Understanding and Improving Azimuthal AVO Analysis

CREWES Technical Talk 11/14/14

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Azimuthal AVO

What it is:

- Analysis of incidence angle and azimuthal amplitude variations of reflection coefficients;
- Measures change in elastic properties at an interface.

Why it's used:

- Better vertical resolution than propagation methods (e.g. VVAZ, S-wave splitting);
- Usually used to attempt to characterize natural fractures or differential stress in a reservoir.

Motivation

Want an azimuthal AVO technique that

- Works for general anisotropy (no rock physics assumptions in the initial stage);
- Has no ambiguities (solutions are unique);
- Is easy to understand.

Isotropic AVO

Shuey (1985) wrote the linearized weak contrast PP reflection coefficient in the form

$$R_{PP}^{iso}(\theta) = A + B\sin^2\theta + C\tan^2\theta\sin^2\theta$$

And Thomsen (1990) used this form to describe the effect of common seismic parameters V_P,V_S, ρ, and μ on the reflection coefficient:

$$R_{PP}^{iso}(\theta) = \frac{1}{2}\left[\frac{\Delta V_P}{\overline{V}_P} + \frac{\Delta\rho}{\overline{\rho}}\right] + \frac{1}{2}\left[\frac{\Delta V_P}{\overline{V}_P} - \left(\frac{2\overline{V}_S}{\overline{V}_P}\right)\frac{\Delta\mu}{\overline{\mu}}\right]\sin^2\theta + \frac{1}{2}\frac{\Delta V_P}{\overline{V}_P}\tan^2\theta\sin^2\theta$$

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Intercept Gradient Curvature

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Anisotropic Reflection Coefficient

Thomsen (1993) derived the linearized PP reflection coefficient for small contrast weak anisotropy along a vertical plane:

$$
\begin{aligned}
R_{PP}(\theta) = & \frac{1}{2}\left[\frac{\Delta Z_0}{\overline{Z}_0}\right] \\
& + \frac{1}{2}\left[\frac{\Delta V_{P_0}}{\overline{V}_{P_0}} - \left(\frac{2\overline{V}_{S_0}}{\overline{V}_{P_0}}\right)\frac{\Delta\mu_0}{\overline{\mu}_0} + (\delta_2 - \delta_1)\right]\sin^2\theta \\
& + \frac{1}{2}\left[\frac{\Delta V_{P_0}}{\overline{V}_{P_0}} - (\delta_2 - \delta_1 - \epsilon_2 + \epsilon_1)\right]\tan^2\theta\sin^2\theta
\end{aligned}
$$

as did Vavryuk and Psencík (1998) using perturbations from background P-wave velocities, α, and S-wave velocities β:

$$
\begin{aligned}
R_{PP}(\theta_P) = & \frac{\rho\Delta A'_{33} + 2\alpha^2\Delta\rho}{4\rho\alpha^2} \\
& + \frac{1}{2}\left[\frac{\Delta A'_{33}}{2\alpha^2} - -\frac{4(\rho\Delta A'_{55} + \beta^2\Delta\rho)}{\rho\alpha^2} + \Delta\delta^{*\prime}\right]\sin^2\theta_P \\
& + \frac{1}{2}\left(\frac{\Delta A'_{33}}{2\alpha^2} + \Delta\epsilon^{*\prime}\right)\sin^2\theta_P\tan^2\theta_P
\end{aligned}
$$

Where

$$
\delta^{*(I)\prime} = \frac{A_{13}^{(I)\prime} + 2A_{55}^{(I)\prime} - A_{33}^{(I)\prime}}{\alpha^2}, \epsilon^{*(I)\prime} = \frac{A_{11}^{(I)\prime} - A_{33}^{(I)\prime}}{2\alpha^2}
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Where

$$\delta^{*(I)\prime} = \frac{A^{(I)\prime}_{13} + 2A^{(I)\prime}_{55} - A^{(I)\prime}_{33}}{\alpha^2}, \epsilon^{*(I)\prime} = \frac{A^{(I)\prime}_{11} - A^{(I)\prime}_{33}}{2\alpha^2}$$

