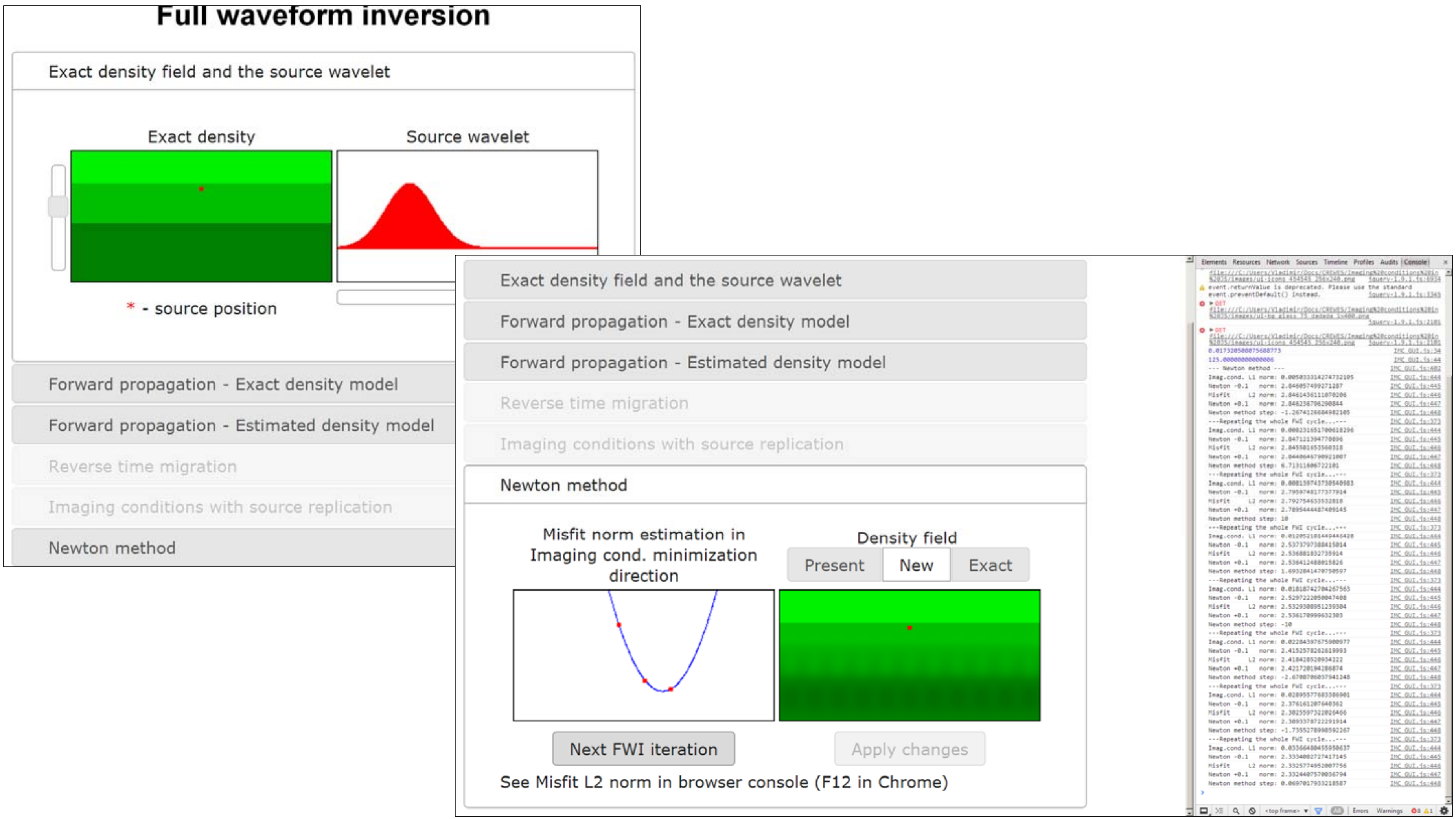
people.ucalgary.ca/~mikel/IMC.htm



Conclusion

- Partial stacking is filtering dependent, lower frequency filtering require more shots in the stack
- Partial stack is cancelling high frequency Gauss noise more effectively in comparison with full stack FWI of the same computational difficulty
- Bulk modulus FWI converges on low frequencies faster than corresponding density FWI

Acknowledgments

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- Michael Lamoureux
- Joe Wong
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- PIMS, NSERC and
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Thank you for your attention