A framework for linear and nonlinear converted wave time-lapse difference AVO

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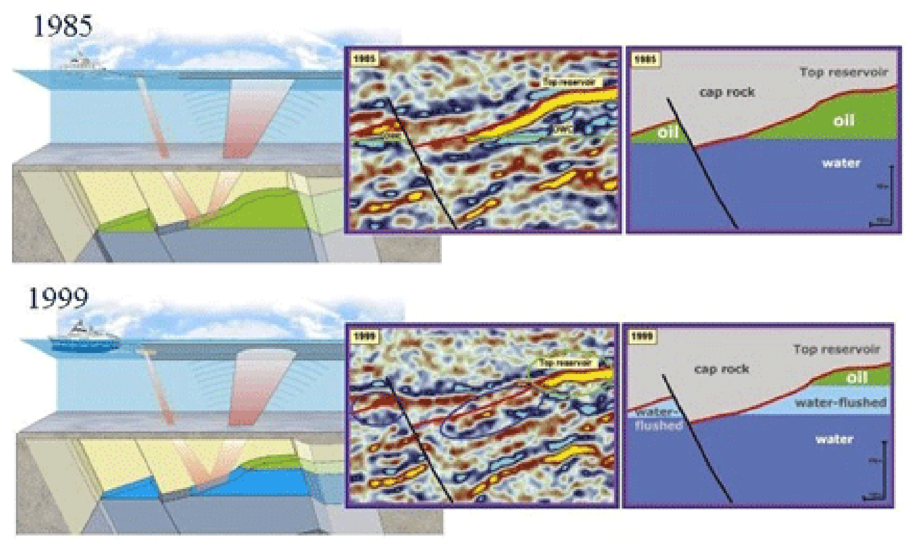
Outline

- Introduction and review
- A framework for converted wave time-lapse AVO
- Numerical example
- Conclusions
- Future work
- Acknowledgments

Time-lapse seismic

- Monitoring changes in reservoir: production, EOR
- Repeated seismic surveys over calendar time
- The baseline and monitor survey
- Changes in seismic parameters

Quantitative time-lapse (4D) seismic
(Image courtesy of Statoil)



