1.5D Internal Multiple Prediction in Plane wave domain

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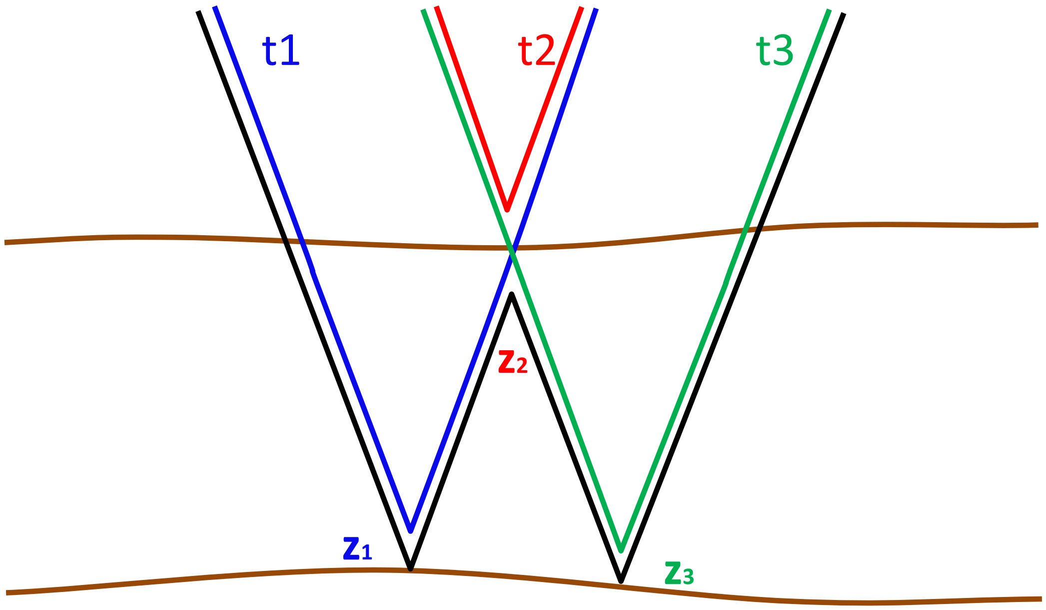
Outline

- ❖ Inverse scattering algorithm
- ❖ Pseudo-depth monotonicity condition
- ❖ IM prediction in plane wave domain
- ❖ Conclusion and Future work
- ❖ Acknowledgements

Inverse scattering series algorithm



$t_{IM} = t1 + t3 - t2$

Actual depth relationship:

$z_1 > z_2$ and $z_2 < z_3$

Lower-Higher-Lower

(Weglein et al., 2003)

2D IM prediction algorithm (Weglein et al., 1997,2003)

$$b_{3_{IM}}(k_g,k_s,\omega)=\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} dk_1 e^{-iq_1(\varepsilon_g-\varepsilon_s)}dk_2 e^{-iq_2(\varepsilon_g-\varepsilon_s)}\int_{-\infty}^{+\infty} dz e^{i(q_g+q_1)z} b_1(k_g,k_1,z)$$

$$\times\int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'} b_1(k_1,k_2,z')\int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2+q_s)z''} b_1(k_2,k_s,z'')$$

$where \qquad q_X=\frac{\omega}{c_0}\sqrt{1-\frac{{k_X}^2{c_0}^2}{\omega^2}}; \qquad k_z=q_g+q_s; \qquad z=\frac{c_0\boldsymbol{t}}{2}$

$z \text{ is the Pseudo} - \text{depth which satisfied: } z>z' \text{ and } z'<z''.$

IM Prediction in 1.5D (*Weglein et al., 1997,2003*)

$$b_{3_{IM}}(k_g,\omega) = \int_{-\infty}^{+\infty} dz e^{ik_z z} b_1(k_g,z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_z z'} b_1(k_g,z') \int_{z'+\epsilon}^{+\infty} dz'' e^{ik_z z''} b_1(k_g,z'')$$

$where \qquad q_X = \frac{\omega}{c_0}\sqrt{1-\frac{{k_X}^2{c_0}^2}{\omega^2}}$

$$k_z = 2q_g = 2q_s\,; \quad k_g = k_s.$$

z is the Pseudo − depth which satisfied: $z > z'$ and $z' < z''$.

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$b_1(k_g, k_s, z)$ Construction

The procedure for getting the input was given by Innanen (2012):

1. Start with the dataset $\mathrm{d}(x_g, x_s, t)$,
2. Fourier Transform from (x_g, x_s, t) to (k_g, k_s, ω),

$$\mathrm{d}(x_g, x_s, t) \rightarrow \mathrm{D}(k_g, k_s, \omega)$$

3. A change of variables from ω to k_z $(where\ k_z = q_g + q_g)$,

$$\mathrm{D}(k_g, k_s, \omega) \rightarrow \mathrm{D}(k_g, k_s, k_z)$$

4.Then scaled by $-i2q_s$,

$$b_1(k_g, k_s, k_z) = -i2q_s\mathrm{D}(k_g, k_s, k_z)$$

5. Inverse Fourier Transform

$$b_1(k_g, k_s, k_z) \rightarrow b_1(k_g, k_s, z)$$

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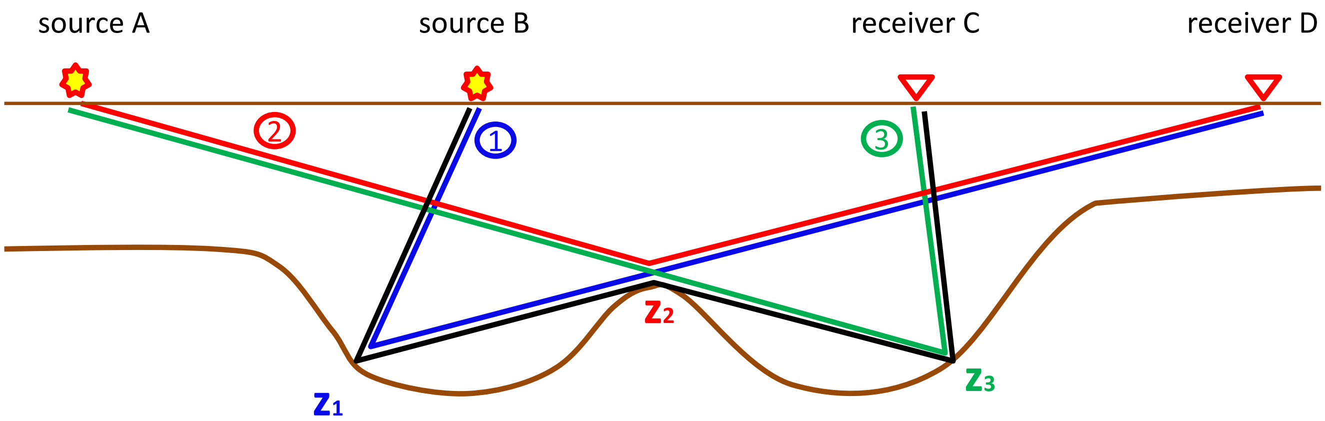
Pseudo-depth monotonicity condition



(Weglein and Nita et al., 2003,2009)

$$z_1^{actual} > z_2^{actual} \quad \Leftrightarrow \quad z_1^{pseudo} > z_2^{pseudo}$$

$$z_1^{actual} > z_2^{actual} \quad \begin{matrix} \Leftarrow \\ \nRightarrow \end{matrix} \quad t_1 > t_2$$

$$z_1^{actual} > z_2^{actual} \quad \Leftrightarrow \quad \tau_1 > \tau_2$$

$$\boldsymbol{z = \frac{c_0 \tau}{2}} \quad ?$$

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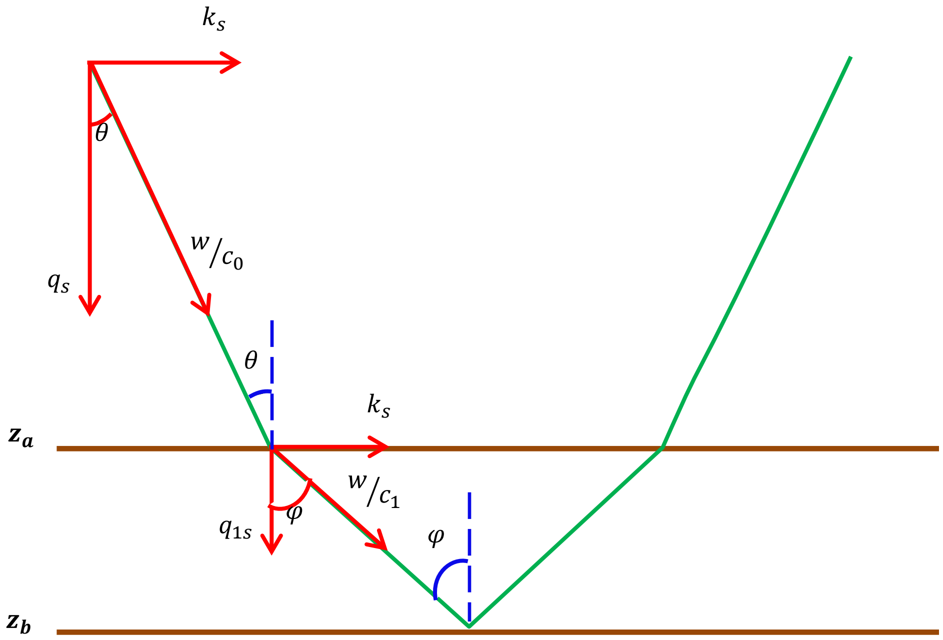
Pseudo-depth monotonicity condition