Curvature Term

Substituting Δε*’ into the equation for a reflection coefficient from Vavryuk and Psencík (1998) results in a curvature of

$$\frac{1}{2}\left(\frac{\Delta A'_{33}}{2\alpha^2}+\frac{\Delta A'_{11}-\Delta A'_{33}}{2\alpha^2}\right)=\frac{\Delta A'_{11}}{4\alpha^2}\approx\frac{1}{2}\frac{\Delta V_{P}{}'_{H}}{V_{P}{}'_{H}}$$

This term

- Is only influenced by a single stiffness coefficient
- Easily relates to the isotropic AVO equations
- Is simple to understand

Curvature compared to Ruger's Equation

The coordinate transformation for A'_{11} along an arbitrary vertical plane at an angle ϕ from the original plane is

$$A'_{11} = A_{11}\cos^4\phi + 4A_{16}\cos^3\phi\sin\phi + 2(A_{12}+2A_{66})\cos^2\phi\sin^2\phi + 4A_{26}\sin^3\phi\cos\phi + A_{22}\sin^4\phi$$

Ruger's curvature is

$$\frac{1}{2}\left[\frac{\Delta\alpha}{\overline{\alpha}} + \Delta\epsilon^{(V)}\cos^4\phi + \Delta\delta^{(V)}\sin^2\phi\cos^2\phi\right]$$

and inserting the weak anisotropy parameters and assuming $\alpha^2 = A_{33}$ it becomes

$$\frac{1}{2}\left[\left(\frac{\Delta A_{33}}{2\alpha^2}\right) + \left(\frac{\Delta A_{11} - \Delta A_{33}}{2\alpha^2}\right)\cos^4\phi + \left(\frac{\Delta A_{13} + 2\Delta A_{55} - \Delta A_{33}}{\alpha^2}\right)\sin^2\phi\cos^2\phi\right] =$$

$$\frac{1}{4\alpha^2}\left[\Delta A_{11}\cos^4\phi + 2(\Delta A_{13} + 2\Delta A_{55})\cos^2\phi\sin^2\phi + \Delta A_{33}\sin^4\phi\right]$$

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as did Vavryuk and Psencík (1998) using perturbations from background P-wave velocities, α, and S-wave velocities β:

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Where

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$$

Gradient along a plane

Substituting the weak anisotropy parameters into Vavrycuk and Psencik’s gradient along a vertical plane results in

$$\frac{1}{2\alpha^2}\left[-2\Delta A'_{55} - 4\beta^2\frac{\Delta\rho}{\rho} + \Delta A'_{13} - \frac{\Delta A'_{33}}{2}\right]$$

The only terms in this gradient that change with azimuth are $\Delta A'_{55}$ and $\Delta A'_{13}$ under the following relations

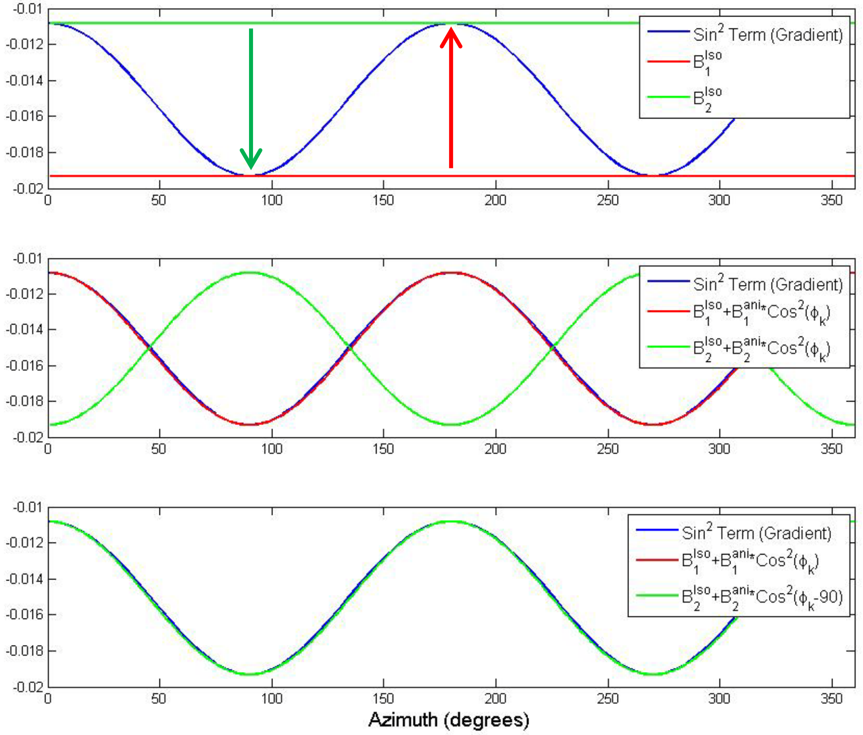
$$A'_{55} = A_{55}\cos^2\phi + 2A_{45}\cos\phi\sin\phi + A_{44}\sin^2\phi,$$
$$A'_{13} = A_{13}\cos^2\phi + 2A_{36}\cos\phi\sin\phi + A_{23}\sin^2\phi.$$

