Suppress Parameter Cross-talk for Elastic Full-waveform Inversion :

Parameterization and Acquisition Geometry

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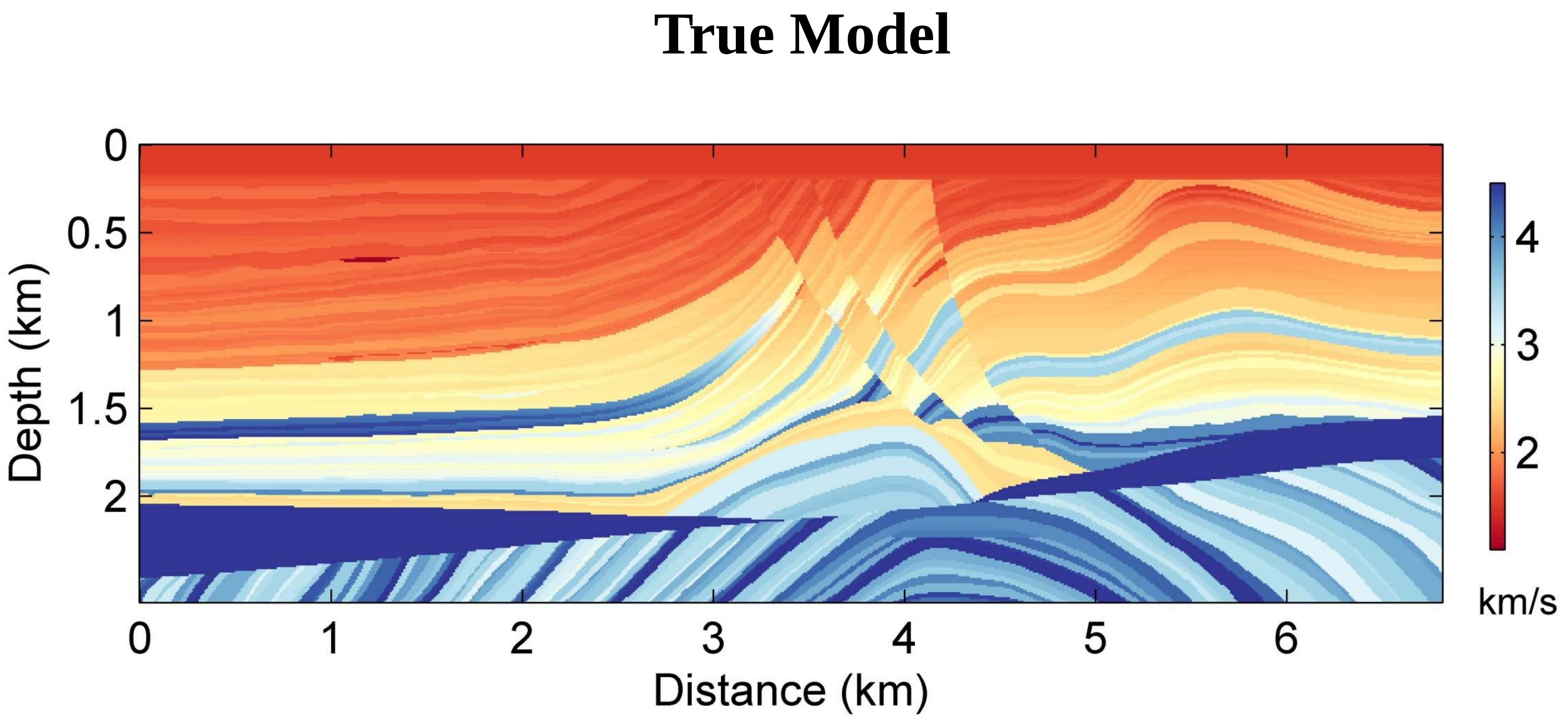
The Challenges of Full-waveform Inversion

- **Extensive Computation Burden** *(Phase-encoding)*
- **Slow Convergence Rate** *(Hessian-free optimization methods)*
- **Lack of Source Signature** *(Source-independent FWI)*
- **Cycle-skipping** *(BLIMP, POCS, Well log data)*
- **Parameter Cross-talk *(Gauss-Newton Hessian, Parameterization, Acquisition Geometry,….)***
- ………

1. Numerical Illustration of Cycle-skipping Problem

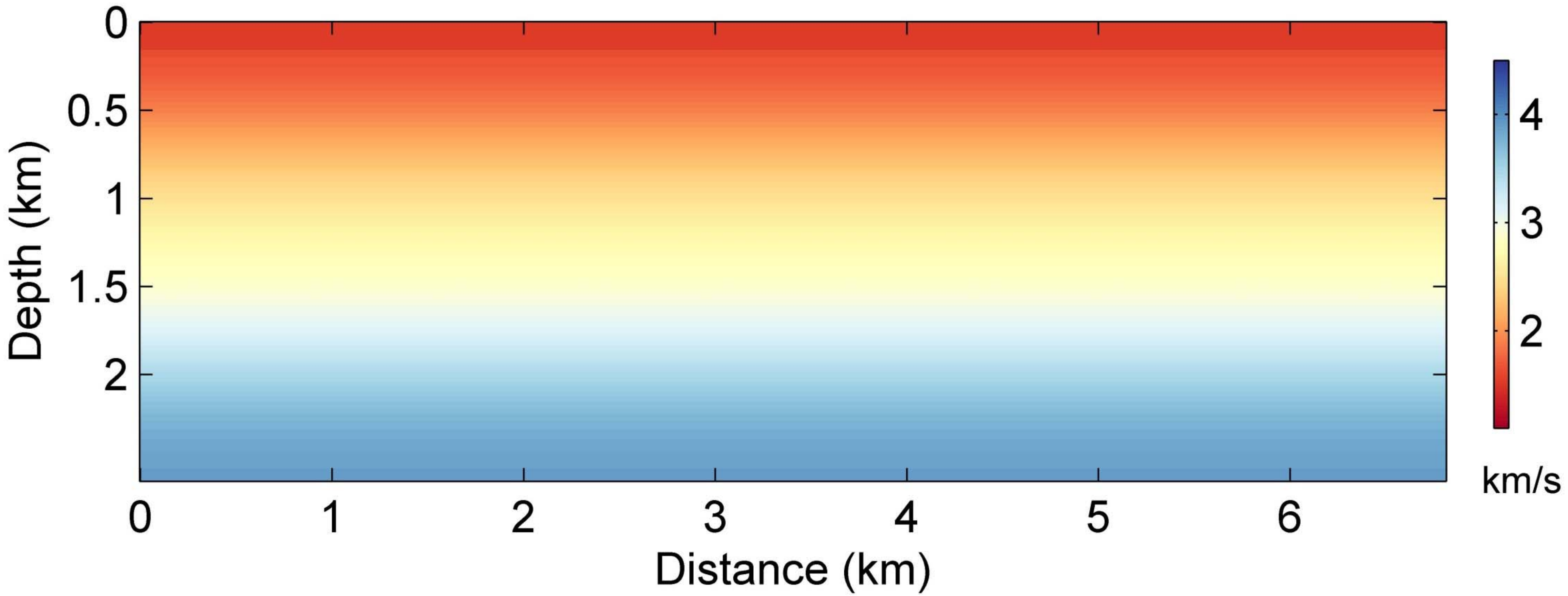
NSERC CRSNG
UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
Department of Geoscience

