

# Iterative multiparameter elastic waveform inversion using prestack Kirchhoff approximation

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# Outline

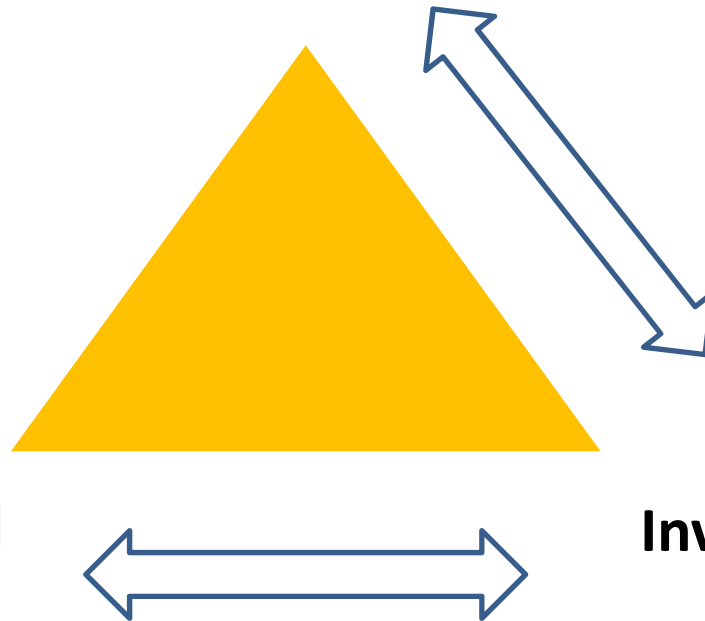
- Theory of inversion
  - Philosophy of Full Waveform Inversion (FWI)
  - Born approximation
  - Kirchhoff approximation
- Numerical implementation
  - Synthetic data
  - Real data
- Future Applications
  - 3D seismic data
- Conclusions

# Background

**Tarantola 's  
Inversion method  
(Iterative)**

**Bleistein 's  
Inversion method  
(Kirchhoff)**

**Beylkin 's  
Inversion method  
(Born)**



# Philosophy of FWI method

Moving from direct inversion to iterative inversion (Tarantola, 1984 & 1986)

$$\begin{array}{ccc} \text{model} & \text{Data} & \\ \downarrow & \downarrow & \\ \boxed{GM = d} & \Rightarrow & M = (G^T G)^{-1} G^T d \end{array}$$

Forward modeling operator

**Problem: How accurate is  $G$  ?**

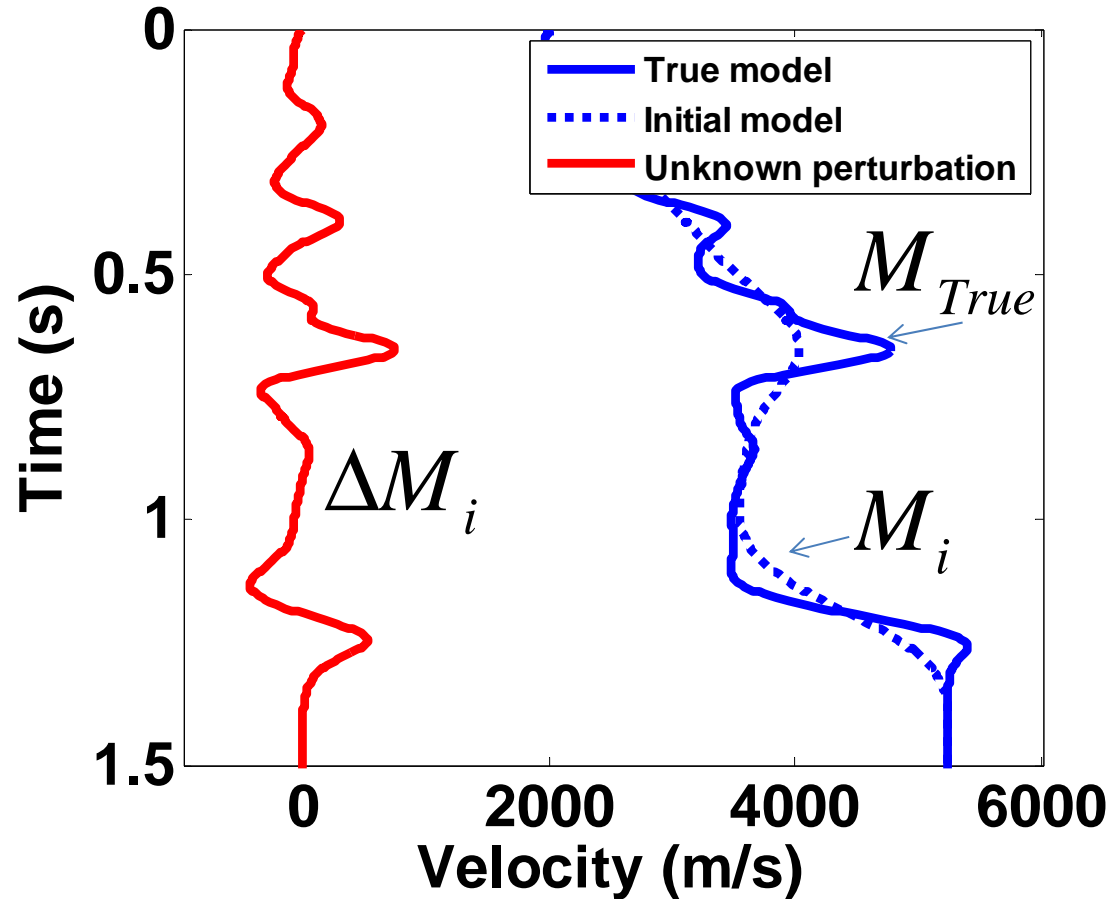
**FWI: Initial guess**  $M_i \longrightarrow G_i \longrightarrow \Delta M_i$

$$\boxed{G_i \Delta M_i = \Delta d_i \quad \Rightarrow \quad \Delta M_i = (G_i^T G_i)^{-1} G_i^T \Delta d}$$

$$M_{i+1} = M_i + \Delta M_i$$

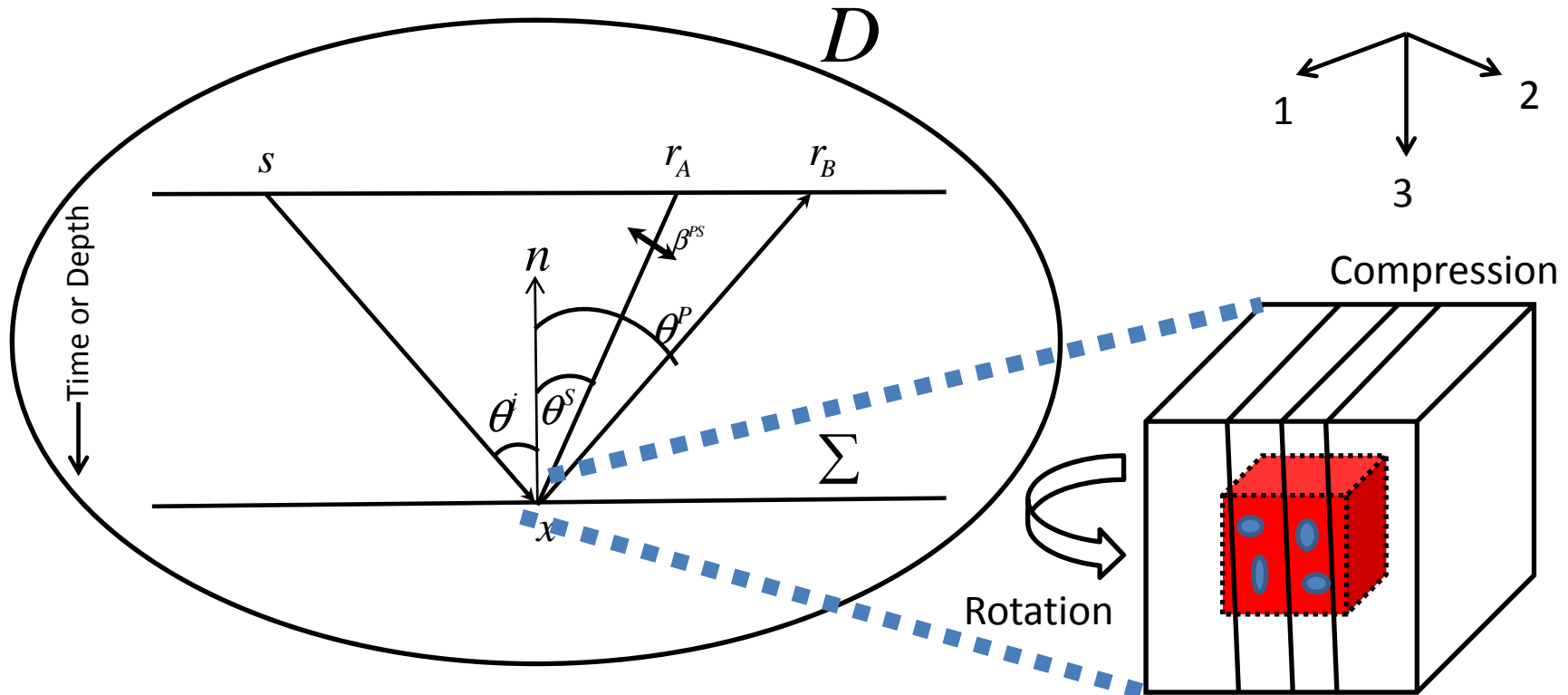
# Model Perturbation in Born approximation

(e.g., Beylkin, 1984, 1990)



# Theory of elasticity and scattering potentials

(e.g., Beylkin, 1985, 1990)



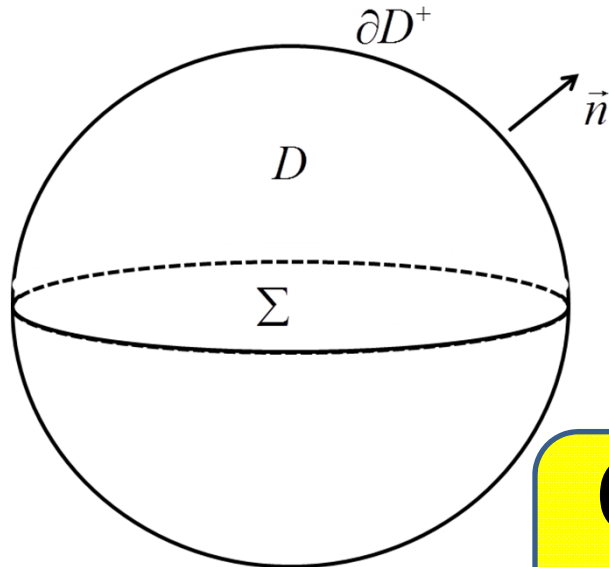
$$S(x) = \left[ \frac{\Delta\rho(x)}{\rho_0(x)} \delta_{ik} + \frac{\Delta c_{ijkl}(x)}{c_{ijkl_0}(x)} p_j^r p_l^s \right] h_k^s h_i^r \quad i, j, k \text{ \& } l = 1, 2, 3$$

$\rho$  = Density     $c_{ijkl}$  = Stiffness tensor     $p$  = Slowness     $h$  = Polarization

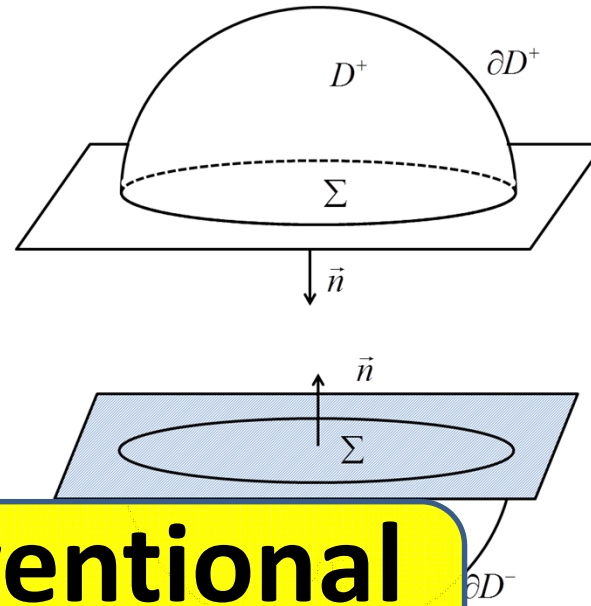
# Kirchhoff approximations

(Bleistein, 1984 & 1986)