Principle of 4D acquisition

Gullfaks field

AVO : Amplitude Versus Offset

Baseline and time-lapse changes

Baseline

$$\Delta V_{Pb} = V_{Pb} - V_{P_0}$$

$$\Delta V_{Sb} = V_{Sb} - V_{S_0}$$

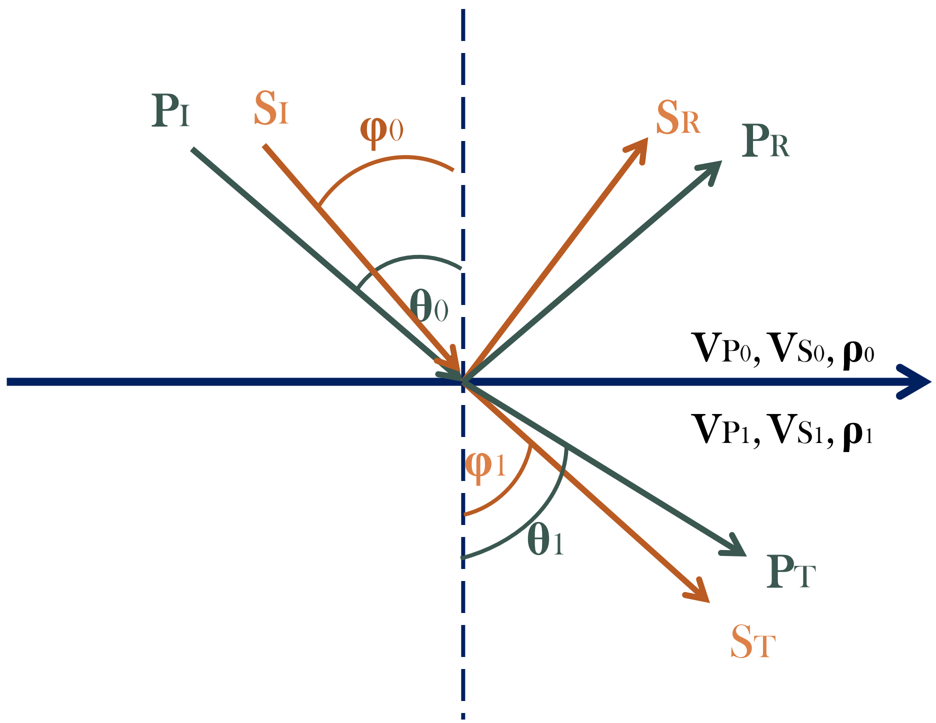
$$\Delta \rho_b = \rho_b - \rho_0$$

Time lapse

$$\delta V_P = V_{Pm} - V_{P_b}$$

$$\delta V_S = V_{Sm} - V_{S_b}$$

$$\delta \rho = \rho_m - \rho_b$$



Landrø et al. 2001

- Landrø's linear equation for ΔR_{PP}

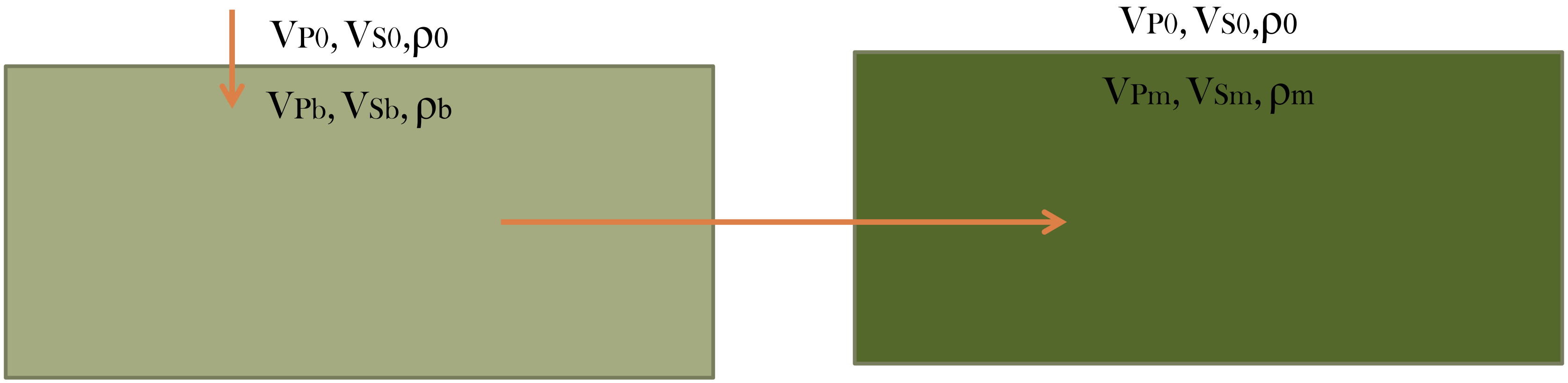
$$\Delta R_{PP}(\theta)=\frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)-2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho}+2\frac{\delta V_S}{V_S}\right)\sin^2\theta+\frac{\delta V_P}{2V_P}\tan^2\theta$$

- Inaccurate for large time-lapse contrasts
- Independent of the contrast in the baseline survey

Time lapse difference AVO for P-P section

- In plausible large contrast time-lapse scenarios Landrø's approximation requires correction.
- A framework for linear and non linear time-lapse AVO analysis is formulated.
- Agreement of linear term in ΔR_{PP} with Landrø's work.
- Higher order approximations made corrections in ΔR_{PP}.
- Physical model validated the importance of low order interpretable nonlinear corrections.

A time-lapse problem



Baseline Survey

Monitor Survey

$$b_{VP}=1-\frac{V_{P_0}^2}{V_{P_{BL}}^2},\quad b_{VS}=1-\frac{V_{S_0}^2}{V_{S_{BL}}^2},\quad b_{\rho}=1-\frac{\rho_0}{\rho_{BL}},$$

$$a_{VP}=1-\frac{V_{P_{BL}}^2}{V_{P_M}^2},\quad a_{VS}=1-\frac{V_{S_{BL}}^2}{V_{S_M}^2},\quad a_{\rho}=1-\frac{\rho_{BL}}{\rho_M},$$