90-degree Ambiguity

Ruger writes the gradient as

$$B(\phi_k) = B^{iso} + B^{ani}\cos^2(\phi_k - \phi_{sym})$$

which has 3 variables and is nonunique with 2 solutions.

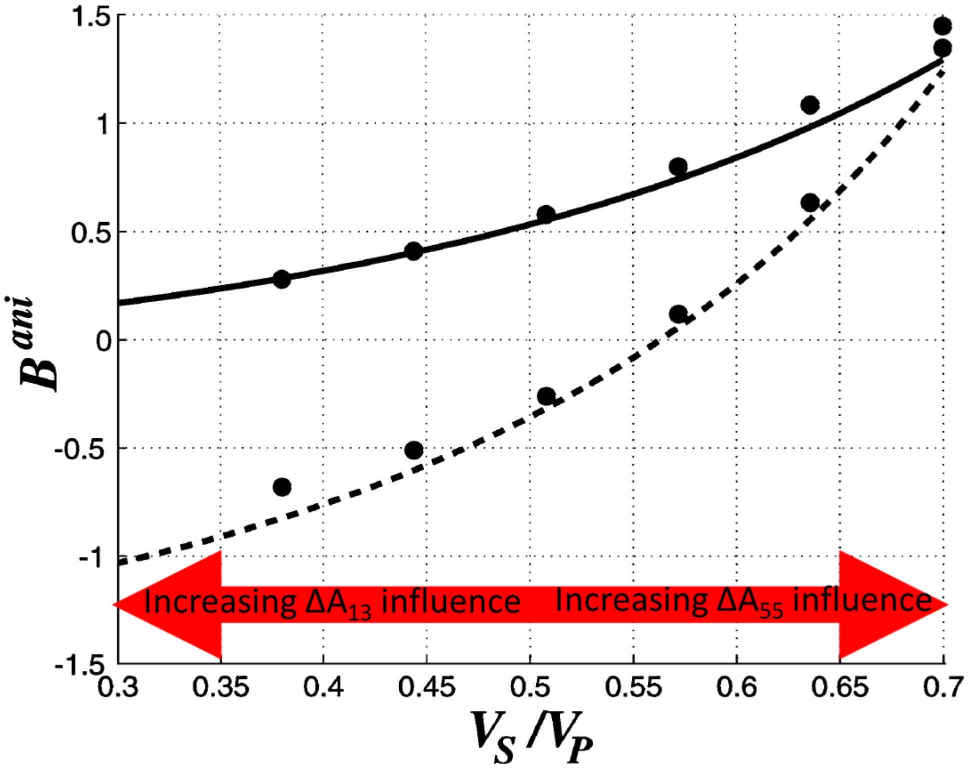


Azimuthal AVO gradient (blue) with two possible solutions (green and red). It is unclear if minima or maxima correspond to the symmetry axis.

Ambiguity as Minima/Maxima

The azimuthal gradient is a combination of gradients along individual vertical planes so its minima and maxima are minima and maxima of the quantity

