AVAZ inversion for fracture orientation and fracture density on physical model data

Faranak Mahmoudian

Gary Margrave

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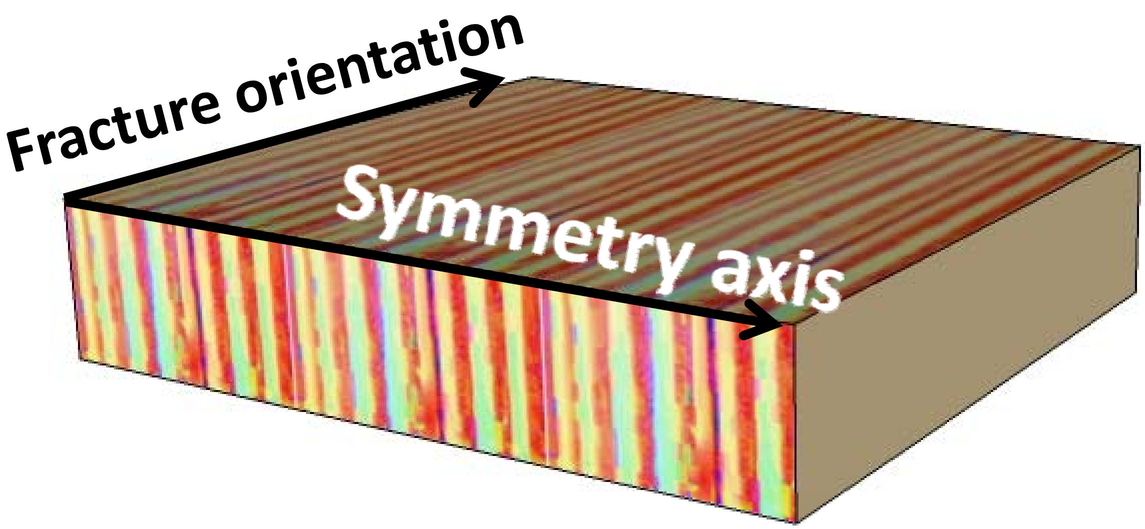
Objective

Inversion of prestack PP amplitudes
(different incident angles and azimuths)
for fracture orientation and fracture density.

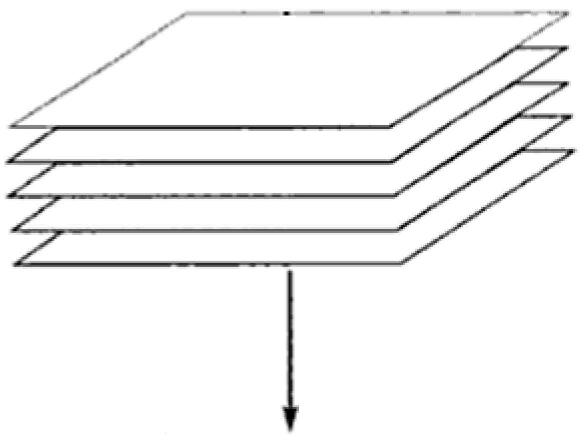


Fracture orientation: direction of fracture planes
Fracture density: number of fractures in unit area

Outline

- Anisotropic models
- Previous work
- Jenner’s method for fracture orientation
- Implementation on physical model data
- Conclusions
- Acknowledgements

VTI (vertical transverse isotropy)



- Horizontal isotropic planes
- Vertical symmetry axis
- 5 parameters to describe the medium (α, β, ε, δ, γ)

α = P-vetical velocity

β = S-vertical velocity

$$\varepsilon = \frac{V_P(fast) - V_P(slow)}{V_P(slow)}$$

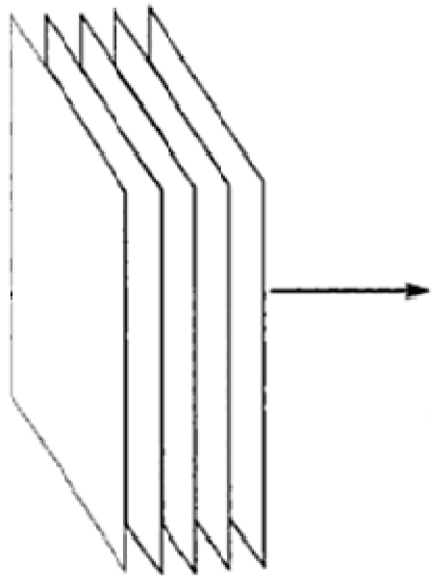
$$\gamma = \frac{V_S(fast) - V_S(slow)}{V_S(slow)}$$

Dominates the small angle wave propagation →

$$\delta = \frac{(A_{13} + A_{55})^2 - (A_{33} - A_{55})}{2A_{33}(A_{33} - A_{55})}$$

HTI (horizontal transverse isotropy)

Simple model to describe vertical fractures



- Vertical isotropic plane
- Horizontal symmetry axis
- (α, β, ε, δ, γ) to describe the medium