The Phenomena of Cycle-skipping



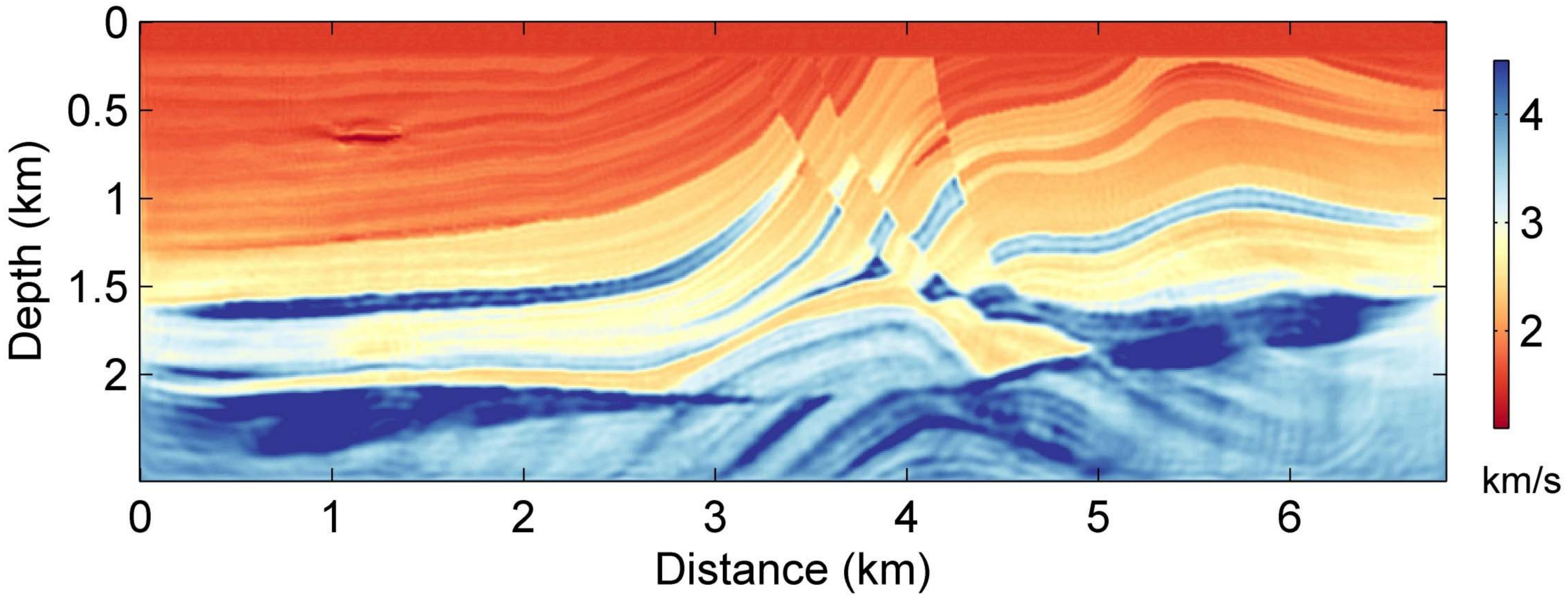
The Phenomena of Cycle-skipping

Initial Model



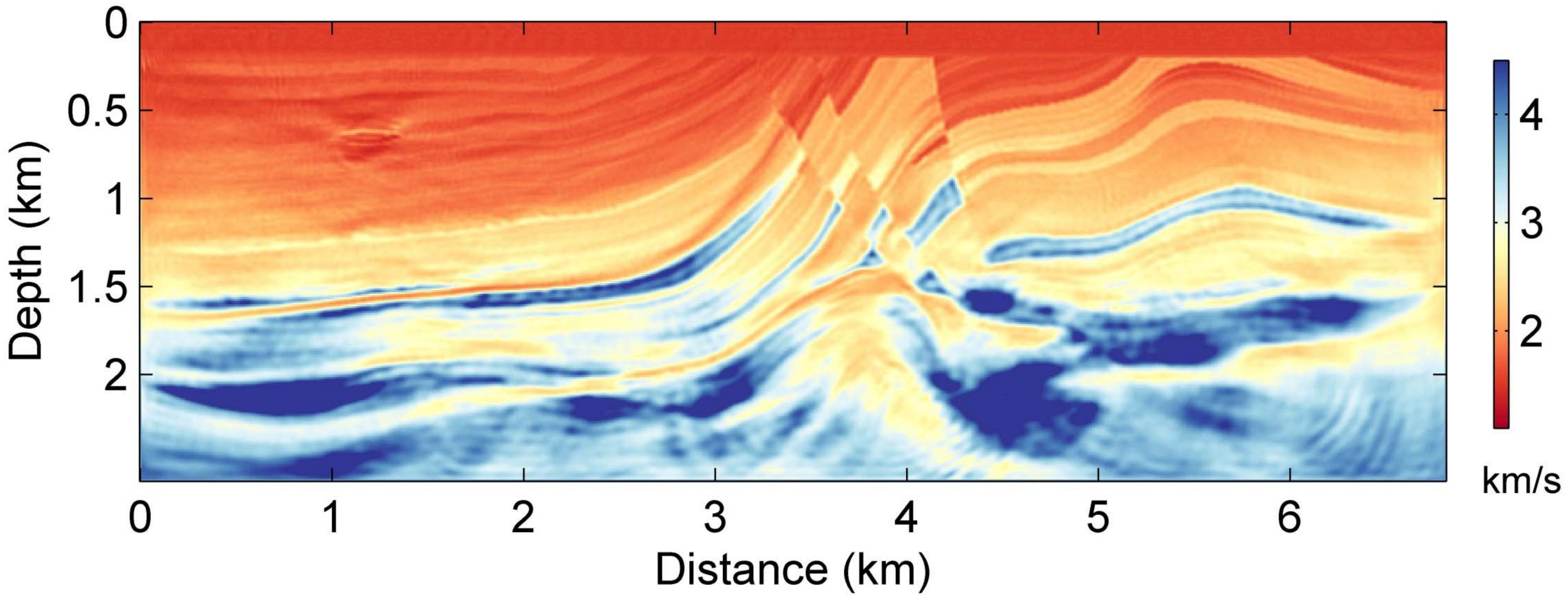
The Phenomena of Cycle-skipping

1 Hz -- 30 Hz



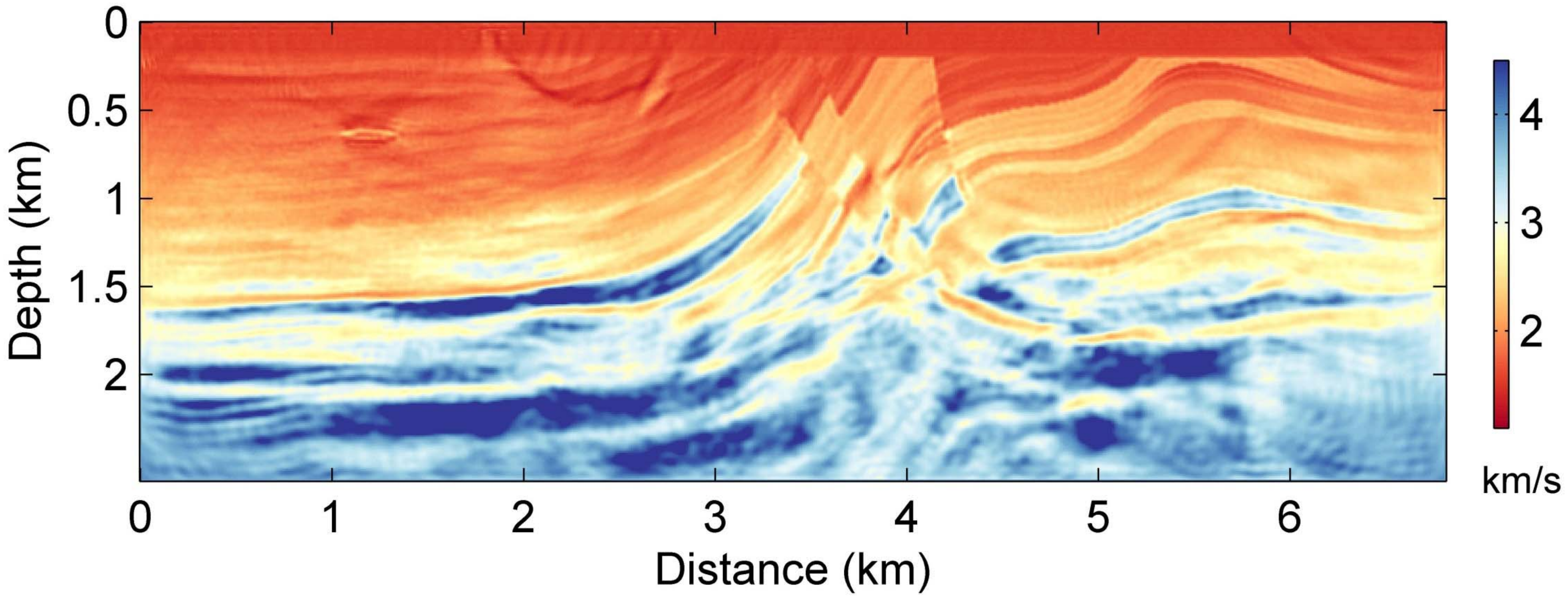
The Phenomena of Cycle-skipping

3 Hz -- 30 Hz



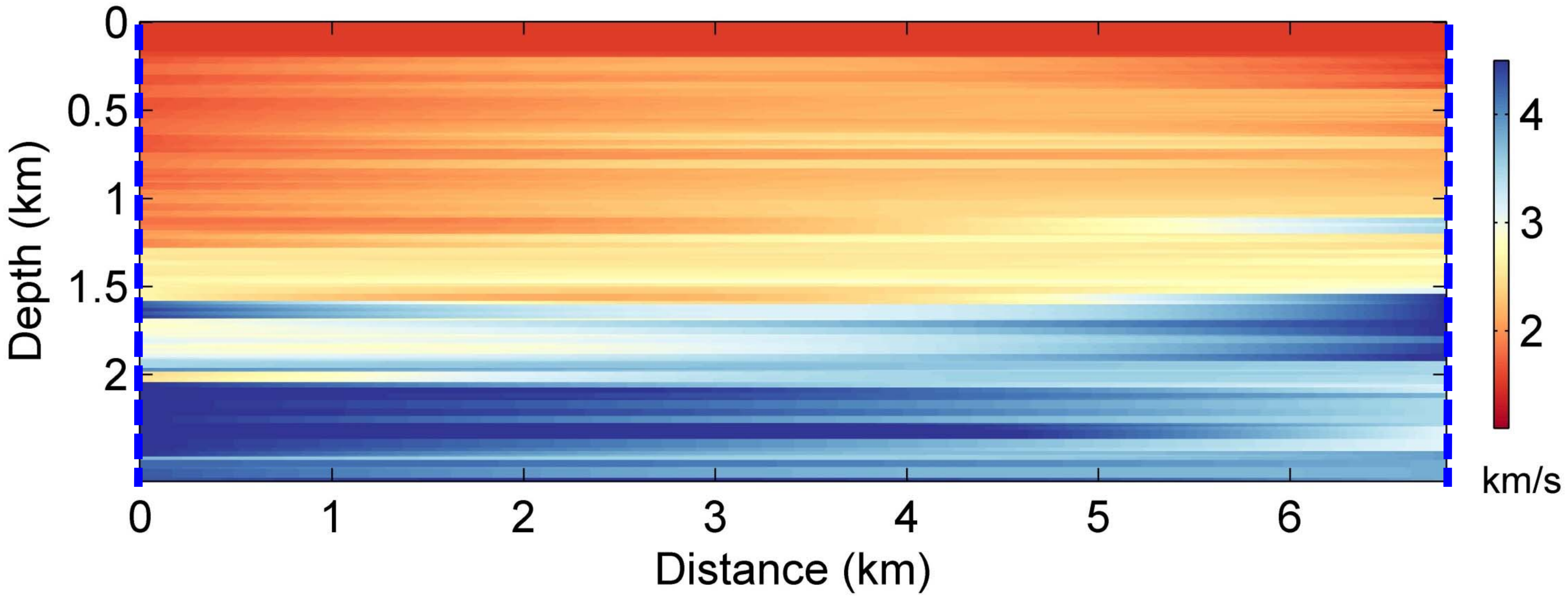
The Phenomena of Cycle-skipping

6 Hz -- 30 Hz



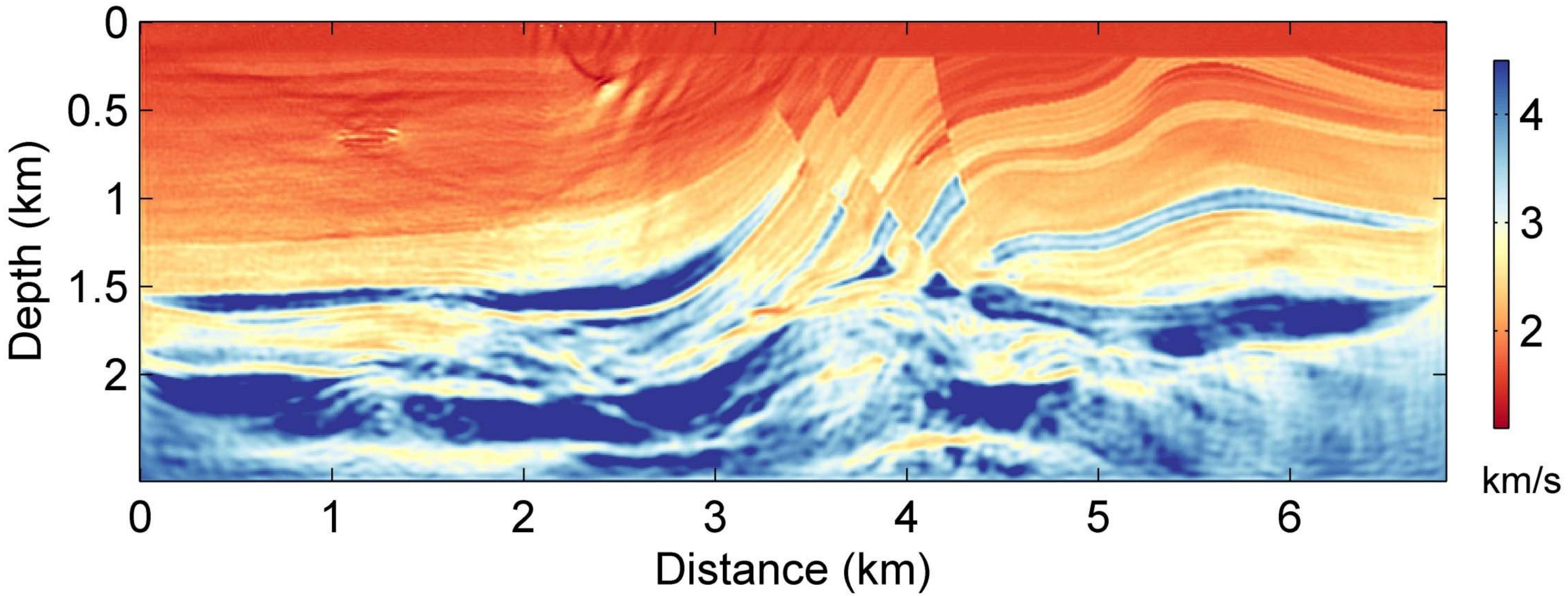
The Phenomena of Cycle-skipping

Well Log Data Model



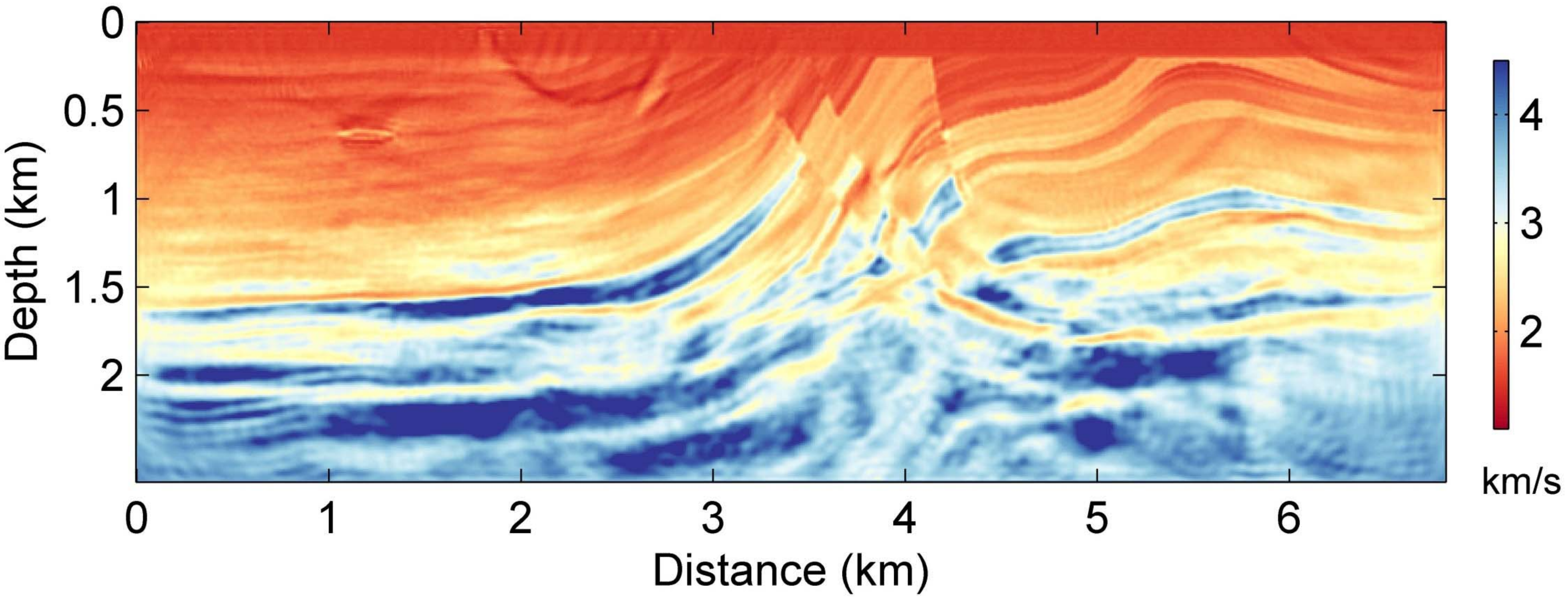
The Phenomena of Cycle-skipping

6 Hz -- 30 Hz with Well Log Data Regularization



The Phenomena of Cycle-skipping

6 Hz -- 30 Hz without Well Log Data Regularization



2. Suppressing Parameter Cross-talk in Elastic FWI

The Parameter Cross-talk Problem in Multi-parameter FWI

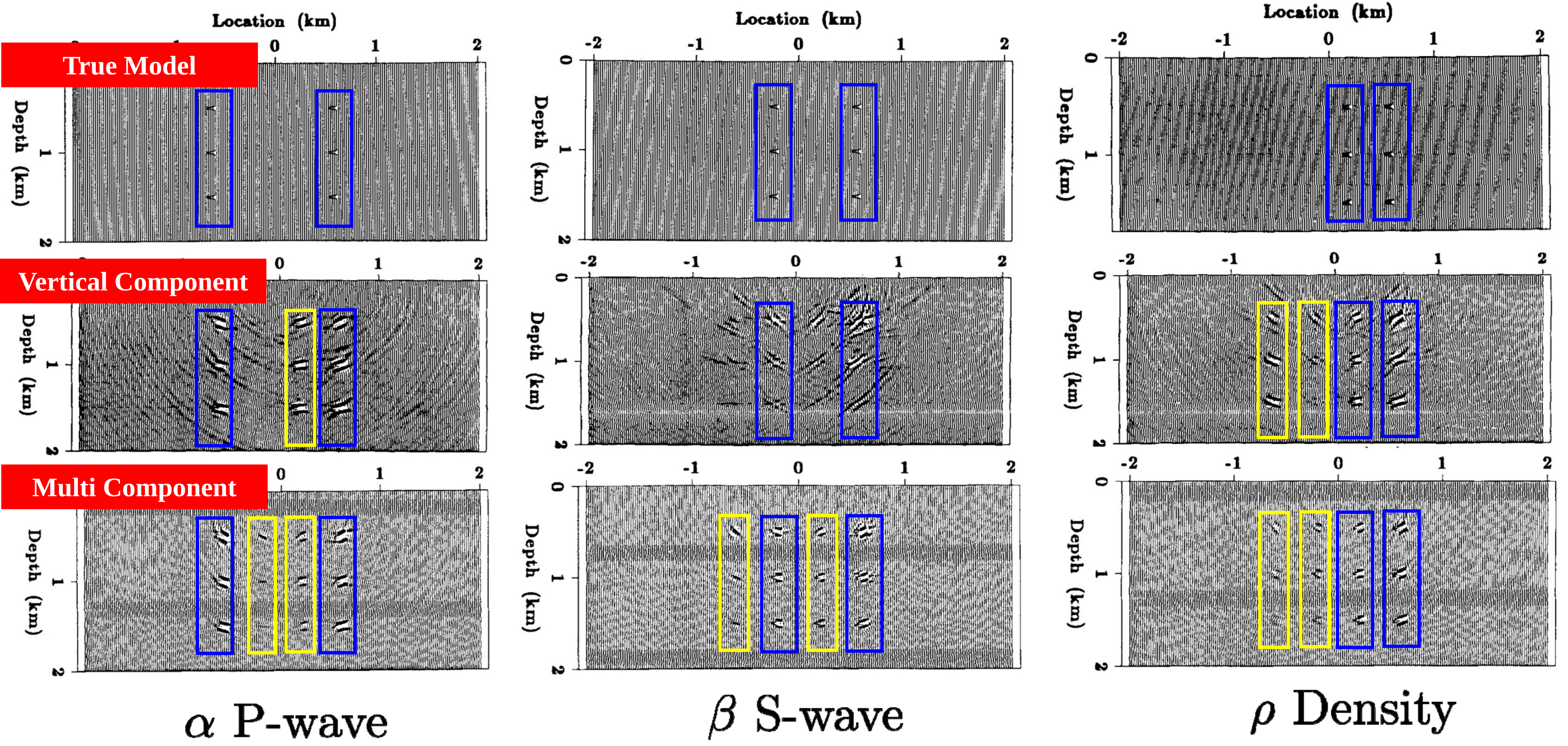
- **Short-term FWI is employed for building high-resolution velocity model for seismic imaging.**

- **Longer-term FWI is expected to unlock the ultimate potential of seismic data by solving for more elastic parameters tied to the rock physics for reservoir characterization.**

  - **The coupling effects of different elastic parameters results in parameter cross-talk.**
  - **The scattering patterns are used to explain the parameter cross-talk.**
  - ***Multi-parameter Gauss-Newton Hessian and its parameter-type approximation (Innanen, 2014).***
  - ***Parameterizations.***
  - ***Acquisition geometry.***

Illustrations of the Parameter Cross-talk in Elastic FWI

Mora, P., 1987, Nonlinear two-dimensional elastic inversion of multioffset seismic data: *Geophysics*, 53, 1425-1436.



Illustrations of the Parameter Cross-talk in Elastic FWI

Mora, P., 1987, Nonlinear two-dimensional elastic inversion of multioffset seismic data: *Geophysics*, 53, 1425-1436.

- **Conclusions from Mora's research:**
  - **Parameter cross-talk problem exists in elastic FWI;**
  - **The conjugate gradient method is limited in suppressing parameter cross-talk;**
  - **The role of Gauss-Newton Hessian is not discussed;**
  - **Multi-component data appears to improve spatial resolution but also reduce parameter resolution;**
  - **The influences of parameterization and acquisition geometry are not discussed.**

2. 1 Suppressing Parameter Cross-talk with Multi-parameter Gauss-Newton Hessian

The Parameter Cross-talk Problem in FWI

- **The search direction for steepest-descent multi-parameter FWI:**

$$\Delta \mathbf{m} = -\mathbf{g}$$

$$\begin{bmatrix} \Delta \mathbf{m}_1 \\ \Delta \mathbf{m}_2 \\ \Delta \mathbf{m}_3 \end{bmatrix} = - \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix}$$

The Parameter Cross-talk Problem in FWI

- **The search direction for Gauss-Newton multi-parameter FWI:**

$$\tilde{\mathbf{H}}\Delta\mathbf{m} = -\mathbf{g}$$

$$\begin{bmatrix} \tilde{\mathbf{H}}_{11} & \tilde{\mathbf{H}}_{12} & \tilde{\mathbf{H}}_{13} \\ \tilde{\mathbf{H}}_{21} & \tilde{\mathbf{H}}_{22} & \tilde{\mathbf{H}}_{23} \\ \tilde{\mathbf{H}}_{31} & \tilde{\mathbf{H}}_{32} & \tilde{\mathbf{H}}_{33} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{m}_1 \\ \Delta\mathbf{m}_2 \\ \Delta\mathbf{m}_3 \end{bmatrix} = - \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix}$$

$$\tilde{\mathbf{H}}_{\mathrm{xy}}(\mathbf{r}, \mathbf{r}') = \frac{\partial \mathbf{u}^{\dagger}}{\partial \mathbf{m}_{\mathrm{x}}(\mathbf{r})} \frac{\partial \mathbf{u}}{\partial \mathbf{m}_{\mathrm{y}}(\mathbf{r}')}$$

The Parameter Cross-talk Problem in FWI

- **The search direction for Gauss-Newton multi-parameter FWI:**

$$\tilde{\mathbf{H}}\Delta\mathbf{m} = -\mathbf{g}$$

$$\begin{bmatrix} \tilde{\mathbf{H}}_{11} & \tilde{\mathbf{H}}_{12} & \tilde{\mathbf{H}}_{13} \\ \tilde{\mathbf{H}}_{21} & \tilde{\mathbf{H}}_{22} & \tilde{\mathbf{H}}_{23} \\ \tilde{\mathbf{H}}_{31} & \tilde{\mathbf{H}}_{32} & \tilde{\mathbf{H}}_{33} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{m}_1 \\ \Delta\mathbf{m}_2 \\ \Delta\mathbf{m}_3 \end{bmatrix} = - \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix}$$

$$\tilde{\mathbf{H}}_{\mathbf{xy}}(\mathbf{r}, \mathbf{r}') = \frac{\partial \mathbf{u}^\dagger}{\partial \mathbf{m}_\mathbf{x}(\mathbf{r})} \frac{\partial \mathbf{u}}{\partial \mathbf{m}_\mathbf{y}(\mathbf{r}')}$$

Off-diagonal blocks measure the coupling effects of different physical parameters.

The Parameter Cross-talk Problem in FWI

- **The search direction with parameter-type Hessian approximation (Innanen, 2014):**

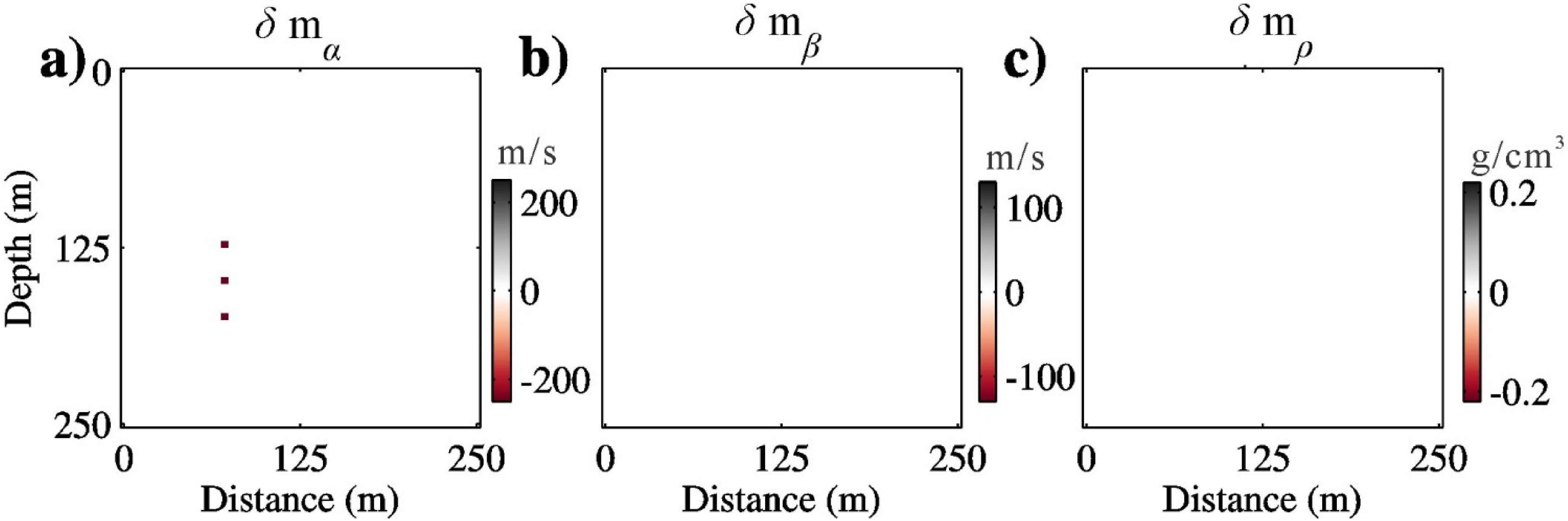
$$\tilde{\mathbb{H}}\Delta\mathbf{m} = -\mathbf{g}$$

$$\begin{bmatrix} \tilde{\mathbb{H}}_{11} & \tilde{\mathbb{H}}_{12} & \tilde{\mathbb{H}}_{13} \\ \tilde{\mathbb{H}}_{21} & \tilde{\mathbb{H}}_{22} & \tilde{\mathbb{H}}_{23} \\ \tilde{\mathbb{H}}_{31} & \tilde{\mathbb{H}}_{32} & \tilde{\mathbb{H}}_{33} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{m}_1 \\ \Delta\mathbf{m}_2 \\ \Delta\mathbf{m}_3 \end{bmatrix} = - \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix}$$

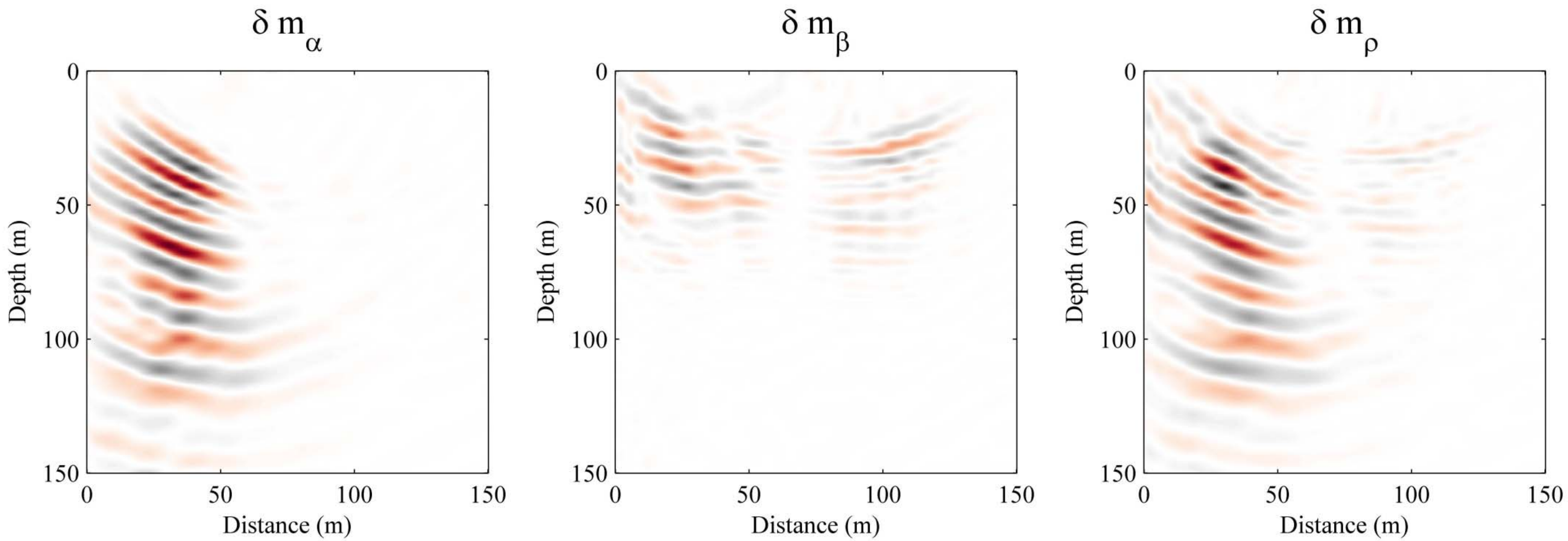
$$\tilde{\mathbb{H}}_{\mathrm{xy}}(\mathbf{r}) = \frac{\partial \mathbf{u}^{\dagger}}{\partial \mathbf{m}_{\mathrm{x}}(\mathbf{r})} \frac{\partial \mathbf{u}}{\partial \mathbf{m}_{\mathrm{y}}(\mathbf{r})}$$

The parameter-type Hessian approximation is formed by all diagonal elements of the blocks.