A general framework for time-lapse AVO

- Zoeppritz equations for baseline and monitoring targets:

$$P_{BL}\begin{bmatrix}R_{PP}\\R_{PS}\\T_{PP}\\T_{PS}\end{bmatrix}=b_{BL} \quad R_{PS}^{BL}(\theta)=\frac{\det(P_P)}{\det(P)}$$

$$P_{M}\begin{bmatrix}R_{PP}\\R_{PS}\\T_{PP}\\T_{PS}\end{bmatrix}=b_{M} \quad R_{PS}^{M}(\theta)=\frac{\det(P_P)}{\det(P)}$$

$$S_{BL}\begin{bmatrix}R_{SS}\\R_{SP}\\T_{SS}\\T_{SP}\end{bmatrix}=c_{BL} \quad R_{SP}^{BL}(\varphi)=\frac{\det(S_S)}{\det(S)}$$

$$S_{M}\begin{bmatrix}R_{SS}\\R_{SP}\\T_{SS}\\T_{SP}\end{bmatrix}=c_{M} \quad R_{SP}^{M}(\varphi)=\frac{\det(S_S)}{\det(S)}$$

Time-lapse AVO

- Deriving ΔRPS(θ) and ΔRPS(φ) from Zoeppritz equations

$$\Delta R_{PS}(\theta) = R_{PS}^{(M)}(\theta) - R_{PS}^{(BL)}(\theta) \qquad \Delta R_{SP}(\varphi) = R_{SP}^{(M)}(\varphi) - R_{SP}^{(BL)}(\varphi)$$

- Expand ΔRPP(θ) in orders of perturbation parameters

$$\Delta R_{PS}(\theta) = \Delta R_{PS}^{(1)}(\theta) + \Delta R_{PS}^{(2)}(\theta) + \Delta R_{PS}^{(3)}(\theta) + \ldots$$

$$\Delta R_{SP}(\varphi) = \Delta R_{SP}^{(1)}(\varphi) + \Delta R_{SP}^{(2)}(\varphi) + \Delta R_{SP}^{(3)}(\varphi) + \ldots$$

