geoconvention
Calgary · Canada · June 20-22 2022

Amplitude-encoding acoustic FWI using different bases

He Liu, Daniel Trad and Kristopher Innanen
University of Calgary

Summary

Full waveform inversion (FWI) is a promising tool to estimate high-resolution velocity models, but it suffers from expensive data acquisition and processing costs. To overcome these challenges, FWI using super-shot or blended data with source-encoding strategies have been proposed to accelerate FWI process. In previous work, we have presented the synthetic examples using amplitude-encoding strategy with Hartley and cosine bases. In both static- and dynamic-encoding experiments, amplitude-encoding strategy shows great performance and the latter one requires less number of super-shots. In this follow-up work, we further adopt sine and random polarity bases as the encoding functions. With dynamic-encoding process, the inversion results also show comparable imaging quality and convergency as in the conventional FWI case with further reduced computational effort.

Amplitude-encoding acoustic FWI

In conventional acoustic FWI, the objective function (data misfit function) is given by

$$E(\boldsymbol{m}) = \frac{1}{2}\mathbf{\Delta p}^{\dagger}\mathbf{\Delta p} = \frac{1}{2}||\mathbf{p}_{\mathrm{cal}} - \mathbf{p}_{\mathrm{obs}}||^2 \tag{1}$$

where $\Delta\mathbf{p}$, $\mathbf{p}_{\mathrm{obs}}$ and $\mathbf{p}_{\mathrm{cal}}$ denote the data misfit, the observed and the simulated data, respectively. In amplitude-encoding FWI, shot gathers are blended into super-shot gathers by

$$\mathbf{p}^{\mathrm{sup}} = \mathbf{Bp} \tag{2}$$

where B is the amplitude encoding matrix, which is defined as

$$\mathbf{B} = \begin{bmatrix} b^{1,1} & \cdots & b^{Nsig,1} \\ \vdots & \ddots & \vdots \\ b^{1,N_{sup}} & \cdots & b^{Nsig,N_{sup}} \end{bmatrix}_{N_{sup}\times N_{sig}} \tag{3}$$

where *Nsup* is the number of the super-shots and *Nsig* is the number of the individual shots (*Nsup* < *Nsig*).

The ratio between *Nsig* and *Nsup* is the factor by which the computational cost is reduced. Since usually *Nsup* is much smaller than *Nsig*, the encoding FWI would achieve much better efficiency due to the reduction of data dimension. Then the encoding objective function is given by:

$$E(\mathbf{m}) = \frac{1}{2}\mathbf{\Delta p}^{\dagger}\mathbf{\Delta p} = \frac{1}{2}||\mathbf{p}_{\mathrm{cal}} - \mathbf{p}_{\mathrm{obs}}||^2 = \frac{1}{2}(p_{cal} - p_{obs})\mathbf{B}^{\mathrm{T}}\mathbf{B}(p_{cal} - p_{obs}) \tag{4}$$

The matrix $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ is referred to as the crosstalk matrix, and when it's equal to the identity matrix, the encoding objective function is equal to the traditional objective function. FWI using blended data would produce the same results as in conventional FWI cases. Previously, we have used Hartley (Tsitsas, 2010) and cosine bases (Hu et al., 2016) as the encoding functions. In this follow-up work, we further adopt sine and random polarity bases defined in equation 5 and 6. The sine basis is defined as (Tsitsas, 2010):

$$\mathrm{b_{m,n}} = \sqrt{\frac{2}{n_{\mathrm{sig}}}} \sin\left(\frac{\left(m+\frac{1}{2}\right)\left(n+\frac{1}{2}\right)\pi}{n_{\mathrm{sig}}}\right) \tag{5}$$

The random polarity basis can be expressed as (Krebs et al., 2009):

$$\mathrm{b_{m,n}} = 1 \text{ or } -1$$

where *m* = 1,…, *Nsig* is the shot-index, *n* = 1,…, *Nsup* is the super-shot index, and *nsig* is the periodization index, which we set to be half of *Nsig*.

Synthetic examples

In previous work, we have already presented the synthetic examples using Hartley and cosine bases (Liu et al., 2021). In this follow-up work, we use the same Marmousi model shown in Fig 1a to generate 140 shots. The initial model for inversion is shown in Fig 1b.