(Nita et al., 2009)

$$v_x = {c_0}/{sin\theta} \qquad v_z = {c_0}/{cos\theta}$$

Intercept time: $\tau = {2z}/{v_z} = \frac{2zcos\theta}{c_0}$

$$q_s z_a = \frac{wcos\theta}{c_0} * z_a = w\frac{\tau_a}{2} = q_g z_a$$

$$\boldsymbol{k_z z_a = w\tau_a} \qquad \textbf{(1)}$$

$$\boldsymbol{\gamma_1(z_b - z_a) = w(\tau_b - \tau_a)} \qquad \textbf{(2)}$$

$$where\ \ \gamma_1 = q_{1s} + q_{1g} = \frac{2wcos\varphi}{c_1}$$

$$\boldsymbol{k_z z_a + \gamma_1(z_b - z_a) = w\tau_a} \qquad \textbf{(3)}$$

$$\boldsymbol{k_z z_b{}' = w\tau_b} \qquad \textbf{(4)}$$

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IM prediction algorithm in (k_x, z) domain (Weglein et al.,1997,2003)

$$b_{3_{IM}}(k_g, k_s, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} dk_1 e^{-iq_1(\varepsilon_g - \varepsilon_s)} dk_2 e^{-iq_2(\varepsilon_g - \varepsilon_s)} \int_{-\infty}^{+\infty} dz e^{i(q_g + q_1)z} b_1(k_g, k_1, z)$$

$$\times \int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'} b_1(k_1, k_2, z') \int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2+q_s)z''} b_1(k_2, k_s, z'')$$

where $q_X = \frac{\omega}{c_0}\sqrt{1 - \frac{{k_X}^2 {c_0}^2}{\omega^2}}$

$k_x \rightarrow p$

$k_z z \rightarrow w\tau$

IM prediction algorithm in (p, z) domain

$$b_{3_{IM}}(p_g, p_s, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} dp_1 e^{-iq_1(\varepsilon_g - \varepsilon_s)} dp_2 e^{-iq_2(\varepsilon_g - \varepsilon_s)} \int_{-\infty}^{+\infty} dz e^{i(q_g + q_1)z} b_1(p_g, p_1, z)$$

$$\times \int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1 + q_2)z'} b_1(p_1, p_2, z') \int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2 + q_s)z''} b_1(p_2, p_s, z'')$$

$where \qquad q_X = \frac{\omega}{c_0} \sqrt{1 - {p_X}^2 {c_0}^2}$

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IM prediction algorithm in (p, τ) domain

$$b_{3_{IM}}(p_g, p_s, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} dp_1 e^{-i\omega(\tau_{1g} - \tau_{1s})} dp_2 e^{-i\omega(\tau_{2g} - \tau_{2s})} \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} b_1(p_g, p_1, \tau)$$

$$\times \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'} b_1(p_1, p_2, \tau') \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''} b_1(p_2, p_s, \tau'')$$

where $\tau_{XY} = \frac{\omega\varepsilon_Y}{c_0}\sqrt{1 - {p_X}^2{c_0}^2}$

ε_g *and* ε_g *are the source and receiver depth.*

p_X *is the horizontal slowness.*

1.5D IM prediction algorithm in plane wave domain

(p, z) domain : (Nita and Weglein, 2009)

$$b_{3_{IM}}(p_g,\omega) = \int_{-\infty}^{+\infty} dz e^{i2q_g z} b_1(p_g,z) \int_{-\infty}^{z-\epsilon} dz' e^{-i2q_g z'} b_1(p_g,z') \int_{z'+\epsilon}^{+\infty} dz'' e^{i2q_g z''} b_1(p_g,z'')$$

$$where \qquad q_X = \frac{\omega}{c_0}\sqrt{1 - {p_X}^2 {c_0}^2}$$

(p, τ) domain : (Coates and Weglein, 1996)

$$b_{3_{IM}}(p_g,\omega) = \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} b_1(p_g,\tau) \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'} b_1(p_g,\tau') \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''} b_1(p_g,\tau'')$$

$b_1(p,z)$ Construction

Similar to the procedure was given by Innanen (2012):

1. Start with the dataset $\mathrm{d}(x,t)$,
2. Tau-p Transform from (x,t) to (p,τ), (codes from UA)

$$\mathrm{d}(x,t) \rightarrow D_1(p,\tau)$$

3. Fourier Transform from τ to ω,

$$D_1(p,\tau) \rightarrow \mathrm{D}(p,\omega)$$

4. A change of variables from ω to k_z $(where\ k_z = q_g + q_g)$,

$$\mathrm{D}(p,\omega) \rightarrow \mathrm{D}(p,k_z)$$

5. Then scaled by $-i2q_s$,

$$b_1(p,k_z) = -i2q_s\mathrm{D}(p,k_z)$$

6. Inverse Fourier Transform

$$b_1(p,k_z) \rightarrow b_1(p,z\ )$$

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$\boldsymbol{b_1(p,\tau)}$ Construction

1. Start with the dataset $\mathrm{d}(x,t)$,
2. Tau-p Transform from (x,t) to (p,τ), (codes from UA)

$$\mathrm{d}(x,t) \rightarrow D_1(p,\tau)$$

3. Fourier Transform from τ to ω,

$$D_1(p,\tau) \rightarrow \mathrm{D}(p,\omega)$$

4.Then scaled by $-i2q_s$,

$$b_1(p,\omega) = -i2q_s \mathrm{D}(p,\omega\ )$$

5. Inverse Fourier Transform

$$b_1(p,\omega) \rightarrow b_1(p,\tau\ )$$

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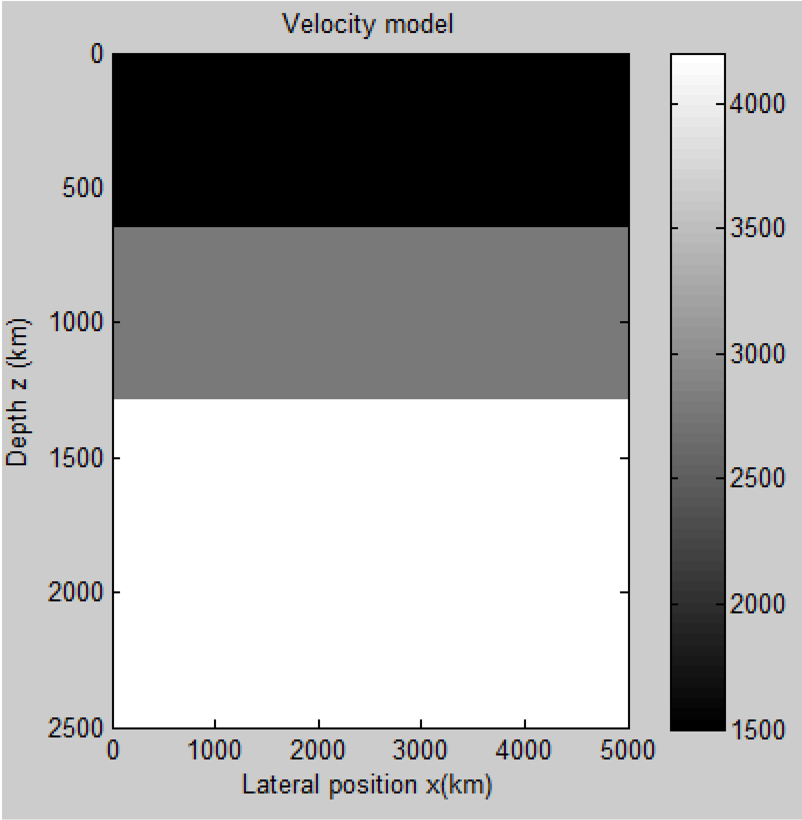
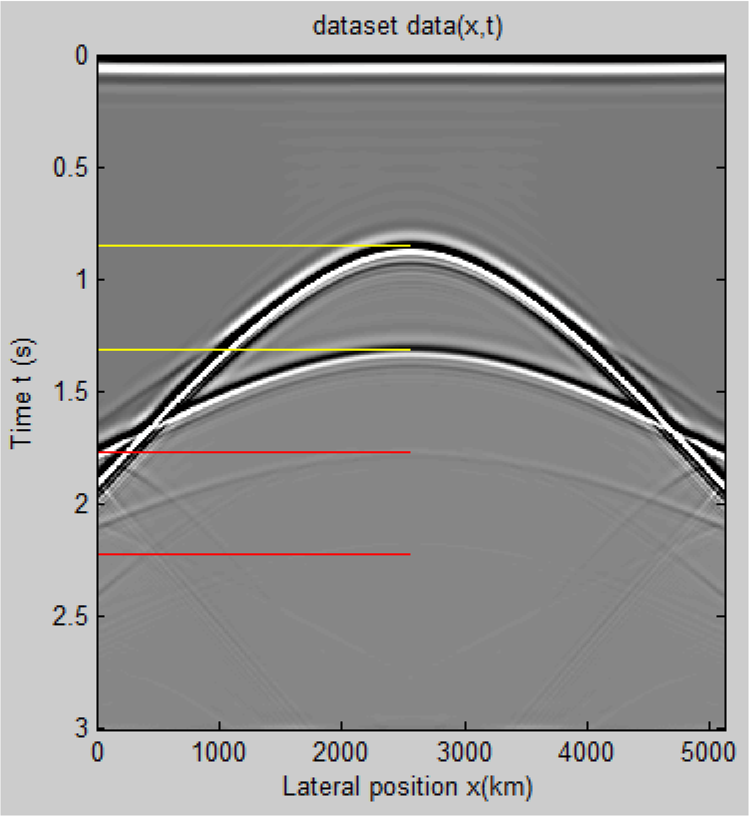
IM prediction algorithm in (p, z) domain --- $b_1(p,z)$





