# Application of amplitude-encoding strategy to elastic full waveform inversion

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## Summary

Elastic full waveform inversion (EFWI) is a powerful tool to characterize the underground P- and S-wave velocity profiles by matching the synthetic and observed waveform data, which requires super expensive data acquisition and processing costs. We have presented the synthetic examples of acoustic FWI using amplitude-encoding strategy, which requires much less calculation effort and provides inversion results with ignorable crosstalk noises. In this work, we further apply this strategy to EFWI using cosine basis, which provides comparable inversion results with conventional method with half reduced calculation time.

## Amplitude-encoding FWI

Using amplitude-encoding strategy, the individual shots are blended into super-shots by

$$\mathbf{p}^{\text{sup}} = \mathbf{B}\mathbf{p} \tag{1}$$

where $\mathbf{p}$ denotes the waveform data and B denotes the amplitude encoding matrix, which is defined as

$$\mathbf{B} = \begin{bmatrix} b^{1,1} & \cdots & b^{Nsig,1} \\ \vdots & \ddots & \vdots \\ b^{1,N_{sup}} & \cdots & b^{Nsig,N_{sup}} \end{bmatrix}_{N_{sup}\times N_{sig}} \tag{2}$$

where *Nsup* is the number of the super-shots and *Nsig* is the number of the individual shots (*Nsup* < *Nsig*).
Taking the least-square norm, the encoding objective function can be written as

$$E(\mathbf{m}) = \frac{1}{2}\boldsymbol{\Delta}\mathbf{p}^{\dagger}\boldsymbol{\Delta}\mathbf{p} = \frac{1}{2}||\mathbf{p}_{\text{cal}} - \mathbf{p}_{\text{obs}}||^2 = \frac{1}{2}(p_{cal} - p_{obs})\mathbf{B}^{\mathrm{T}}\mathbf{B}(p_{cal} - p_{obs}) \tag{3}$$

where $\Delta\mathbf{p}$, $\mathbf{p}_{\text{obs}}$ and $\mathbf{p}_{\text{cal}}$ are the data misfit, the observed and the simulated data, respectively.

Since the data dimension is reduced by a factor determined by the ratio between *Nsig* and *Nsup,* the calculation efficiency can be improved. We need to notice that the matrix $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ is referred to as the crosstalk matrix, and the off-diagonal elements are not zero, so crosstalk noises will be introduced eventually. However, choosing appropriate number of super-shots, the crosstalk noises will be mitigated very well (Liu et al., 2021). In this work, we specifically use cosine basis as the encoding function to perform EFWI, which is defined by (Tsitsas, 2010; Hu et al., 2016)

$$b_{m,n} = \sqrt{\frac{2}{n_{\text{sig}}}}\cos\left(\frac{\pi}{n_{\text{sig}}}\frac{(2m\%n_{\text{sig}}+1)(2n+1)}{4}\right) \tag{4}.$$

where *m* = 1,…, *Nsig* is the shot-index, *n* = 1,…, *Nsup* is the super-shot index, and *nsig* is the periodization index, which we set to be half of *Nsig*.

## Synthetic examples

We use a subsampled Marmousi II elastic model with a distance of 4300 m and a depth of 1500 m in a grid of 430 by 150 cells with 10 meters size each. This model consists of a 200 m thick water layer above. The true and initial models are shown in Fig 1, we only perform EFWI for vp and vs in this work.

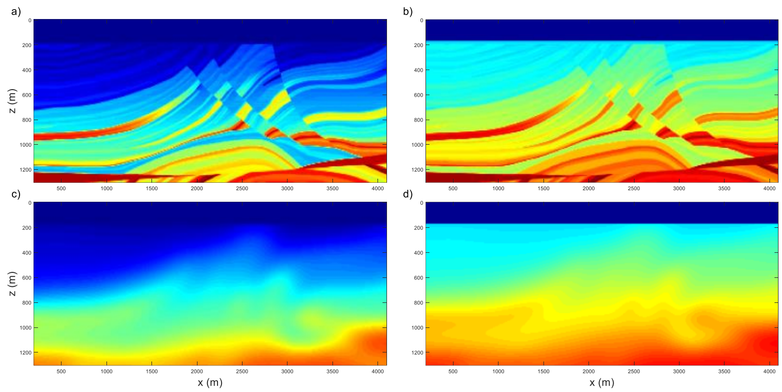


*Fig 1: The subsampled Marmousi II model: a) and b) true vp and vs models; c) and d) initial vp and vs models.*

We generate synthetic shot gathers for 80 explosive sources and deploy 400 two–component receivers. The central frequency is 10 Hz. The sources and receivers are evenly distributed at depth 20 and 30 meters, respectively. In this experiment, we blend all 80 individual shots into 40 super-shots, the encoding and crosstalk matrices are shown in Fig 2. In the conventional FWI case, all the sources are fired individually and shot gathers are recorded separately, so the encoding and crosstalk matrices are both identity matrices. While in the amplitude-encoding case, we apply different amplitude weights (shown in Fig 2b) to the shot gathers to compose super-shots, whose number is reduced by half compared to the conventional case.

