Dreamlet based efficient Gaussian packet migration

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CREWES Friday Tech Talk

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OUTLINE

- Introduction
- Dreamlet
  - *What is Dreamlet transform?*
  - *Sparse representation of seismic data*
  - *Examples*
- Gaussian packet
  - *Initial parameters and propagation*
  - *Dreamlet based Gaussian packet migration*
  - *Examples*
- Conclusions

Wavefield propagation and migration method

- High-frequency approximation based
  - Kirchhoff (Schneider, 1978; Gray and May, 1994; Audebert et al., 1997)
  - Gaussian beam (Cerveny, et al., 1982; Hill, 1990; Gray, 2005; Popov et al., 2010; …)
  - **Gaussian packet** (Babich and Ulin, 1984; Klimes, 1989; Zacek, 2004)
- Wave equation based
  - Finite difference
  - RTM
  - Phase-shift (Gazdag, 1978; Stolt, 1978), phase screen (Stoffa et al., 1990; Wu and Huang,1992), GSP(de Hoop et al., 2000; Xie et al., 2000; Jin et al., 2002; Wu, 2003), PSPI …
  - Beamlet basd one-way (Steinberg, 1993; Wu, Wang, and Luo, 2008), **Dreamlet based one-way**
  - ……

Why Drumbeat-Beamlet (Dreamlet)?

- Seismic data $u(x,t)$

$$\sin\theta = v\xi / \omega, \;\; \cos\theta = v\varsigma / \omega$$

$$\varsigma = \pm\sqrt{(\omega / v)^2 - \xi^2}$$

- Time and space localization
  - Position and direction information
  - Impulse width
- Sparse and efficient representation of seismic data

Drumbeat-Beamlet (Dreamlet)

- What is Dreamlet?