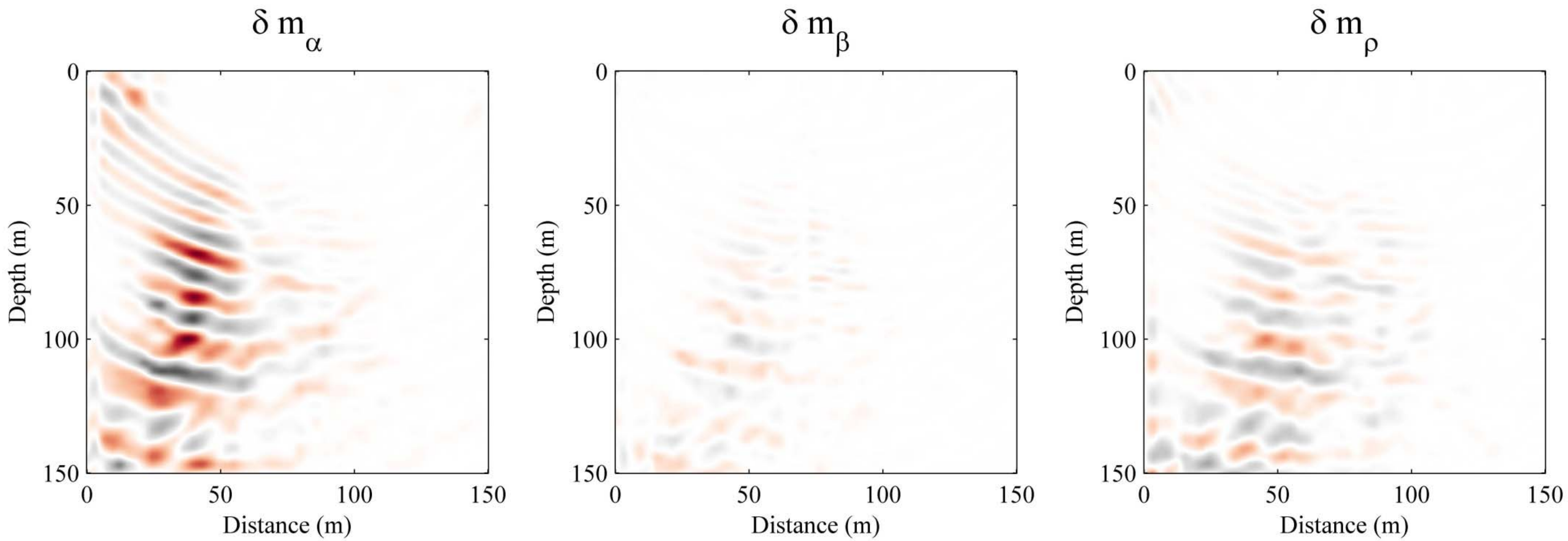
True Model:


a)
b)
c)
δm_α
δm_β
δm_ρ
Depth (m)
0
125
250
Distance (m)
m/s
200
0
-200
m/s
100
0
-100
g/cm³
0.2
0
-0.2

Gradient Updates:

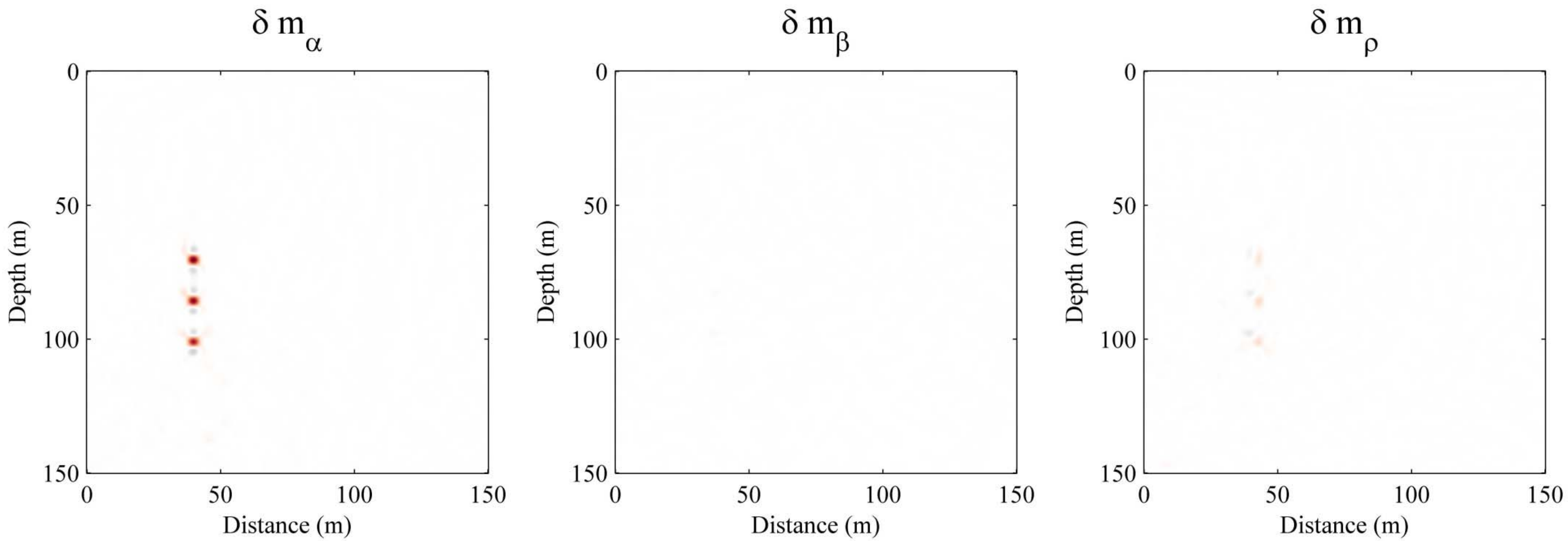


Parameter-type Hessian Preconditioning:



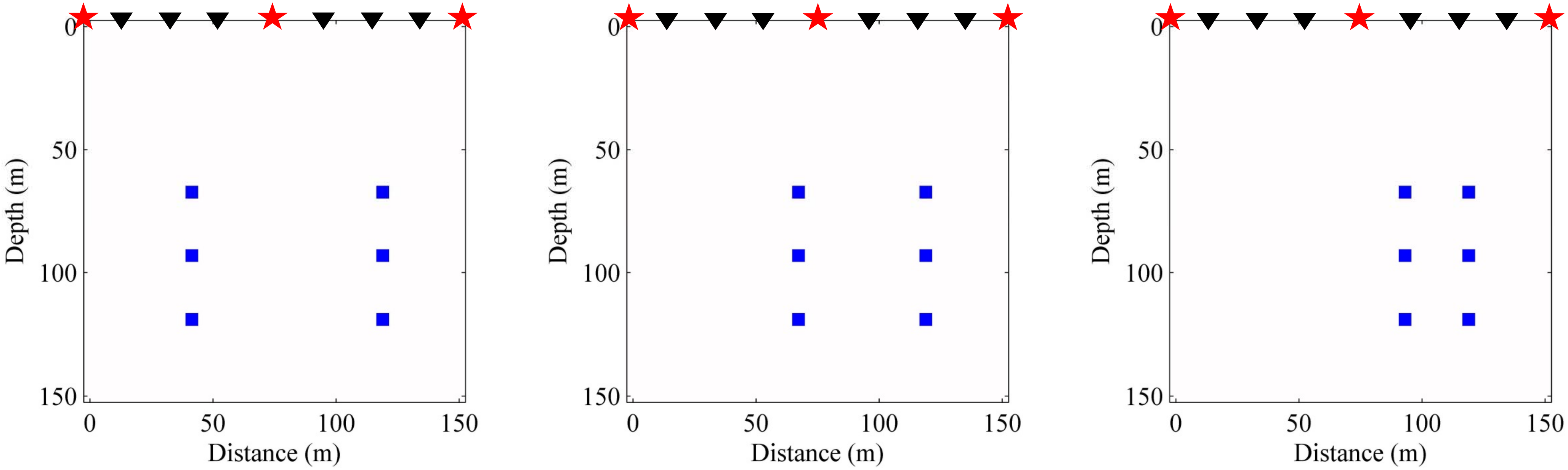
Numerical Illustrations

Gauss-Newton Updates:



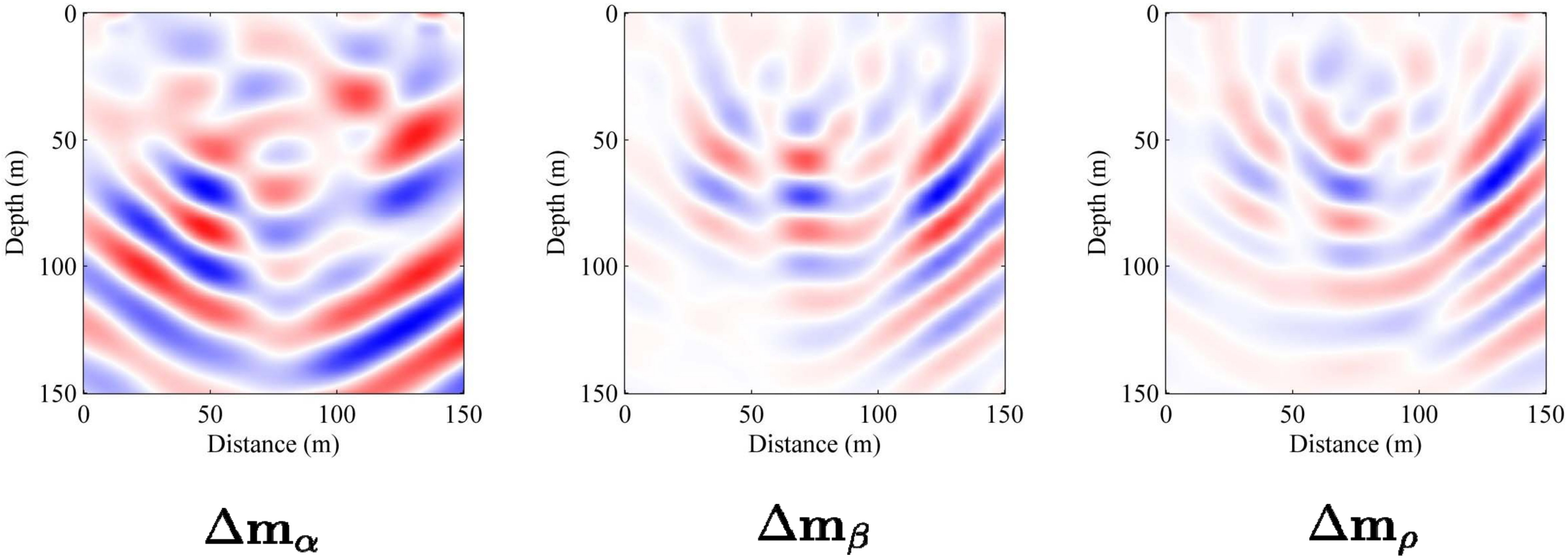
Numerical Illustrations

True Model:

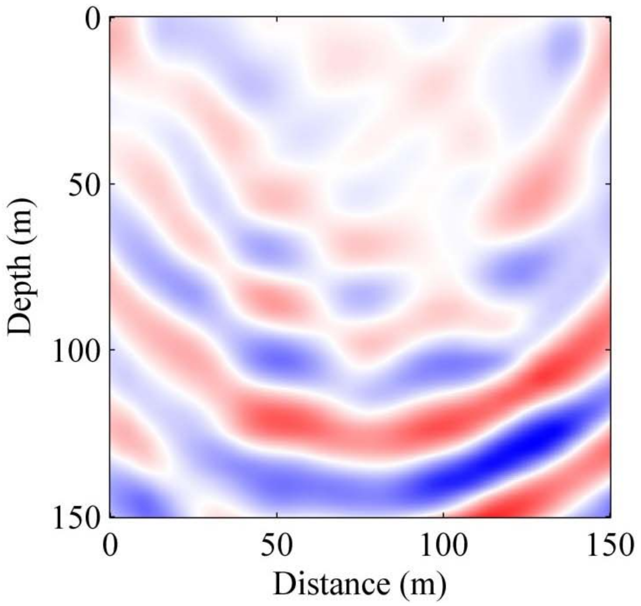


$\Delta \mathbf{m}_\alpha$ $\Delta \mathbf{m}_\beta$ $\Delta \mathbf{m}_\rho$

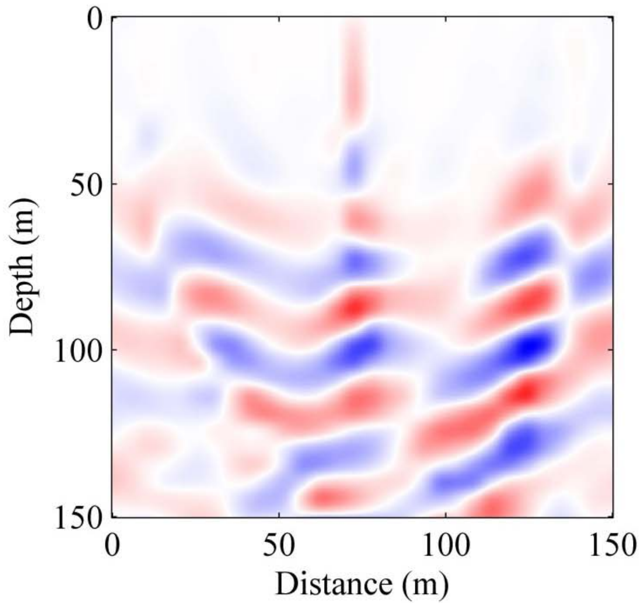
Reflection survey (vertical componnet and steepest descent):



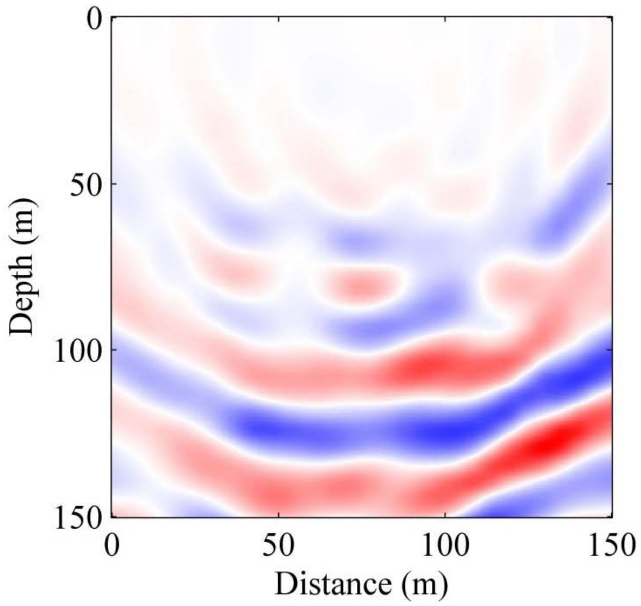
Reflection survey (vertical component and parameter-type):



$\Delta \mathbf{m}_{\alpha}$



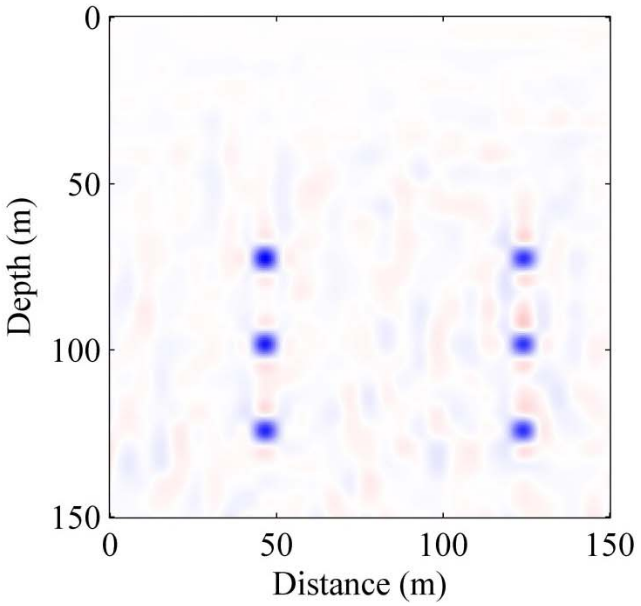
$\Delta \mathbf{m}_{\beta}$



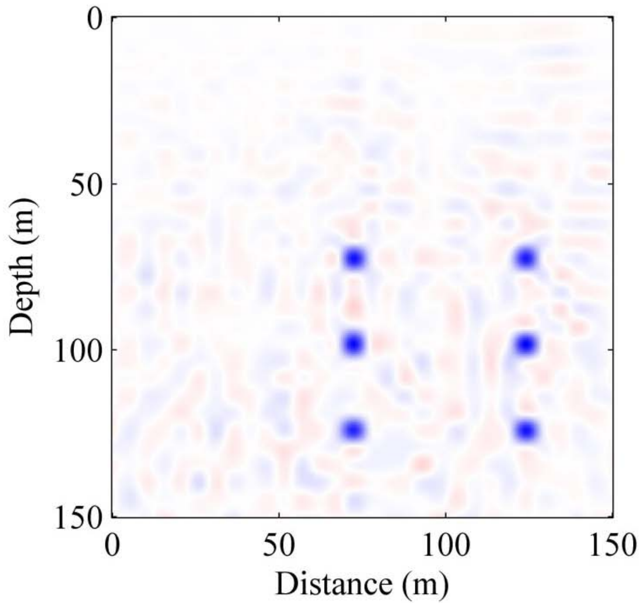
$\Delta \mathbf{m}_{\rho}$

Numerical Illustrations

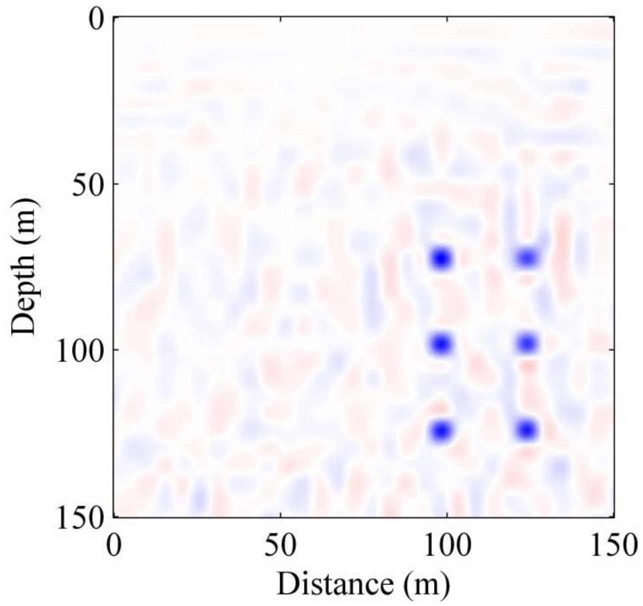
Reflection survey (vertical component and Gauss-Newton):