α = P-vetical velocity

β = S-vertical velocity

$$\varepsilon = \frac{V_{Px} - V_{Pz}}{V_{Pz}}$$

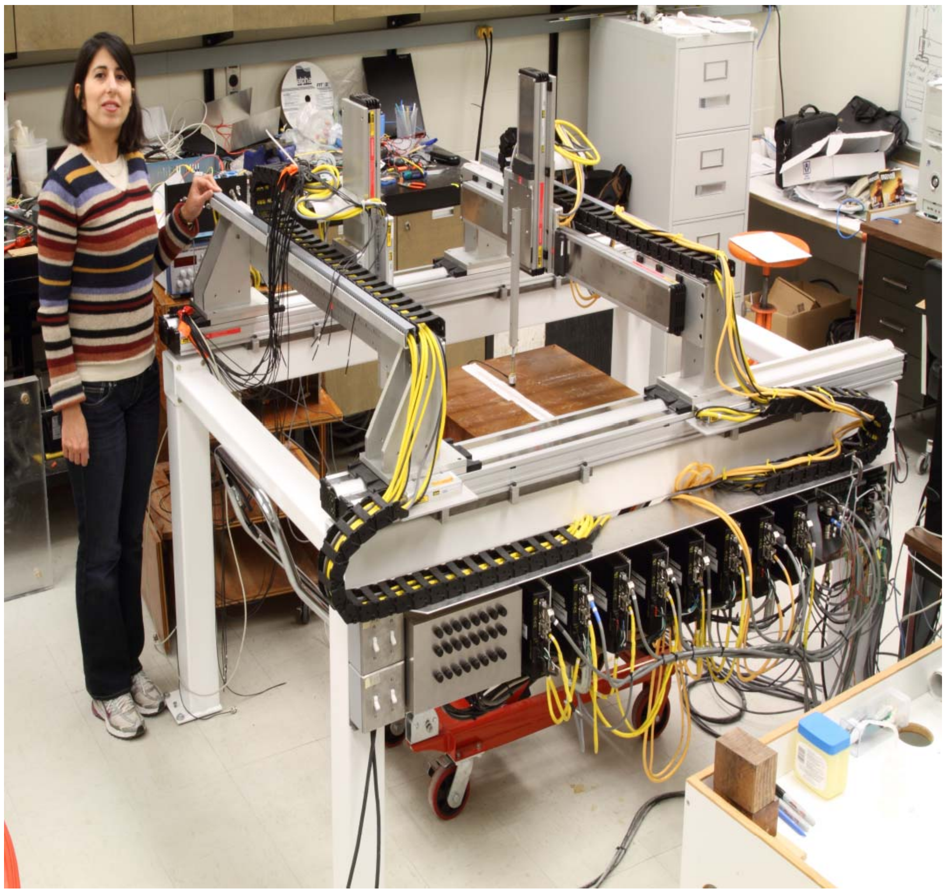
(Shear-wave splitting parameter) directly related to fracture density →

$$\gamma = \frac{V_{Sx} - V_{Sz}}{V_{Sz}} \quad \text{:Sv-wave in } (x_2, x_3) \text{ plane}$$

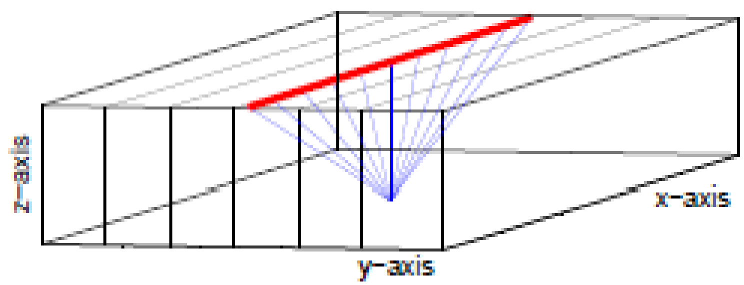
$$\delta = \frac{(A_{13} + A_{55})^2 - (A_{33} - A_{55})}{2A_{33}(A_{33} - A_{55})}$$

Determining elastic constants of phenolic layer
(2010 work)