$$[-2\Delta A'_{55} + \Delta A'_{13}]$$



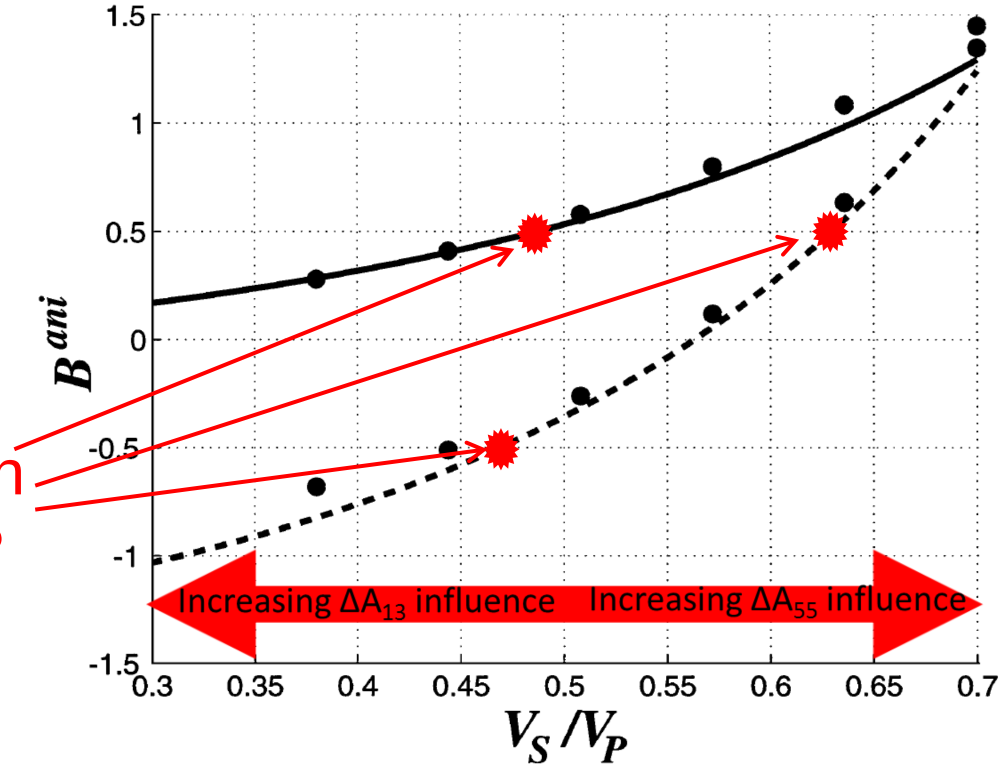
Change in anisotropic gradient with Vs/Vp ratio for aligned vertical dry (dashed line) and wet (solid line) fractures. After Bakulin et al. (2000).

Ambiguity Example

Here is an example of an ambiguity that would be caused if B^{ani} had a measured magnitude of 0.5.

Values of 0.5 and -0.5 are both possibilities.

Which One?



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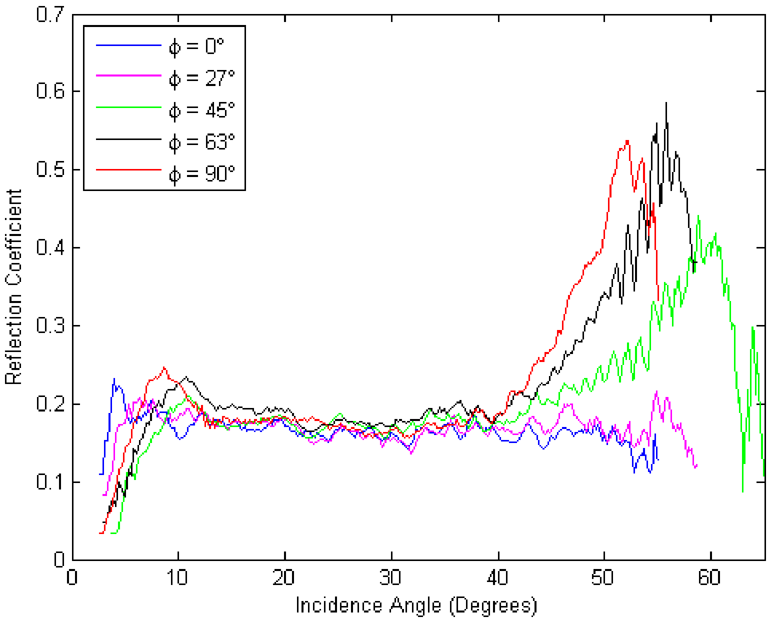
Ways to constrain gradient

- Knowledge of liquid presence or lack of liquid presence + Vp/Vs ratio
- Approximate fracture direction from another method or from geology
- Azimuthal change in critical angle
- Azimuthal AVO curvature