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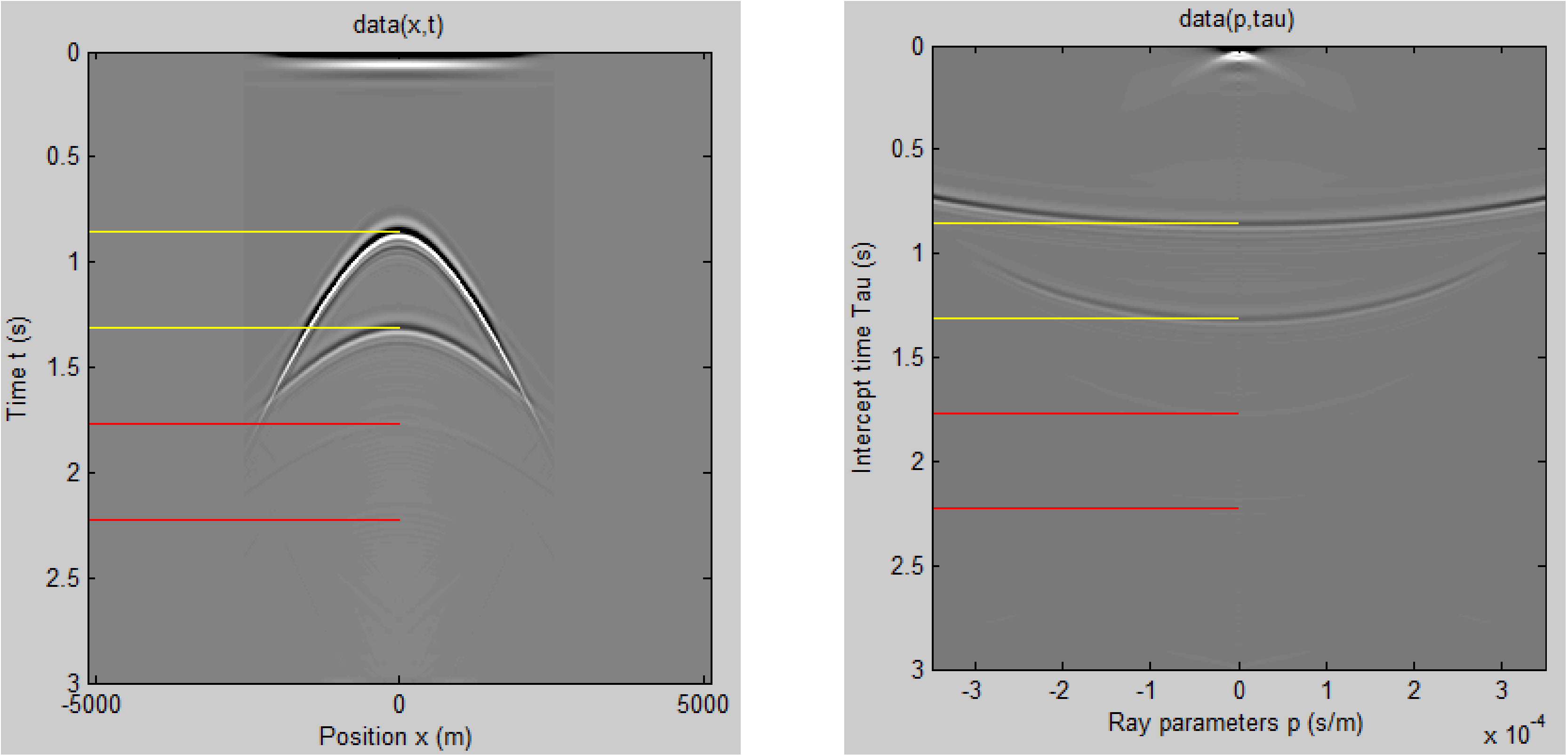
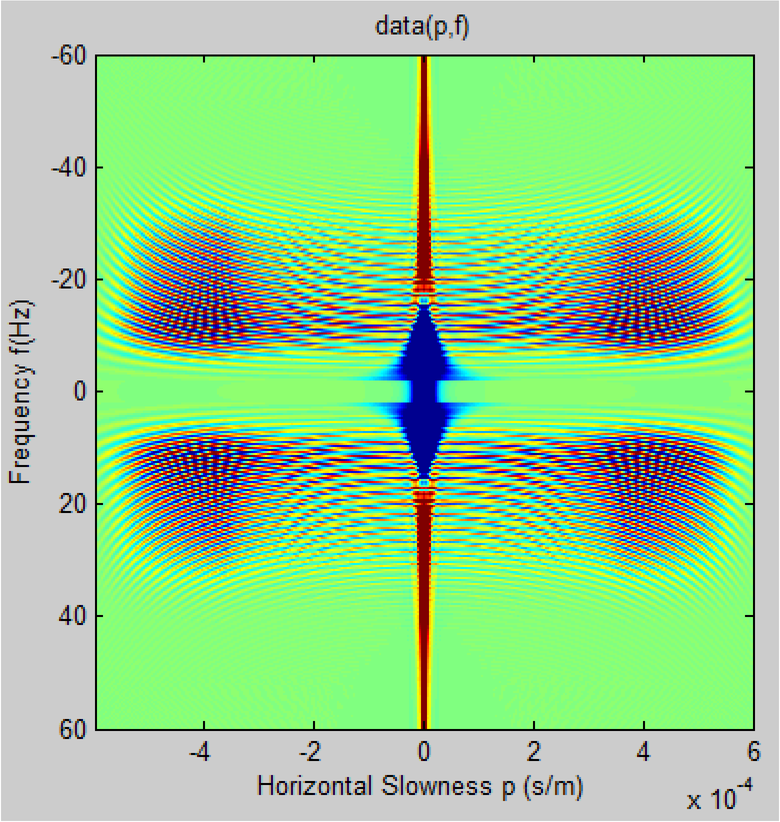
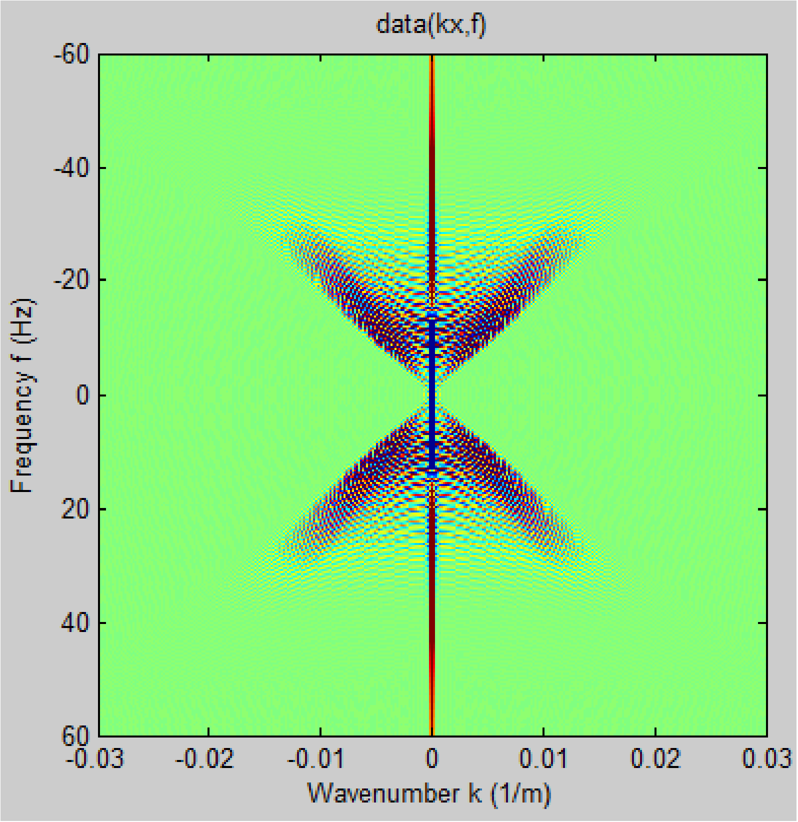
IM prediction algorithm in (p, z) domain --- $b_1(p,z)$