Phenolic layer ≈ HTI

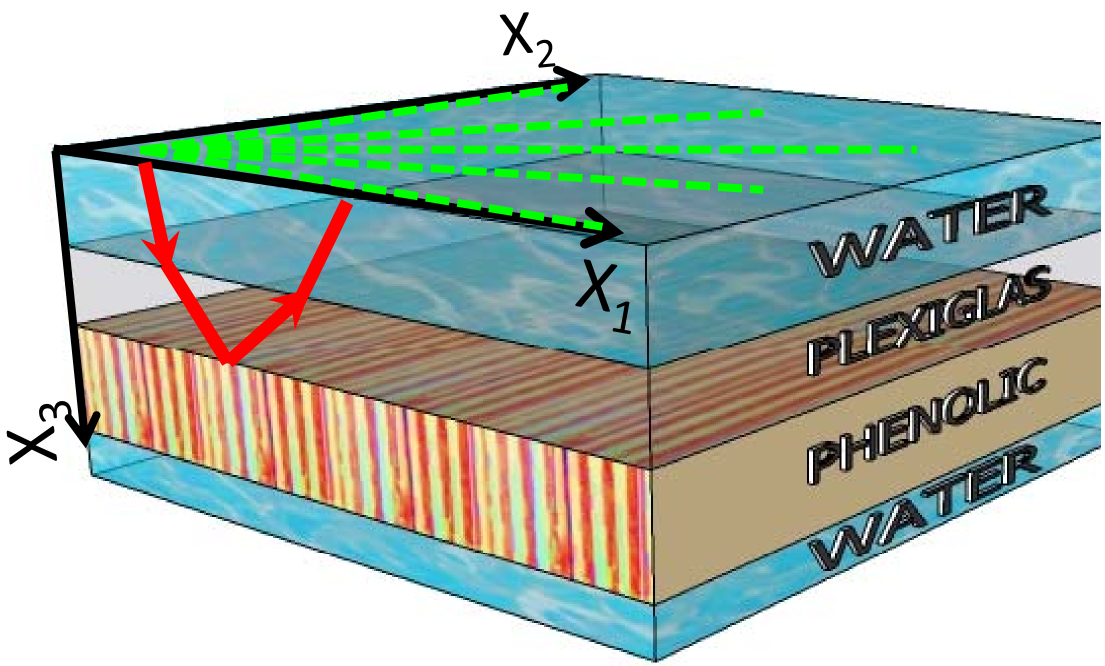


- Transmission shot gathers on single layer
- Traveltime inversion
- True (ε, δ, γ)



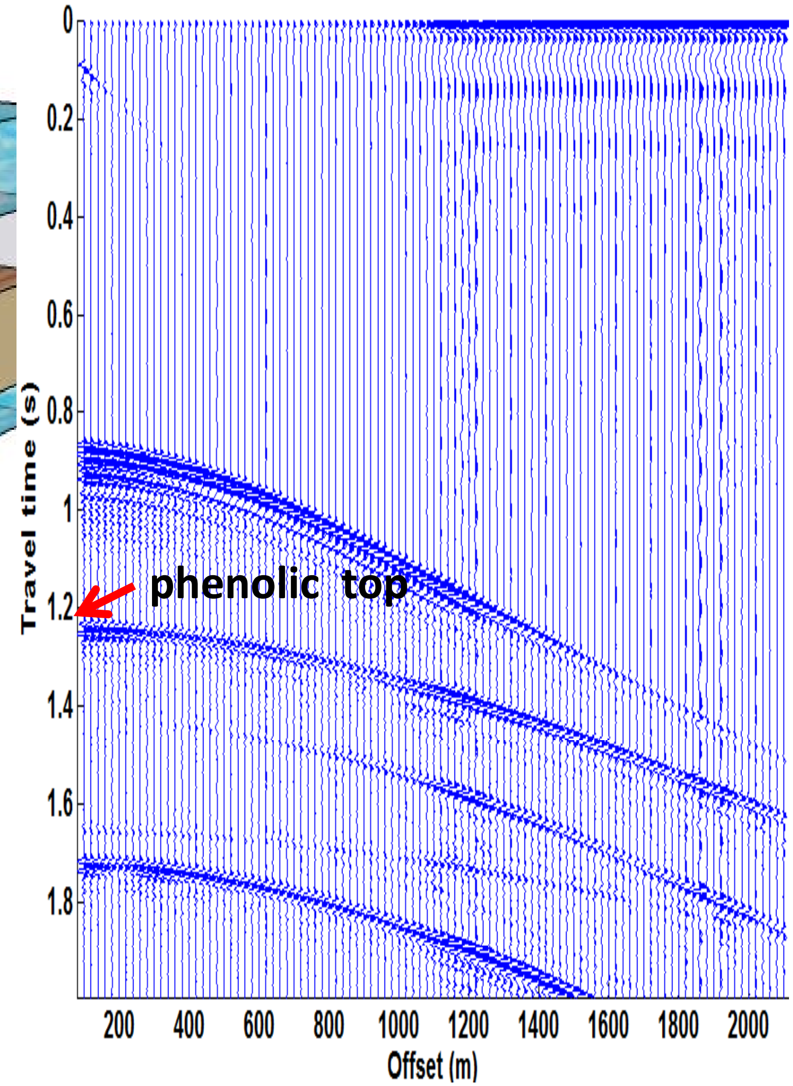
AVAZ inversion for (ε , δ , γ)

(2011 work)