Faranak’s data

Faranak collected and processed azimuthal reflection data using a model with the following approximate stiffnesses:

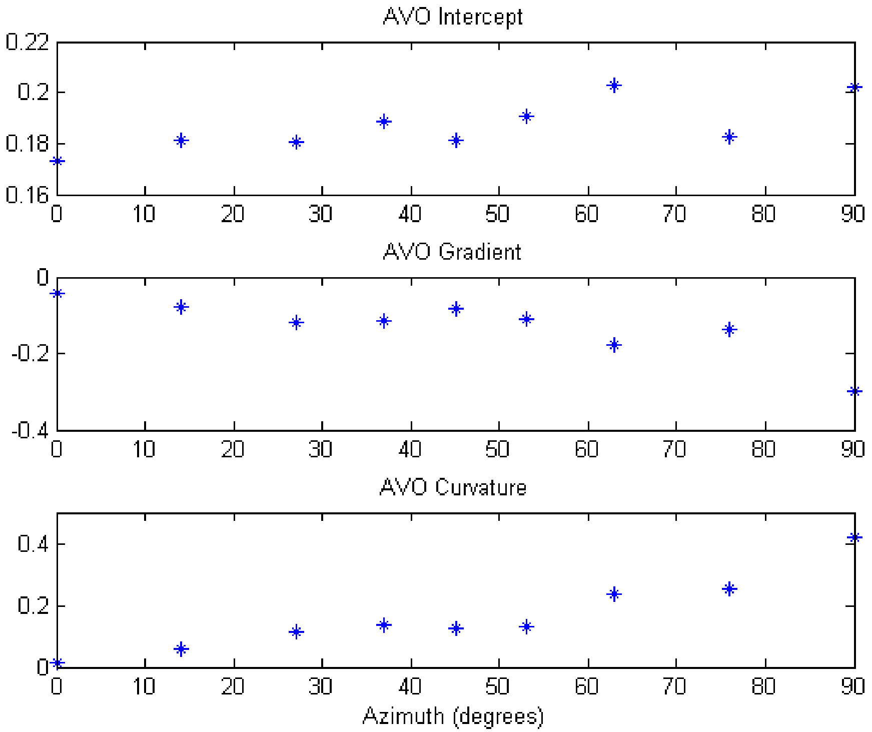
$$
\begin{vmatrix}
8.70 \pm 0.49 & 4.68 \pm 0.21 & 5.07 \pm 0.21 & 0 & 0 & 0 \\
 & 13.25 \pm 0.49 & 5.13 \pm 0.23 & 0 & 0 & 0 \\
 & & 12.25 \pm 0.49 & 0 & 0 & 0 \\
 & & & 2.89 \pm 0.12 & 0 & 0 \\
 & & & & 2.34 \pm 0.12 & 0 \\
 & & & & & 2.28 \pm 0.12
\end{vmatrix}
$$



Azimuthal reflection data from Mahmoudian (2013).

Intercept, Gradient, and Curvature

- Intercept, gradient, and curvature were estimated.
- Gradient is more negative at 90 degrees but this is a combination of -2$\Delta A'_{55}$ and $\Delta A'_{13}$ and does not indicate fast and slow directions.