$\Delta \mathbf{m}_\alpha$

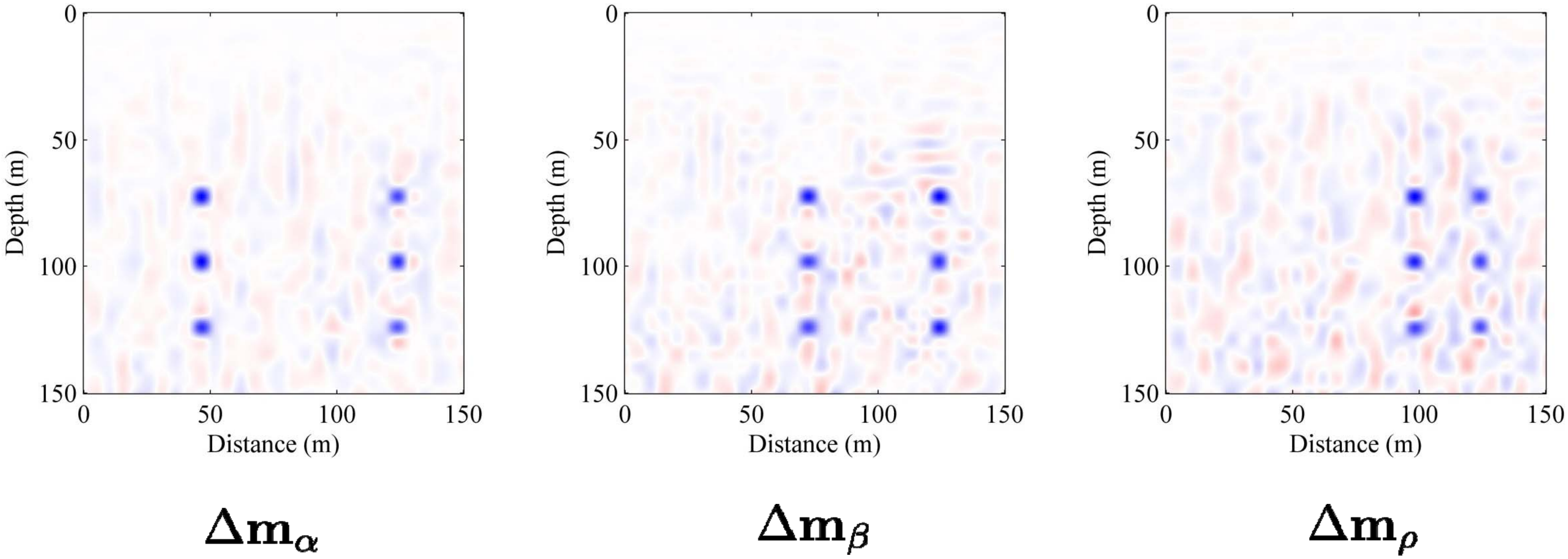


$\Delta \mathbf{m}_\beta$



$\Delta \mathbf{m}_\rho$

Reflection survey (horizontal component and Gauss-Newton):



2. 2 The Influences of Different Parameterizations and Acquisition Geometry

The Parameterization Issue for Elastic FWI

- The parameterization in isotropic and elastic media:

$$\mathbf{m}_1,\ \mathbf{m}_2,\ \rho$$

Velocity Parameterization: $\alpha,\ \beta,\ \rho$

Lamé Constants Parameterization: $\lambda,\ \mu,\ \rho$

Impedance Parameterization: $I_{\mathrm{p}},\ I_{\mathrm{s}},\ \rho$

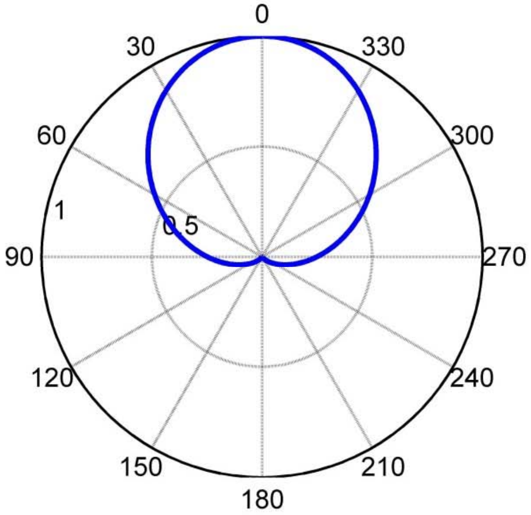
Other Parameterization

- **Which parameter class is most efficient for elastic FWI?**

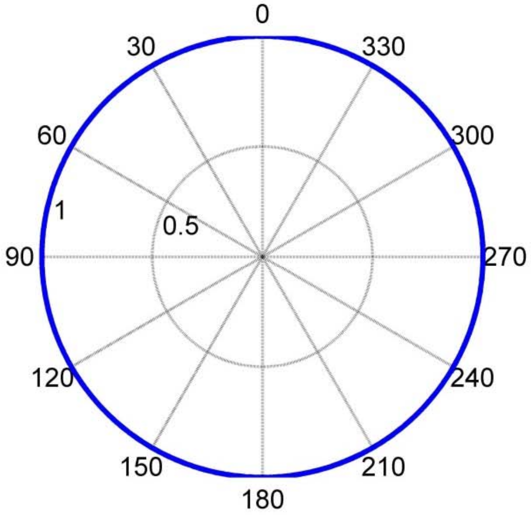
- **A good choice of parameterization will gives three different *scattering patterns* which are as *different* as possible, to allow easy identification of the parameters.**

P-P Scattering Patterns for Different Parameterizations

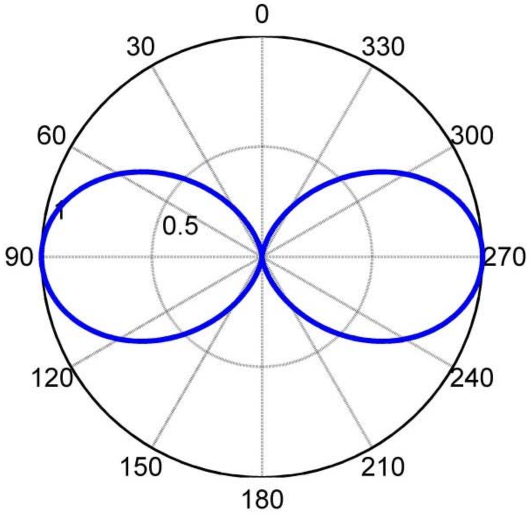
Velocity Parameterization: $\alpha,\ \beta,\ \rho$



$$\frac{\partial \mathbf{u}}{\partial \rho}$$



$$\frac{\partial \mathbf{u}}{\partial \alpha}$$

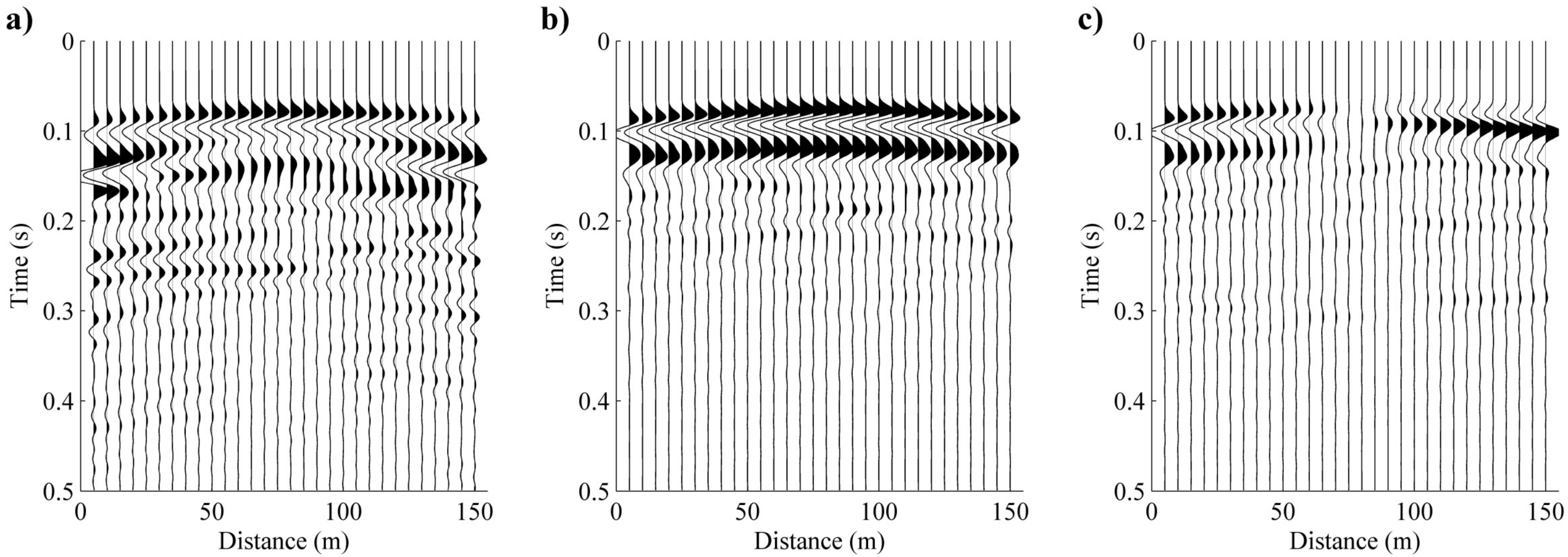


$$\frac{\partial \mathbf{u}}{\partial \beta}$$

Stolt, R. H. and A. B. Weglein. , 2012, Seismic imaging and inversion, Cambridge.

P-P Scattering Patterns for Different Parameterizations

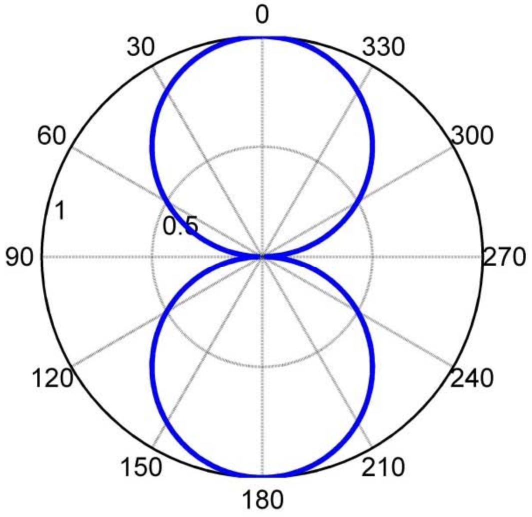
Velocity Parameterization: α, β, ρ



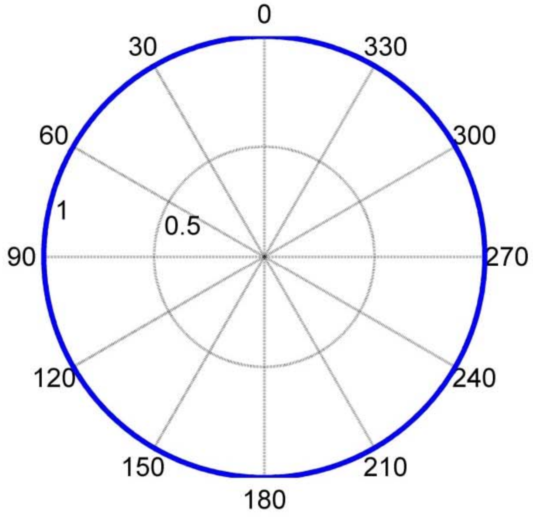
$$\frac{\partial \mathbf{u}^z}{\partial \rho} \qquad \frac{\partial \mathbf{u}^z}{\partial \alpha} \qquad \frac{\partial \mathbf{u}^z}{\partial \beta}$$

P-P Scattering Patterns for Different Parameterizations

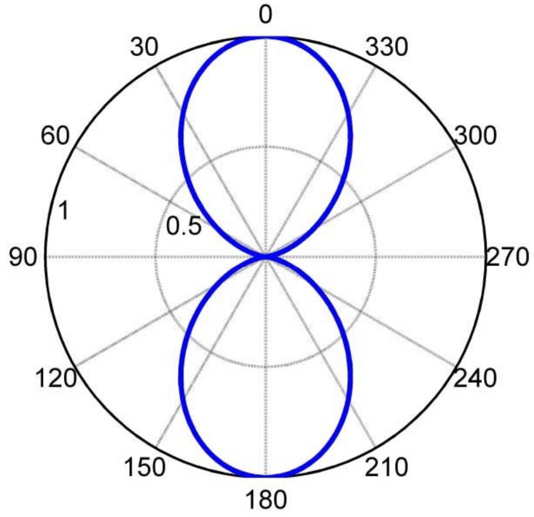
Lamé Constants Parameterization: $\lambda,\ \mu,\ \rho$



$$\frac{\partial \mathbf{u}}{\partial \rho}$$



$$\frac{\partial \mathbf{u}}{\partial \lambda}$$

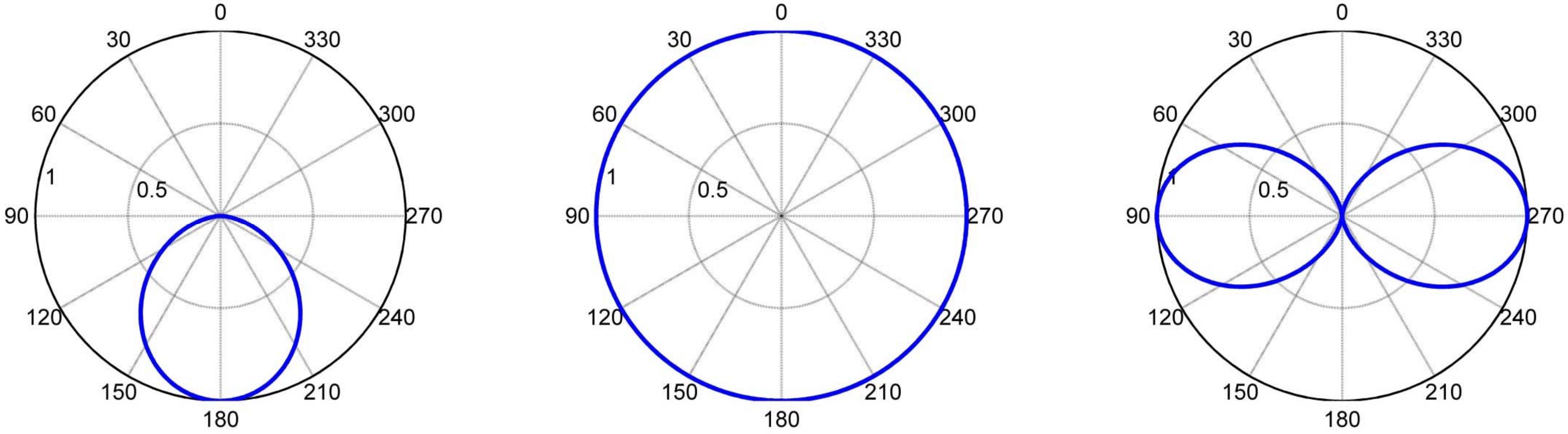


$$\frac{\partial \mathbf{u}}{\partial \mu}$$

Stolt, R. H. and A. B. Weglein. , 2012, Seismic imaging and inversion, Cambridge.

P-P Scattering Patterns for Different Parameterizations

Impedance Parameterization: $I_{\mathrm{p}},\ I_{\mathrm{s}},\ \rho$

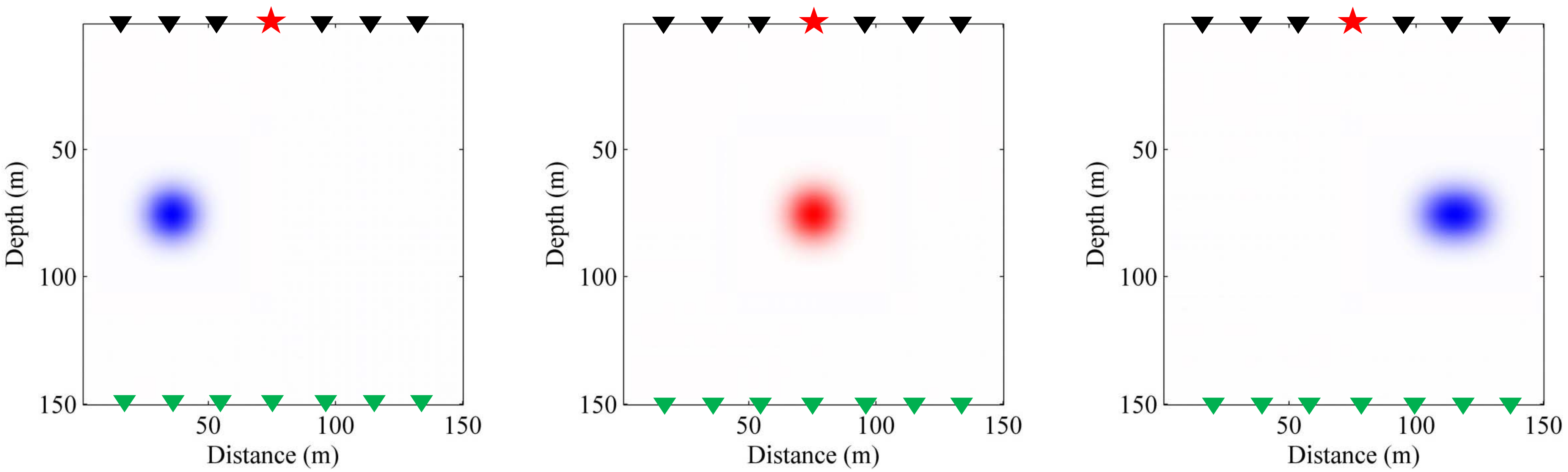


$$\frac{\partial \mathbf{u}}{\partial \rho} \qquad \frac{\partial \mathbf{u}}{\partial I_{\mathrm{p}}} \qquad \frac{\partial \mathbf{u}}{\partial I_{\mathrm{s}}}$$

Stolt, R. H. and A. B. Weglein. , 2012, Seismic imaging and inversion, Cambridge.