Numerical example

- Baseline survey

  V_{P0}=2000 m/s

  V_{S0}=1500 m/s

  ρ_0=2000 kg/m^3

  V_{Pbl}=3000 m/s

  V_{Sbl}=1700 m/s

  ρ_{bl}=2100 kg/m^3

- Monitor survey

  V_{P0}=2000 m/s

  V_{S0}=1500 m/s

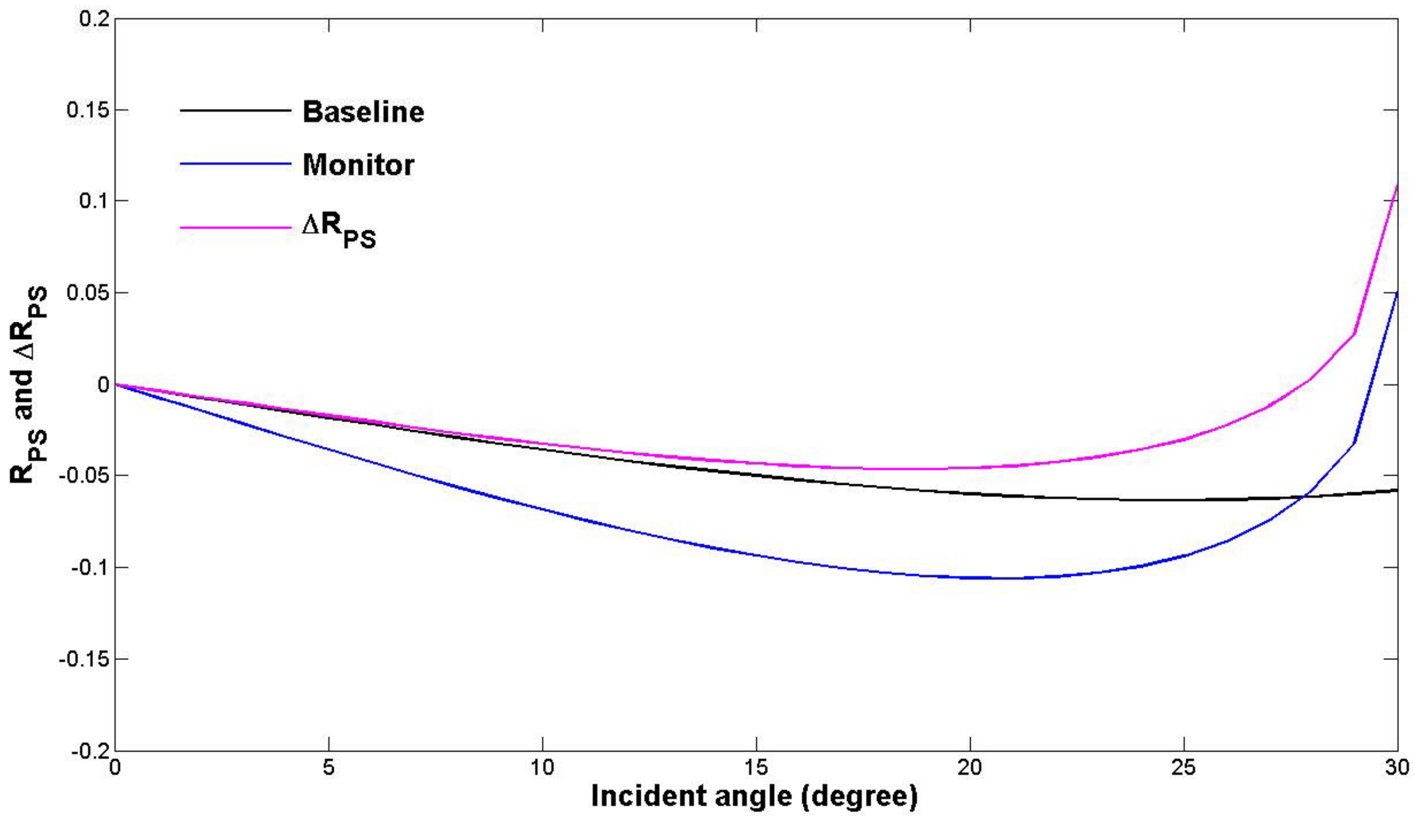
  ρ_0=2000 kg/m^3

  V_{Pm}=4000 m/s

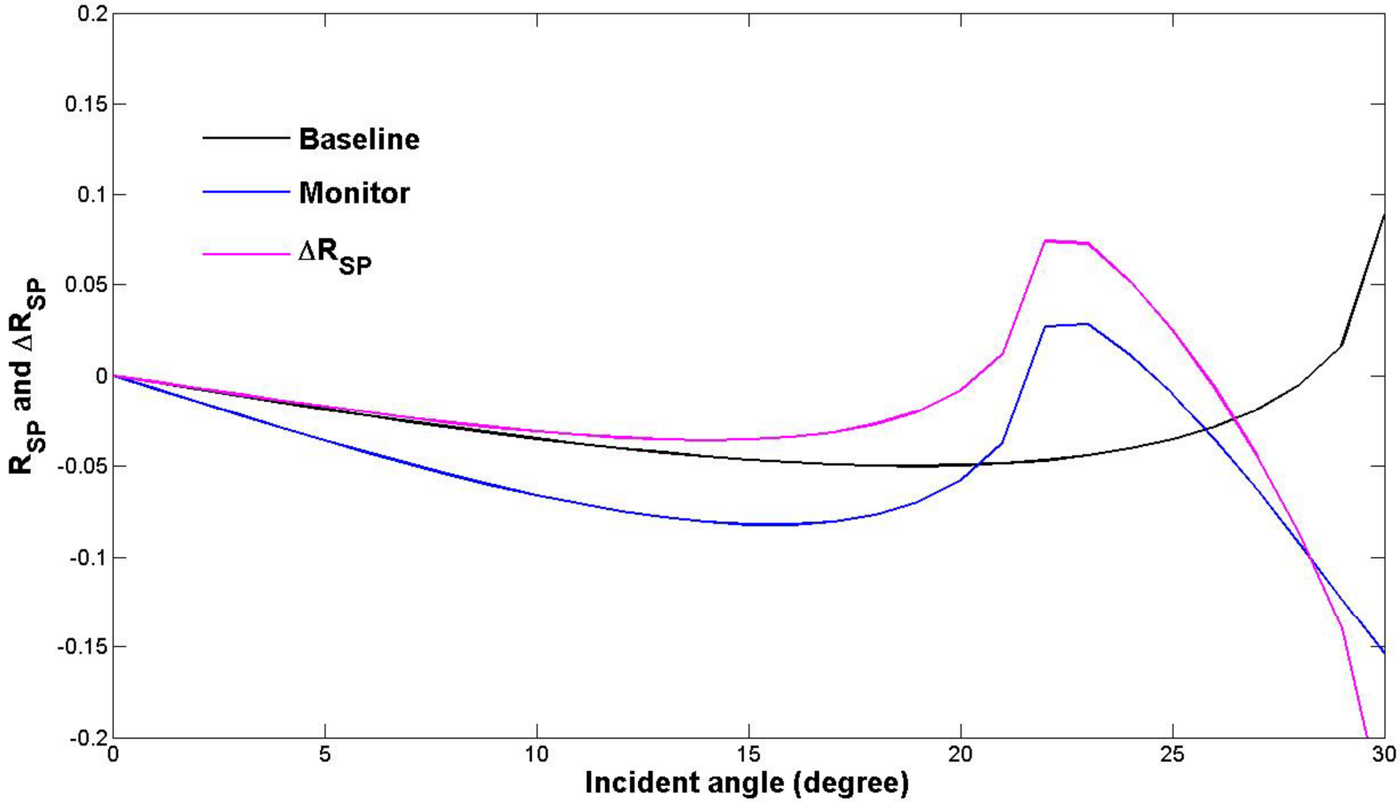
  V_{Sm}=1900 m/s

  ρ_m=2300 kg/m^3

R_{PS} for baseline and monitoring survey



R_{SP} for baseline and monitoring survey



Linear and higher order ΔR_{PS}

$$\Delta R_{PS}^{(1)}(\theta) = k_1^1\, a_{VS} + k_2^1\, a_\rho$$

$$\Delta R_{PS}^{(2)}(\theta) = k_1^2\, a_{VS}^2 + k_2^2\, a_\rho^2 + k_3^2 a_\rho b_\rho + k_4^2\, a_{VS} b_{VS} + k_5^2 \left(a_{VP} a_{VS} + a_{VP} b_{VS} + b_{VP} a_{VS} \right)$$
$$+ k_6^2 \left(a_{VP} a_\rho + a_{VP} b_\rho + b_{VP} a_\rho + a_{VS} a_\rho + b_{VS} a_\rho + a_{VS} b_\rho \right)$$