Born approximation



Kirchhoff approximation



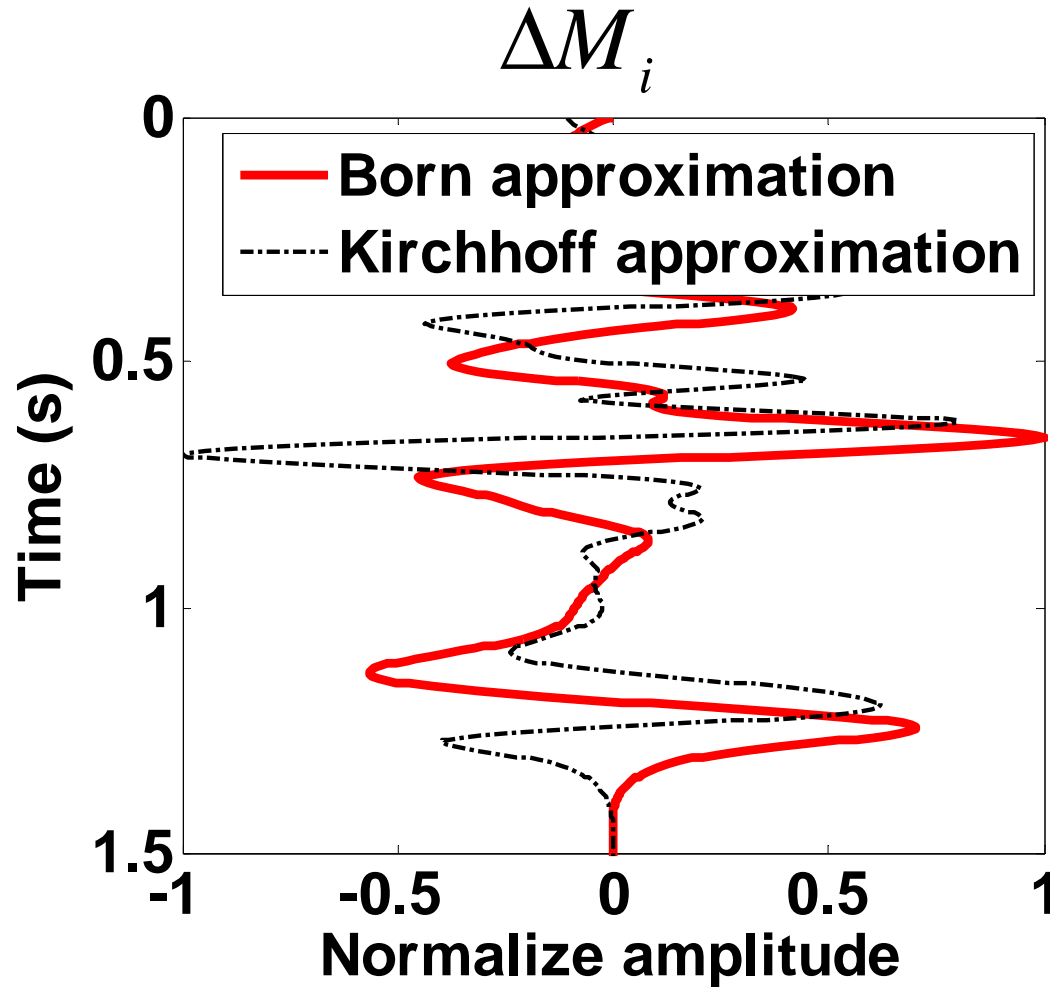
**Conventional  
AVO inversion**

$$R(x) = \left[ \frac{\Delta \rho^+(x) - \Delta \rho^-(x)}{\rho_0(x)} + \frac{\Delta c_{ijkl}(x)}{c_{ijkl_0}(x)} p_j^r p_l^s \right] h_k^s h_i^r$$

$$R = \infty \frac{\partial}{\partial n} S \quad \Rightarrow \quad R\left(\frac{\Delta v_p}{v_p}, \frac{\Delta v_s}{v_s}, \frac{\Delta \rho}{\rho}, \dots, \theta\right)$$

# Model perturbation

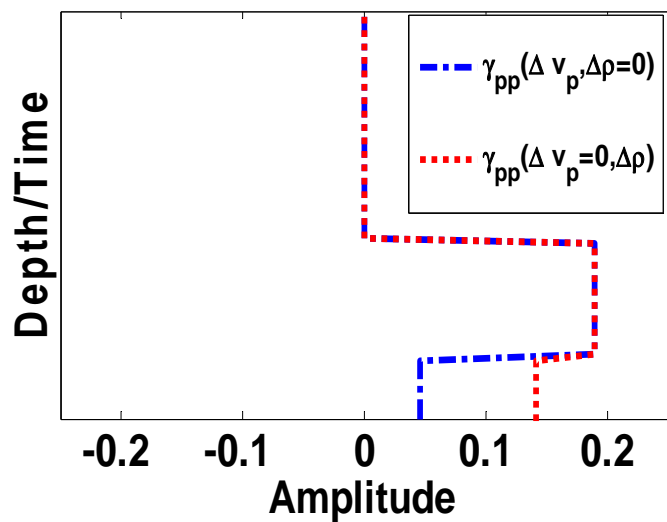
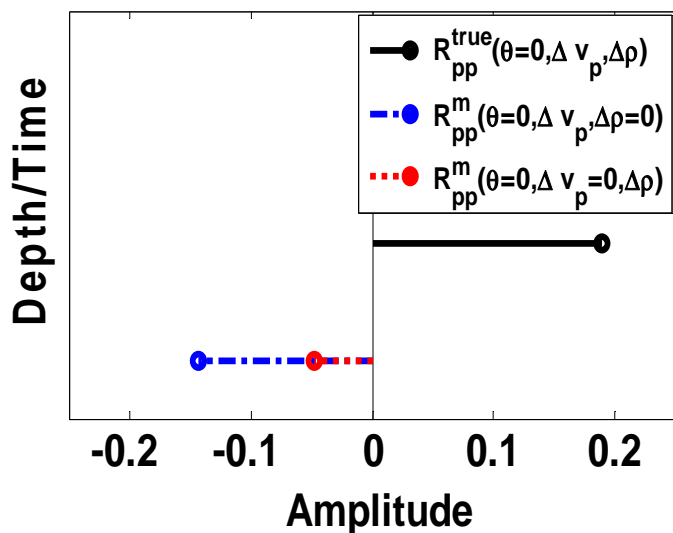
(e.g., Bleistein vs Beylkin)



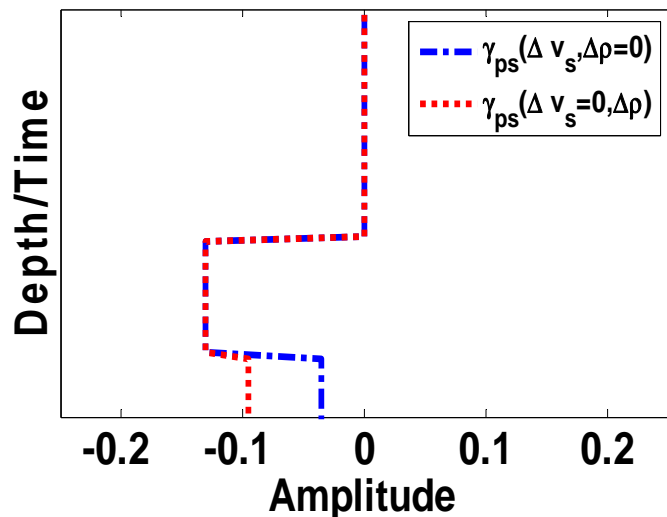
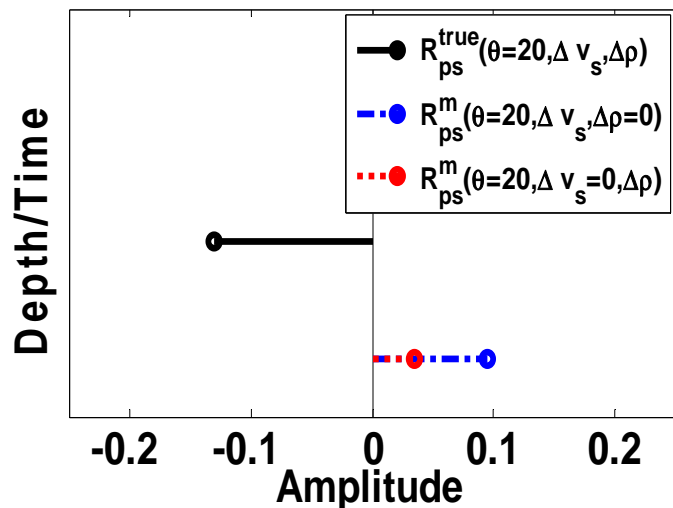