Numerical Illustrations

True Model:



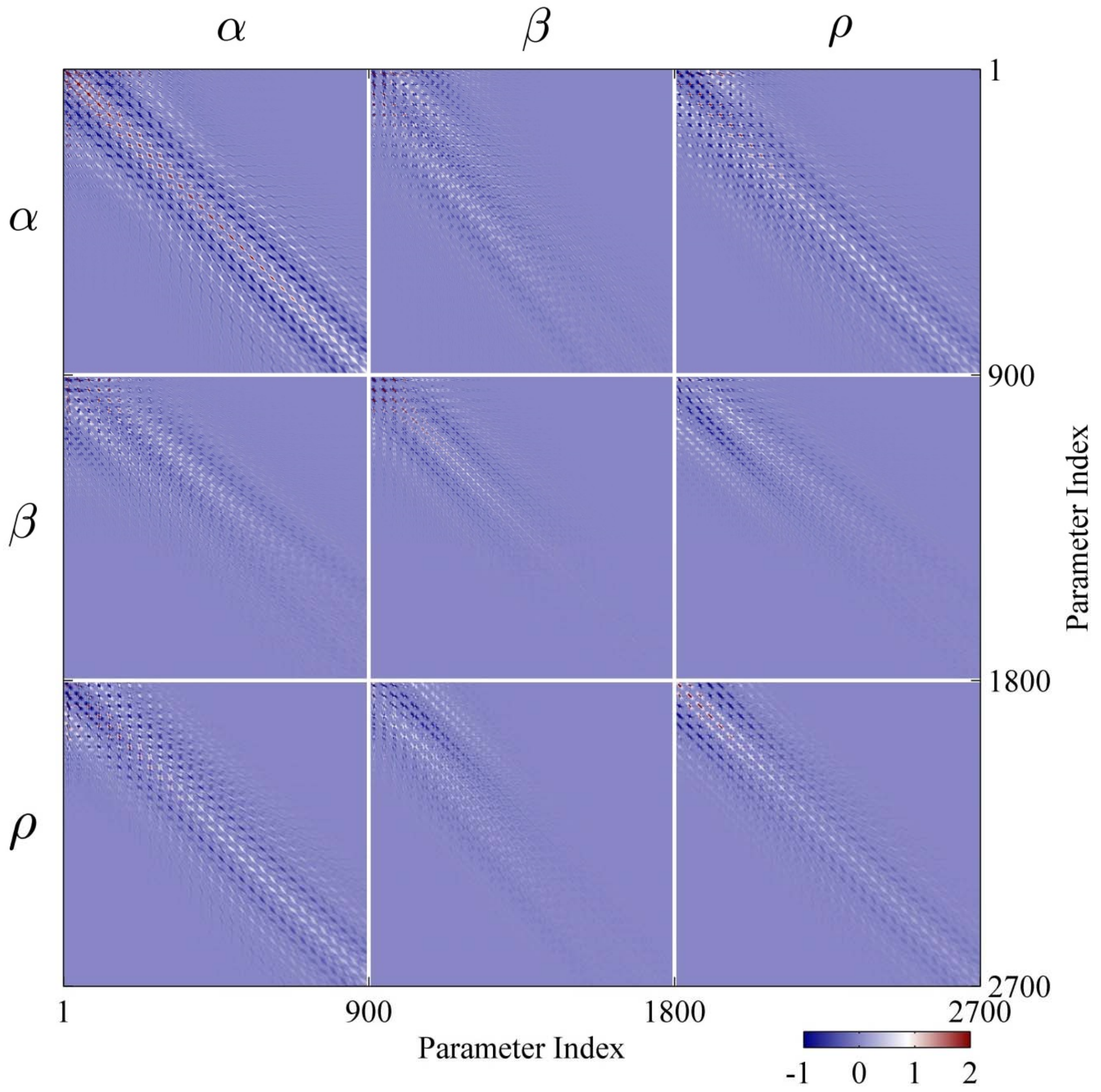
$\Delta \mathbf{m}_\alpha$ $\Delta \mathbf{m}_\beta$ $\Delta \mathbf{m}_\rho$

★ **Source** ▼ **Reflection Survey** ▼ **Transmission Survey**

Gauss-Newton Hessian Approximation

Reflection survey:

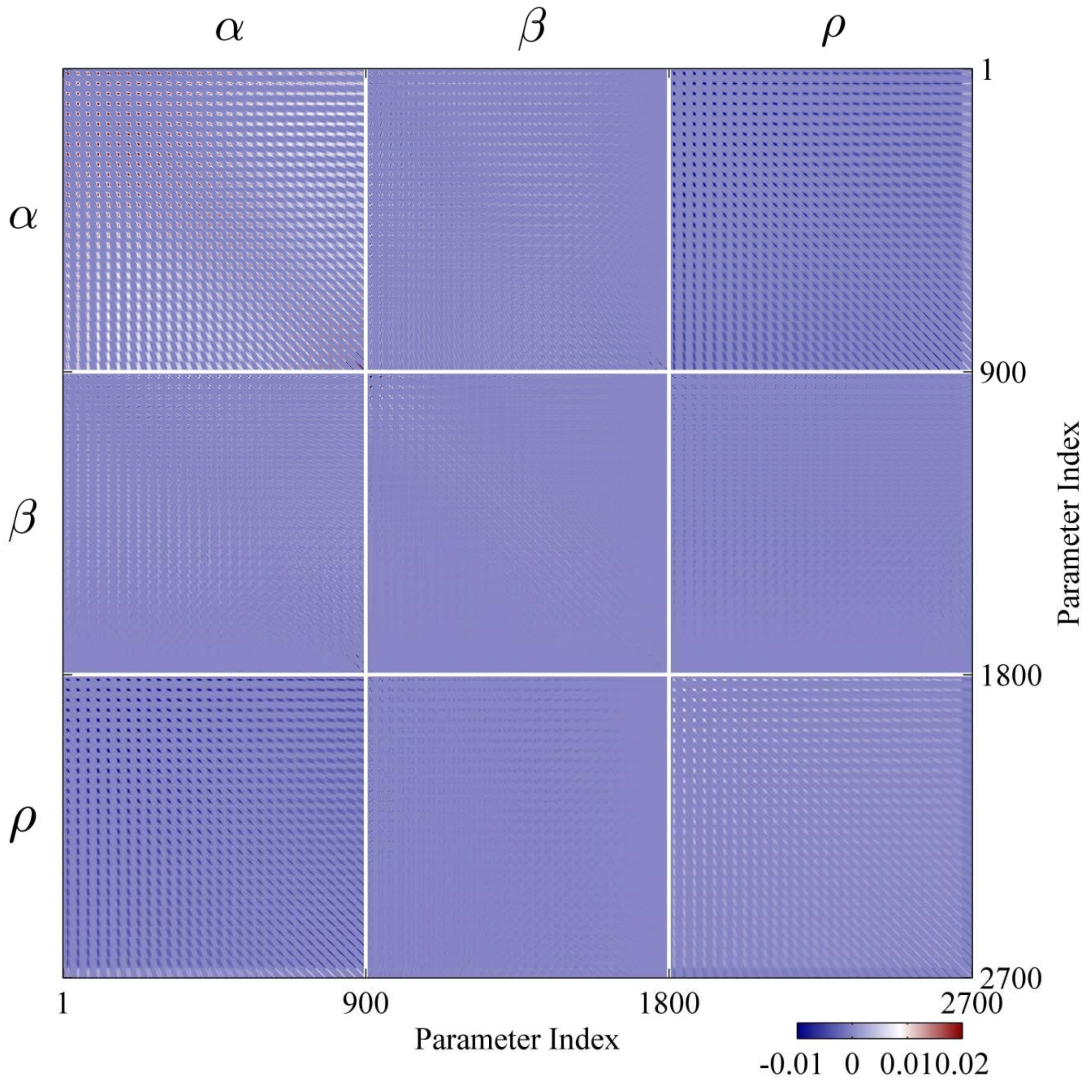
$$\tilde{\mathbf{H}}^{\mathbf{z}}_{\mathrm{vel}} = \begin{bmatrix} \tilde{\mathbf{H}}^{\mathbf{z}}_{\alpha\alpha} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\alpha\beta} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\alpha\rho} \\ \tilde{\mathbf{H}}^{\mathbf{z}}_{\beta\alpha} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\beta\beta} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\beta\rho} \\ \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho\alpha} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho\beta} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho\rho} \end{bmatrix}$$



Gauss-Newton Hessian Approximation

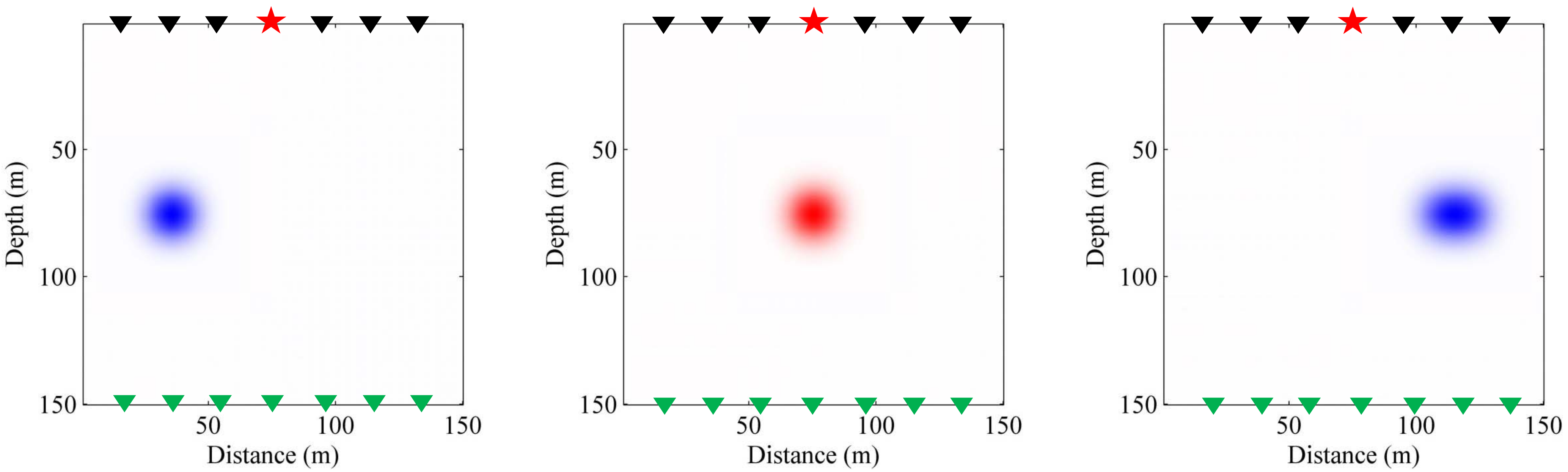
Transmission survey:

$$\tilde{\mathbf{H}}^{\mathrm{z}}_{\mathrm{vel}} = \begin{bmatrix} \tilde{\mathbf{H}}^{\mathrm{z}}_{\alpha\alpha} & \tilde{\mathbf{H}}^{\mathrm{z}}_{\alpha\beta} & \tilde{\mathbf{H}}^{\mathrm{z}}_{\alpha\rho} \\ \tilde{\mathbf{H}}^{\mathrm{z}}_{\beta\alpha} & \tilde{\mathbf{H}}^{\mathrm{z}}_{\beta\beta} & \tilde{\mathbf{H}}^{\mathrm{z}}_{\beta\rho} \\ \tilde{\mathbf{H}}^{\mathrm{z}}_{\rho\alpha} & \tilde{\mathbf{H}}^{\mathrm{z}}_{\rho\beta} & \tilde{\mathbf{H}}^{\mathrm{z}}_{\rho\rho} \end{bmatrix}$$



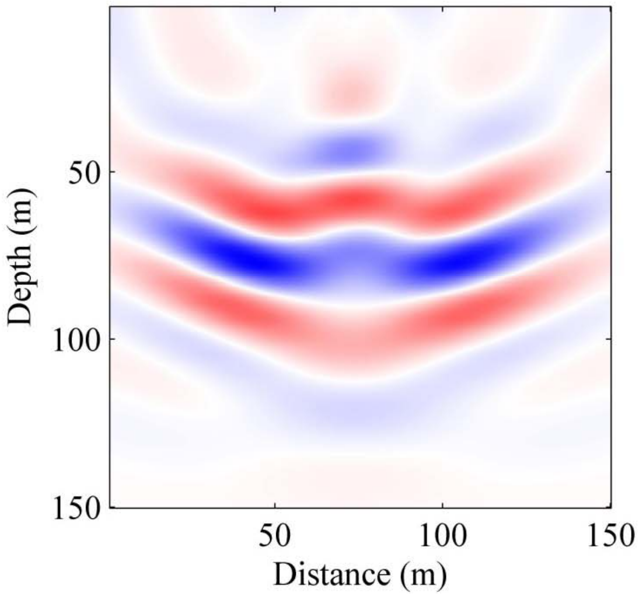
Numerical Illustrations

True Model:

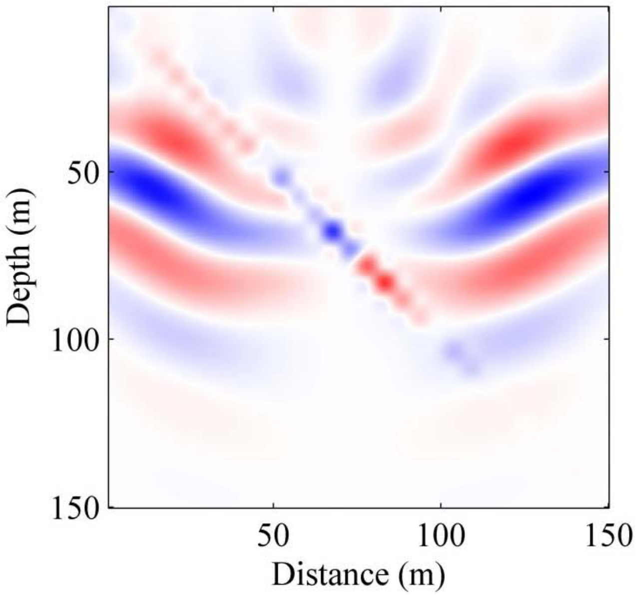


$\Delta \mathbf{m}_{\alpha}$ $\Delta \mathbf{m}_{\beta}$ $\Delta \mathbf{m}_{\rho}$

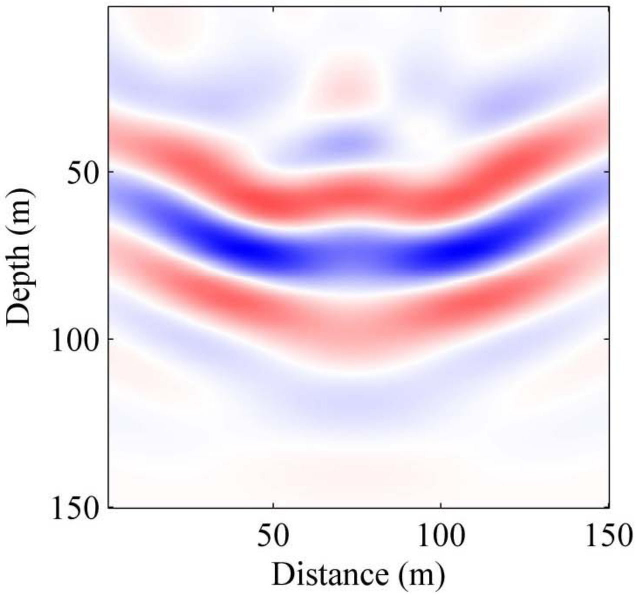
Reflection Survey (Gradient Updates):



$\Delta \mathbf{m}_\alpha$



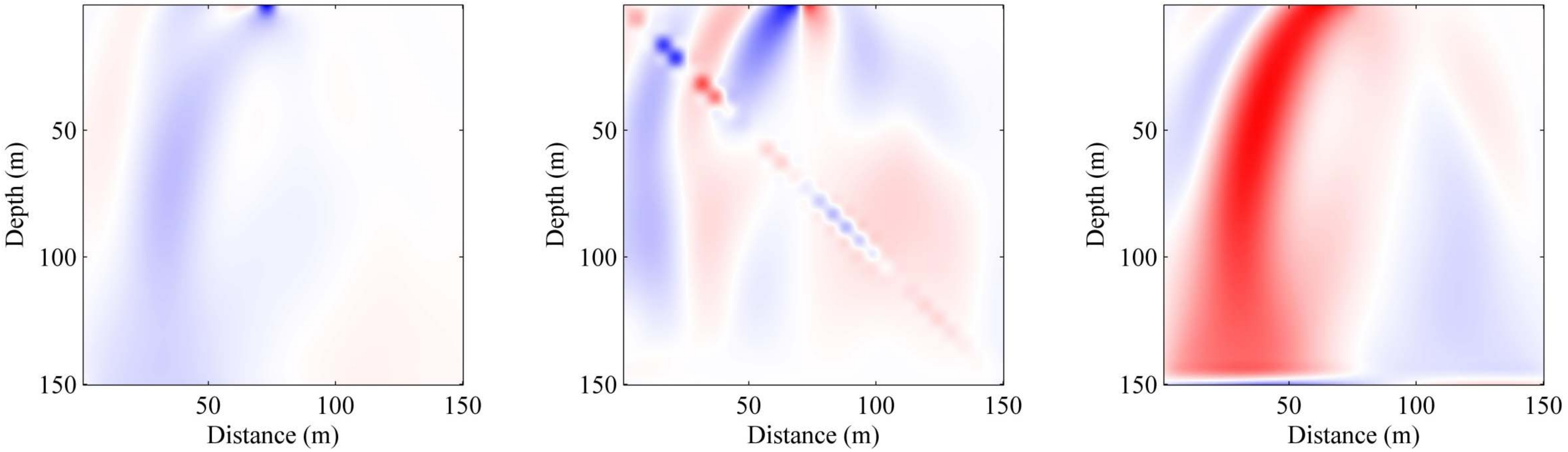
$\Delta \mathbf{m}_\beta$



$\Delta \mathbf{m}_\rho$

Numerical Illustrations

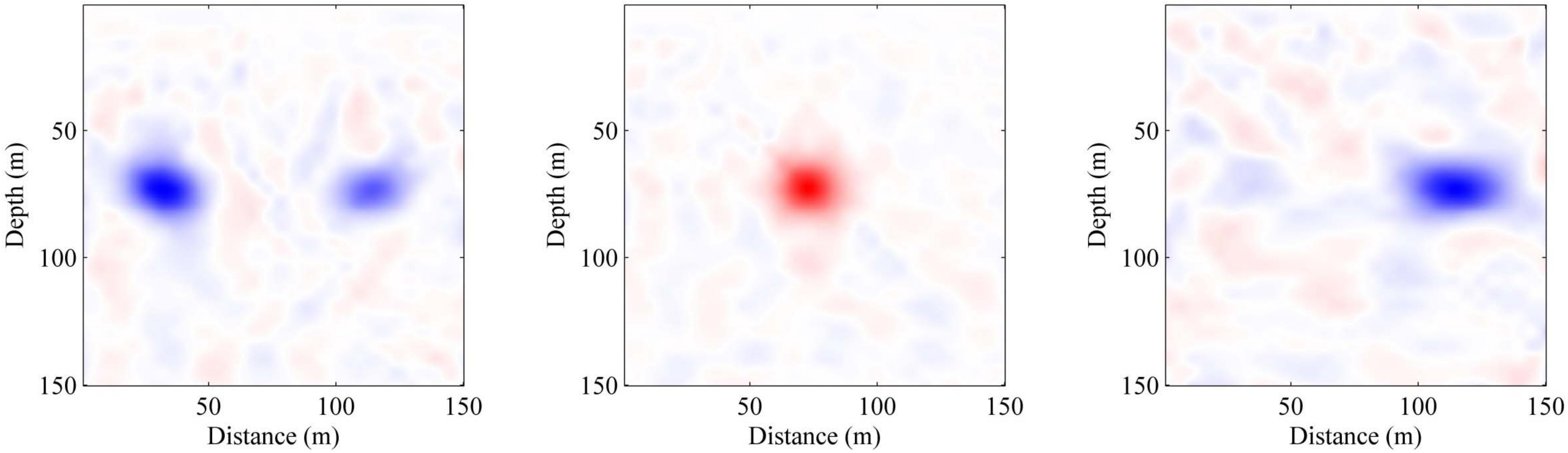
Transmission Survey (Gradient Updates):