$$\Delta R_{PS}^{(3)}(\theta) = k_1^3\, a_{VS}^3 + k_2^3\, a_\rho^3 + k_3^3 \left(b_{VS} a_{VS}^2 + a_{VS} b_{VS}^2 \right) + k_4^3 \left(a_{VS} a_\rho^2 + a_{VS} b_\rho^2 + b_{VS} a_\rho^2 \right)$$
$$+ k_5^3 \left(a_\rho b_{VS}^2 + a_\rho a_{VS}^2 + b_\rho b_{VS}^2 \right) + k_6^3 \left(b_\rho a_{VP}^2 + a_\rho b_{VP}^2 + a_\rho a_{VP}^2 + a_{VP} b_\rho^2 + a_{VP} a_\rho^2 + b_{VP} a_\rho^2 \right)$$
$$+ k_7^3 \left(b_{VS} a_{VP}^2 + a_{VS} b_{VP}^2 + a_{VS} a_{VP}^2 + a_{VS} b_{VP} b_{VS} + a_{VP} b_{VS} a_{VS} \right) + \ldots$$

All coefficients are functions of $\sin\theta$ and V_{S0}/V_{P0}

Linear and higher order ΔRSP

$$\Delta R_{SP}^{(1)}(\varphi) = l_1^1\, a_{VS} + l_2^1\, a_{\rho}$$

$$\Delta R_{SP}^{(2)}(\varphi) = l_1^2\, \mathrm{a}_{VS}^2 + l_2^2\, \mathrm{a}_{\rho}^2 + l_3^2 a_{\rho} b_{\rho} + l_4^2\, a_{VS} b_{VS} + l_5^2 \left(a_{VP} a_{VS} + a_{VP} b_{VS} + b_{VP} a_{VS} \right)$$
$$+ l_6^2 \left(a_{VP} a_{\rho} + a_{VP} b_{\rho} + b_{VP} a_{\rho} + a_{VS} a_{\rho} + b_{VS} a_{\rho} + a_{VS} b_{\rho} \right)$$