IM prediction algorithm in (p, z) domain --- $b_1(p, z)$





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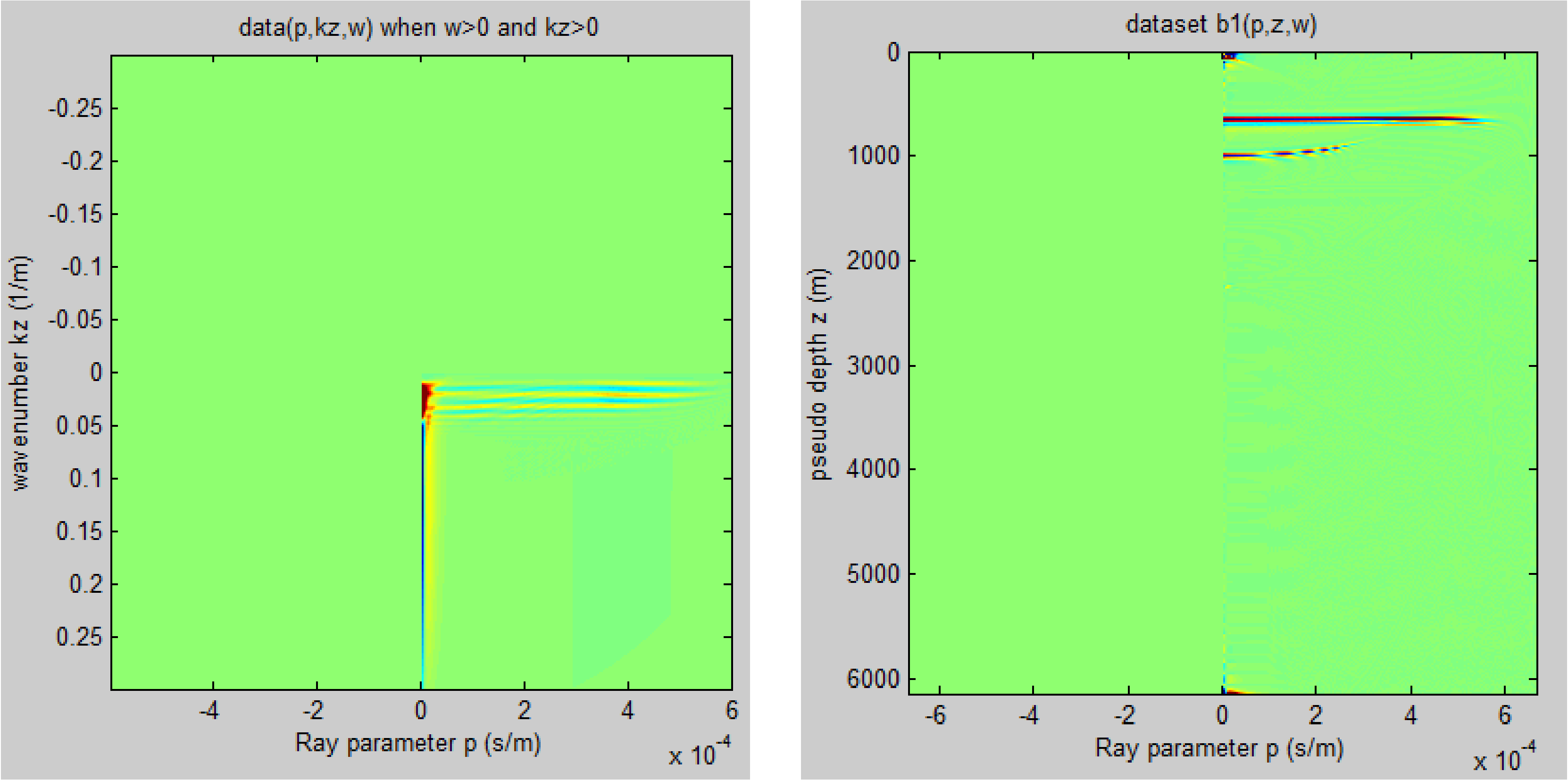
IM prediction algorithm in (p, z) domain --- $b_1(p, z)$



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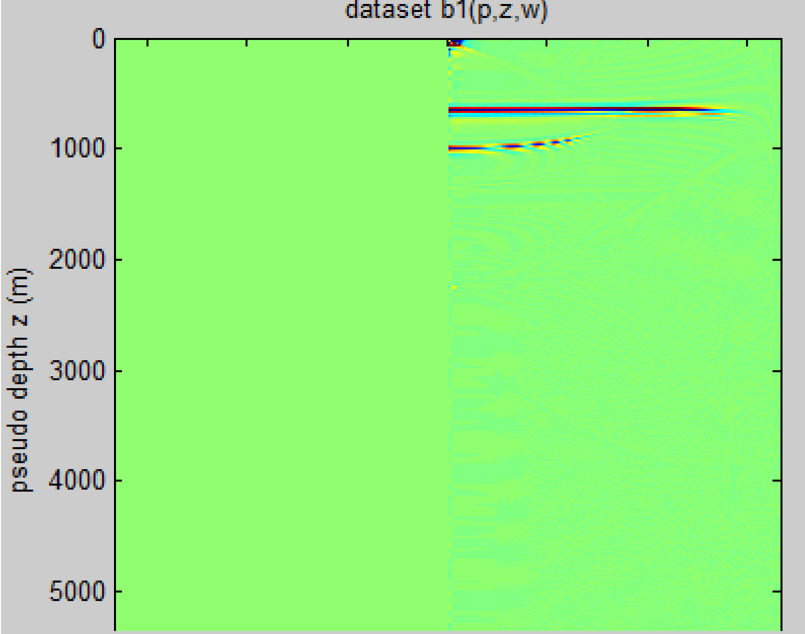
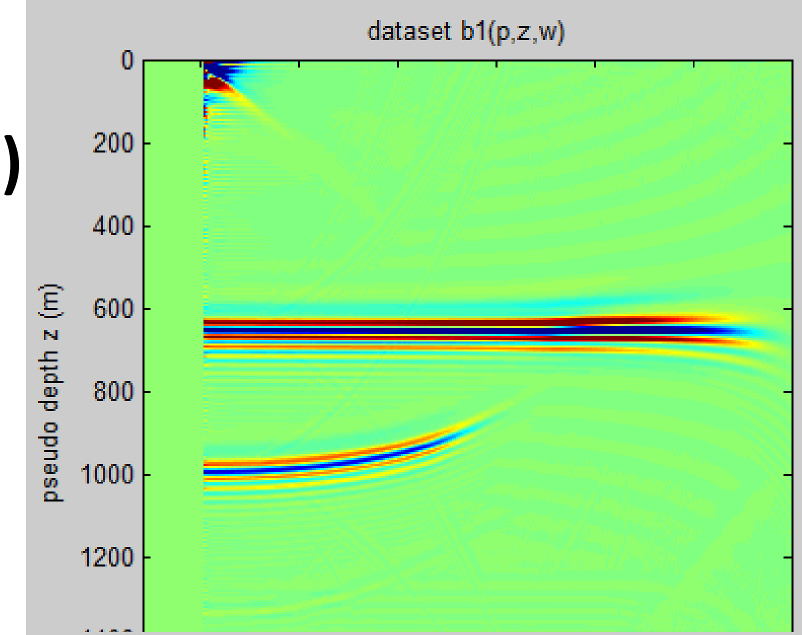
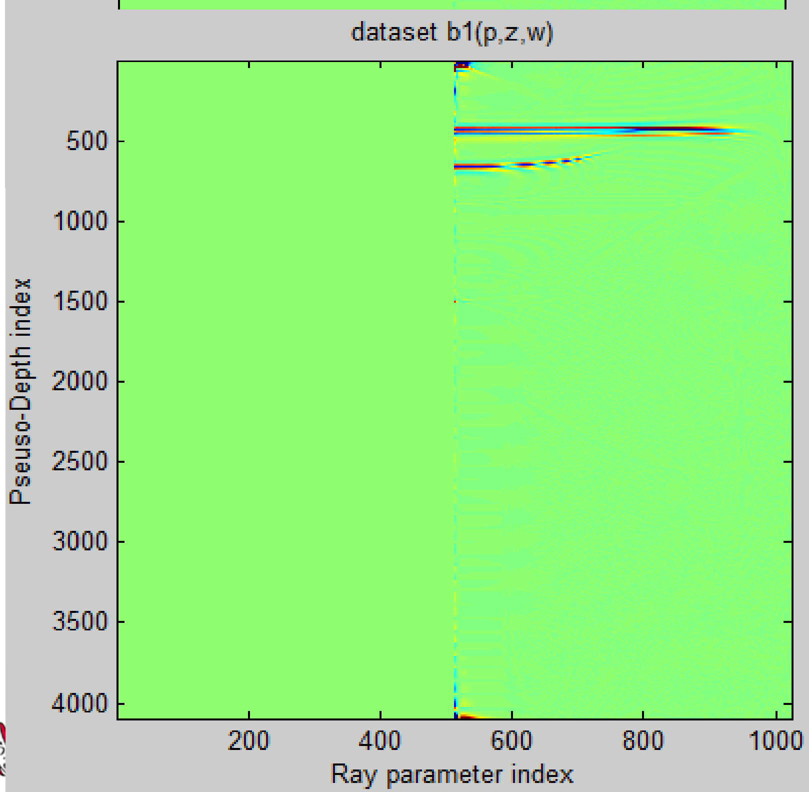
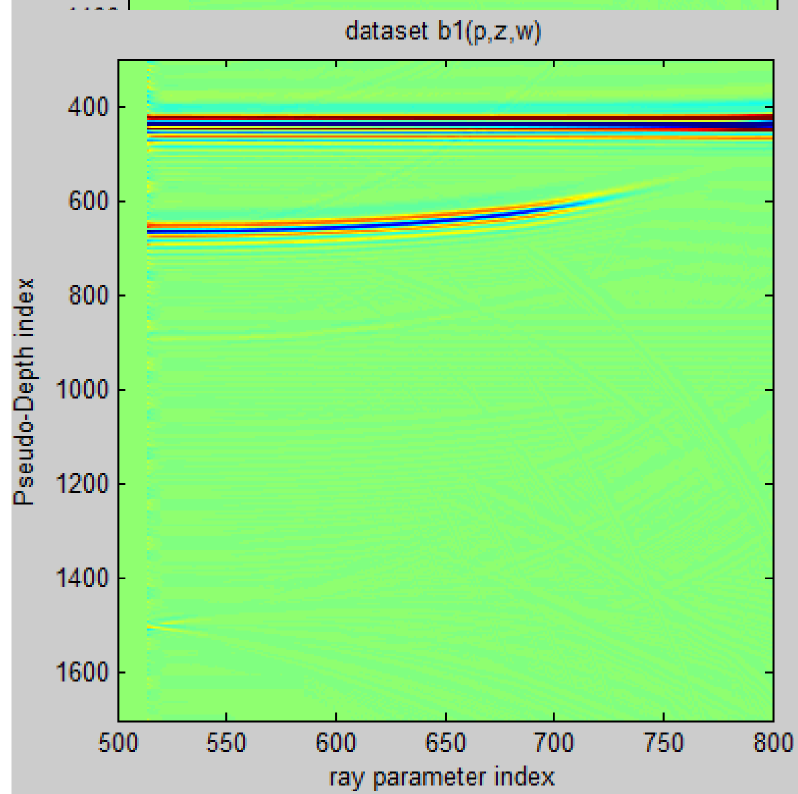