# Tarantola's FWI method, elastic properties and gradient estimation

a) P-to-P data



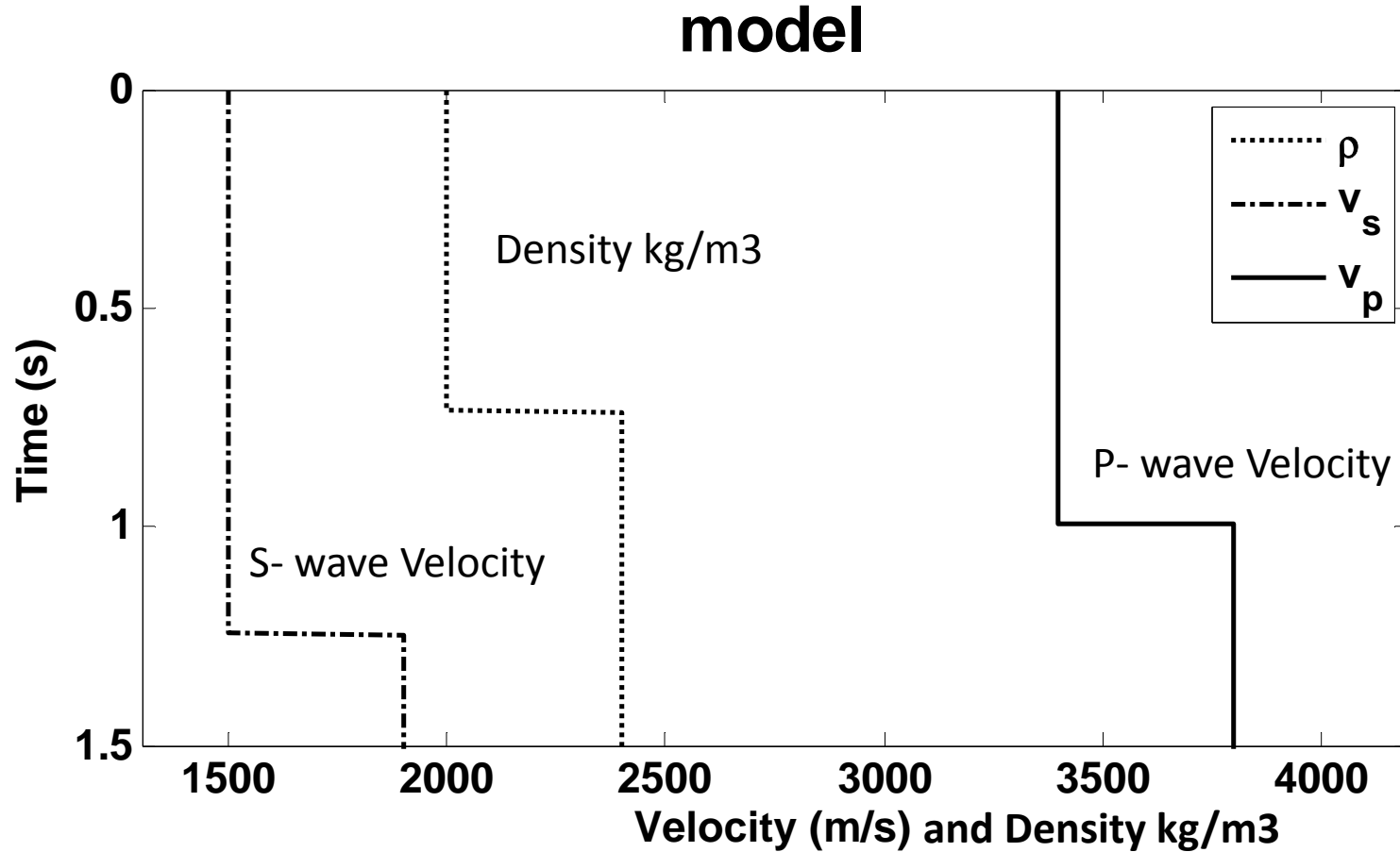
b) P-to-S data



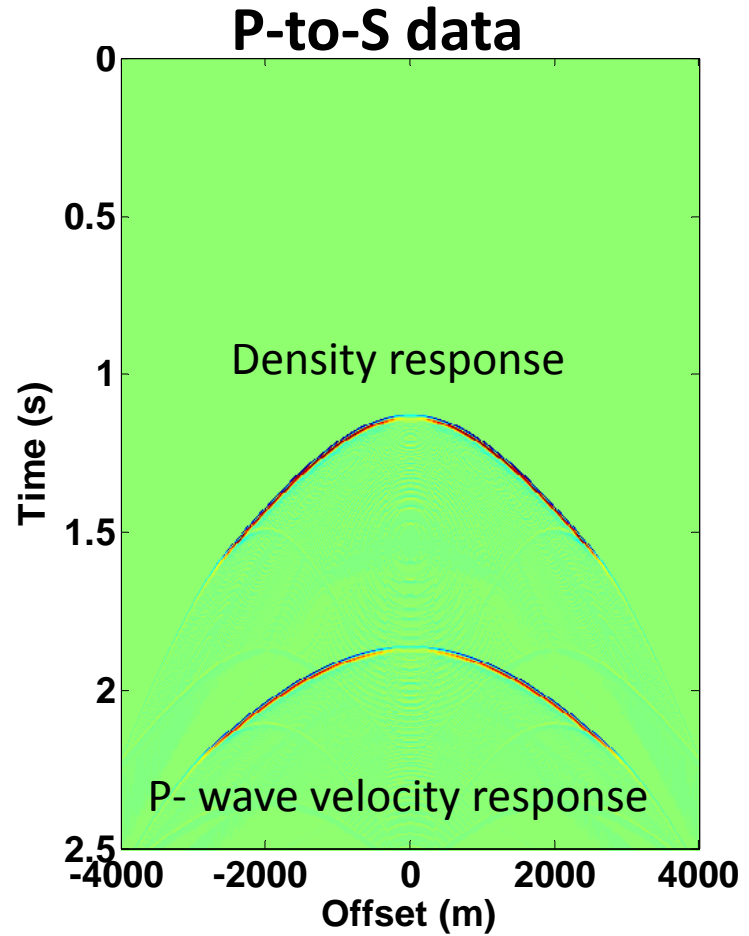
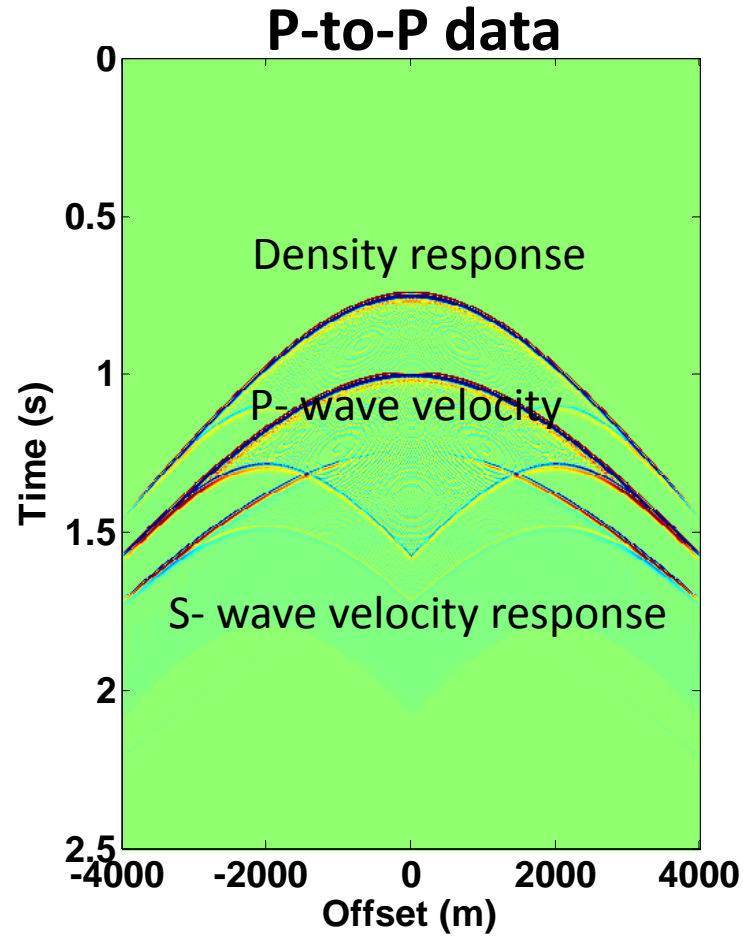
See e.g., Innanen (2011) for analytical expression of gradient functions in FWI

# **Numerical examples**

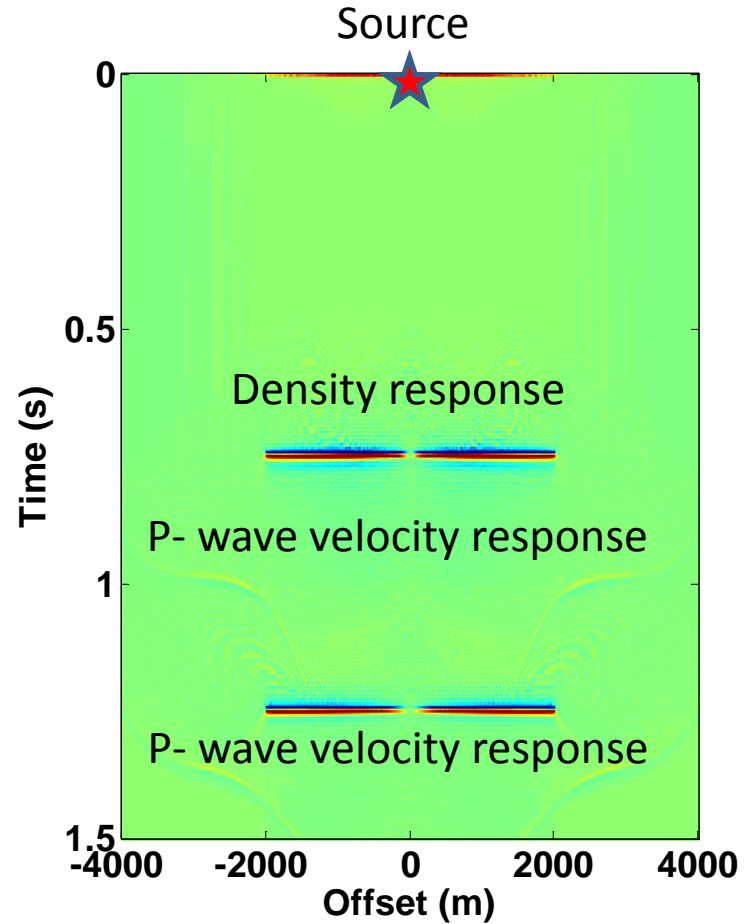
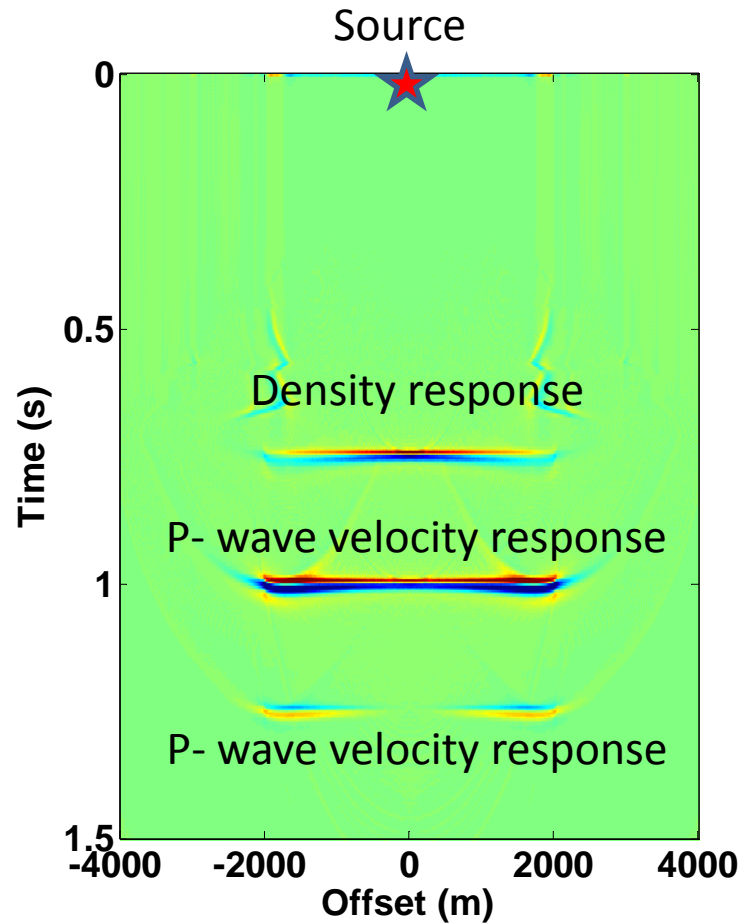
# P-P and P-S radiation pattern