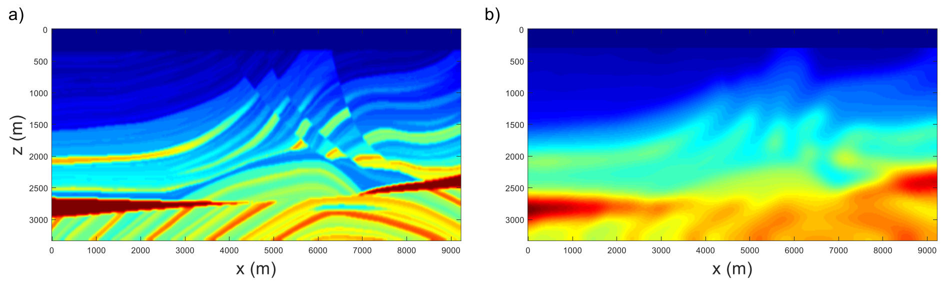

Fig. 1: a) The true down sampled Marmousi model; b) The initial model.

For conventional FWI, all the sources are fired individually and shot gathers are recorded separately. For amplitude encoding FWI, we apply different amplitude weights to the shot gathers to compose super-shots. For comparison, in the experiments, we use Hartley, cosine, sine and random polarity as the encoding functions. The crosstalk matrices are shown in Fig 2, which show how close they are to an identity matrix. We can notice that there are two main diagonals in sine basis case, and only one main diagonal in the random polarity basis case.

We have previously presented the static- and dynamic-encoding results using Hartley and cosine bases, both of which show great performance as in the conventional case (Liu et al., 2021). For dynamic-encoding, at first, we still compose the shot gathers into 70 super-shots and run 25 iterations, then we compose the shot gathers into 35 super-shots and run another 25 iterations using the updated velocity model by the first step. Likewise, we then use 14 super-shots and 7 super-shots for 25 iterations each. So overall, we also update the velocity model 100 times. In this paper, for brevity, we only present the dynamic-encoding inversion results in Fig 3. We can see the inversion results using sine and random polarity bases shown in Fig 3c and 3d are of the same imaging quality as in previous cases.

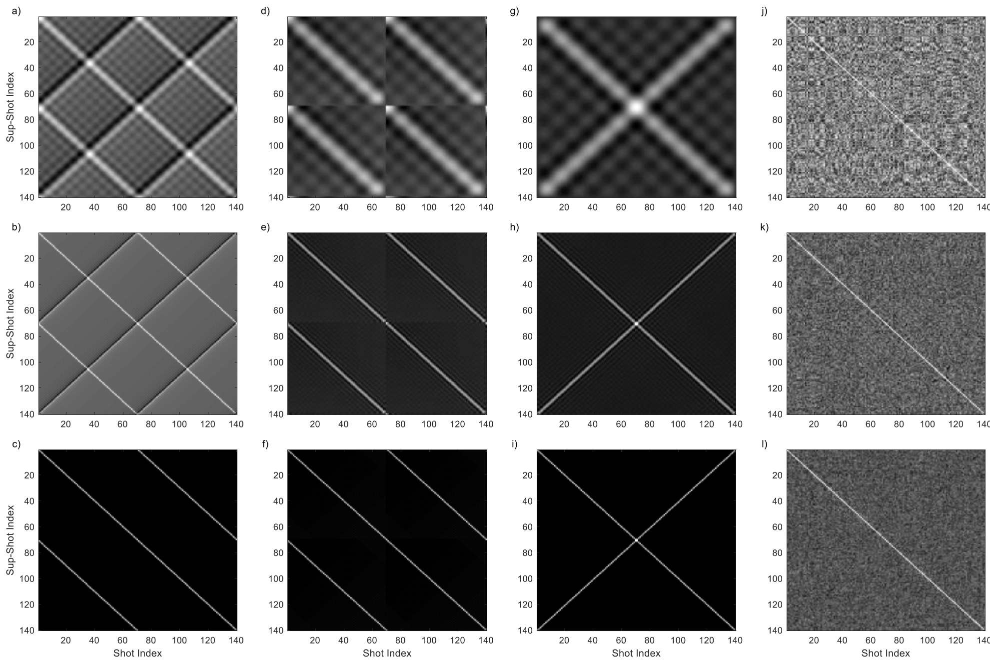

Fig 2: Crosstalk matrices: columns from left to right are by Hartley, cosine, sine and random polarity bases; rows from up to down are for 7, 35 and 70 super-shots, respectively.