dataset b1(p,z,w)
pseudo depth z (m)
0
1000
2000
3000
4000
5000

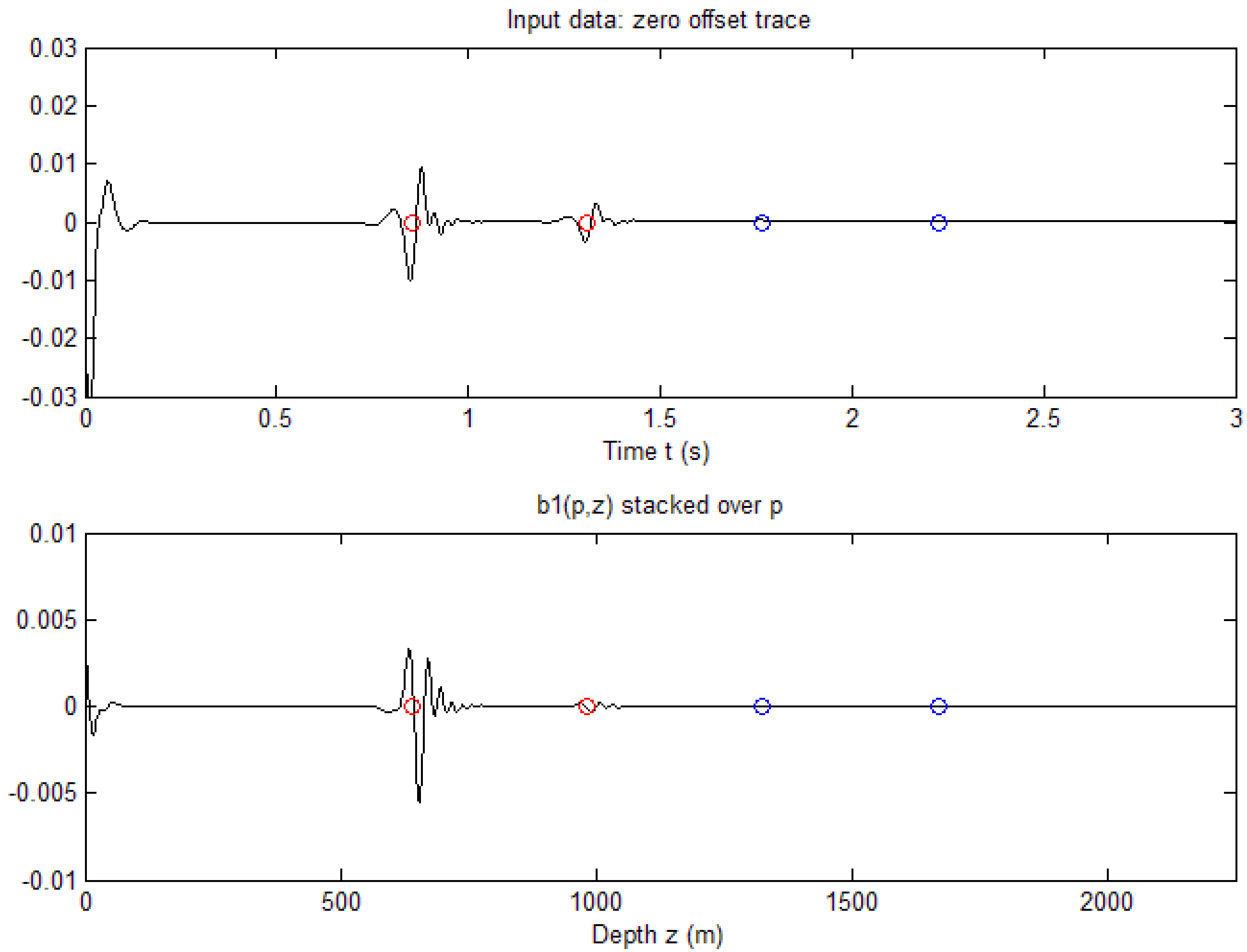
(p, z)


dataset b1(p,z,w)
pseudo depth z (m)
0
200
400
600
800
1000
1200

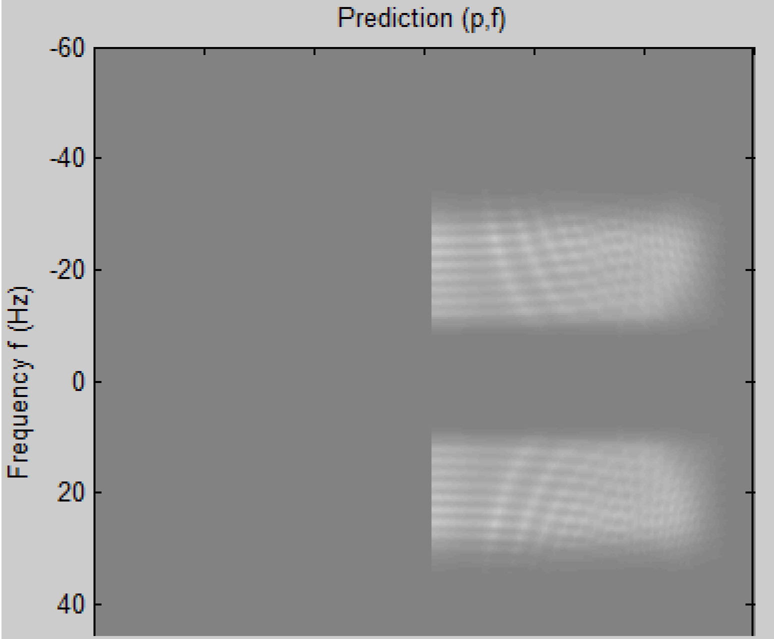

dataset b1(p,z,w)
Pseuso-Depth index
500
1000
1500
2000
2500
3000
3500
4000
200
400
600
800
1000
Ray parameter index

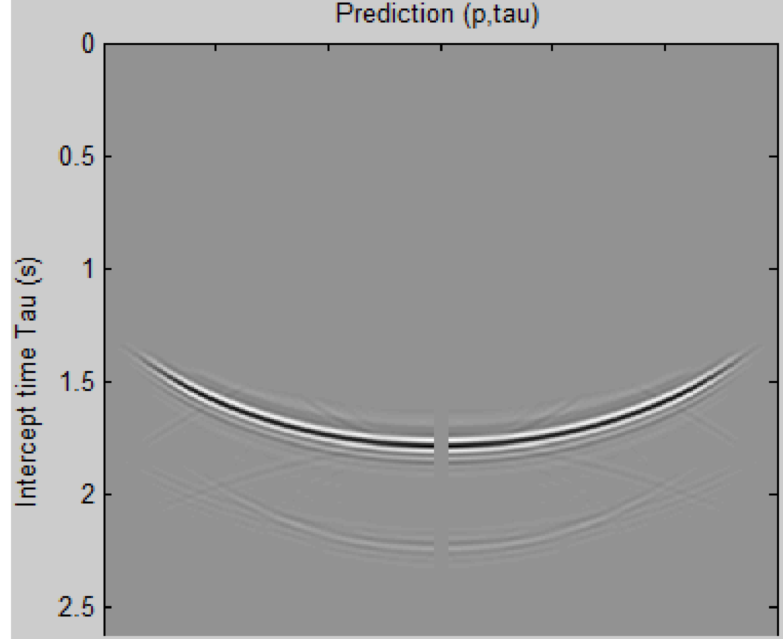

dataset b1(p,z,w)
Pseudo-Depth index
400
600
800
1000
1200
1400
1600
500
550
600
650
700
750
800
ray parameter index