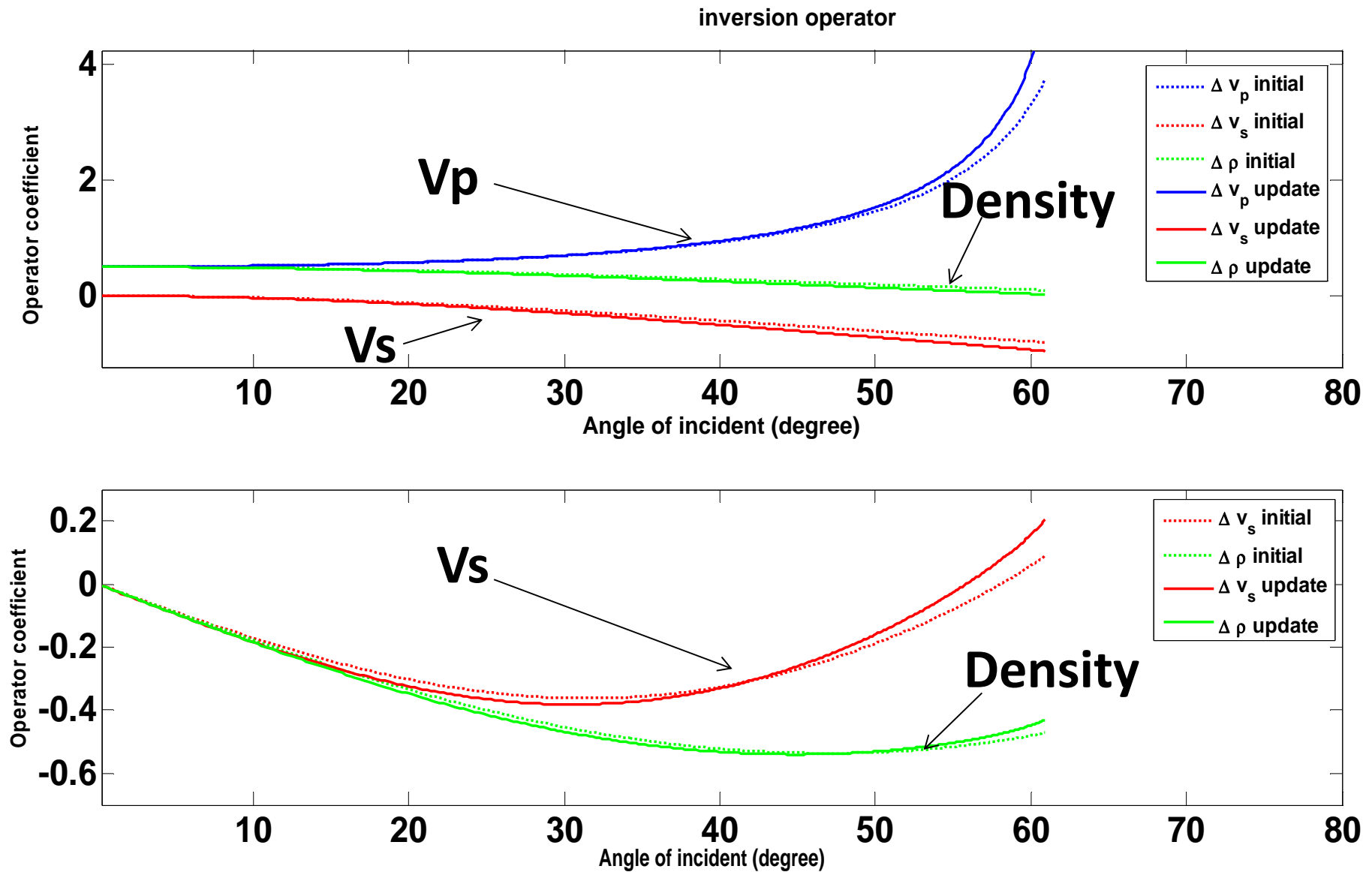
# P-P and P-S radiation pattern



# P-P and P-S radiation pattern

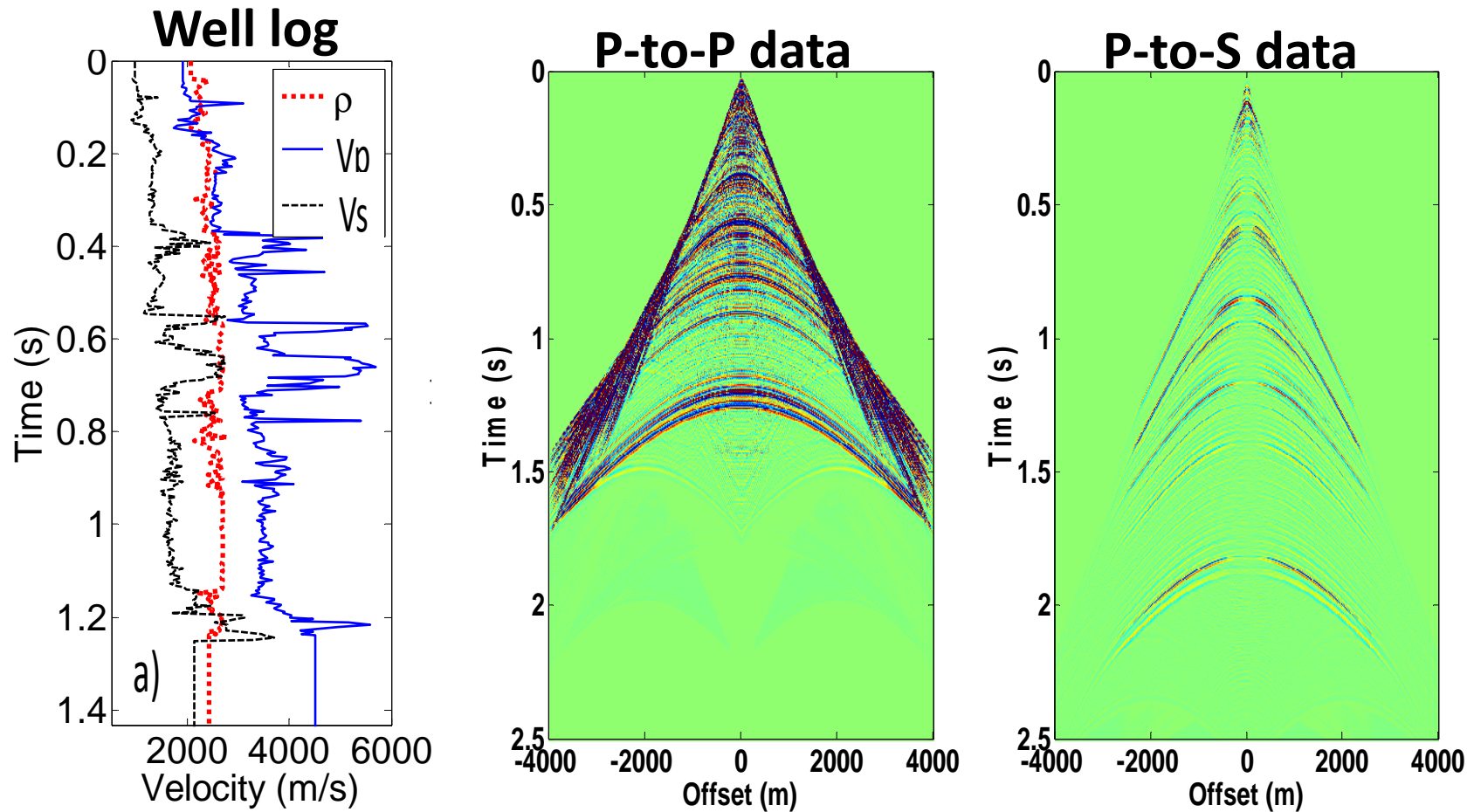


# Effects of accurate model for AVO inversion

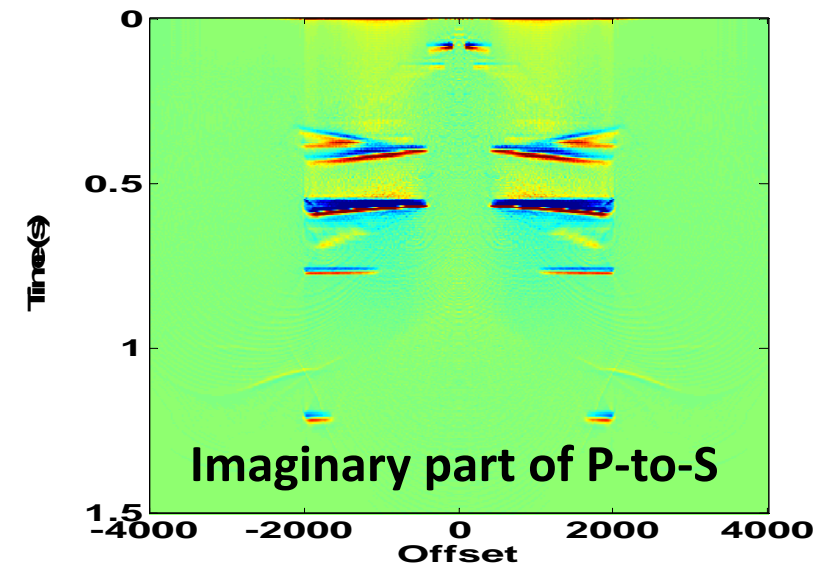
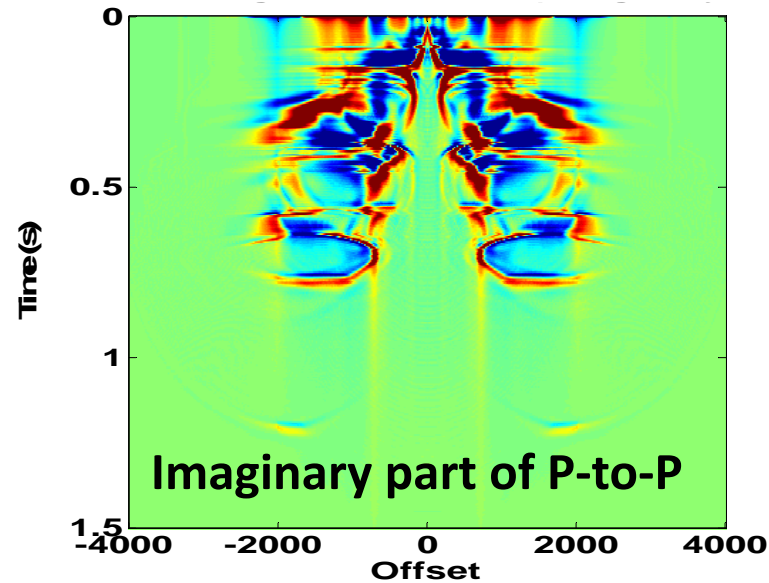
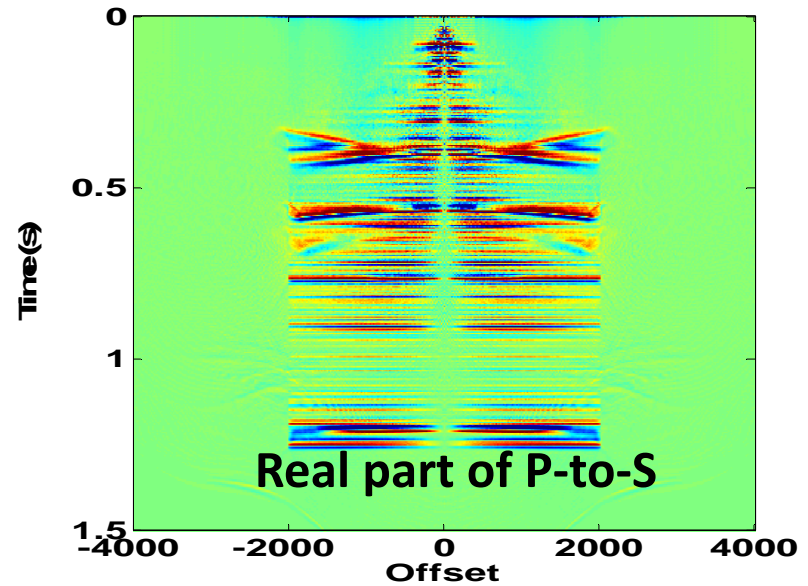
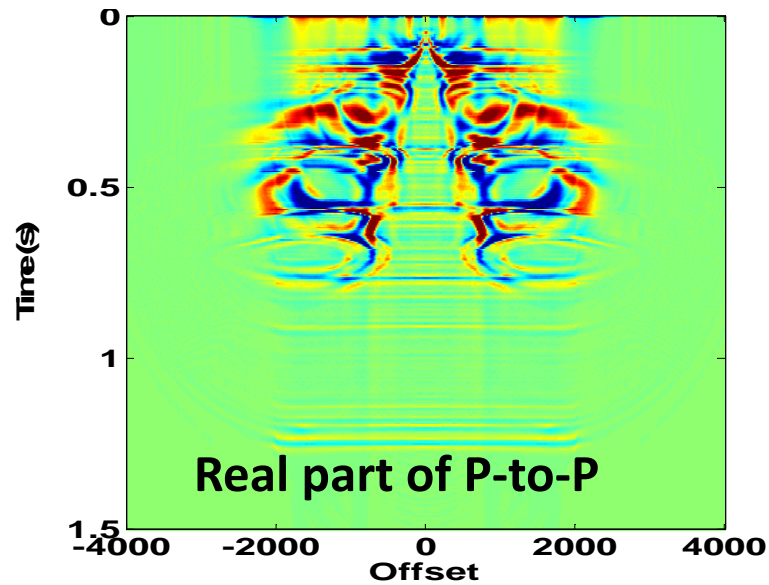