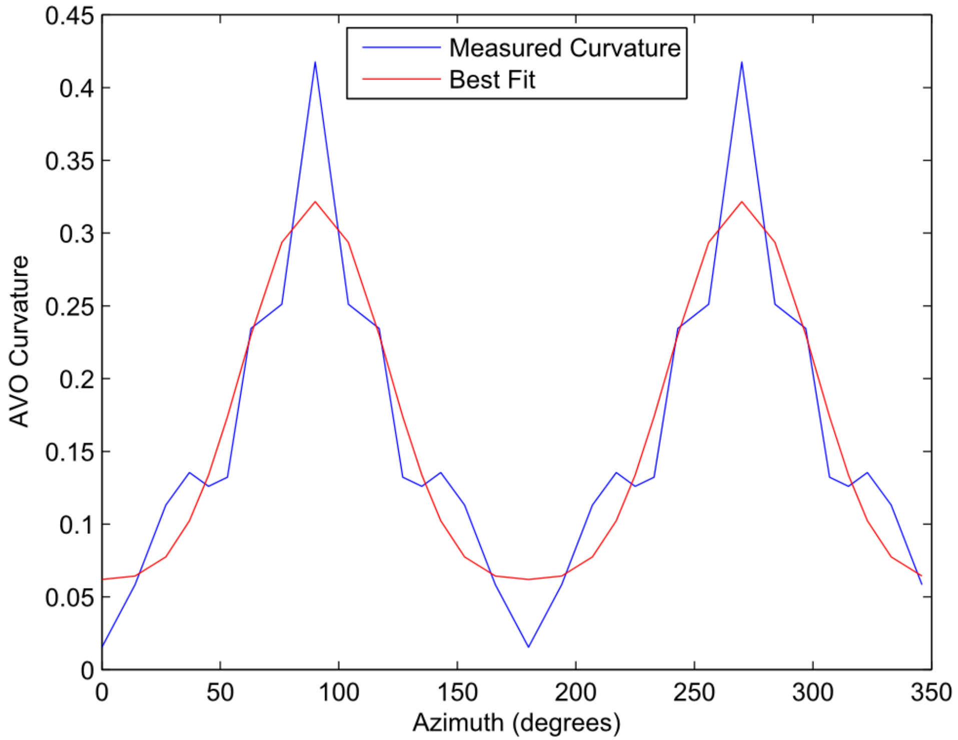


Azimuthal variations in the AVO intercept, gradient, and curvature for the physical modeling dataset.

Fitting the Curvature

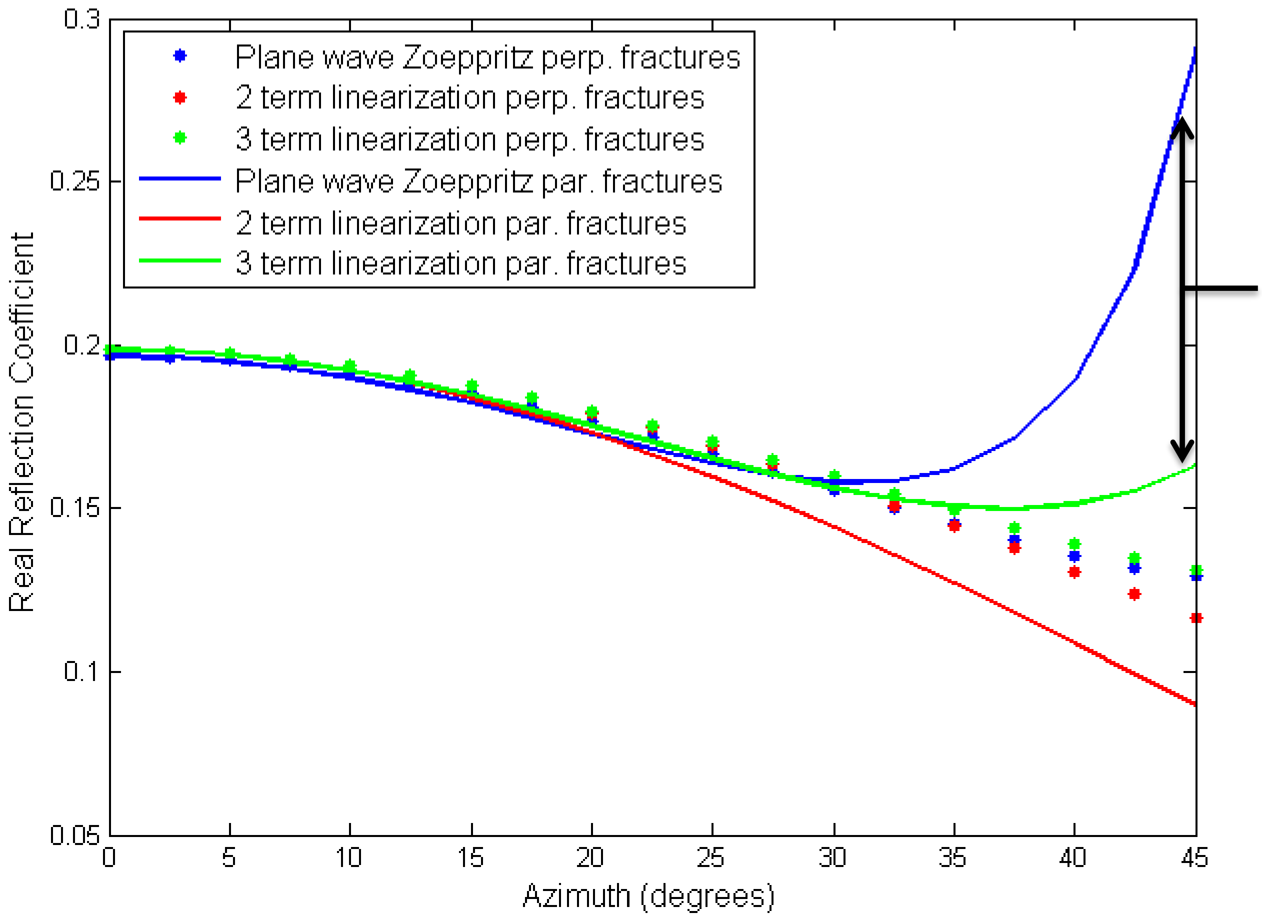
Fitting the curvature resulted in finding 0º and 90º as the azimuths having the biggest separation between ΔA_{11} and ΔA_{22}.