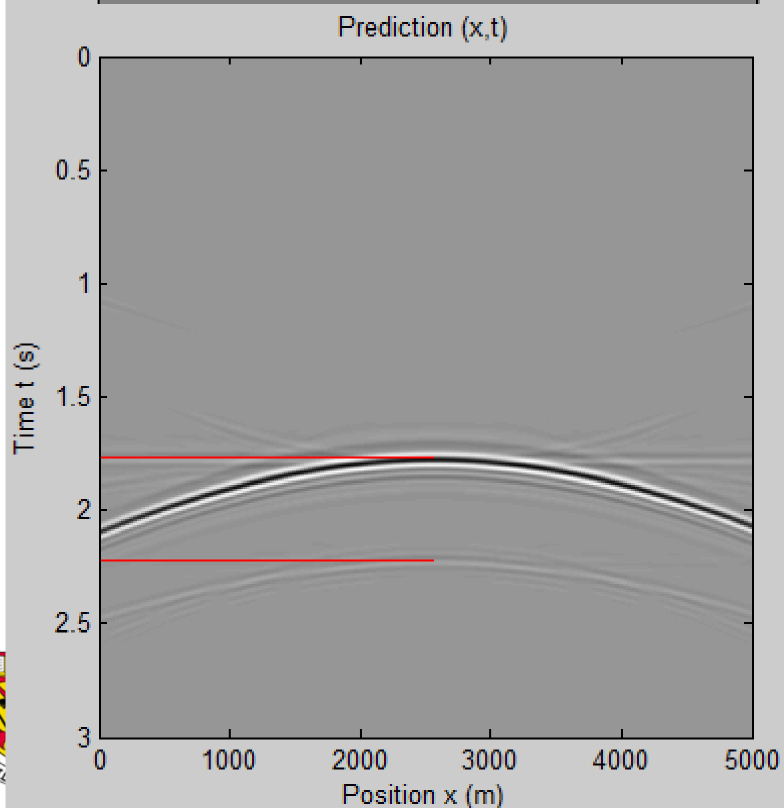
IM prediction algorithm in (p, z) domain --- $b_1(p,z)$

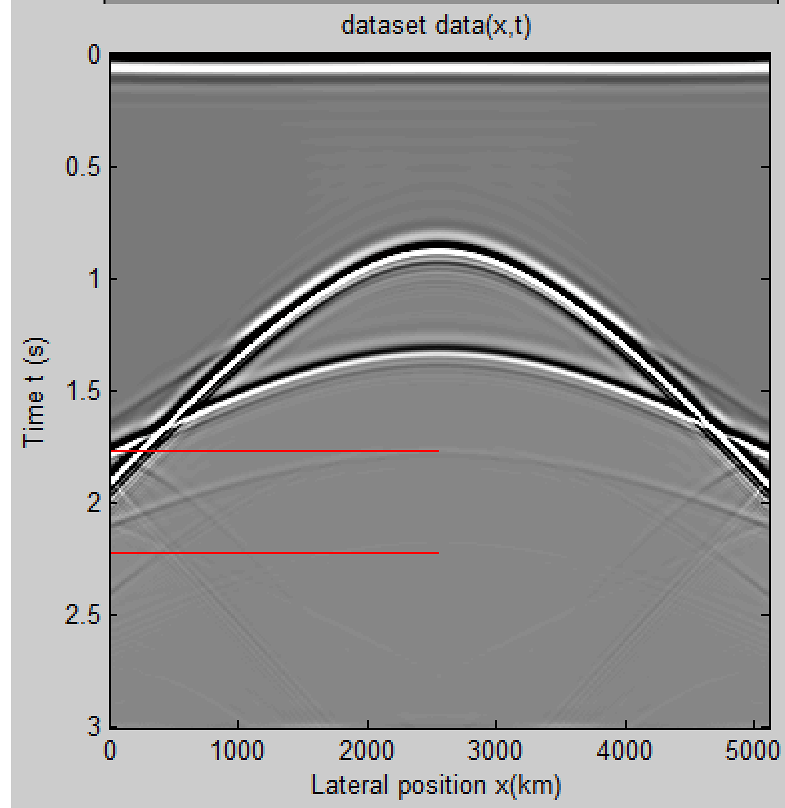


(p, z)

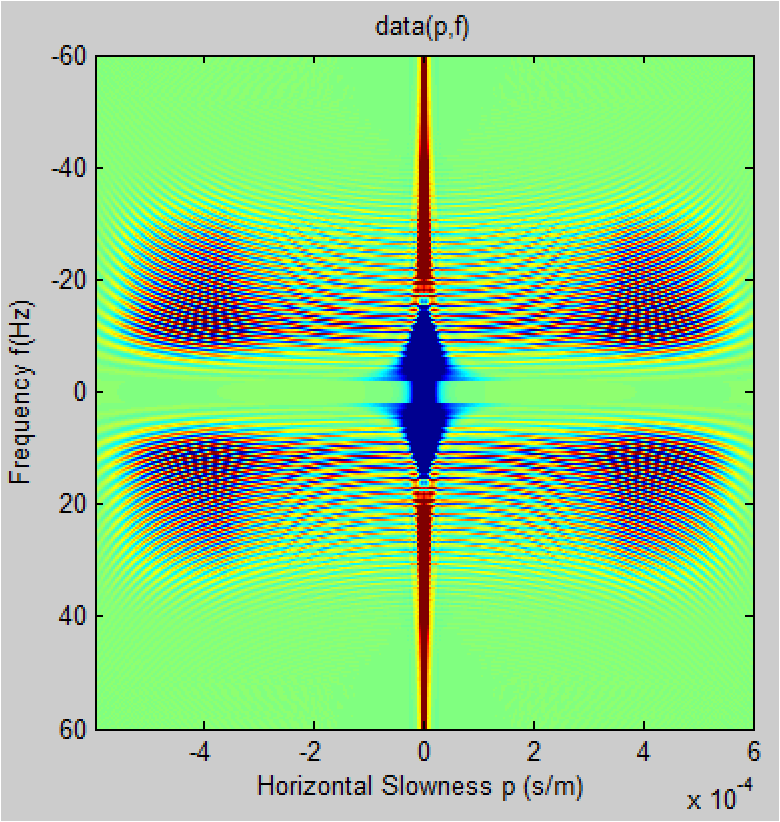

Prediction (p,f)
Frequency f (Hz)
-60
-40
-20
0
20
40

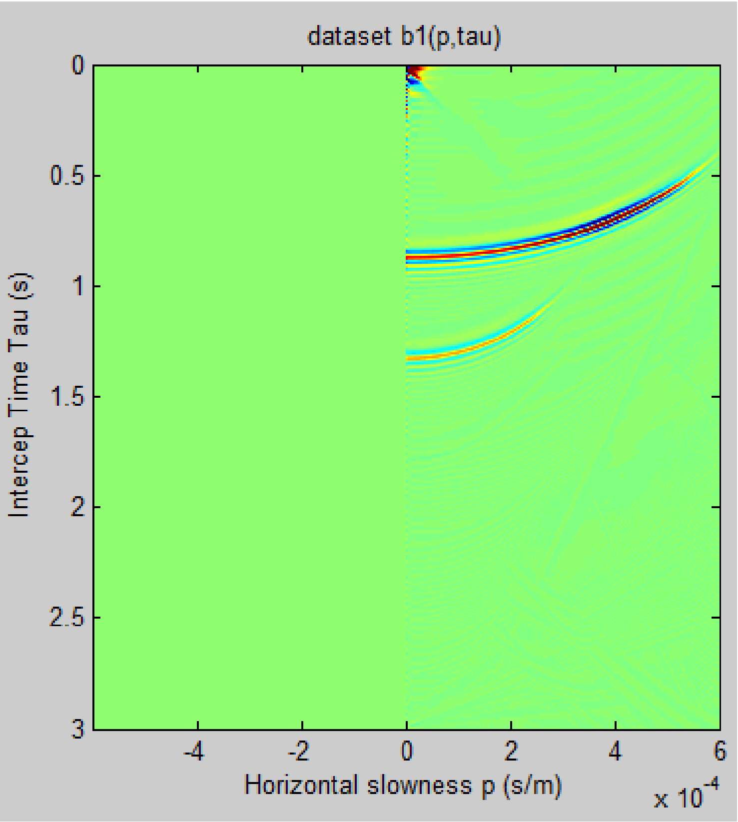

Prediction (p,tau)
Intercept time Tau (s)
0
0.5
1
1.5
2
2.5


Prediction (x,t)
Time t (s)
0
0.5
1
1.5
2
2.5
3
0
1000
2000
3000
4000
5000
Position x (m)


dataset data(x,t)
Time t (s)
0
0.5
1
1.5
2
2.5
3
0
1000
2000
3000
4000
5000
Lateral position x(km)