# Synthetic example (well log from NEBC)

## P-to-P and P-to-S wavefield



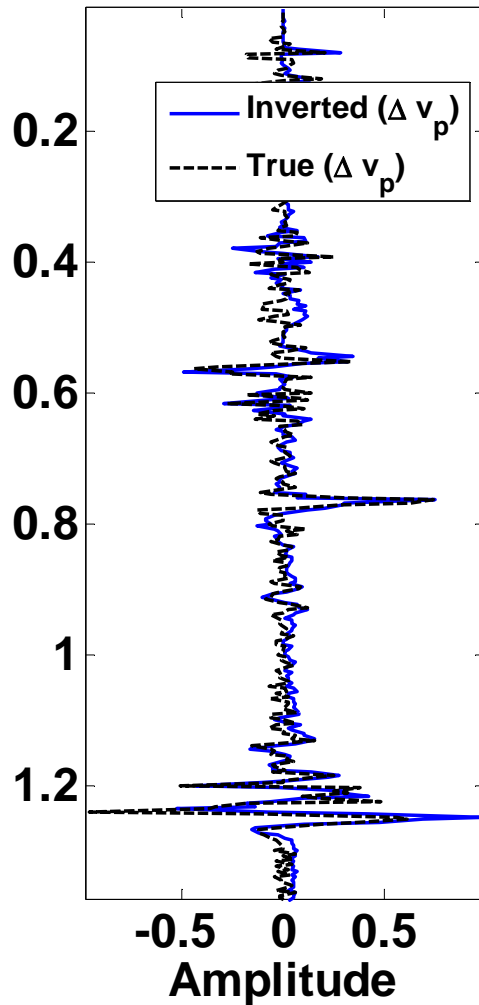
# AVO and common image gather



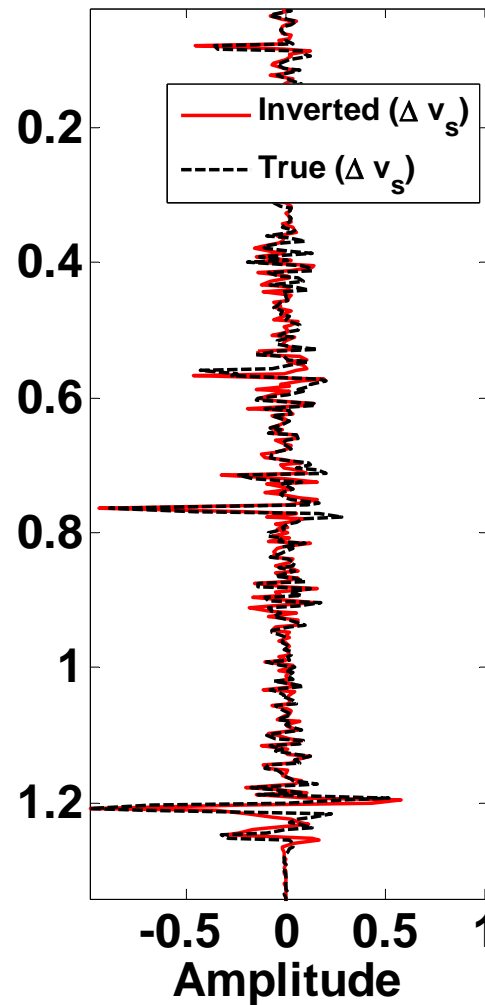


# Synthetic multiparameter inversion

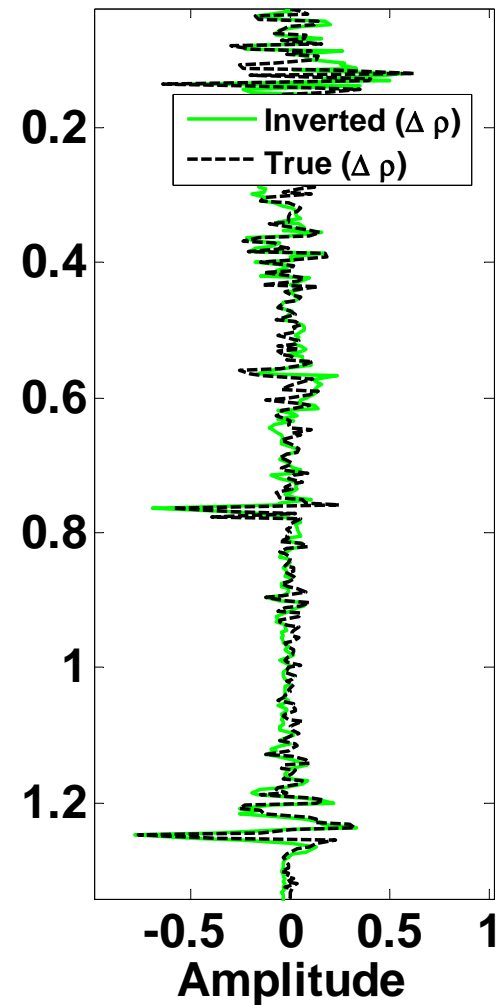
$V_p$  inversion



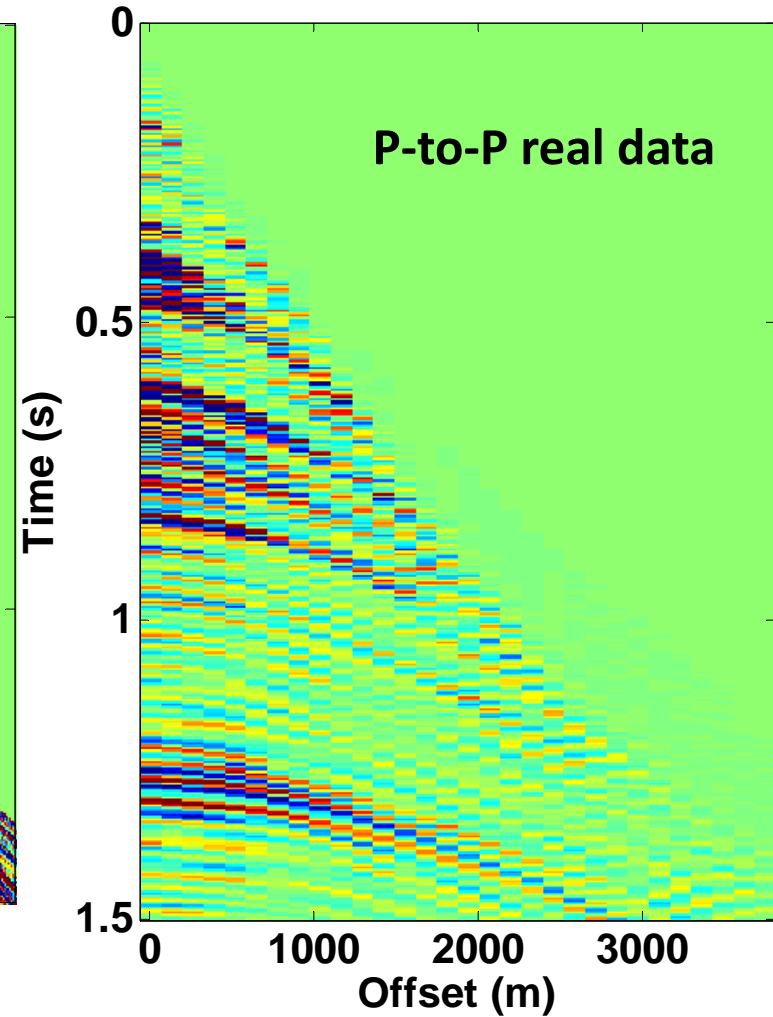
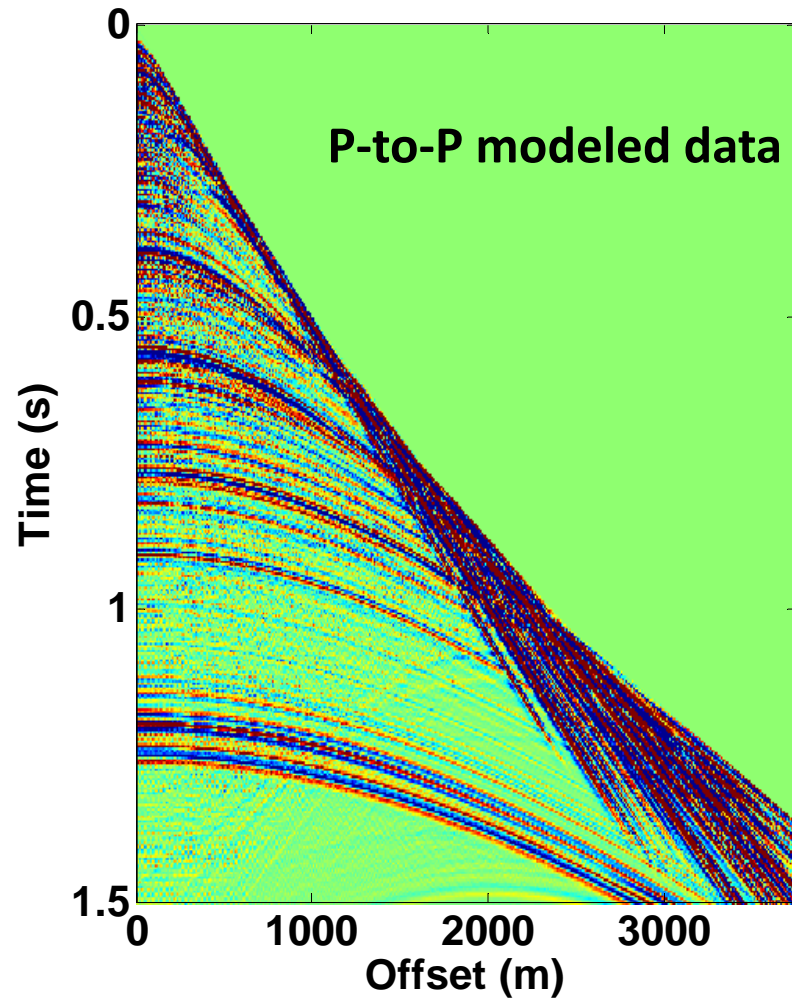
$V_s$  inversion



Density inversion

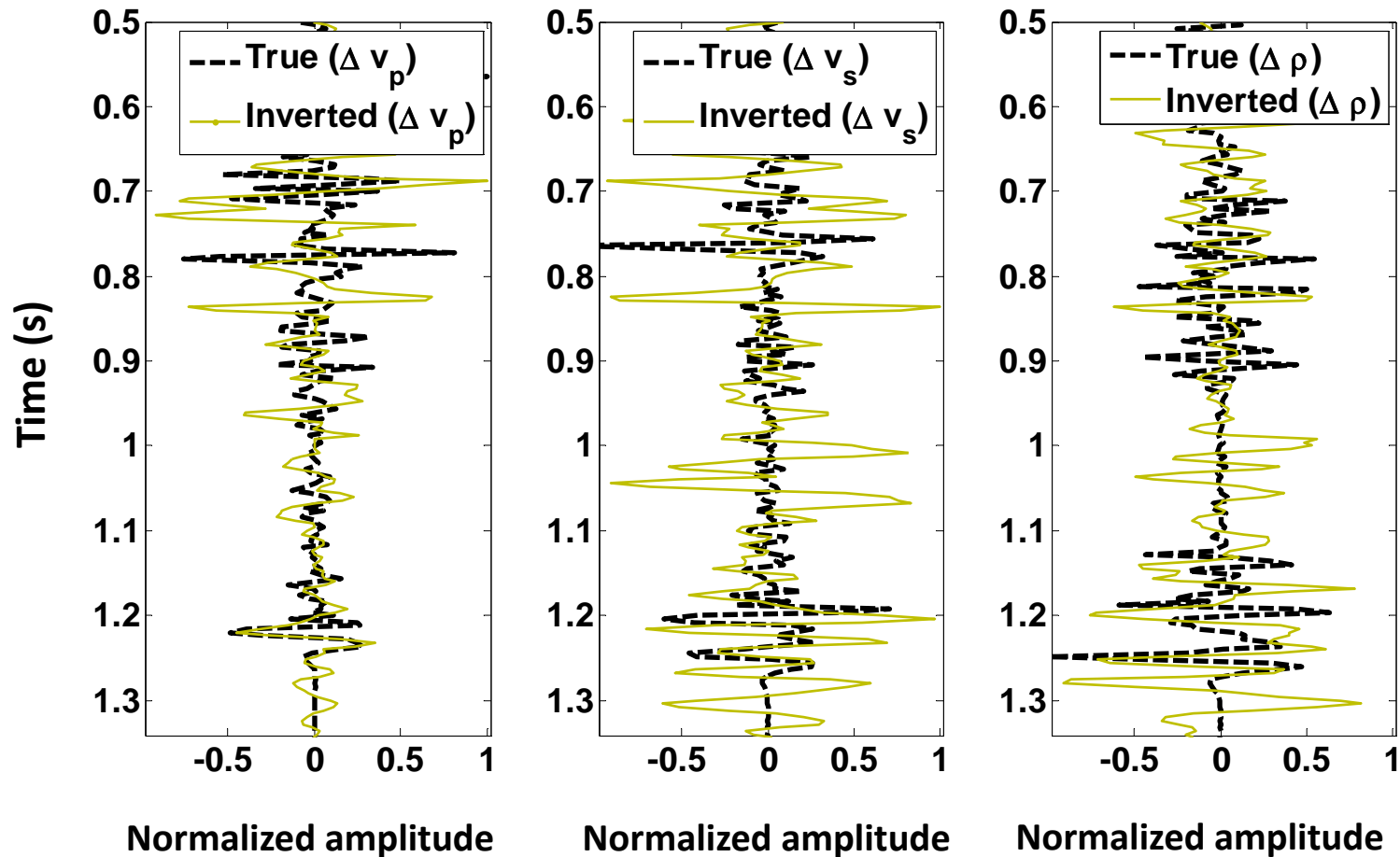


# Real data: Multiparameter inversion P-to-P wavefield only



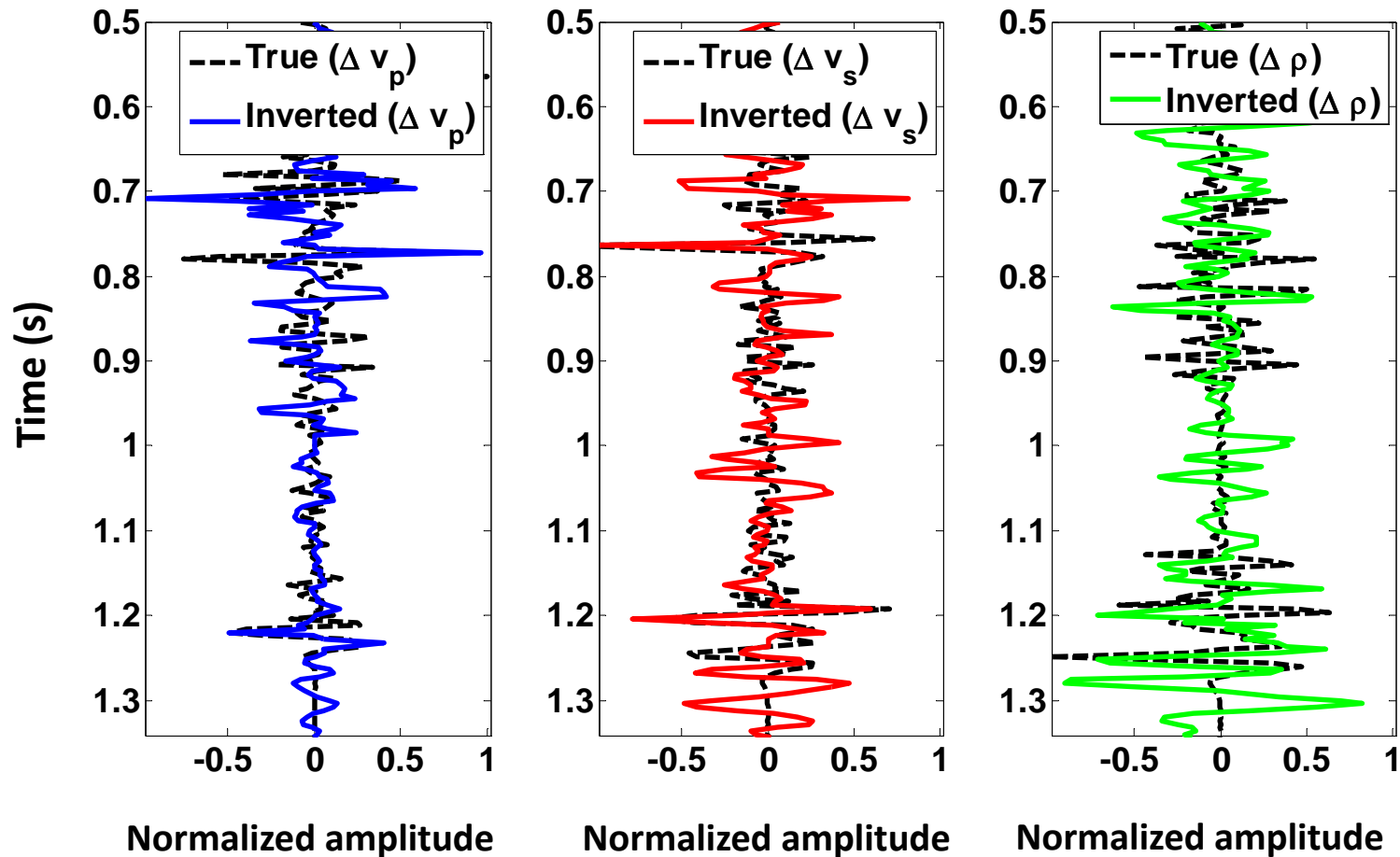
# Real data: Multiparameter inversion P-to-P wavefield only