Drumbeat ⊗ Beamlet

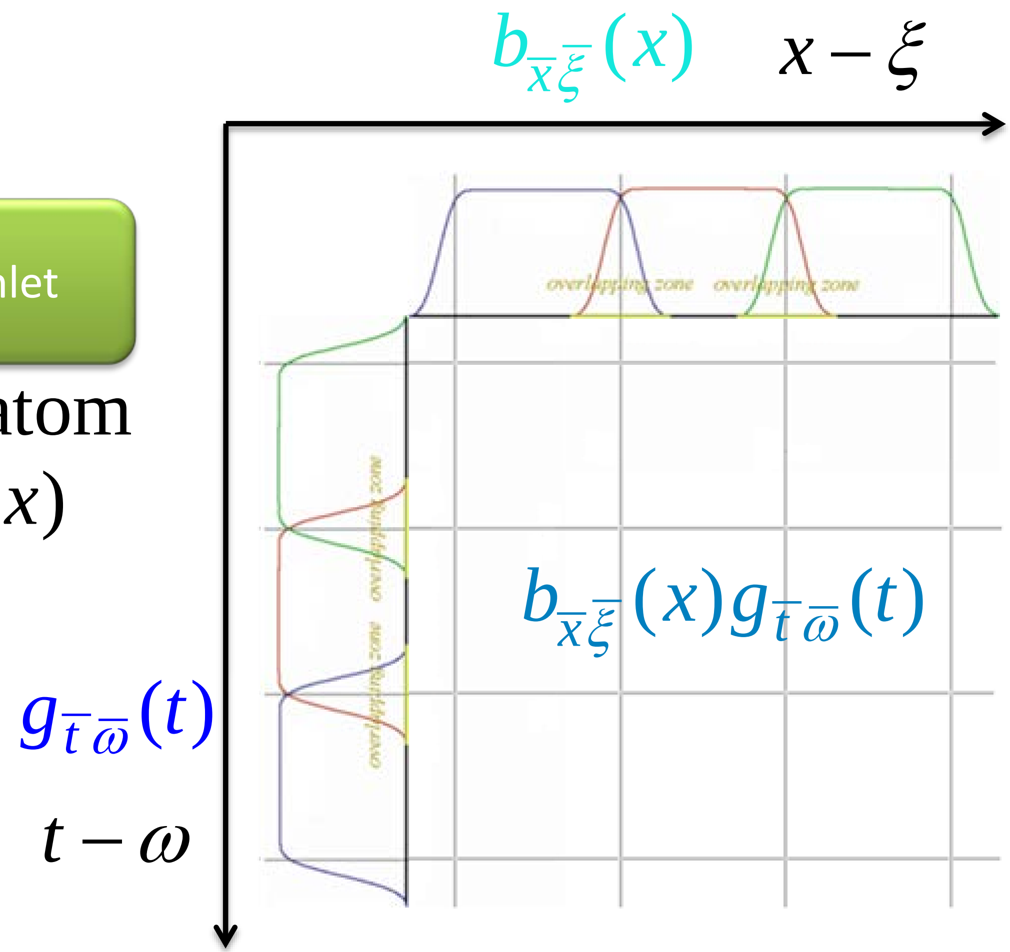
$t-\omega$ atom

$g_{\bar{t}\bar{\varpi}}(t)$

$x-\xi$ atom

$b_{\bar{x}\bar{\xi}}(x)$

Drumbeat-beamlet (Dreamlet)



Orthogonal bases (LCB) and tight frame (Gabor frame) can