IM prediction algorithm in (p, τ) domain --- $b_1(p,\tau)$





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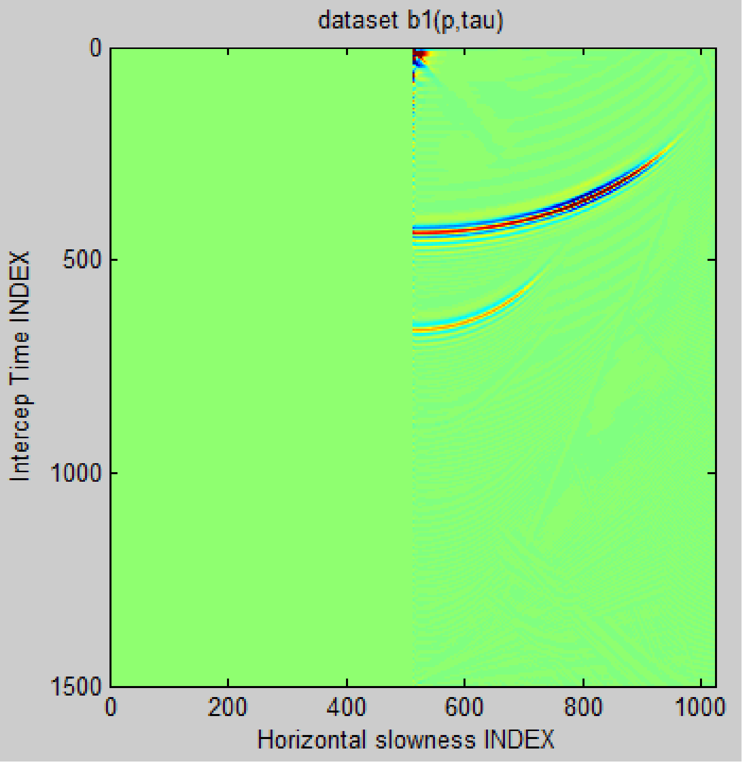
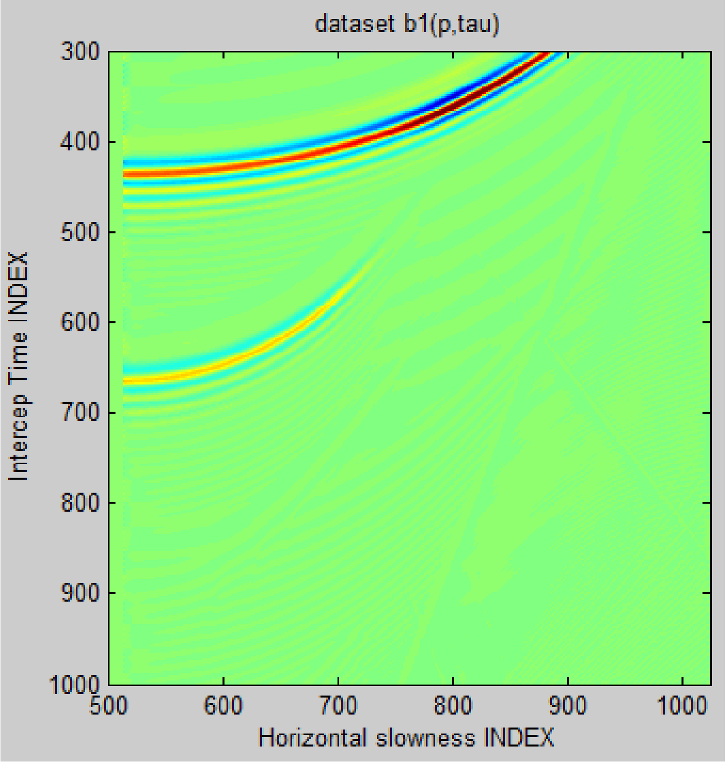
IM prediction algorithm in (p, τ) domain --- $b_1(p,\tau)$





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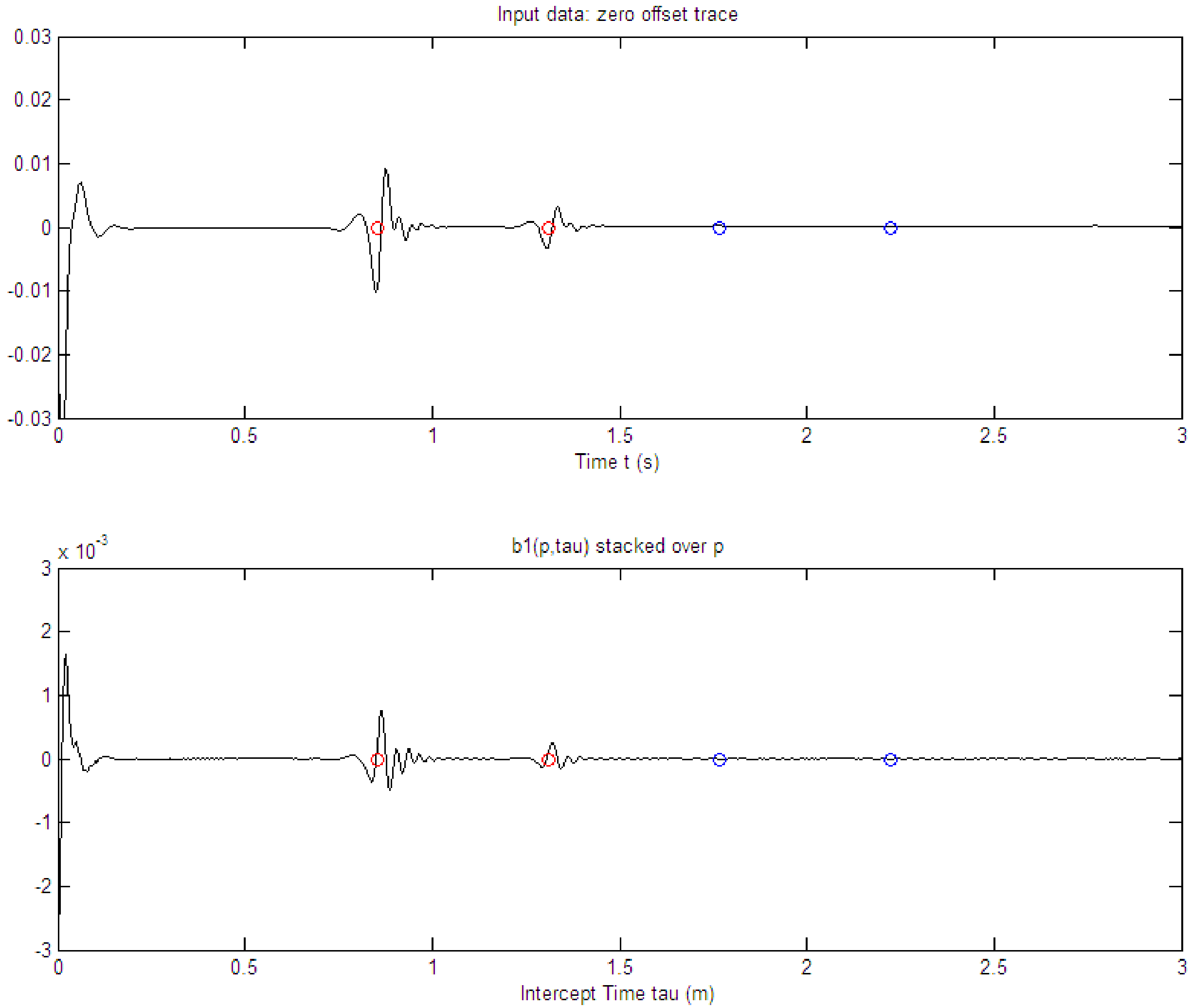
IM prediction algorithm in (p, τ) domain --- $b_1(p, \tau)$