## Initial model result



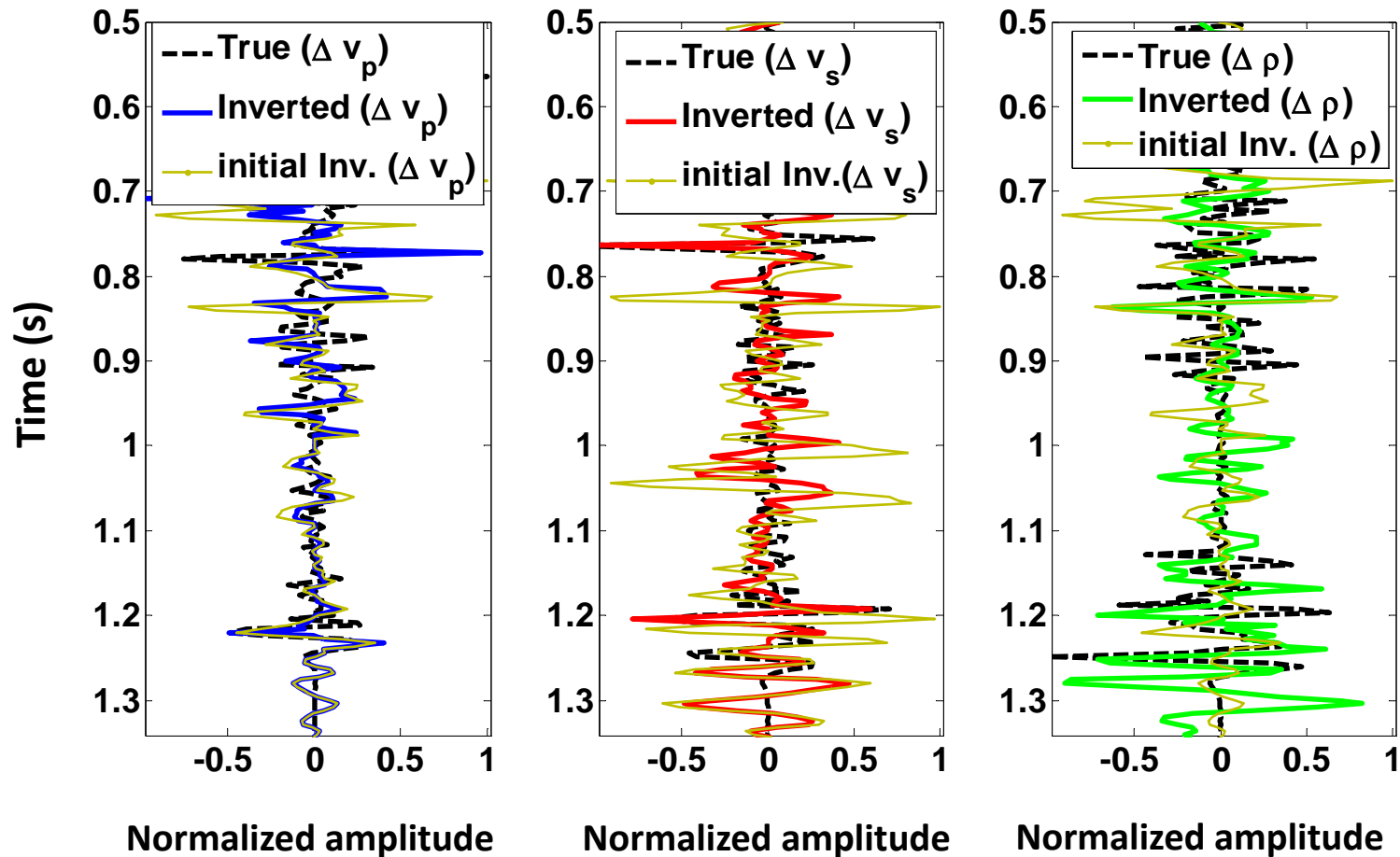
# Real data: Multiparameter inversion P-to-P wavefield only

## Updated model result



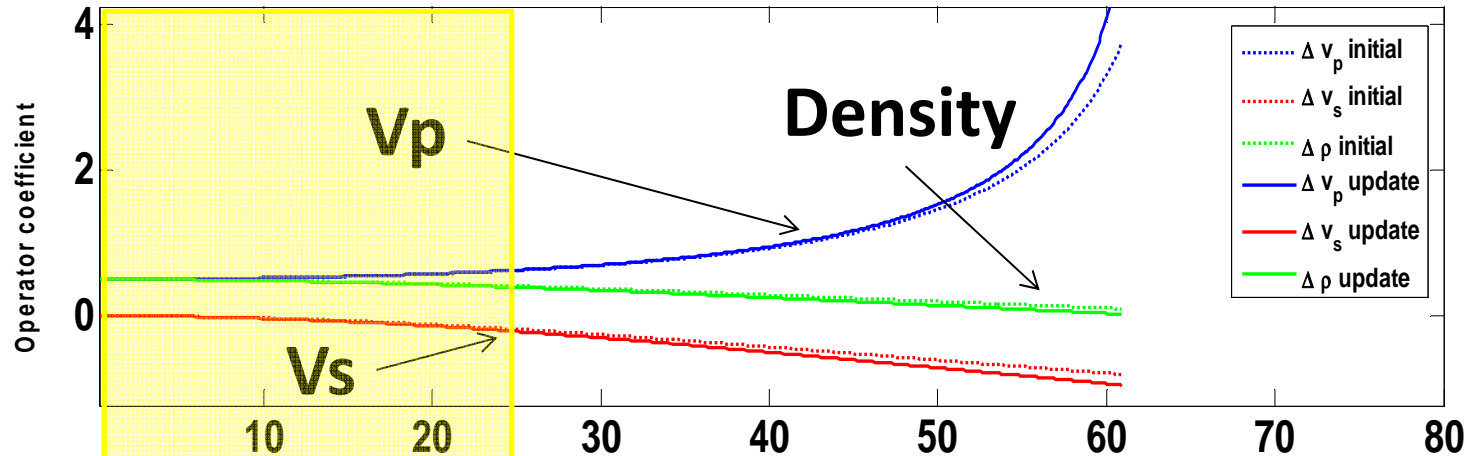
# Real data: Multiparameter inversion P-to-P wavefield only

## Initial vs updated model result

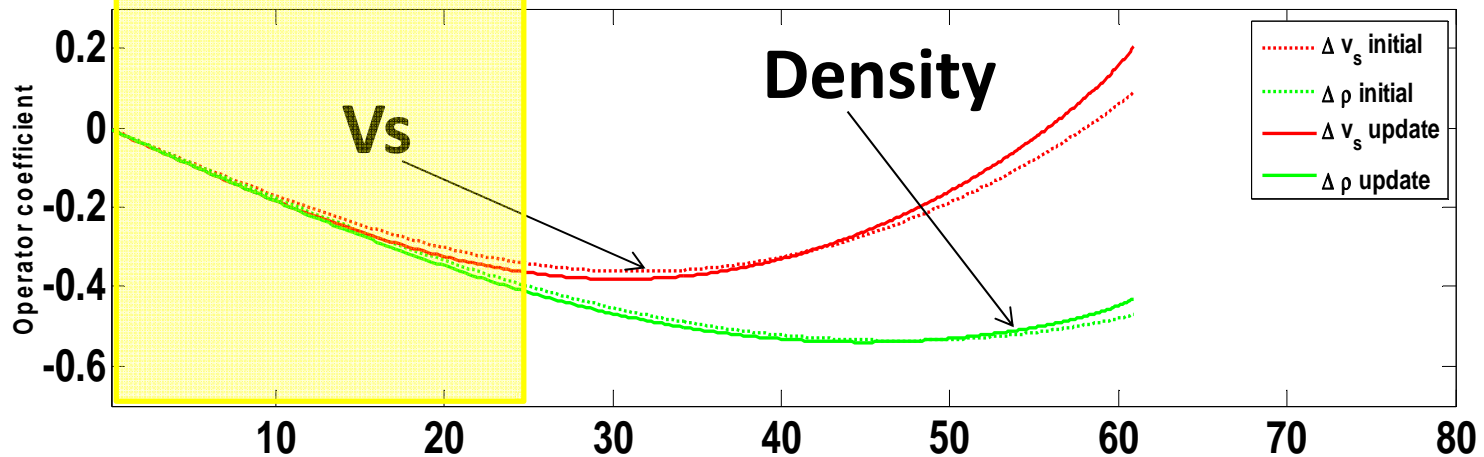


# Real data: Multiparameter inversion P-to-P and P-to-S wavefield (Limited offset)

## A sample P-to-P inversion operator

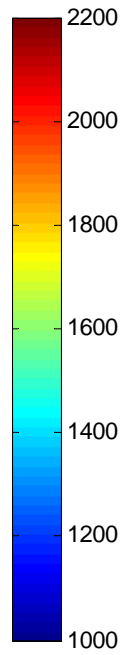
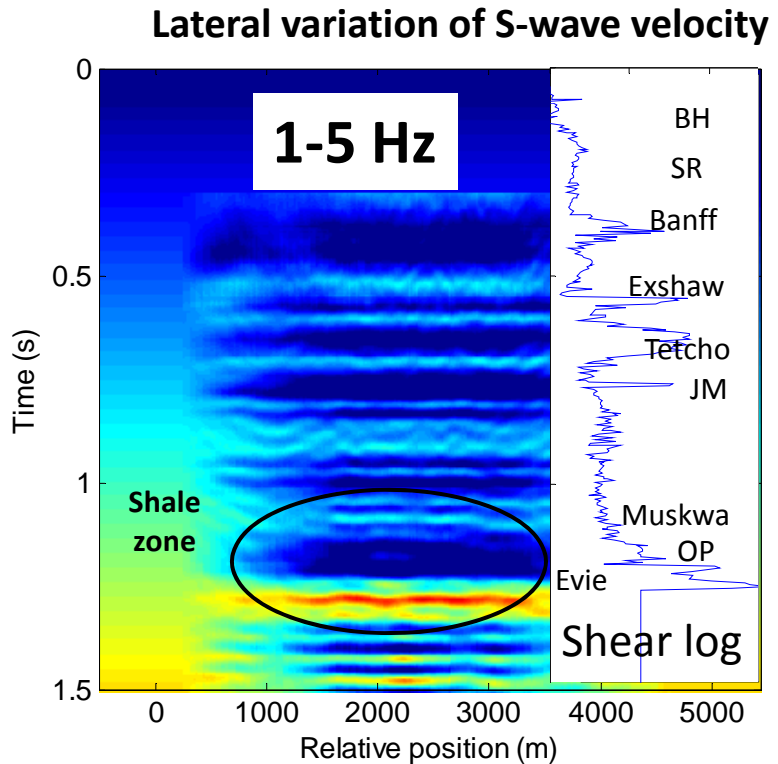
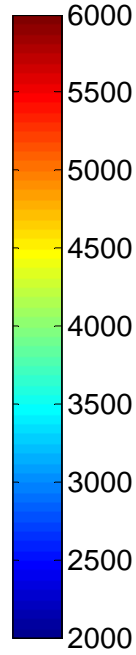
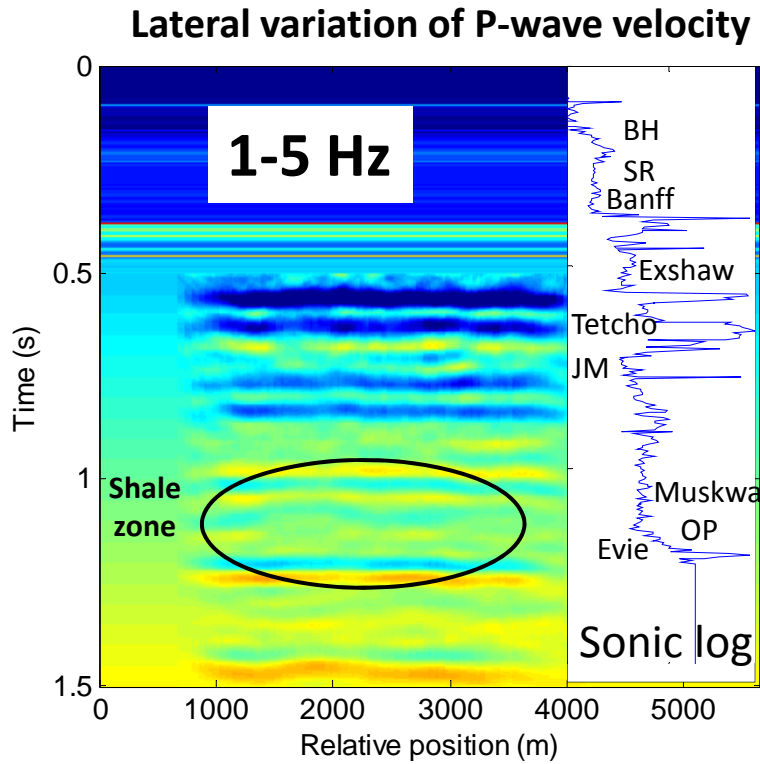