$$\Delta R_{SP}^{(3)}(\varphi) = l_1^3\, \mathrm{a}_{VS}^3 + l_2^3 \left(\mathrm{a}_{\rho}^3 + a_{VS} b_{\rho} b_{VS} + a_{VS} a_{\rho} b_{VS} + a_{\rho} \mathrm{b}_{VS}^2 + a_{\rho} \mathrm{a}_{VS}^2 + a_{\rho} \mathrm{b}_{VS}^2 \right)$$
$$+ l_3^3 \left(a_{VS} b_{\rho} b_{VP} + a_{VS} a_{\rho} b_{VP} + b_{VS} b_{\rho} a_{VP} + b_{VS} a_{\rho} a_{VP} + b_{VS} a_{\rho} b_{VP} + a_{VS} b_{\rho} a_{VP} + a_{VS} a_{\rho} a_{VP} \right)$$
$$+ l_4^3 \left(a_{VS} b_{\rho} a_{\rho} + b_{VS} b_{\rho} a_{\rho} \right) + l_5^3 \left(a_{VS} \mathrm{a}_{\rho}^2 + b_{VS} \mathrm{a}_{\rho}^2 + a_{VS} \mathrm{b}_{\rho}^2 \right) + l_6^3 \left(b_{\rho} \mathrm{a}_{\rho}^2 + a_{\rho} \mathrm{b}_{\rho}^2 \right)$$
$$+ l_7^3 \left(b_{VP} \mathrm{a}_{\rho}^2 + a_{VP} \mathrm{b}_{\rho}^2 + a_{VP} \mathrm{a}_{\rho}^2 + b_{\rho} \mathrm{b}_{VP}^2 + a_{\rho} \mathrm{a}_{VP}^2 + a_{\rho} \mathrm{b}_{VP}^2 \right) + \ldots$$

All coefficients are functions of sinφ and V_{S0}/V_{P0}

Relative seismic parameter changes

$$\frac{\Delta V_P}{V_P}=2\frac{V_{Pb}-V_{P0}}{V_{Pb}+V_{P0}},\quad \frac{\Delta V_S}{V_S}=2\frac{V_{Sb}-V_{S0}}{V_{Sb}+V_{S0}},\quad \frac{\Delta\rho}{\rho}=2\frac{\rho_b-\rho_0}{\rho_b+\rho_0}$$

$$\frac{\delta V_P}{V_P}=2\frac{V_{Pm}-V_{Pb}}{V_{Pm}+V_{Pb}},\quad \frac{\delta V_S}{V_S}=2\frac{V_{Sm}-V_{Sb}}{V_{Sm}+V_{Sb}},\quad \frac{\delta\rho}{\rho}=2\frac{\rho_m-\rho_b}{\rho_m+\rho_b}$$

Time-lapse AVO in terms of relative seismic parameter changes

$$b_{VP} = 2\left(\frac{\Delta V_P}{V_P}\right) - 2\left(\frac{\Delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\Delta V_P}{V_P}\right)^3 - \ldots$$

...

$$a_{VP} = 2\left(\frac{\delta V_P}{V_P}\right) - 2\left(\frac{\delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\delta V_P}{V_P}\right)^3 - \ldots$$

...

$$\Delta R_{PS}^{(1)}(\theta) = \left(-2\frac{V_S}{V_P}\sin\theta\right)\frac{\delta V_S}{V_S} - \frac{1}{2}\left(2\frac{V_S}{V_P}+1\right)\sin\theta\frac{\delta\rho}{\rho}$$

$$\Delta R_{SP}^{(1)}(\varphi) = \left(-2\frac{V_S}{V_P}\sin\varphi\right)\frac{\delta V_S}{V_S} - \frac{1}{2}\left(2\frac{V_S}{V_P}+1\right)\sin\varphi\frac{\delta\rho}{\rho}$$