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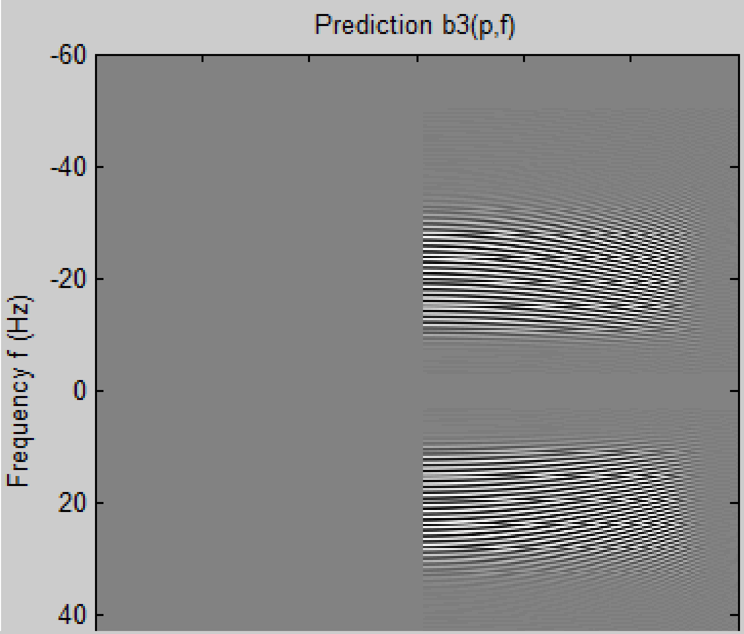
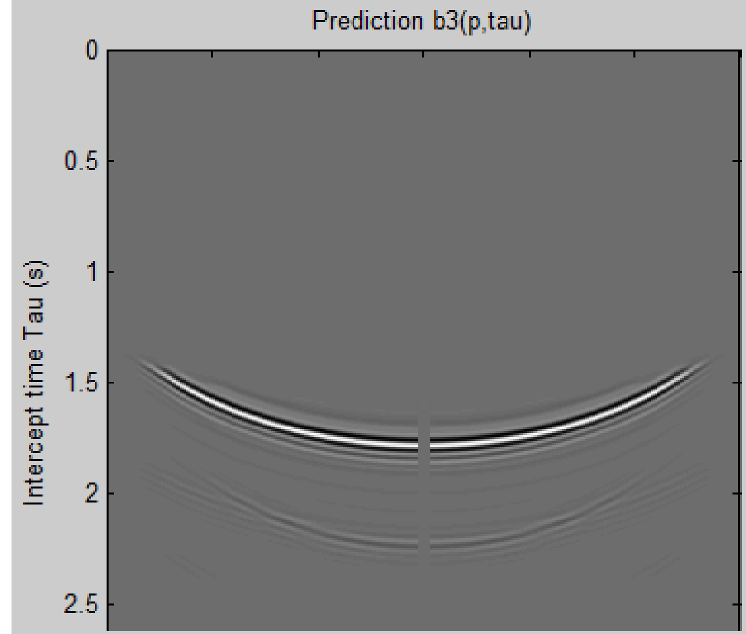
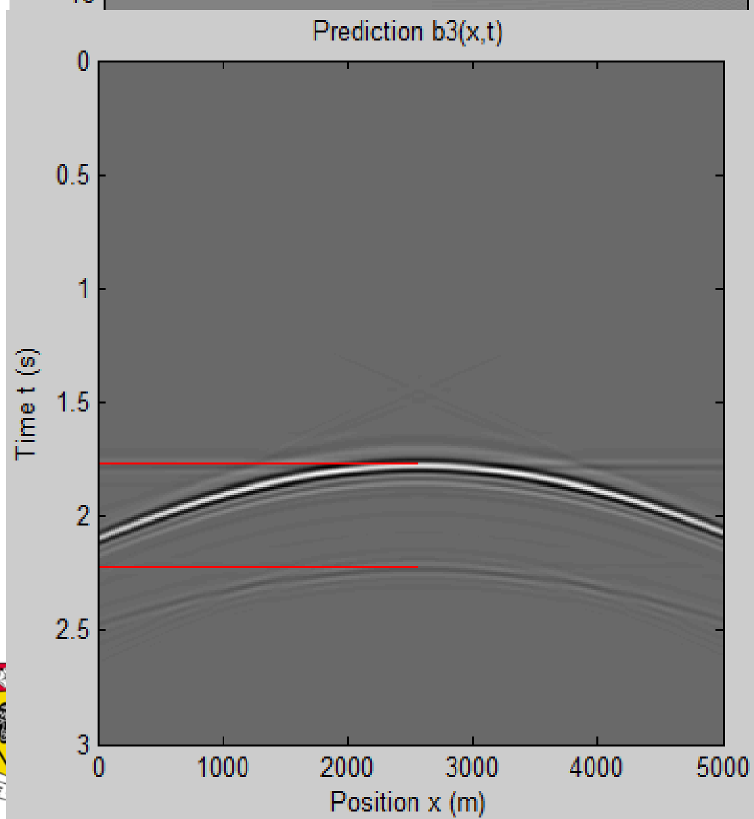
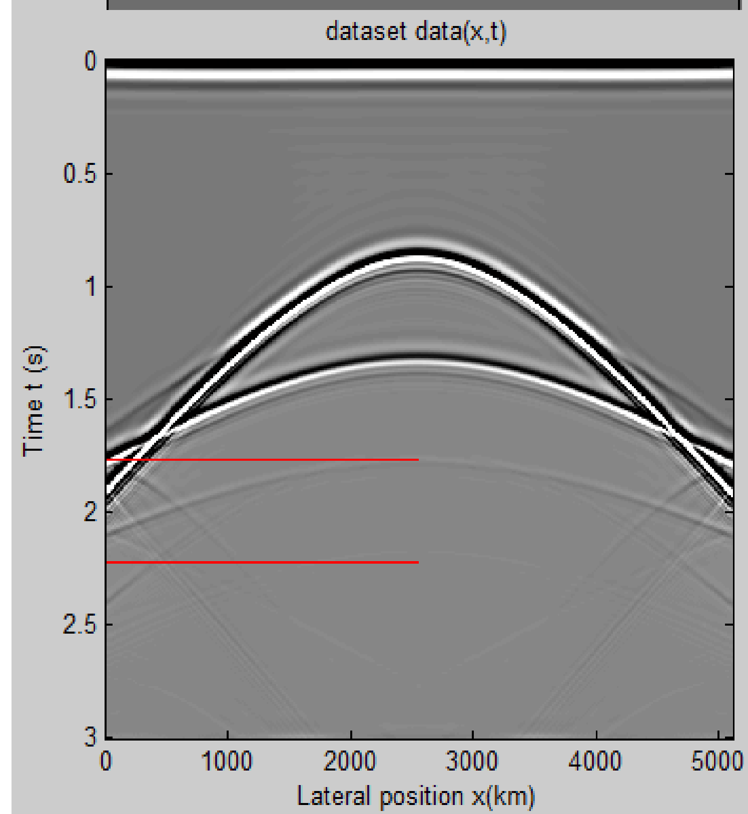
(p, τ)


Prediction b3(p,f)
Frequency f (Hz)
-60
-40
-20
0
20
40

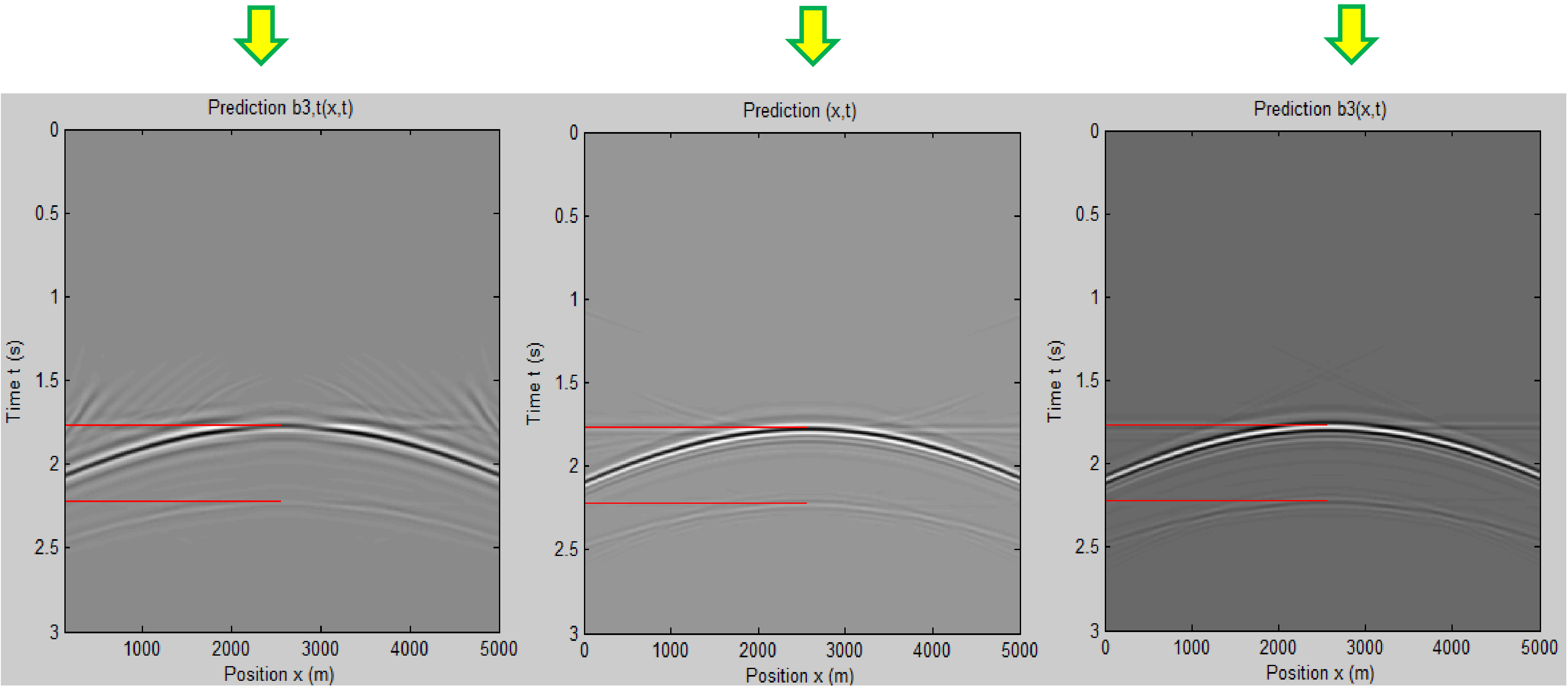

Prediction b3(p,tau)
Intercept time Tau (s)
0
0.5
1
1.5
2
2.5


Prediction b3(x,t)
Time t (s)
0
0.5
1
1.5
2
2.5
3
0
1000
2000
3000
4000
5000
Position x (m)


dataset data(x,t)
Time t (s)
0
0.5
1
1.5
2
2.5
3
0
1000
2000
3000
4000
5000
Lateral position x(km)

Conclusion

Integral of $b_1(k_x, z)$ ***Integral of*** $b_1(p, z)$ ***Intergral of*** $b_1(p, \tau)$



Conclusion and Future Work

Inverse Scattering Series algorithm also works for predicting IM in plane wave domain, either (p, z) domain or (p, τ) domain, with the same accuracy to (k_x, z) domain, even better.

Especially for (p, τ) domain , it may save time because no resampling needed in the processing.

Easier to get rid of Free-surface multiples in (p, τ) domain , and prepare for IM prediction.

Consider the effect of the epsilon value and dip.

A real 2D IM prediction.

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Acknowledgements

- Dr. Kristopher Innanen
- Penny Pan, Raul Cova
- All CREWES Staff and Students
- All CREWES Sponsors

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Question ?