## A sample P-to-S inversion operator



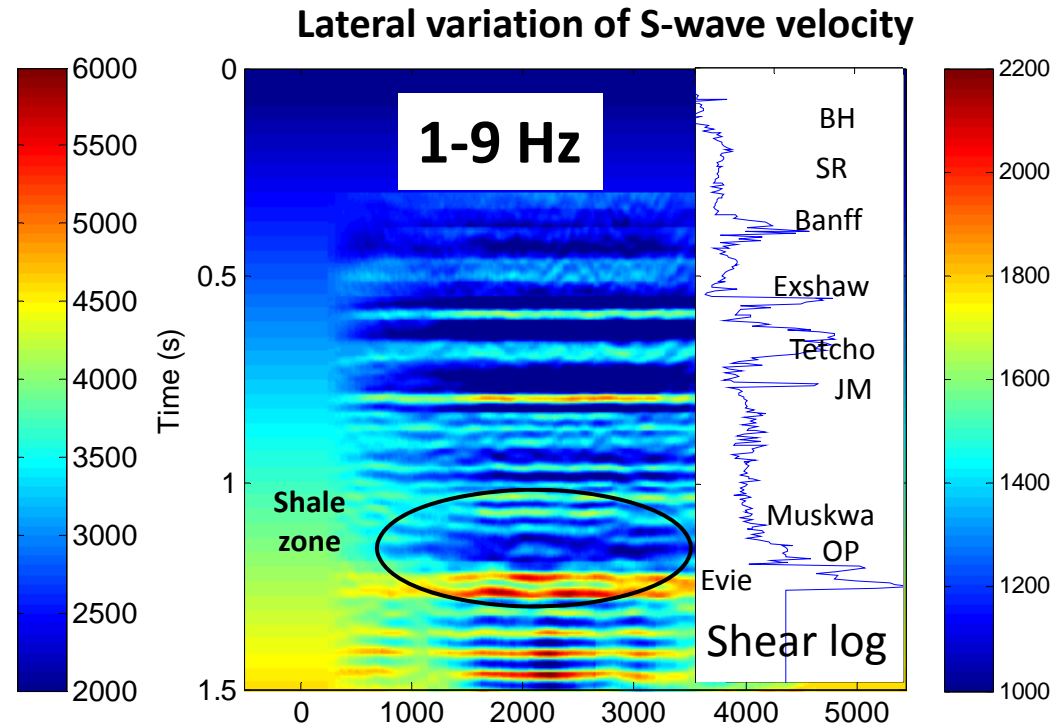
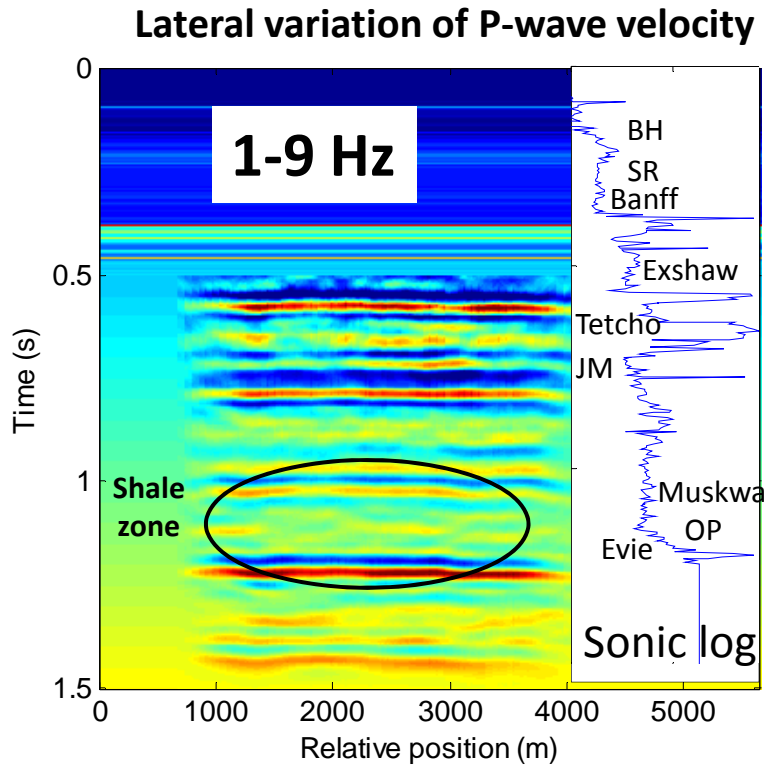
Angle of incident (degree)

# Real data example: P-P and P-Sv Waveform Inversion, NE-BC



- P- wave Velocity inversion using P-P data for the frequency of **1-5 Hz** (left image)
- S- wave velocity inversion using P-Sv data for the frequency of **1-5 Hz** (right image)

# Real data example: P-P and P-Sv Waveform Inversion, NE-BC

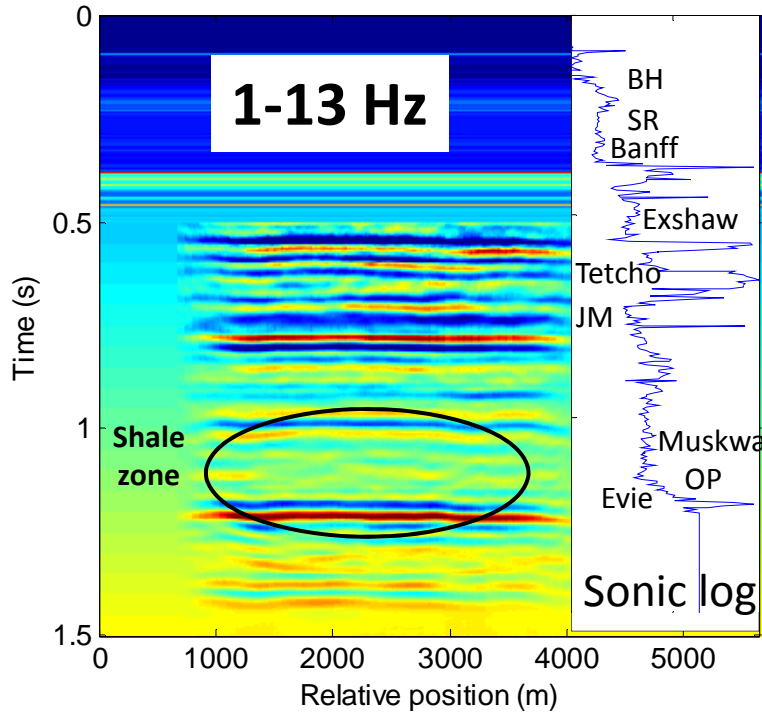


- P- wave Velocity inversion using P-P data for the frequency of **1-9 Hz** (left image)
- S- wave velocity inversion using P-Sv data for the frequency of **1-9 Hz** (right image)

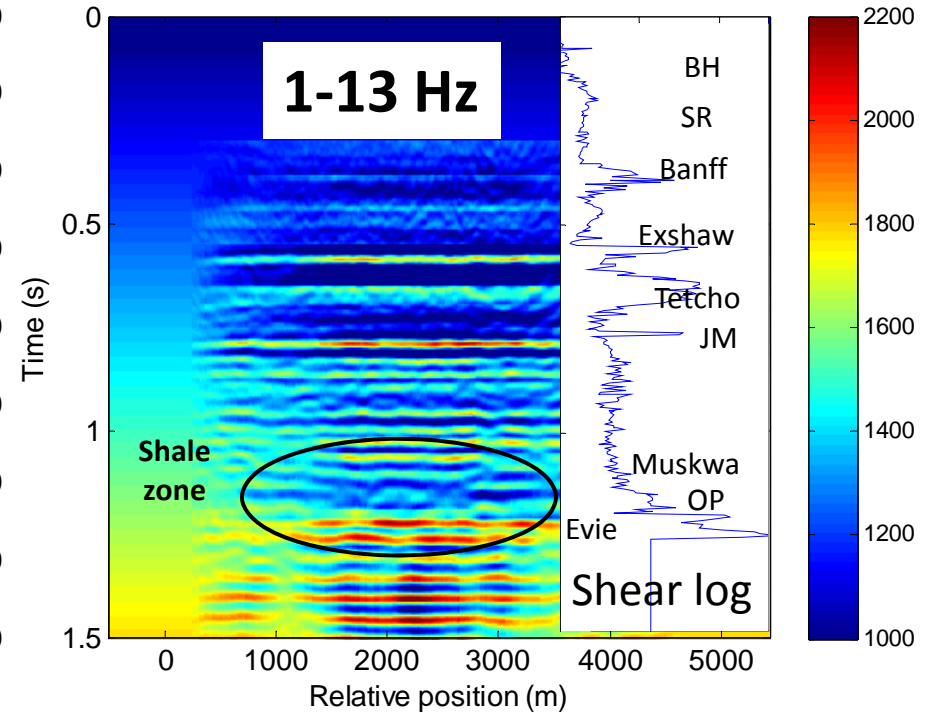


# Real data example: P-P and P-Sv Waveform Inversion, NE-BC

Lateral variation of P-wave velocity

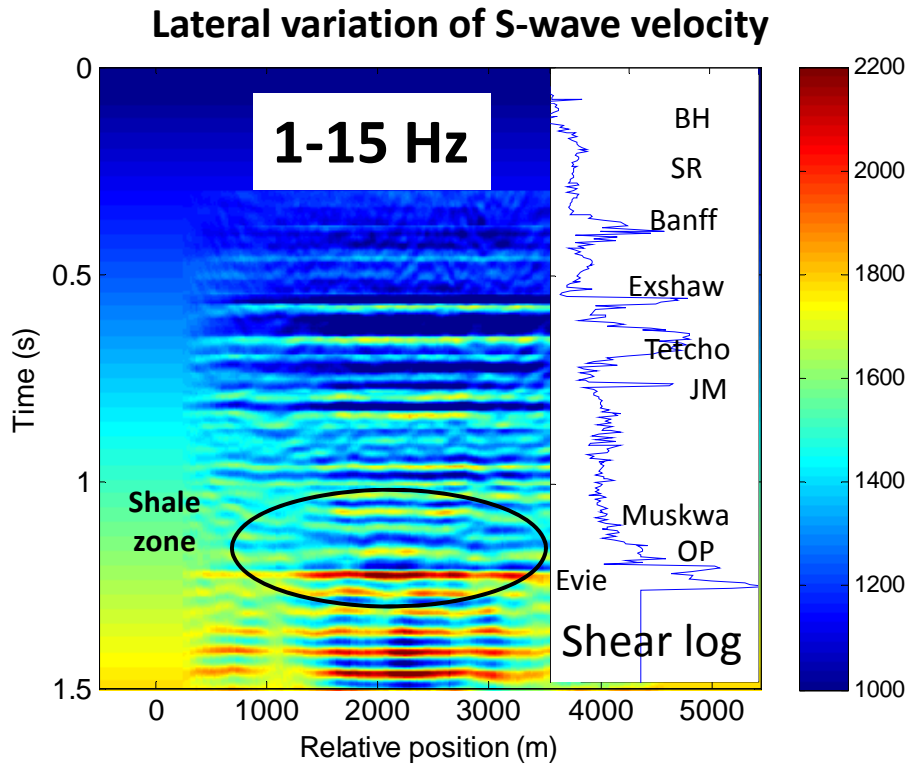
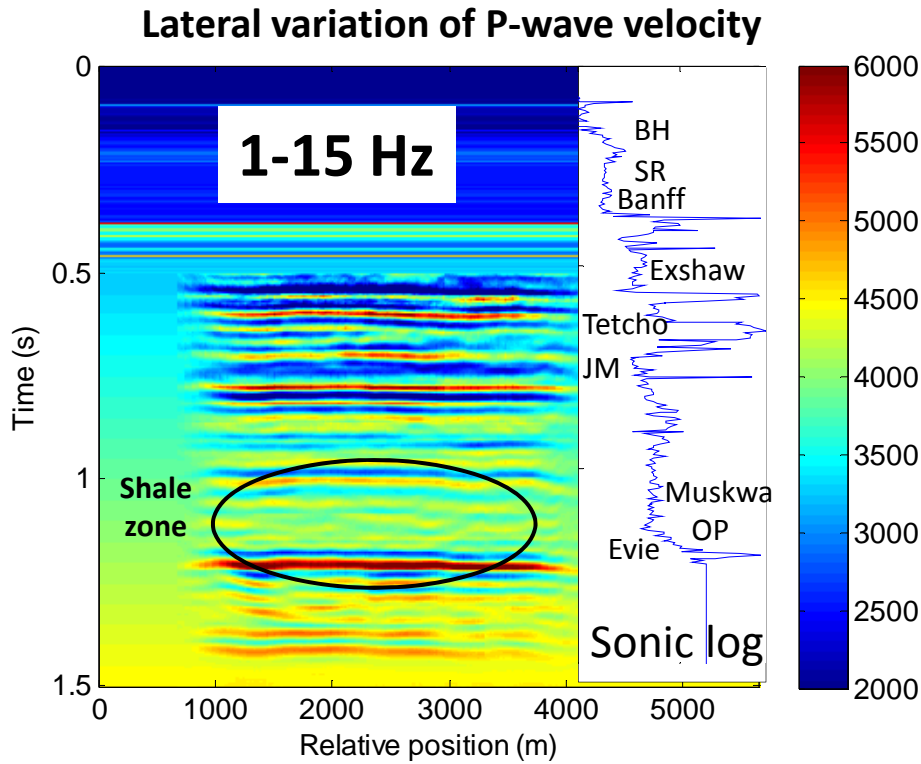