$\Delta \mathbf{m}_\alpha$ $\Delta \mathbf{m}_\beta$ $\Delta \mathbf{m}_\rho$

Numerical Illustrations

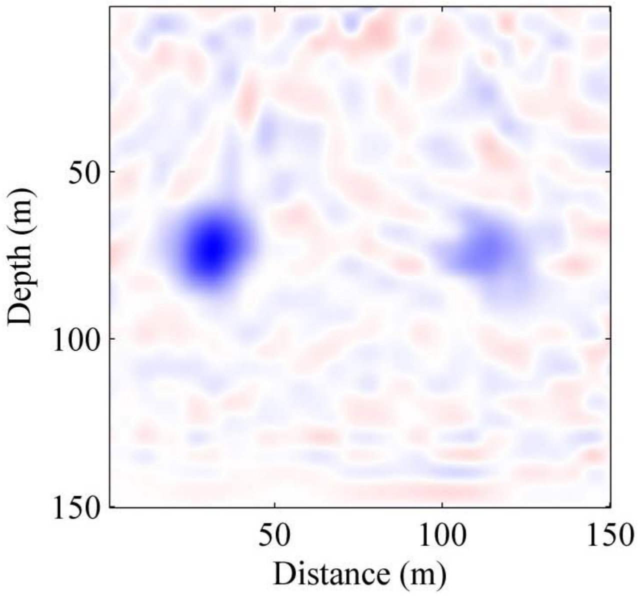
Reflection Survey (Gauss-Newton Updates):



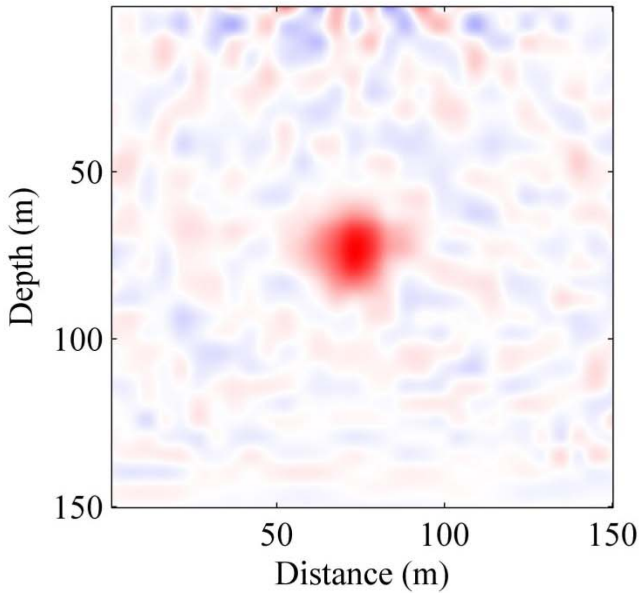
$\Delta \mathbf{m}_\alpha$ $\Delta \mathbf{m}_\beta$ $\Delta \mathbf{m}_\rho$

Numerical Illustrations

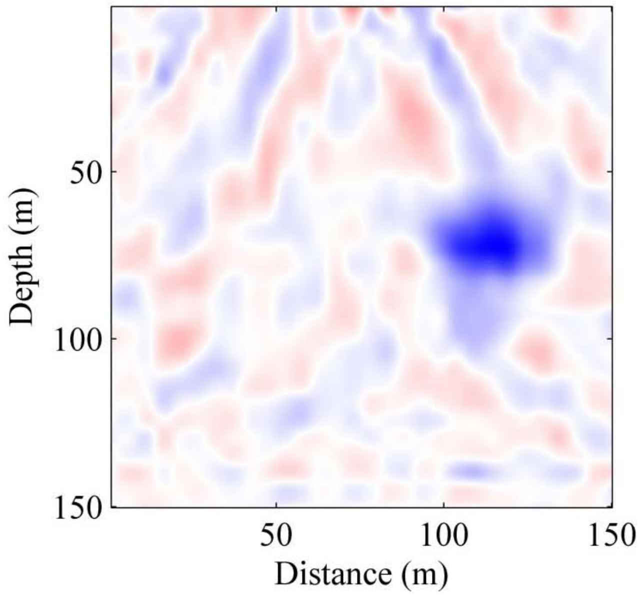
Transmission Survey (Gauss-Newton Updates):



$\Delta\mathbf{m}_{\alpha}$



$\Delta\mathbf{m}_{\beta}$

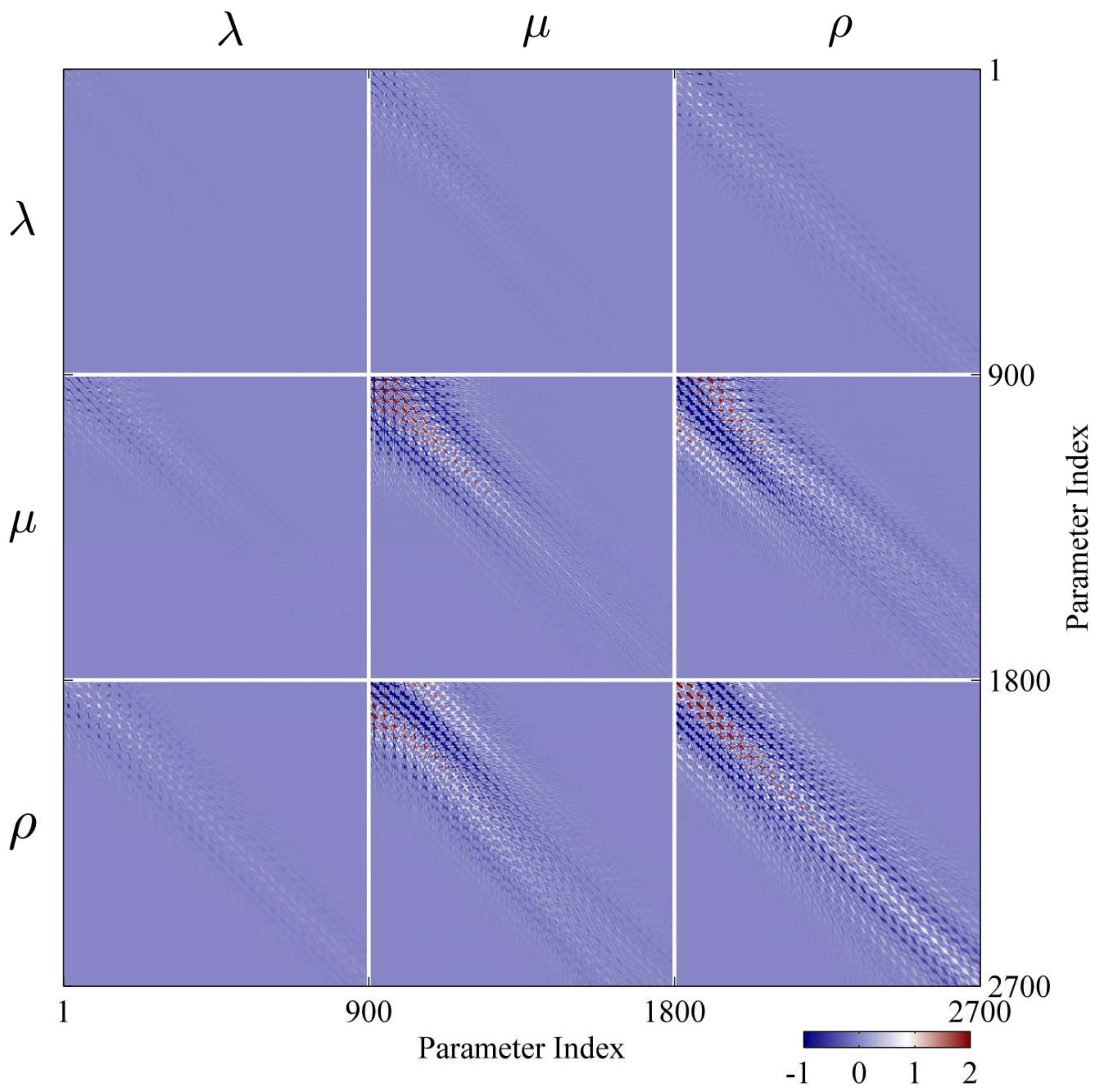


$\Delta\mathbf{m}_{\rho}$

Gauss-Newton Hessian Approximation

Reflection survey:

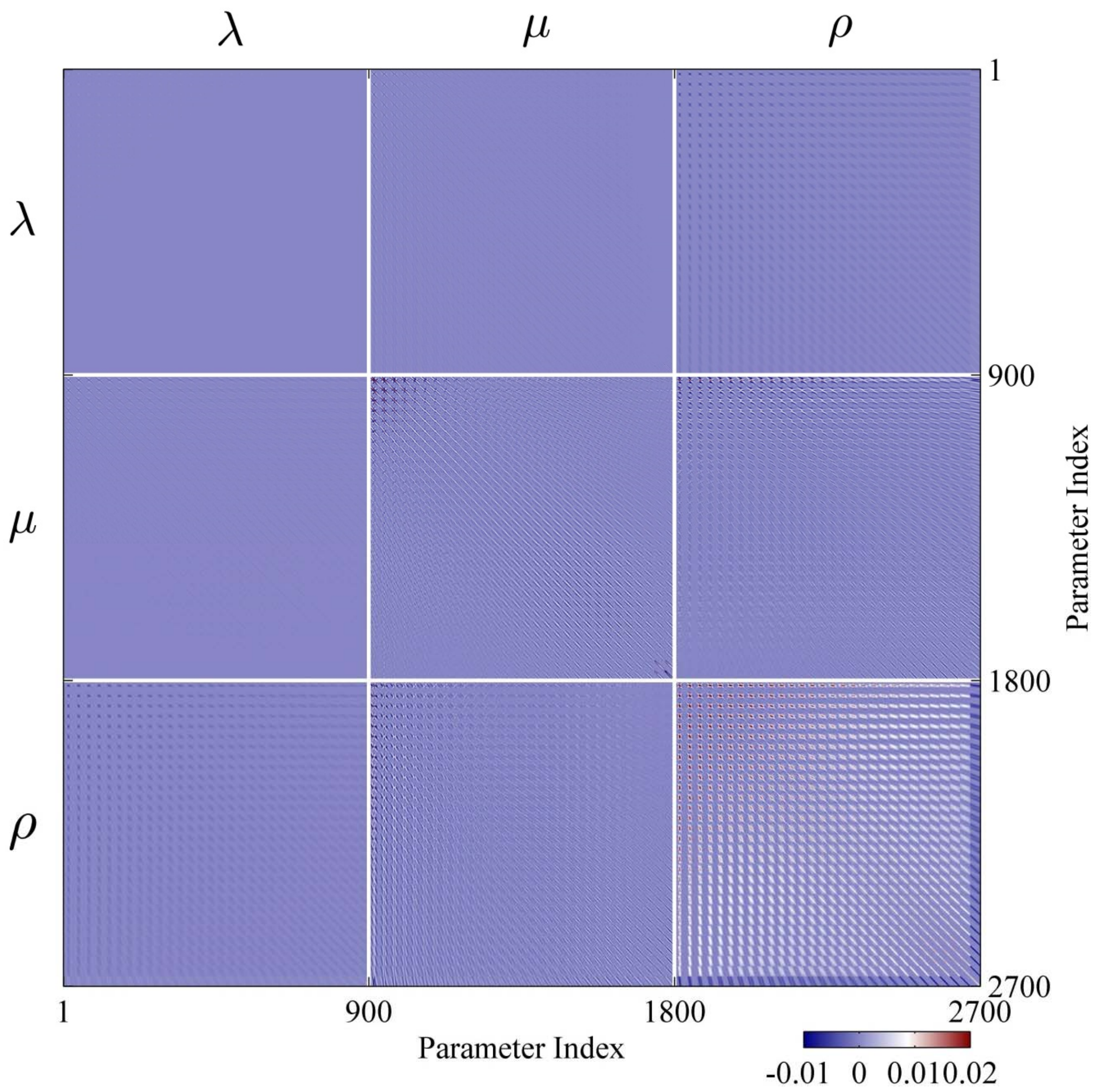
$$\tilde{\mathbf{H}}^{z}_{\text{lame}} = \begin{bmatrix} \tilde{\mathbf{H}}^{z}_{\lambda\lambda} & \tilde{\mathbf{H}}^{z}_{\lambda\mu} & \tilde{\mathbf{H}}^{z}_{\lambda\rho} \\ \tilde{\mathbf{H}}^{z}_{\mu\lambda} & \tilde{\mathbf{H}}^{z}_{\mu\mu} & \tilde{\mathbf{H}}^{z}_{\mu\rho} \\ \tilde{\mathbf{H}}^{z}_{\rho\lambda} & \tilde{\mathbf{H}}^{z}_{\rho\mu} & \tilde{\mathbf{H}}^{z}_{\rho\rho} \end{bmatrix}$$



Gauss-Newton Hessian Approximation

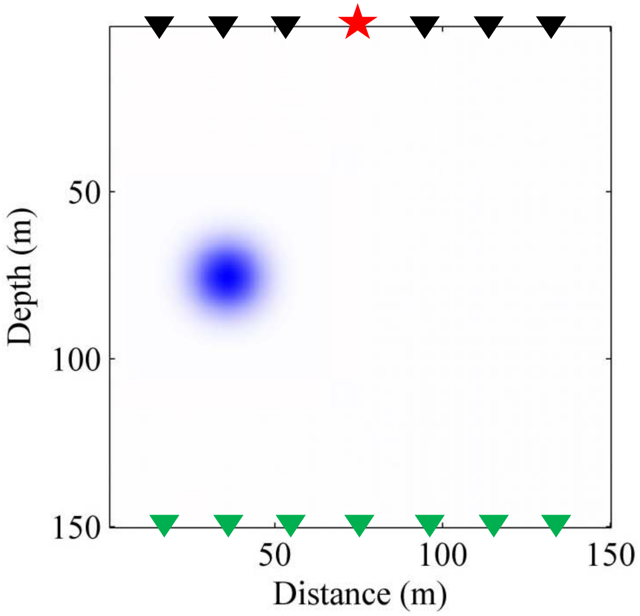
Transmission survey:

$$\tilde{\mathbf{H}}^{\mathbf{z}}_{\text{lame}} = \begin{bmatrix} \tilde{\mathbf{H}}^{\mathbf{z}}_{\lambda\lambda} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\lambda\mu} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\lambda\rho} \\ \tilde{\mathbf{H}}^{\mathbf{z}}_{\mu\lambda} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\mu\mu} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\mu\rho} \\ \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho\lambda} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho\mu} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho\rho} \end{bmatrix}$$

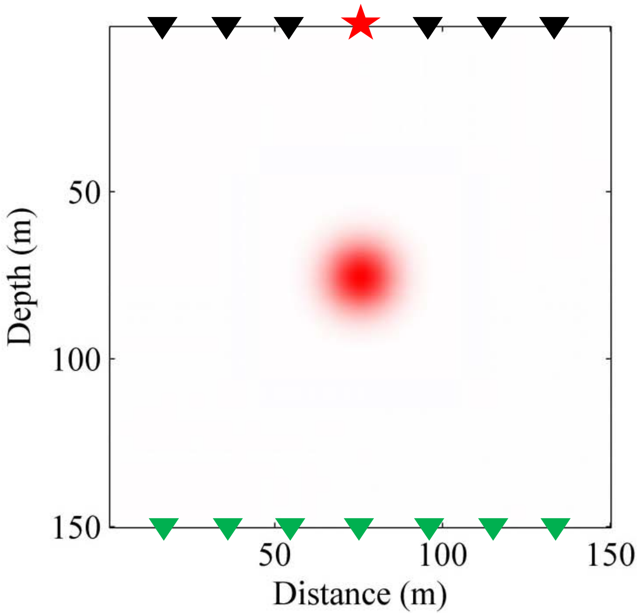


Numerical Illustrations

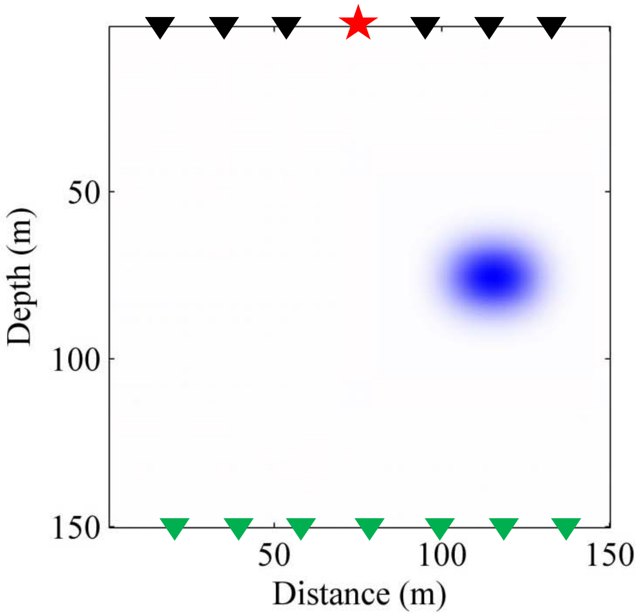
True Model:



$\Delta \mathbf{m}_\lambda$



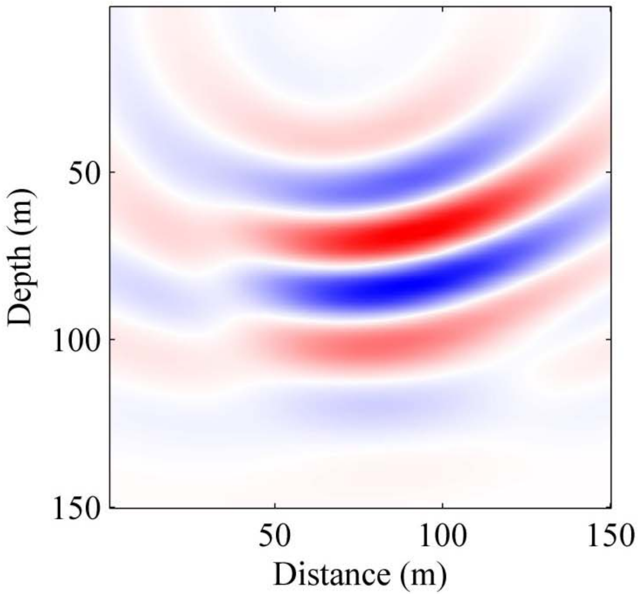
$\Delta \mathbf{m}_\mu$



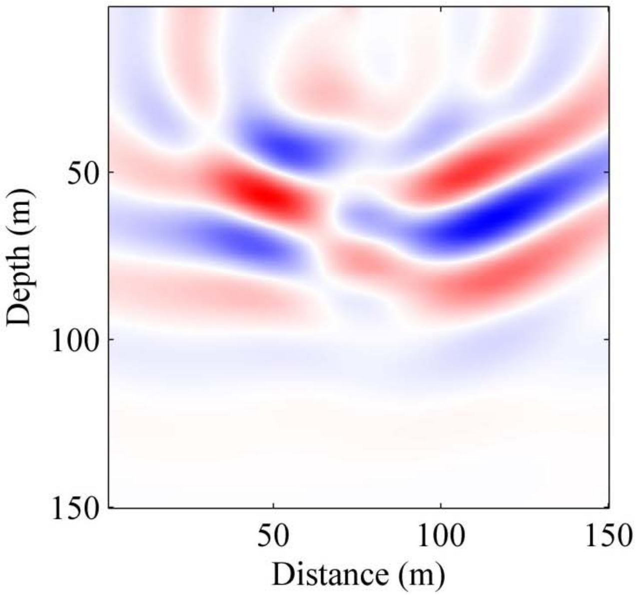
$\Delta \mathbf{m}_\rho$

Numerical Illustrations

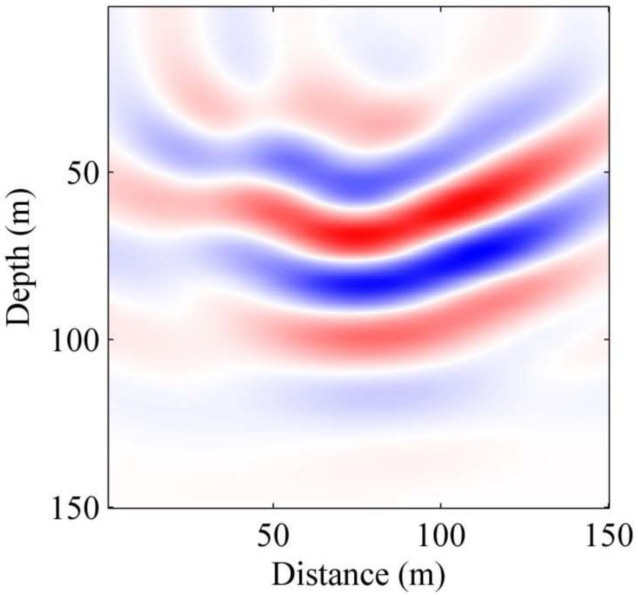
Reflection Survey (Gradient Updates):



$\Delta \mathbf{m}_\lambda$



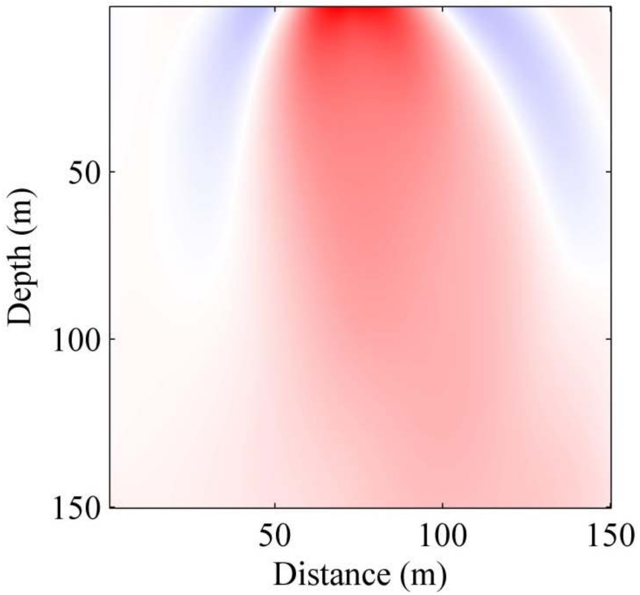
$\Delta \mathbf{m}_\mu$



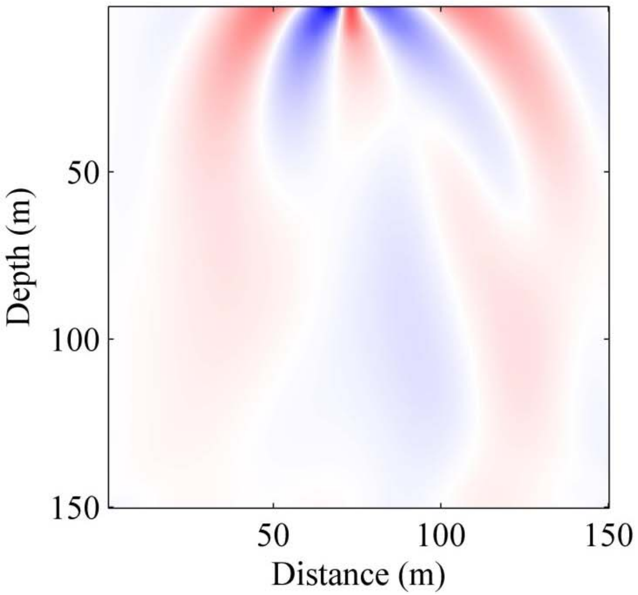
$\Delta \mathbf{m}_\rho$

Numerical Illustrations

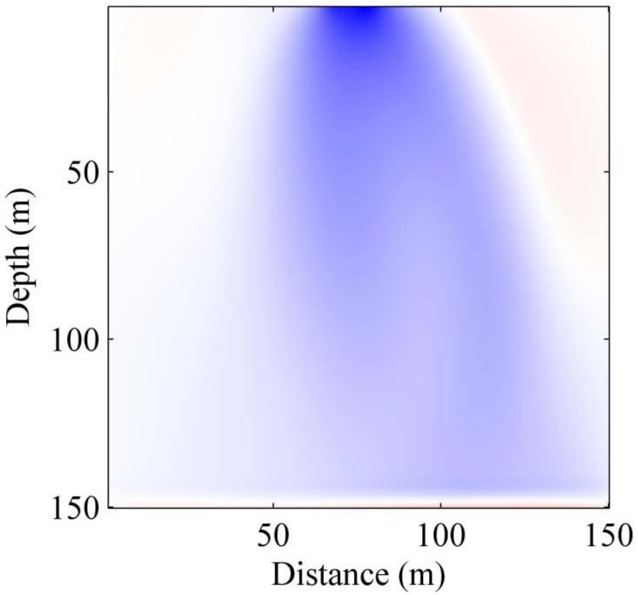
Transmission Survey (Gradient Updates):



$\Delta \mathbf{m}_{\lambda}$

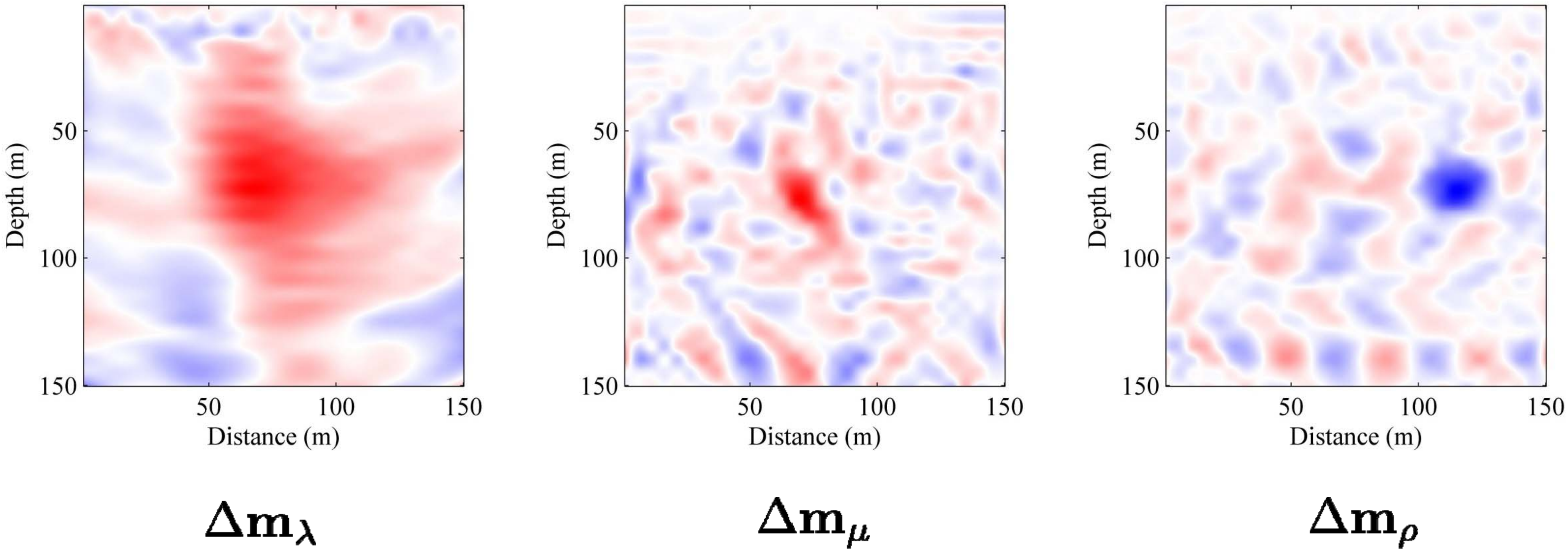


$\Delta \mathbf{m}_{\mu}$



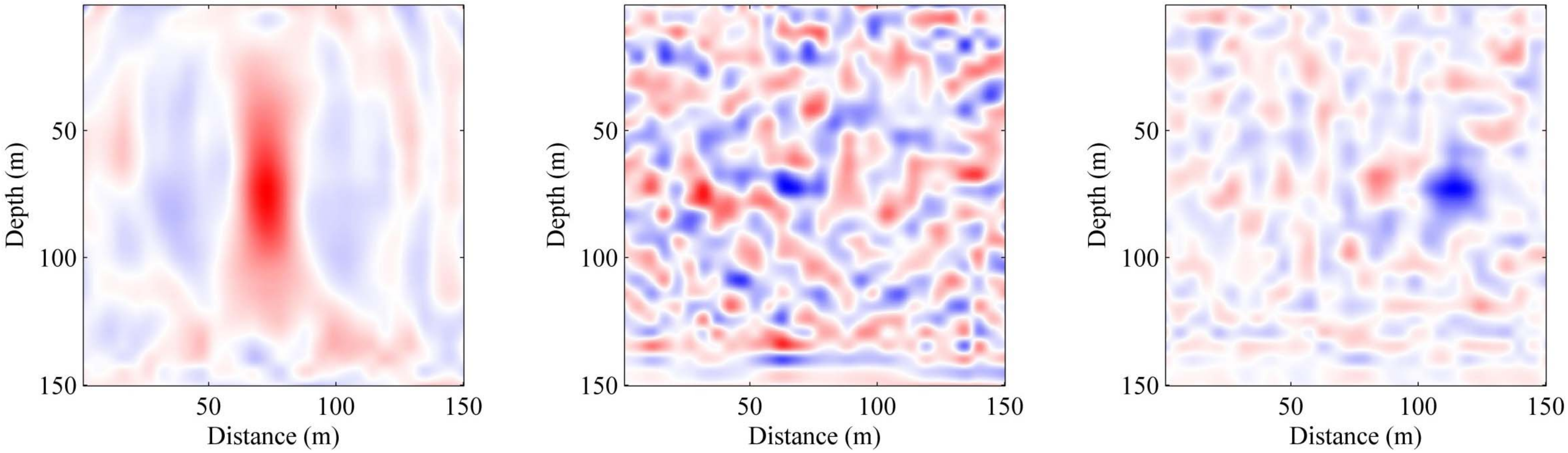
$\Delta \mathbf{m}_{\rho}$

Reflection Survey (Gauss-Newton Updates):



Numerical Illustrations

Transmission Survey (Gauss-Newton Updates):

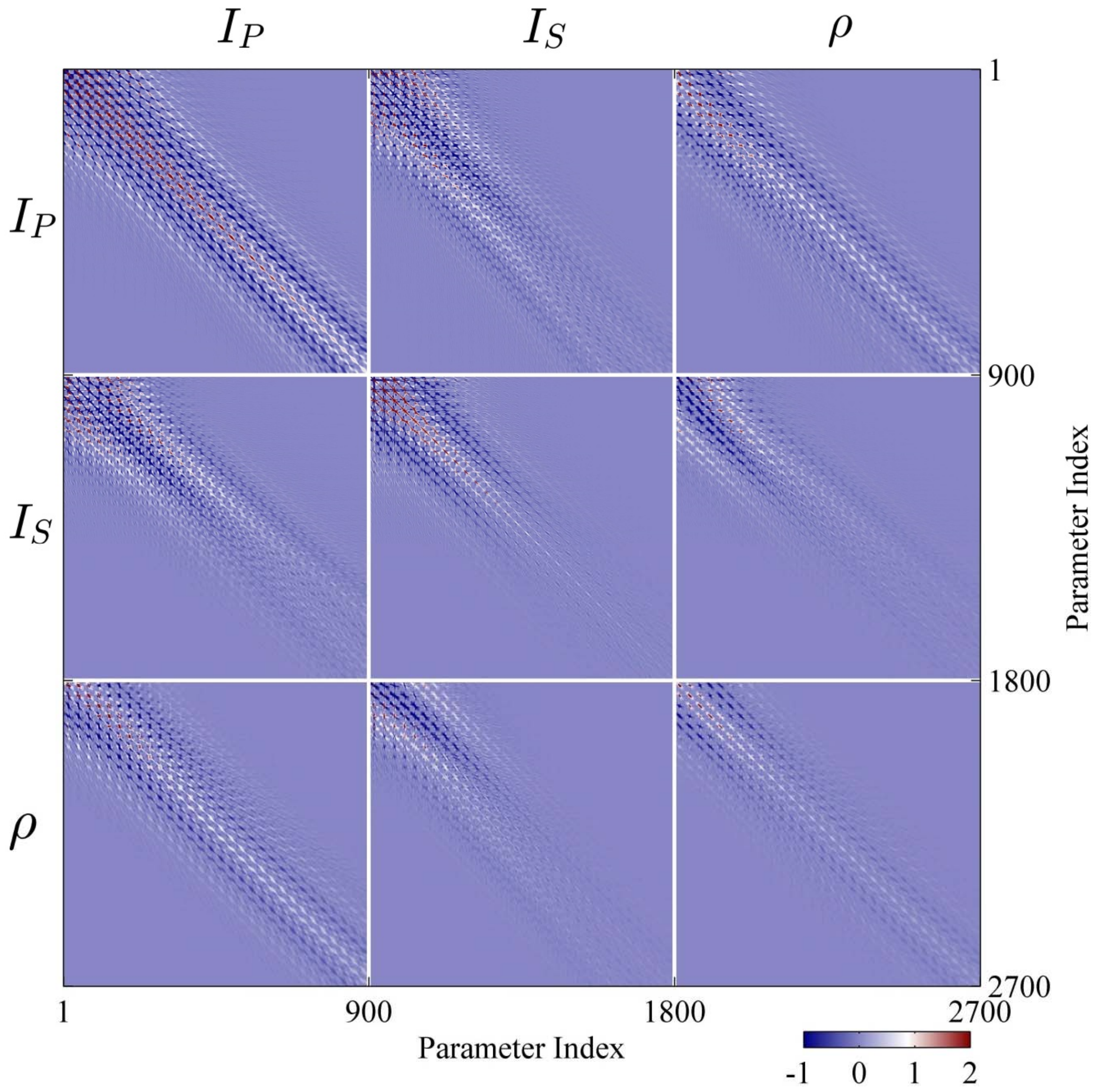


$\Delta \mathbf{m}_\lambda$ $\Delta \mathbf{m}_\mu$ $\Delta \mathbf{m}_\rho$

Gauss-Newton Hessian Approximation

Reflection survey:

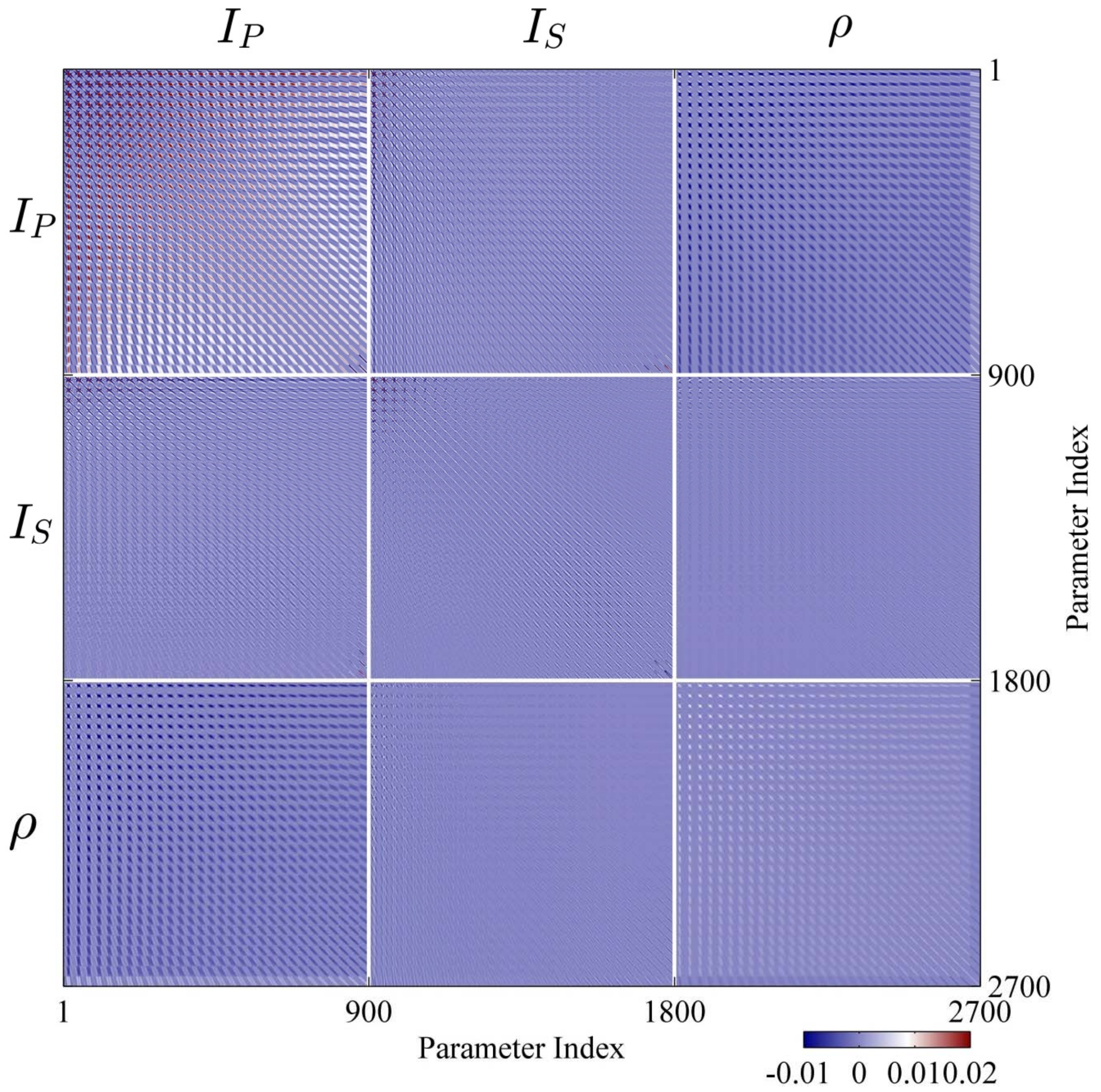
$$\tilde{\mathbf{H}}^{z}_{\mathrm{im}} = \begin{bmatrix} \tilde{\mathbf{H}}^{z}_{I_\mathrm{p} I_\mathrm{p}} & \tilde{\mathbf{H}}^{z}_{I_\mathrm{p} I_\mathrm{s}} & \tilde{\mathbf{H}}^{z}_{I_\mathrm{p} \rho} \\ \tilde{\mathbf{H}}^{z}_{I_\mathrm{s} I_\mathrm{p}} & \tilde{\mathbf{H}}^{z}_{I_\mathrm{s} I_\mathrm{s}} & \tilde{\mathbf{H}}^{z}_{I_\mathrm{s} \rho} \\ \tilde{\mathbf{H}}^{z}_{\rho I_\mathrm{p}} & \tilde{\mathbf{H}}^{z}_{\rho I_\mathrm{s}} & \tilde{\mathbf{H}}^{z}_{\rho \rho} \end{bmatrix}$$