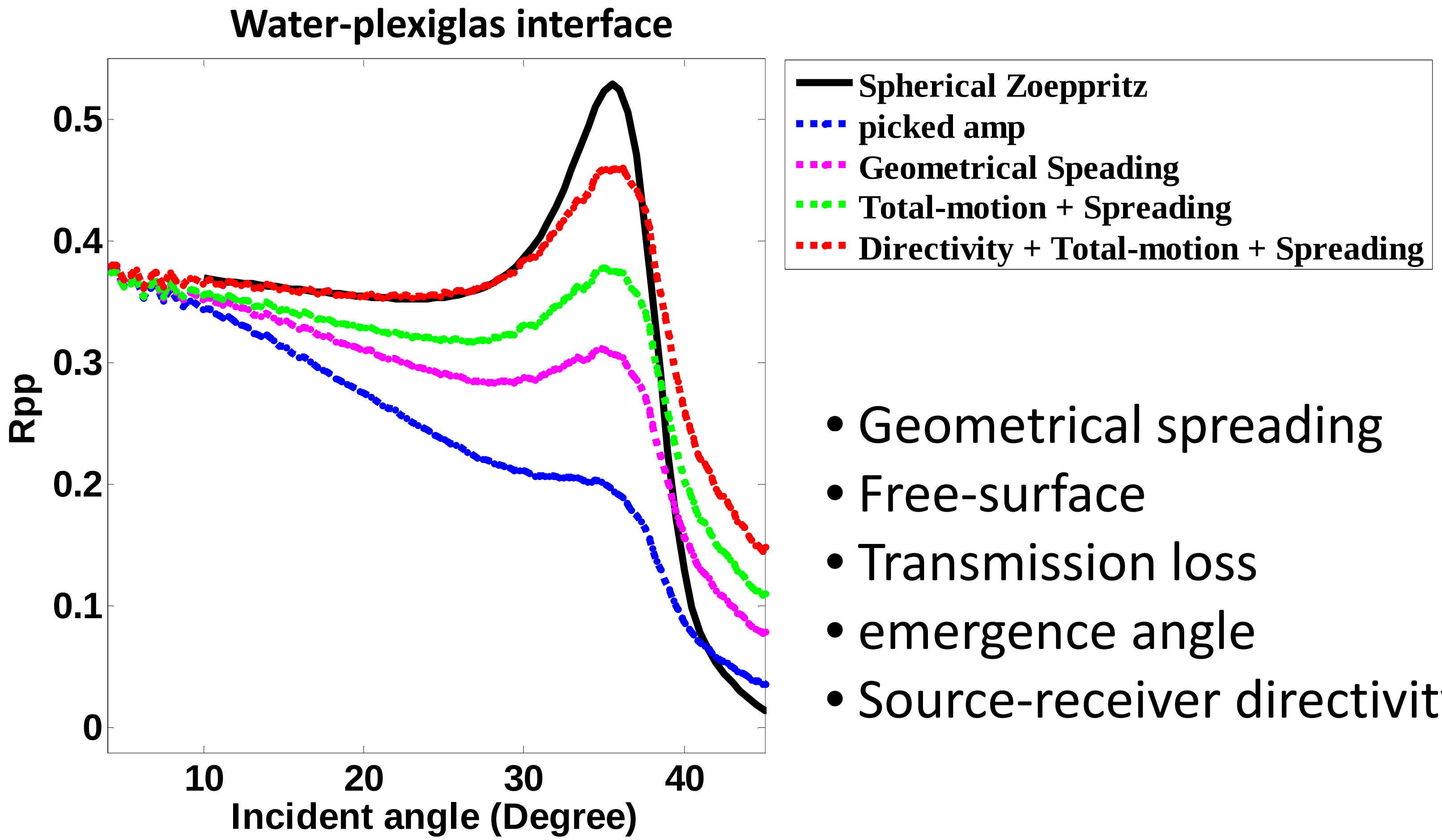


- Acquisition coordinate system along fracture system
- Large offset data

Azimuth 90°

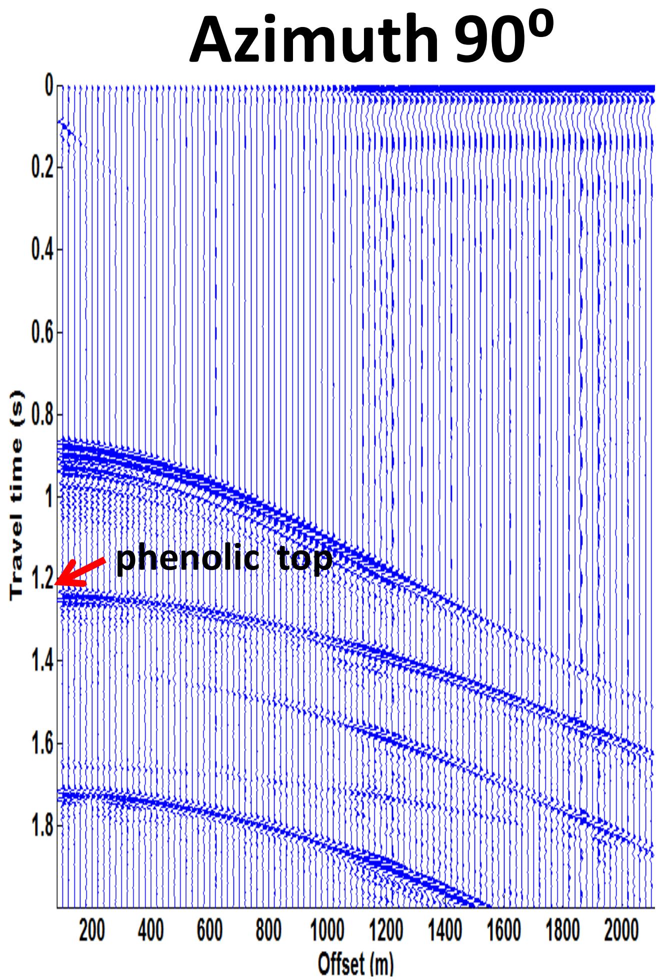


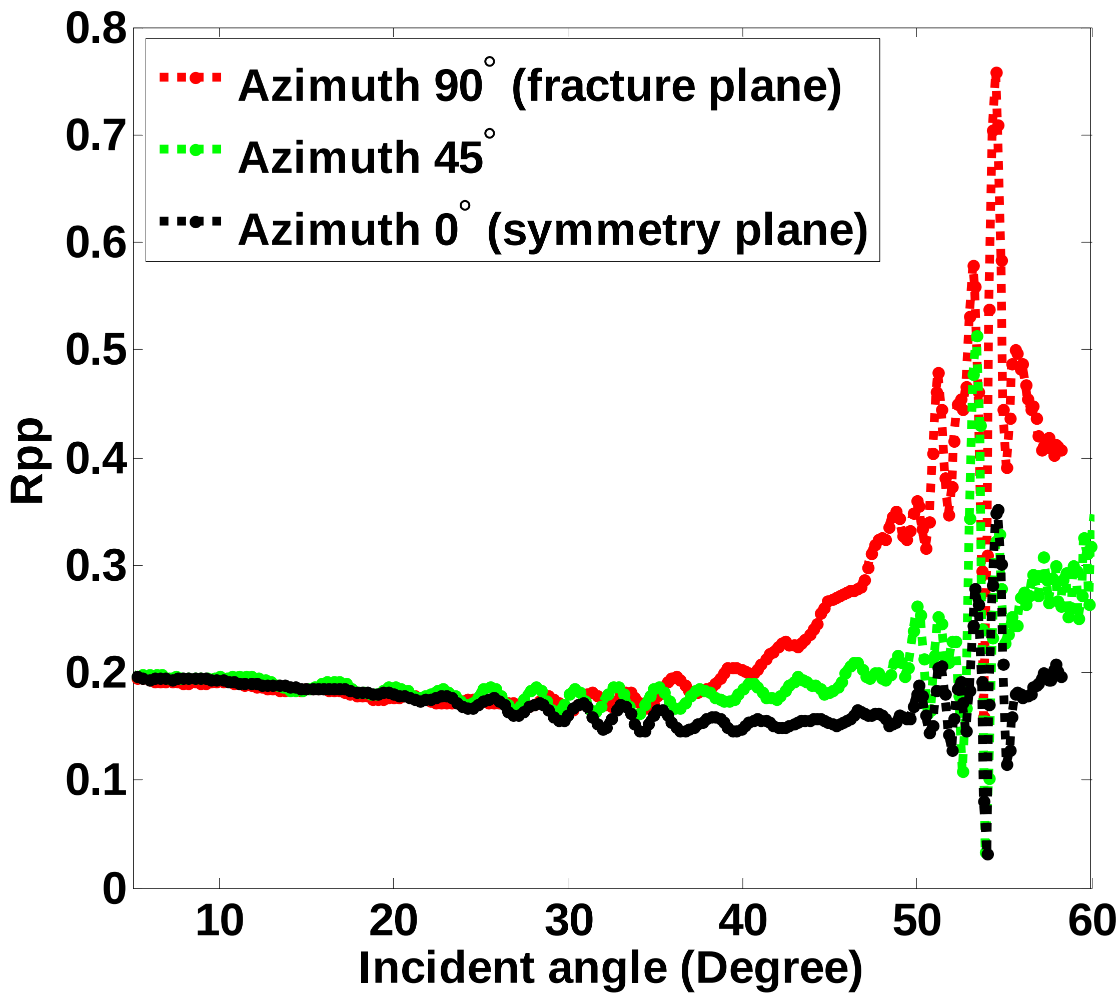
Amplitude corrections



- Geometrical spreading
- Free-surface
- Transmission loss
- emergence angle
- Source-receiver directivity

Phenolic top reflector
corrected amplitudes





HTI: PP reflection coefficient

(Rüger, 1997)