Lateral variation of S-wave velocity



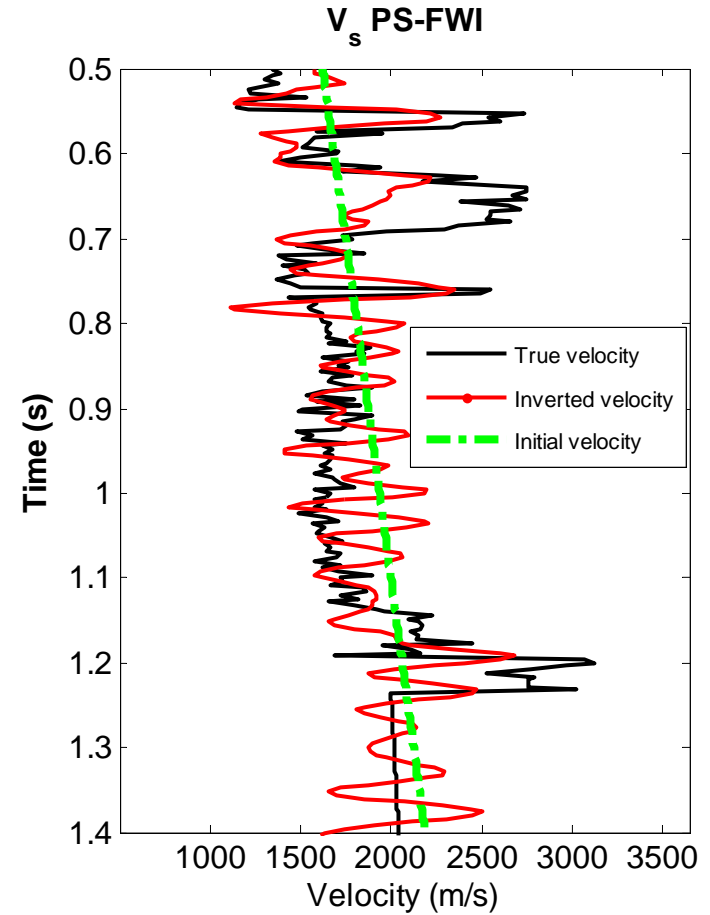
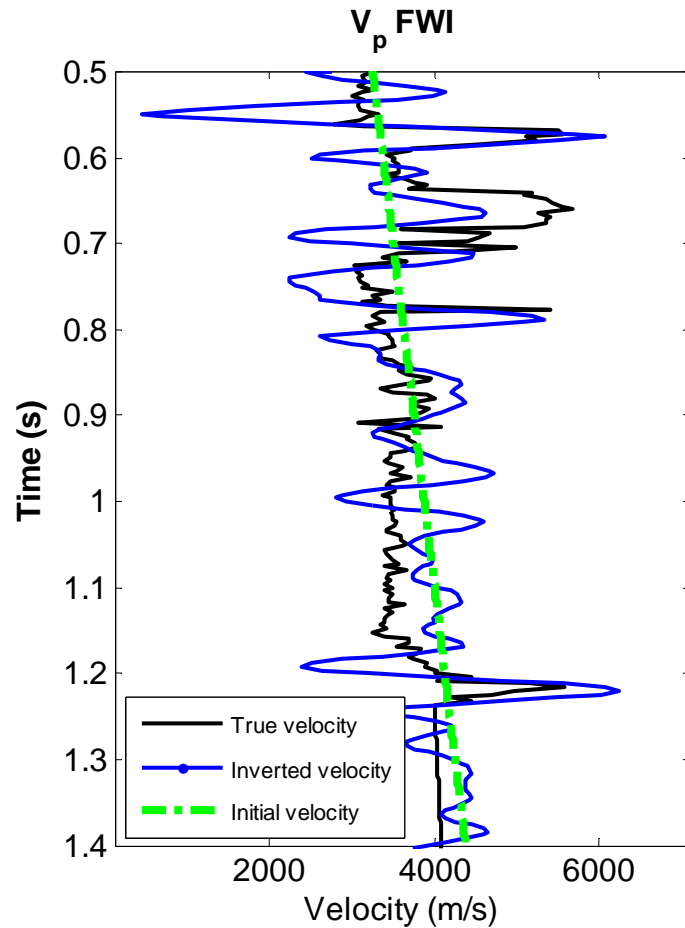
- P- wave Velocity inversion using P-P data for the frequency of 1-13 Hz (left image)
- S- wave velocity inversion using P-Sv data for the frequency of 1-13 Hz (right image)

# Real data example: P-P and P-Sv Waveform Inversion, NE-BC

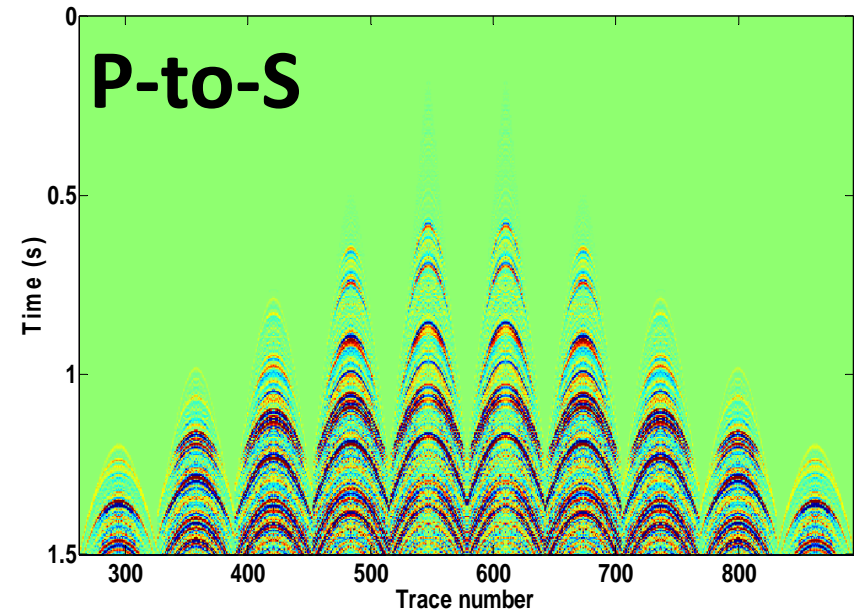
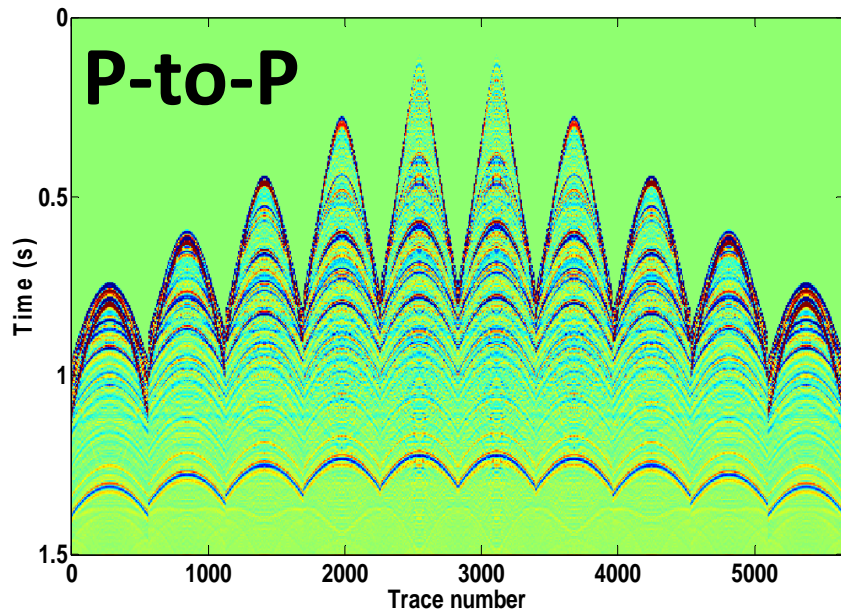
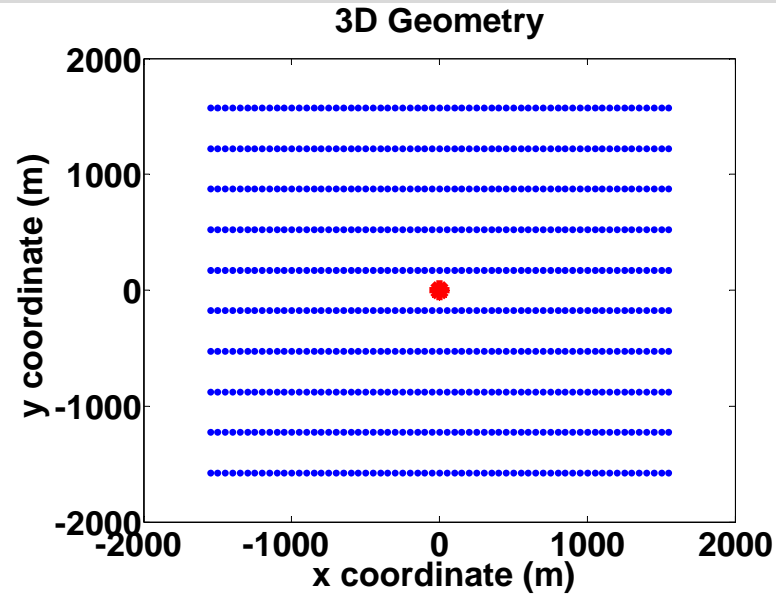


- P- wave Velocity inversion using P-P data for the frequency of **1-13 Hz** (left image)
- S- wave velocity inversion using P-Sv data for the frequency of **1-13 Hz** (right image)

# Real data example: P-P and P-Sv Waveform Inversion, NE-BC

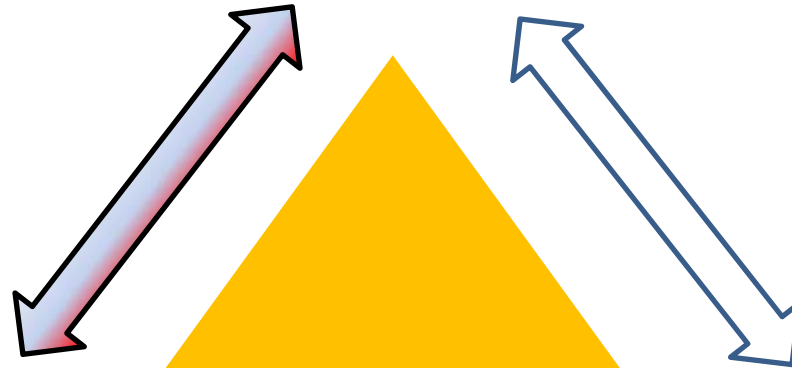


# Future work: 3D data inversion



# Conclusions...

**Tarantola 's  
Inversion method  
(Iterative)**



**Bleistein 's  
Inversion method  
(Kirchhoff)**

**Beylkin 's  
Inversion method  
(Born)**

# Conclusions

- ✓ A standard method is developed to perform FWI method in various types of problems
- ✓ The gradient function is provided by conventional AVO inversion methods
- ✓ Fast and practical

# Acknowledgments

- CREWES Faculty and Sponsors
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**THANK YOU !**