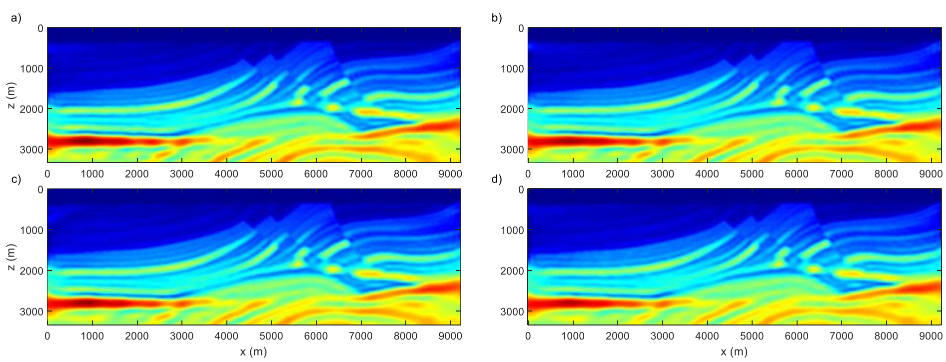

Fig 3: Inversions results using dynamic-encoding concept by different bases: a) Hartley; b) cosine; c) sine and d) random polarity.

We also compare the data misfit and the vertical profiles in the middle of the model in Fig 4. As for the data misfit, when the number of super-shot is reduced during inversion process, there also shows discontinuity in the sine basis case, while the misfit curve in the random polarity basis case is still smooth. As shown in the vertical profiles, the difference in our experiments is so small, so the lines are almost overlapped.

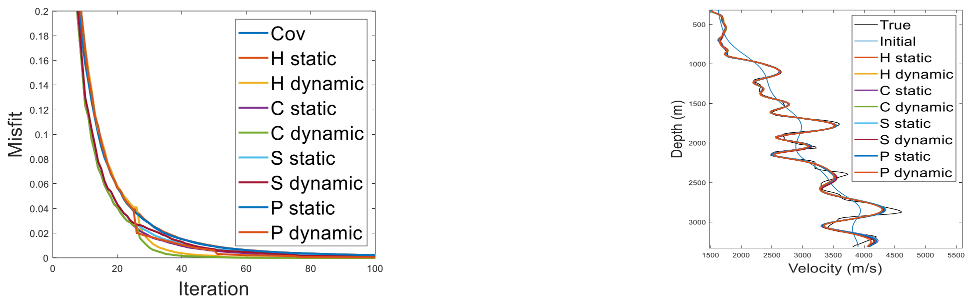

Fig 4: Comparisons between different cases: a) data misfit functions versus iteration; b) vertical profiles in the middle of the model.

Conclusions

In this follow-up work, we use sine and random polarity bases as the encoding functions to perform amplitude-encoding acoustic FWI. The dynamic-encoding inversion results show totally comparable imaging quality as in the conventional and previous amplitude-encoding cases. Amplitude-encoding strategy can mitigate the crosstalk noise very well using all 4 different bases, provide good updated velocity models and convergency.

Acknowledgements

We thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 543578-19. We would also like to thank Dr. Pengliang Yang for his contribution on acoustic FWI program.

References

Liu H, Trad D and Innanen K., 2021, Acoustic FWI using amplitude encoding strategy. 2021 geoconvention virtual event.

Hu, J., Wang, H., Fang, Z., Li, T., and Zhang, J., 2016, Efficient amplitude encoding least-squares reverse time migration using cosine basis: Geophysical prospecting, 64, No. 6, 1483–1497.

Krebs, J. R., Anderson, J. E., Hinkley, D., Neelamani, R., Lee, S., Baumstein, A., and Lacasse, M. D, 2009, Fast full-wavefield seismic inversion using encoded sources. Geophysics, 74(6), WCC177-WCC188.

Tsitsas, N. L., 2010, On block matrices associated with discrete trigonometric transforms and their use in the theory of wave propagation. Journal of Computational Mathematics, 864-878.

Yang, P., Gao, J., and Wang, B., 2015, A graphics processing unit implementation of time-domain full-waveform inversion: Geophysics, 80, No. 3, F31–F39.