Numerical example

- Baseline survey

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  V_{Pm}=4000 m/s

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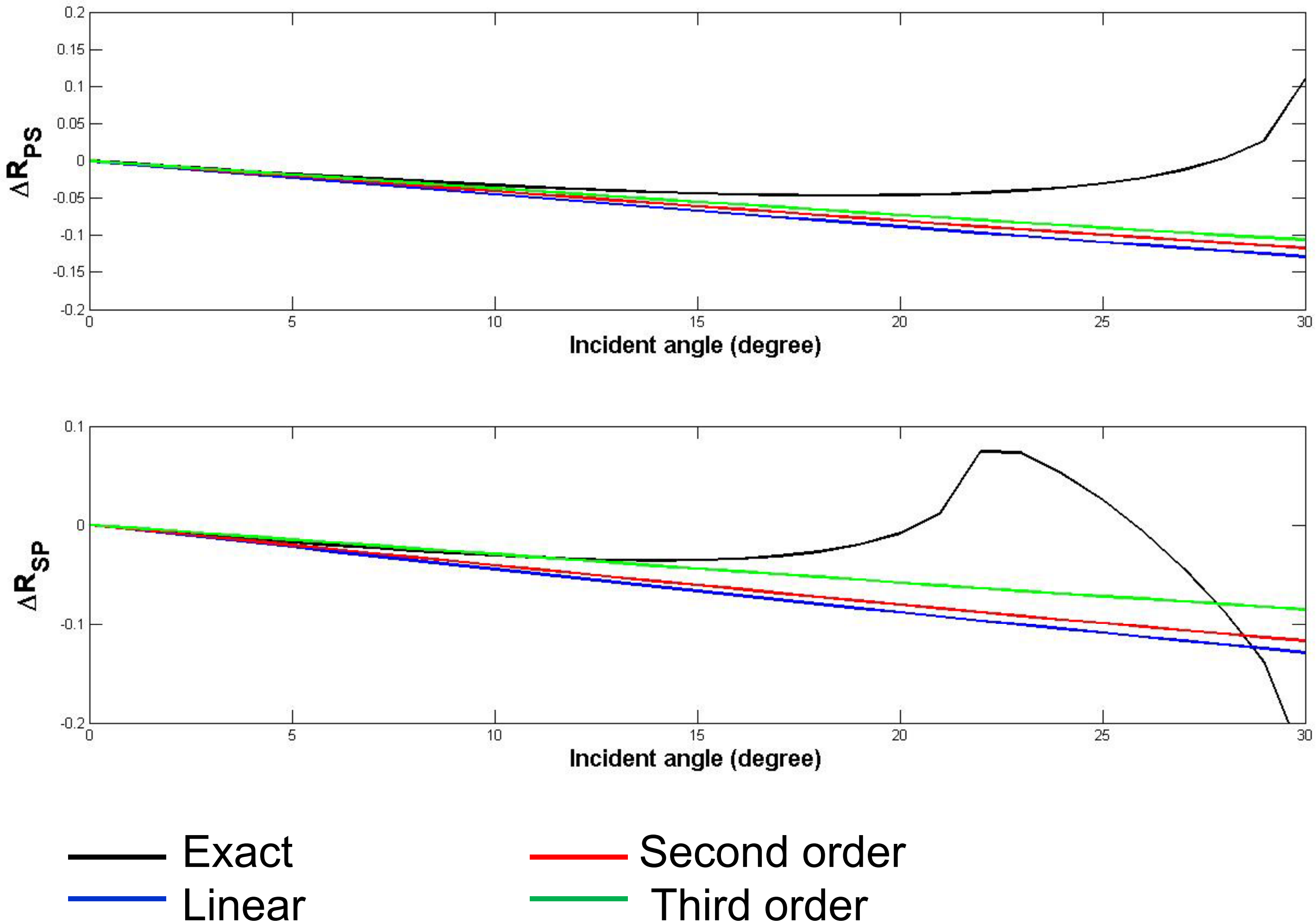
Linear, second, and third order ΔR_{PS} and ΔR_{SP}



Conclusions

- Including higher order terms in ΔR_{PS}, and ΔR_{SP} improves the accuracy of approximating time lapse difference data.
- Higher order terms for ΔR_{PS} and ΔR_{SP} are different, which is not the case in the linear approximation.

Future work

- Further numerical, analytical examination of ΔR_{PP}, ΔR_{PS}, ΔR_{SP}, and ΔR_{SS}
- Validation of time-lapse AVO formula using physical modeling data
- Modeling of inversion of field data example

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Questions