This allows us to determine that 0º is the slow direction.



Best Fit of the coordinate transformation of ΔA_{11} to the curvature.

Need for 2nd order reflection coefficient



Higher order terms cause error

Higher order reflection coefficient

$$
\begin{aligned}
&\frac{1}{4}\left(\delta a33 + 2\,\delta\rho\right) + \\
&\left(C1\ psqr\ \delta a13 - C2\ psqr\ \delta a33 + \frac{1}{8}\,\delta a33^2 - C3\ psqr\ \delta a55 - C4\ psqr\ \delta\rho + \frac{1}{4}\,\delta\rho^2\right) + \\
&\left(\frac{5}{64}\,\delta a33^3 + C5\ psqr^2\ \delta\rho + C6\ psqr\ \delta\rho^2 + \frac{1}{8}\,\delta\rho^3 + C7\ psqr\ \delta a13^2 + \right. \\
&\quad C8\ psqr\ \delta a55^2 + C9\ psqr\ \delta a33^2 - \frac{1}{32}\,\delta\rho\,\delta a33^2 + C10\ psqr^2\ \delta a11 + \\
&\quad C11\ psqr^2\ \delta a55 + C12\ psqr\ \delta\rho\,\delta a55 - C13\ psqr^2\ \delta a13 + C14\ psqr\ \delta\rho\,\delta a13 + \\
&\quad \left. C15\ psqr\ \delta a13\,\delta a33 + C16\ psqr^2\ \delta a33 + C17\ \delta a33\ psqr\ \delta\rho - \frac{1}{16}\,\delta\rho^2\,\delta a33\right)
\end{aligned}
$$

- C's are coefficients containing top layer quantities $a_{11}^{(1)}$, $a_{13}^{(1)}$, $a_{33}^{(1)}$, $a_{55}^{(1)}$, and $\rho^{(1)}$
- Small contrasts: $\delta x = 1 - \dfrac{x^{(1)}}{x^{(2)}}$

Higher order in theta?

- 4^{th} order (and possibly higher) in f(theta) terms
- Would need something that is somewhat linearly independent from other terms so that small noise won't effect it negatively
- Increasing number of variables makes matrix "less overdetermined"

Conclusions

- Analyzing reflections along individual azimuths allows for a simple representation of theory
- Assumptions about symmetry don't need to be made to analyze certain subsurface elastic properties.
- The curvature is only dependent on a single elastic stiffness, while the gradient is dependent on two, leading to ambiguity in interpretation made from the gradient.
- Curvature can be used to determine azimuthal changes in horizontal P-velocity and can be a tool to constrain anisotropy orientation estimates from the gradient.
- Higher order terms are very important for azimuthal AVO but are also very complicated. Use exact formulas?

Possible Future Work

- Analyze curvature in real datasets;
- Calculate and understand higher order terms;
- Incorporate attenuation anisotropy to determine if cracks contain liquid;
- Analyze azimuthal change in critical angles;

Acknowledgements

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Khaled Al Dulaijan

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