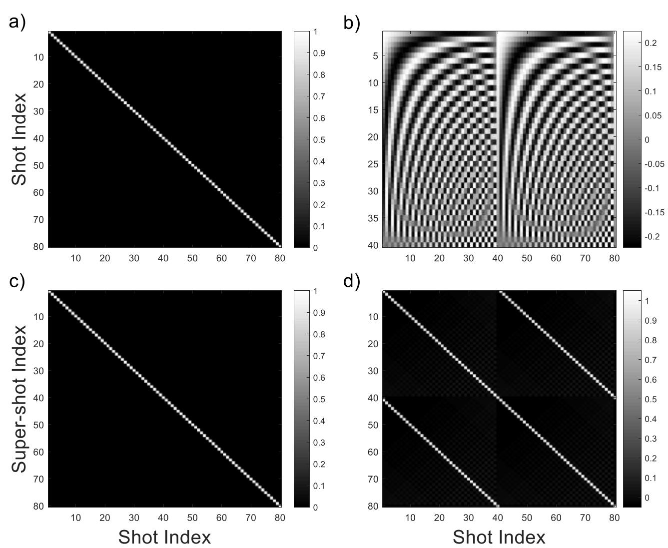


*Fig 2: The amplitude-encoding and crosstalk matrices: a) and c) are for conventional FWI; b) and d) are for amplitude-encoding using cosine basis.*

In this work, we use the IFOS2d software (Bohlen et al., 2016) to do the experiments. Rather than set the iteration times for our tests, we use an abort criterion to control the inversion progress, which is the defined by the relative misfit change within the last two iterations. If the relative change is smaller than one percent, the inversion process stops. The inversion results are shown in Fig 3. The left column are inverted vp models and the right column are inverted vs models displayed under the same scale separately. When we compare the results by both methods, we can barely notice any introduced crosstalk noises, the fine layers with high velocities can be recovered very well.

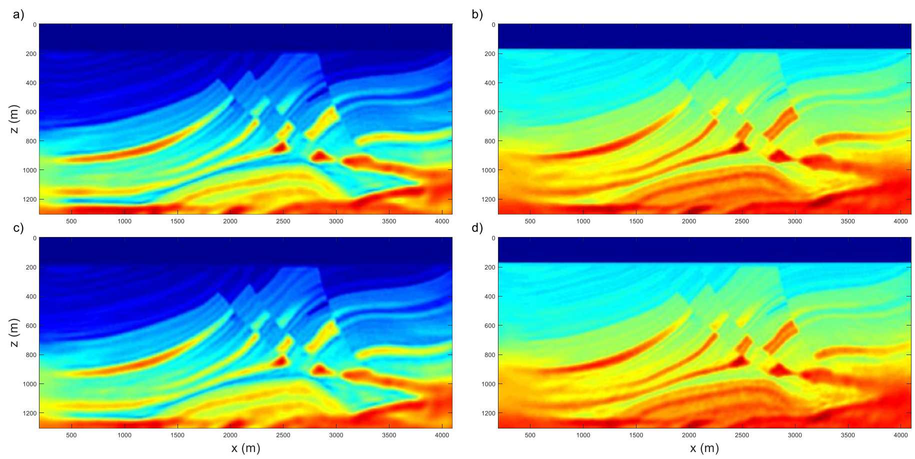


*Fig 3: Inversion results by both conventional and amplitude-encoding FWI: a) and c) are inverted vp models, b) and d) are inverted vs models.*

Additionally, vertical vp and vs profiles at 2.4 km of the initial model and inversion results are compared with the true modes in Fig 4. The black and red lines are the true and initial models, respectively. The blue lines are the vertical profiles by conventional method, which contains a lot of fine details, as for the orange lines by amplitude-encoding FWI, we can see there's minor difference at the fine high-velocity layers. However, it only takes only half of the calculation effort.

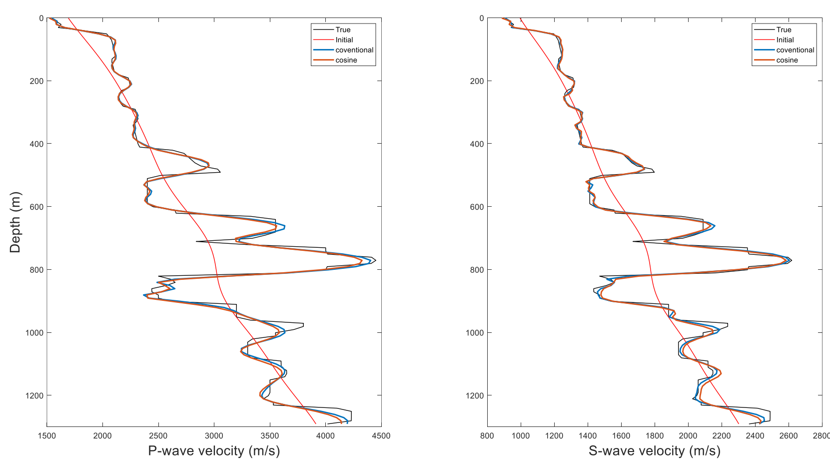


*Fig 4: Depth profiles at distance 2.4 km of the initial model and inversion results are compared with the true model for the Marmousi II model: P-wave velocity (left), S-wave velocity (right).*

## Conclusions

In this work, we apply amplitude-encoding strategy to elastic FWI. We use cosine basis as the encoding function to reduce the data dimension by half. The synthetic examples show that amplitude-encoding strategy can provide inversion results of the same imaging quality as in the conventional case, as well as improve the calculation efficiency.

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