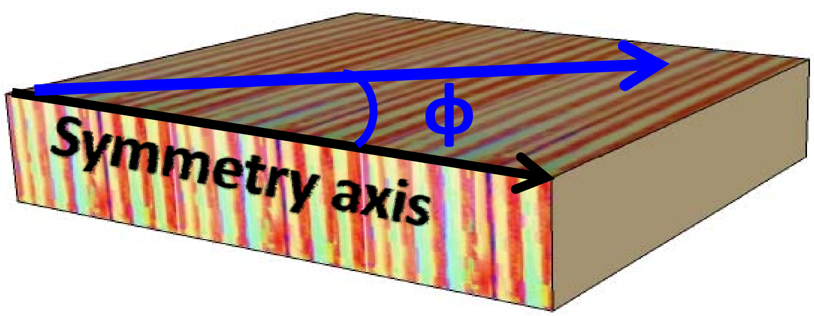
$$R_{PP}^{HTI}(\theta,\varphi) \cong \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} +$$

$$\frac{1}{2}\left(\cos^4\varphi\sin^2\theta\tan^2\theta\right)\Delta\varepsilon + \left(\frac{4\beta^2}{\alpha^2}\cos^2\varphi\sin^2\theta\right)\Delta\gamma +$$

$$\frac{1}{2}\left(\cos^2\varphi\sin^2\theta + \cos^2\varphi\sin^2\varphi\sin^2\theta\tan^2\theta\right)\Delta\delta$$

θ : incident angle

ϕ: angle between source-receiver azimuth and fracture symmetry axis



- **Independent determination of (α , β , ρ)**
- **Inversion for (ε , δ , γ)**

AVAZ inversion

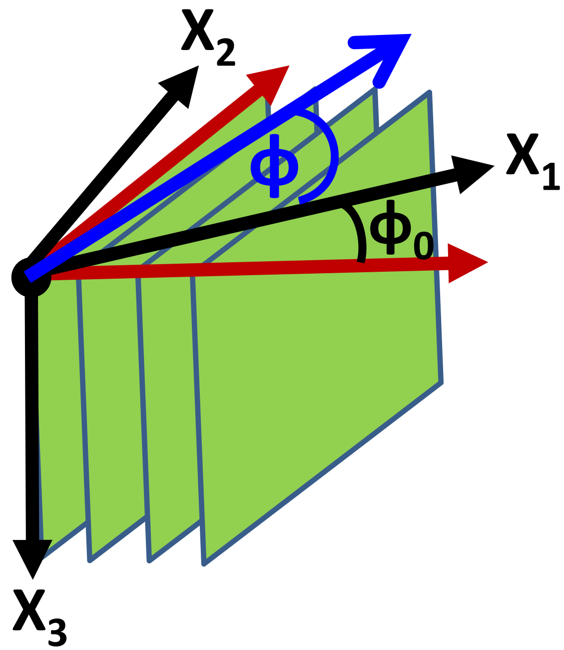
Successful inversion results for (ε, δ, γ)

$$
\begin{matrix} \textbf{Azimuth } \phi_1 \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right. \\ \\ \textbf{Azimuth } \phi_n \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right. \end{matrix}
\begin{bmatrix} A_{1\varphi_1} & B_{1\varphi_1} & C_{1\varphi_1} \\ \vdots & \vdots & \vdots \\ A_{n\varphi_1} & B_{n\varphi_1} & C_{n\varphi_1} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ A_{1\varphi_m} & B_{1\varphi_m} & C_{1\varphi_m} \\ \vdots & \vdots & \vdots \\ A_{n\varphi_m} & B_{n\varphi_m} & C_{n\varphi_m} \end{bmatrix}_{(nm\times 3)}
\begin{bmatrix} \Delta\delta \\ \Delta\varepsilon \\ \Delta\gamma \end{bmatrix}_{(3\times 1)}
=
\begin{bmatrix} R_{11} \\ \vdots \\ R_{n1} \\ \vdots \\ \vdots \\ R_{1m} \\ \vdots \\ R_{nm} \end{bmatrix}_{(nm\times 1)}
$$

$$Gm = d$$

$$m_{est} = (G^T G + \mu)^{-1} G^T d$$

Fracture symmetry axis not known



: fracture system
: acquisition coordinate
ϕ : source-receiver azimuth
ϕ0 : fracture symmetry direction

$$R_{PP}^{HTI}(\theta,\varphi) \cong \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} +$$

$$\frac{1}{2}\left(\cos^4(\varphi-\varphi_0)\sin^2\theta\tan^2\theta\right)\Delta\varepsilon + \left(\frac{4\beta^2}{\alpha^2}\cos^2(\varphi-\varphi_0)\sin^2\theta\right)\Delta\gamma +$$

$$\frac{1}{2}\left(\cos^2(\varphi-\varphi_0)\sin^2\theta + \cos^2(\varphi-\varphi_0)\sin^2(\varphi-\varphi_0)\sin^2\theta\tan^2\theta\right)\Delta\delta$$