Gauss-Newton Hessian Approximation

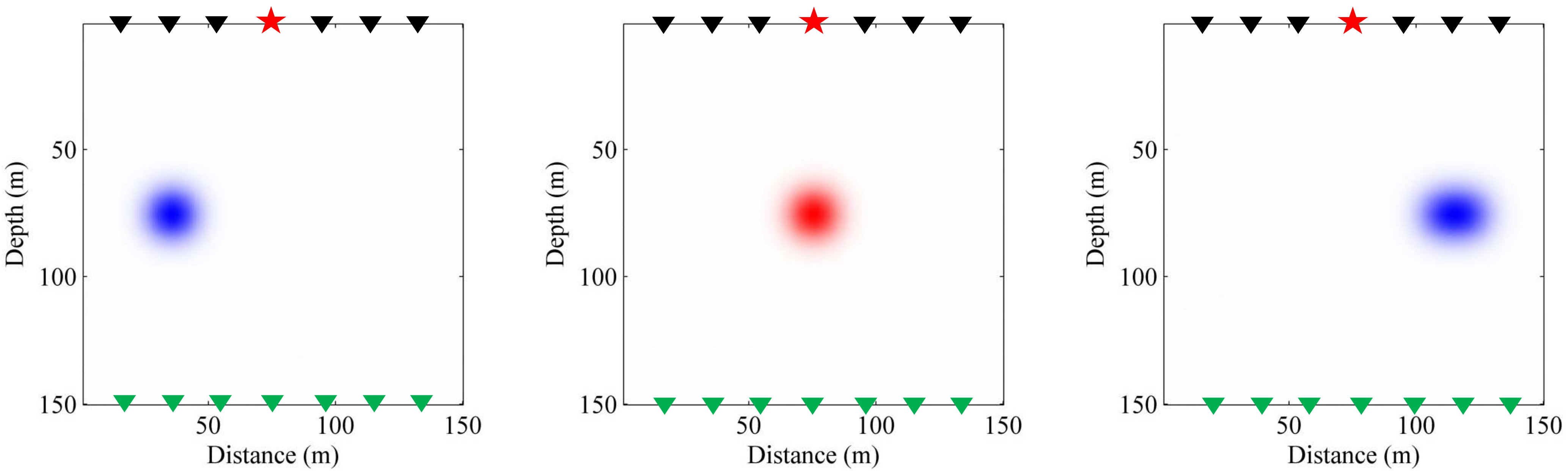
Transmission survey:

$$\tilde{\mathbf{H}}^{\mathbf{z}}_{\mathrm{im}} = \begin{bmatrix} \tilde{\mathbf{H}}^{\mathbf{z}}_{I_\mathrm{p}I_\mathrm{p}} & \tilde{\mathbf{H}}^{\mathbf{z}}_{I_\mathrm{p}I_\mathrm{s}} & \tilde{\mathbf{H}}^{\mathbf{z}}_{I_\mathrm{p}\rho} \\ \tilde{\mathbf{H}}^{\mathbf{z}}_{I_\mathrm{s}I_\mathrm{p}} & \tilde{\mathbf{H}}^{\mathbf{z}}_{I_\mathrm{s}I_\mathrm{s}} & \tilde{\mathbf{H}}^{\mathbf{z}}_{I_\mathrm{s}\rho} \\ \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho I_\mathrm{p}} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho I_\mathrm{s}} & \tilde{\mathbf{H}}^{\mathbf{z}}_{\rho\rho} \end{bmatrix}$$



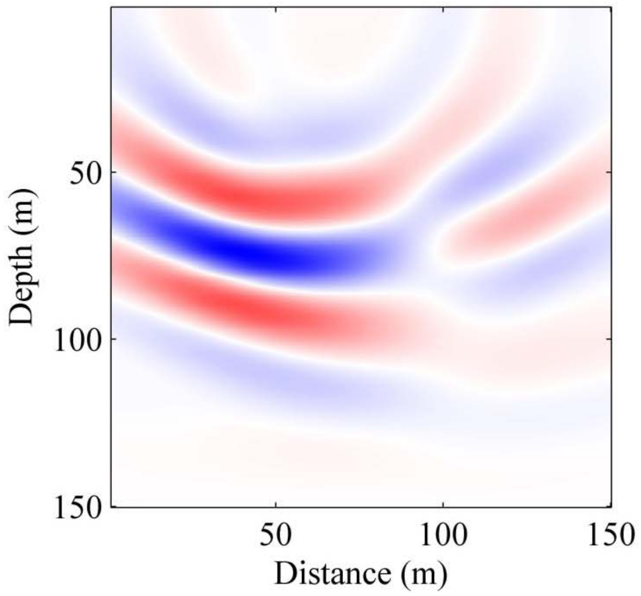
Numerical Illustrations

True Model:

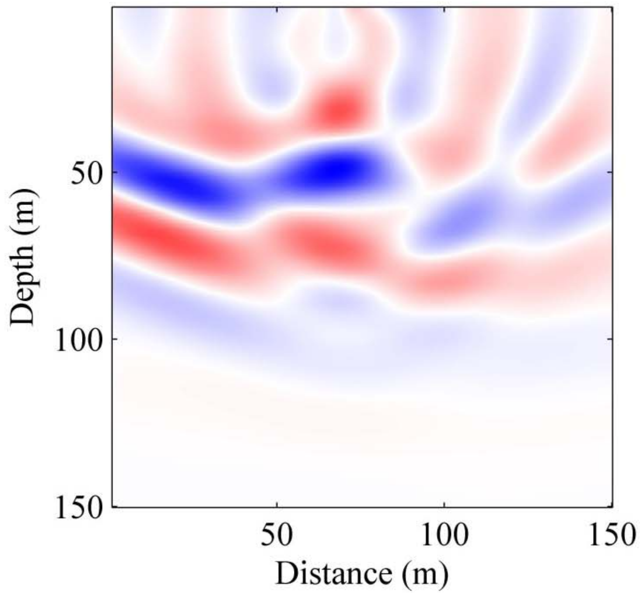


$\Delta \mathbf{m}_{I_p}$ $\Delta \mathbf{m}_{I_s}$ $\Delta \mathbf{m}_{\rho}$

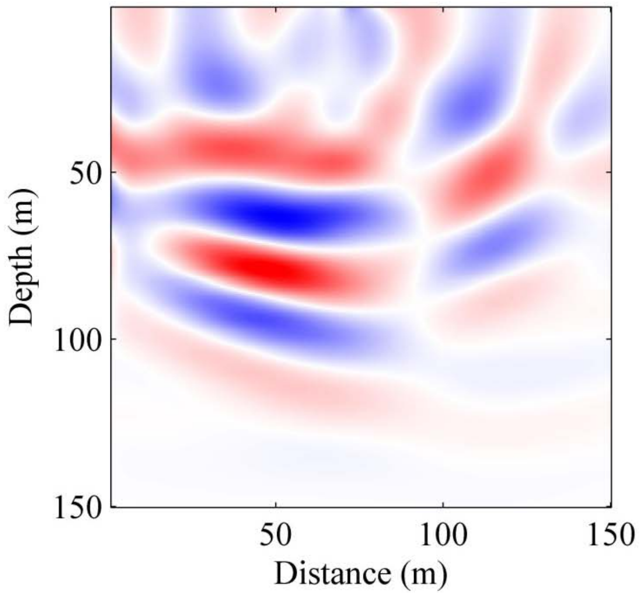
Reflection Survey (Gradient Updates):



$\Delta \mathbf{m}_{I_{\mathrm{p}}}$

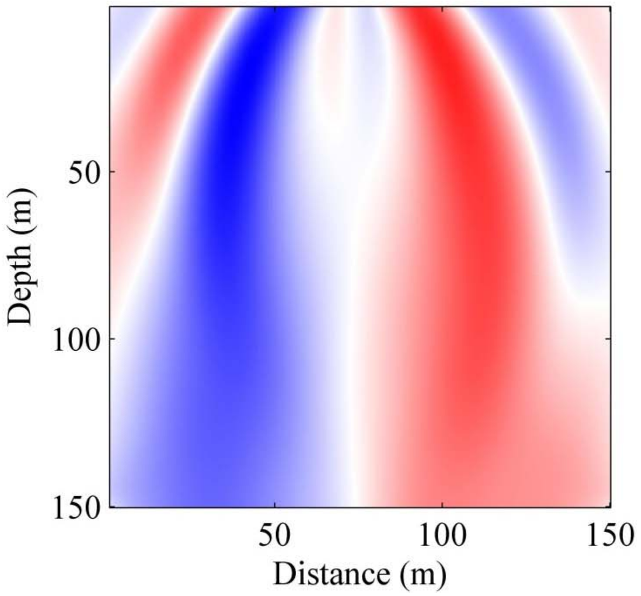


$\Delta \mathbf{m}_{I_{\mathrm{s}}}$

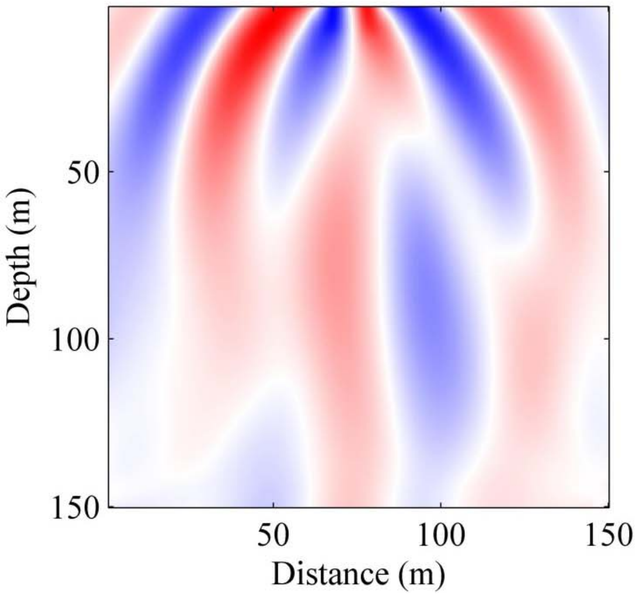


$\Delta \mathbf{m}_{\rho}$

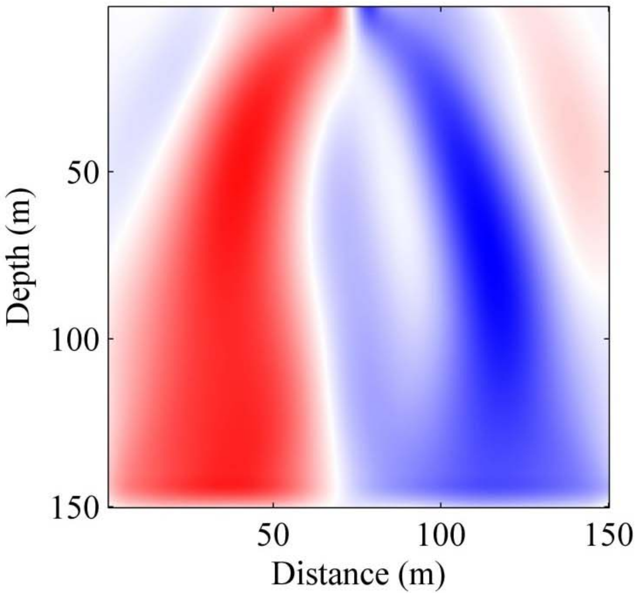
Transmission Survey (Gradient Updates):



$\Delta \mathbf{m}_{I_p}$

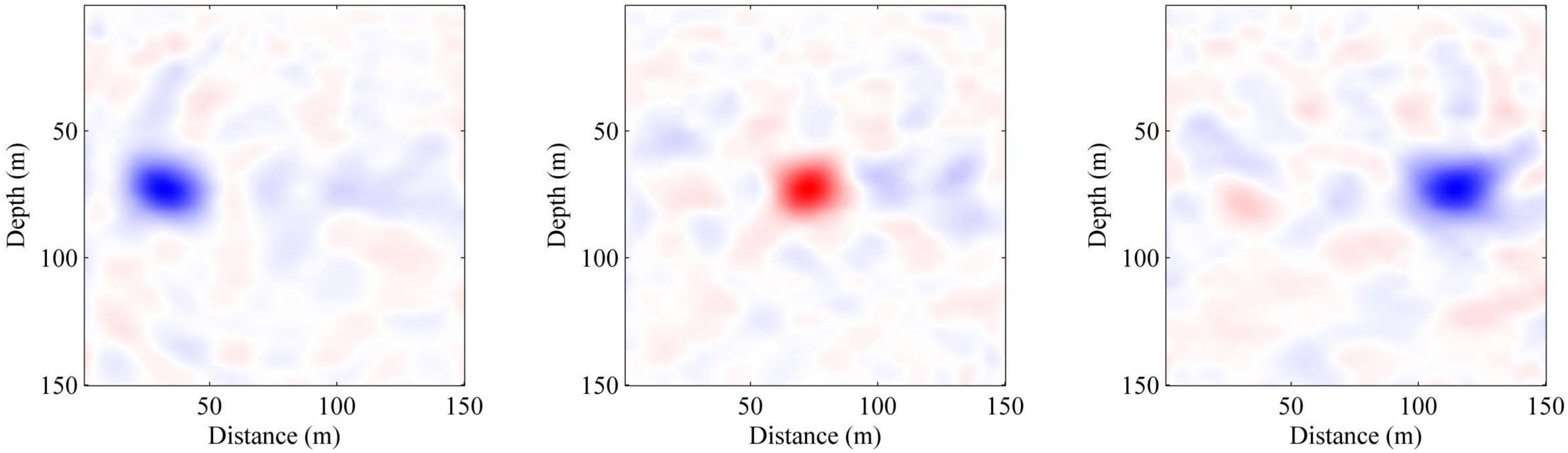


$\Delta \mathbf{m}_{I_s}$



$\Delta \mathbf{m}_{\rho}$

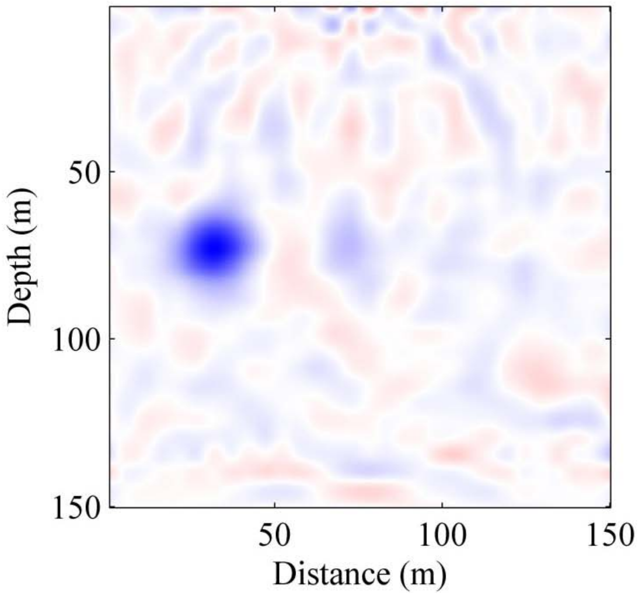
Reflection Survey (Gauss-Newton Updates):



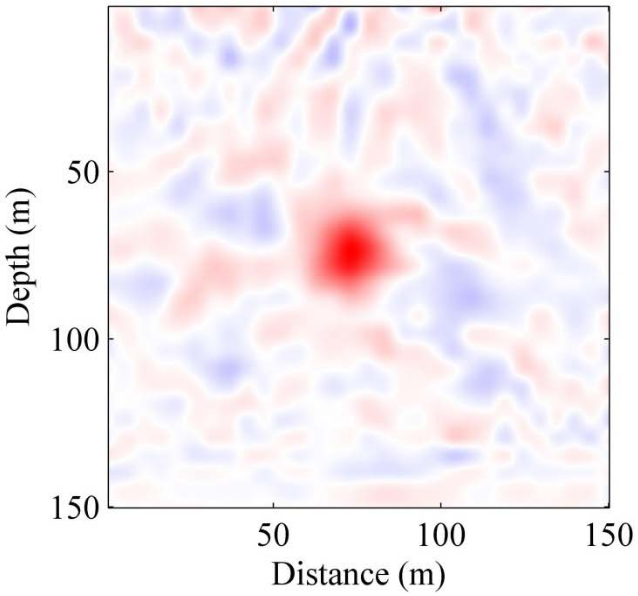
$\Delta \mathbf{m}_{I_p}$ $\Delta \mathbf{m}_{I_s}$ $\Delta \mathbf{m}_{\rho}$

Numerical Illustrations

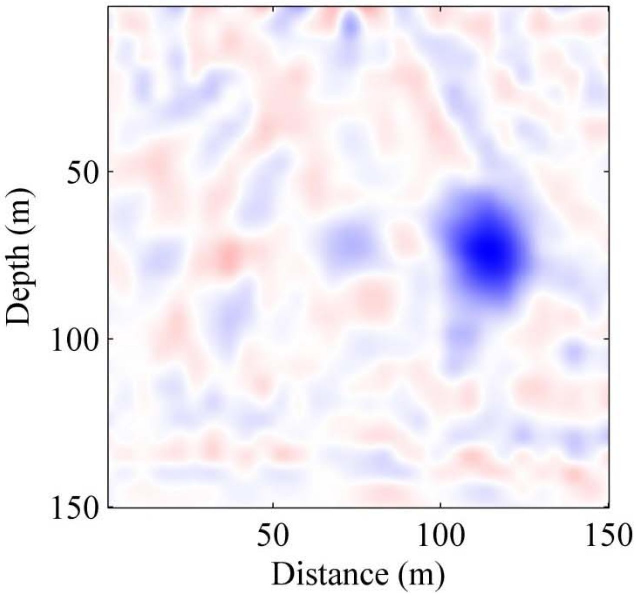
Transmission Survey (Gauss-Newton Updates):



$\Delta \mathbf{m}_{I_\mathrm{p}}$



$\Delta \mathbf{m}_{I_\mathrm{s}}$



$\Delta \mathbf{m}_{\rho}$

Conclusions

- **The parameter cross-talk problem exists in elastic FWI.**
- **The Gauss-Newton Hessian and parameter-type approximation can mitigate parameter cross-talk for elastic FWI.**
- **Proper acquisition geometry should be determined for reducing parameter cross-talk with gradient-based methods.**
- **Appropriate parameter class (impedance parameter class) should be selected for efficient inversion and avoiding parameter cross-talk.**

Acknowledgements

- ❑ **CREWES Sponsors and Researchers**
- ❑ **NSERC**
- ❑ **Junxiao Li**

Thanks!