HTI: PP reflection coefficient

Small incident angle (θ<35º)

$$R_{PP}^{HTI}(\theta,\varphi) \cong \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} + $$

$$+\left(\frac{4\beta^2}{\alpha^2}\Delta\gamma+\frac{1}{2}\Delta\delta\right)\cos^2(\varphi-\varphi_0)\sin^2\theta$$

$$R_{PP}^{HTI}(\theta,\varphi) \cong I + \left[G_1 + G_2\cos^2(\varphi-\varphi_0)\right]\sin^2\theta$$

G_1: Isotropic gradient

G_2: Anisotropic gradient

$$G_2 = \frac{4\beta^2}{\alpha^2}\Delta\gamma + \frac{1}{2}\Delta\delta$$

Estimate fracture orientation

Jenner (2001)

$$R_{PP}^{HTI}(\theta,\varphi) \cong I + \left[G_1 + G_2\cos^2(\varphi - \varphi_0)\right]\sin^2\theta$$

$$R_{PP}^{HTI}(\theta,\varphi) \cong I + \left[G_1\sin^2(\varphi - \varphi_0) + (G_1 + G_2)\cos^2(\varphi - \varphi_0)\right]\sin^2\theta$$

$$\begin{cases} \sin(\varphi - \varphi_0) = \sin\varphi\cos\varphi_0 - \cos\varphi\sin\varphi_0 \\ \cos(\varphi - \varphi_0) = \cos\varphi\cos\varphi_0 + \sin\varphi\sin\varphi_0 \end{cases}$$

$$R_{PP}^{HTI}(\theta,\varphi) \cong I + \left[W_{11}\sin^2\varphi + 2W_{12}\sin\varphi\cos\varphi + W_{22}\cos^2\varphi\right]\sin^2\theta$$

$$G_1 = 0.5\left(W_{11} + W_{22} - \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2}\right)$$

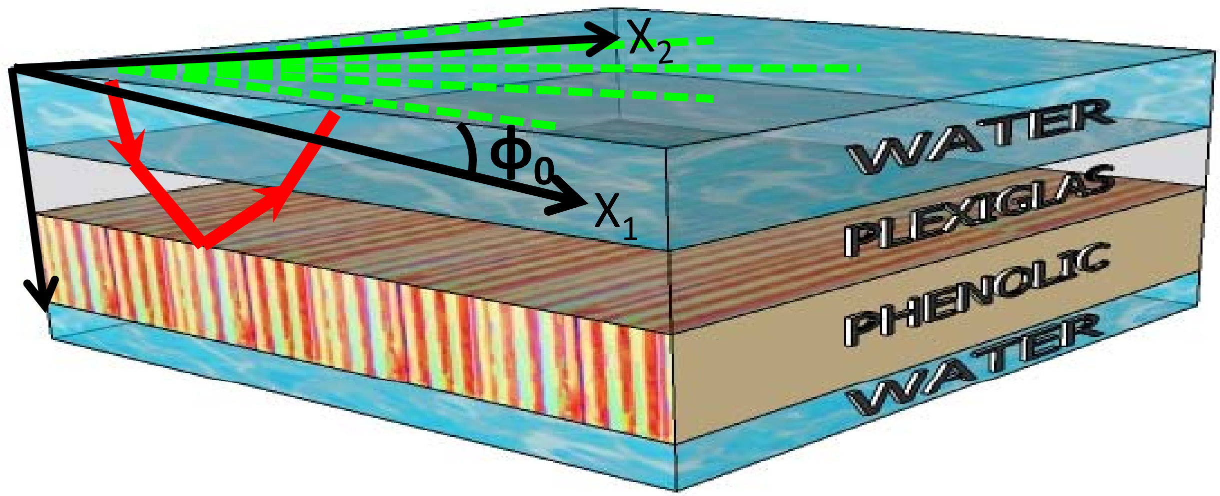
$$G_2 = \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2}$$

$$\varphi_0 = \tan^{-1}\left(\frac{W_{11} - W_{22} + \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2}}{2W_{12}}\right)$$

W11, W22, W33: function of ϕ_0

Test on physical model data, AVAZ inversion
ϕ_0 = 30º

ϕ_0: fracture symmetry axis azimuth



True ϕ_0	30º
Estimate ϕ_0	28.7º

Testing different fracture orientations

ϕ_0: fracture symmetry axis azimuth

True ϕ_0	0^o	5^o	10^o	20^o	30^o	40^o	50^o	60^o	70^o	80^o	90^o
Estimate ϕ_0	1.25^o	3.7^o	8.7^o	18.7^o	28.7^o	38.7^o	48.7^o	58.7^o	68.7^o	78.7^o	88.7^o

Estimate fracture density

γ : directly related to fracture density

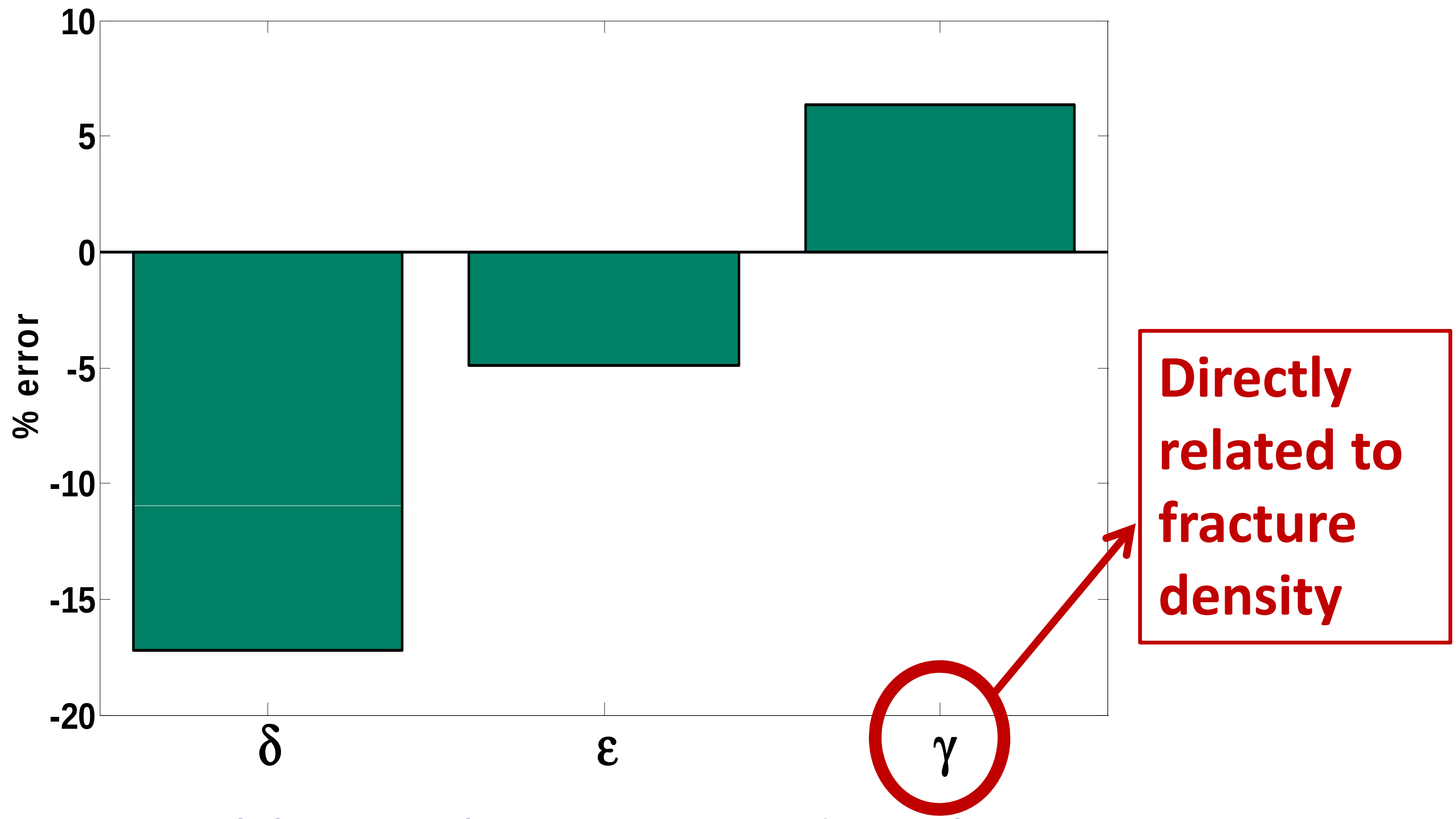
Knowing the fracture orientation,
do the AVAZ inversion of large offset data
for (ε,δ,γ).

$$R_{PP}^{HTI}(\theta,\varphi) \cong \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} +$$

$$\frac{1}{2}\left(\cos^4\varphi\sin^2\theta\tan^2\theta\right)\Delta\varepsilon + \left(\frac{4\beta^2}{\alpha^2}\cos^2\varphi\sin^2\theta\right)\Delta\gamma +$$

$$\frac{1}{2}\left(\cos^2\varphi\sin^2\theta + \cos^2\varphi\sin^2\varphi\sin^2\theta\tan^2\theta\right)\Delta\delta$$

AVAZ inversion of large offset data for (ε, δ, γ)



Favourable results compared to those obtained previously by traveltime inversion

Conclusions

- Rüger equation for HTI, PP reflection coefficient, can be used in an inversion but fracture orientation must be known.
- Jenner's method reformulates to allow a linear inversion to determine fracture orientation.
- Implementation on physical model data gives results consistent with these theories.
- Fracture density can be estimated from AVAZ inversion of large-offset data after determining fracture orientation.

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