be used as drumbeat and beamlet atom

Representation of data

- Two kinds of coefficients
  - Synthesis coefficient:

$$\min \|c\|_1 \text{ s.t. } \mathbf{S}^T c = f$$

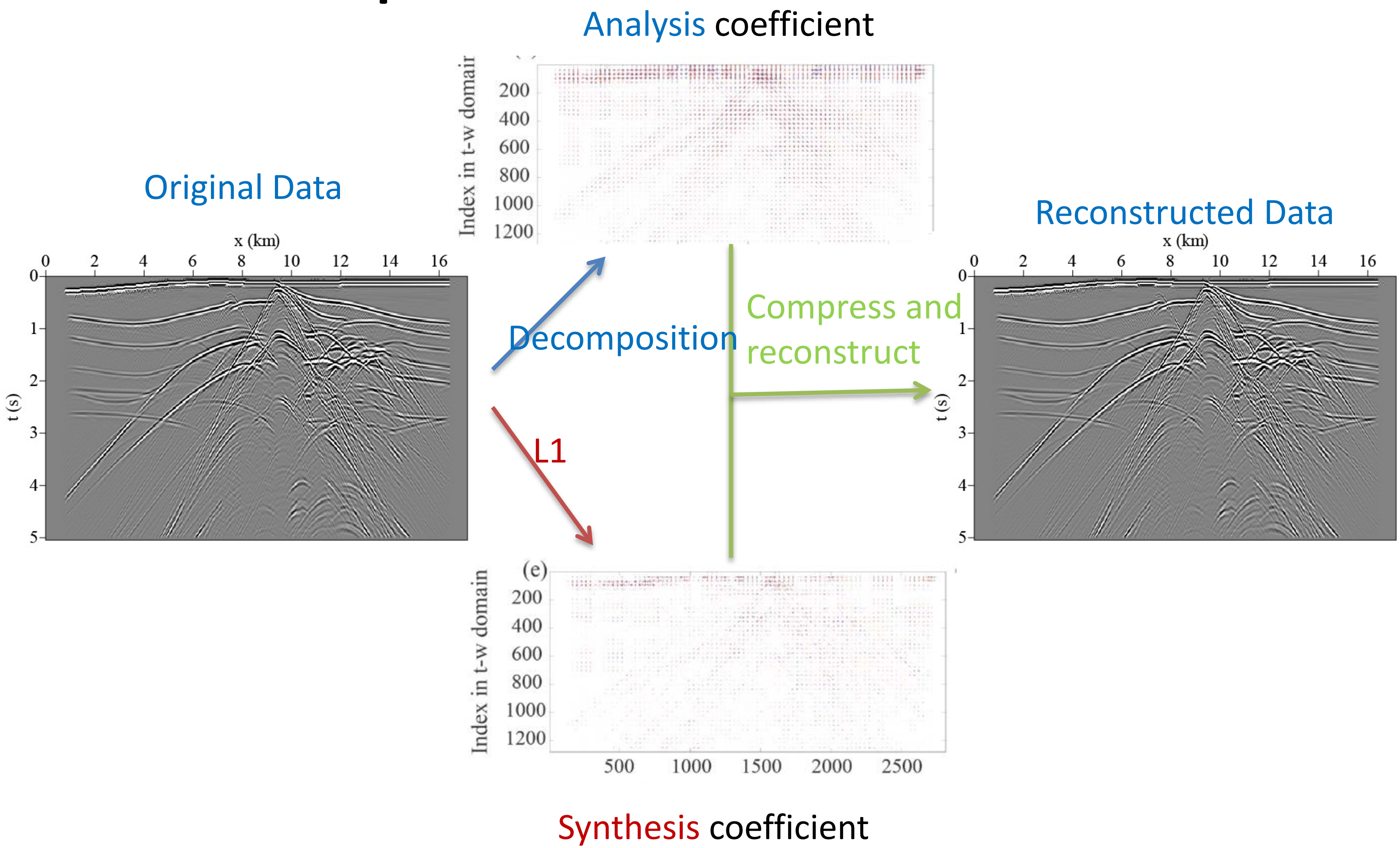
  - Analysis coefficient:

$$c = \mathbf{S} f$$

- Signal-to-Noise ratio (SNR)

$$SNR = -20 \log \frac{\| f - \tilde{f} \|_2}{\| f \|_2}$$

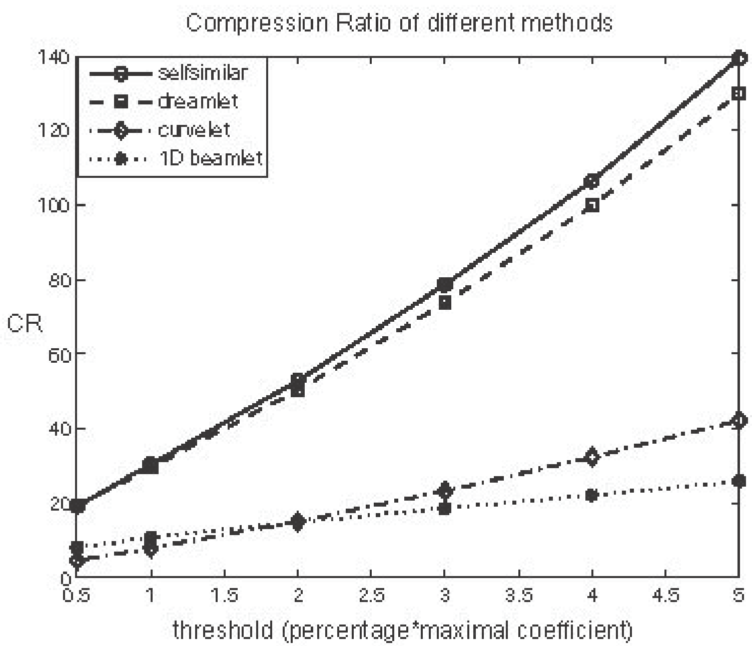
Representation of data



Data compression and imaging

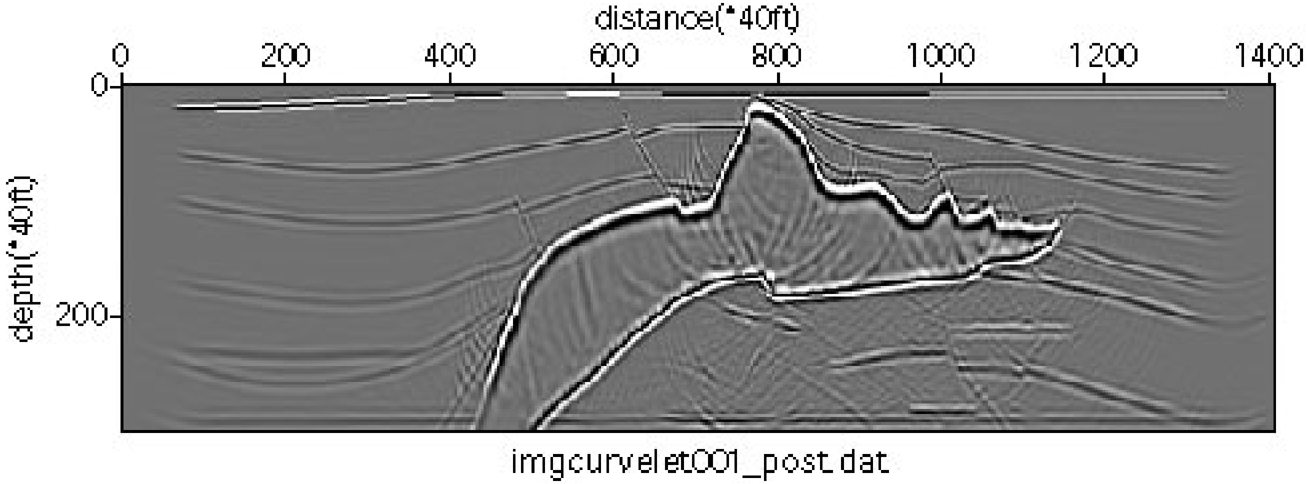
- Compression Ratio (CR)

$$CR = \frac{Uncompressed\ \ Data\ Size}{Compressed\ \ Coefficients\ Size}$$

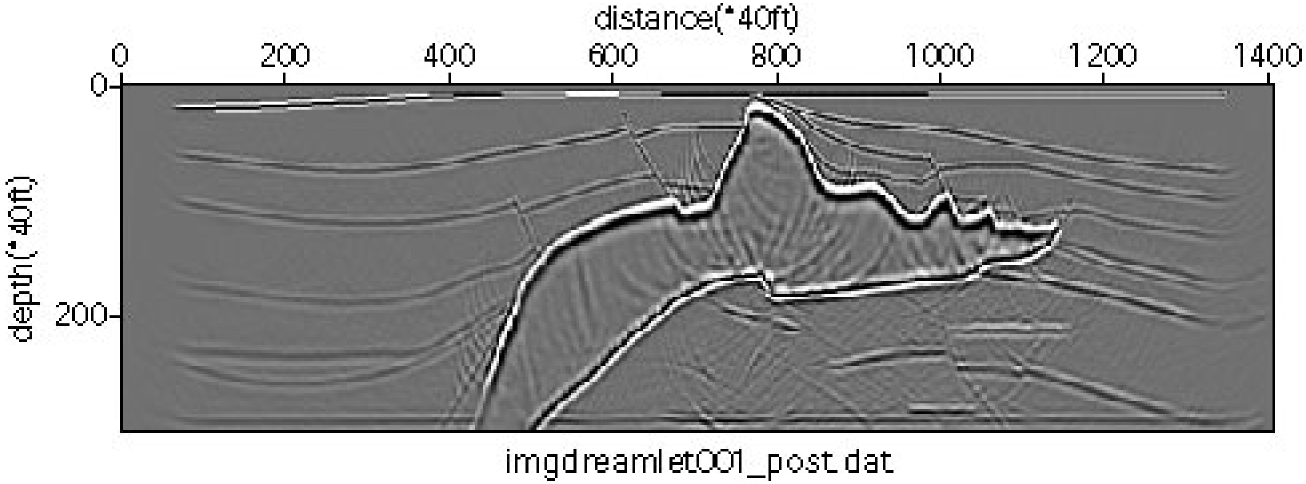


Compression Ratio of different methods, and the threshold is defined from ***0.5% to 5%*** of ***the maximal absolute value of coefficient***.

Data compression and imaging



CR=8:1
Curvelet

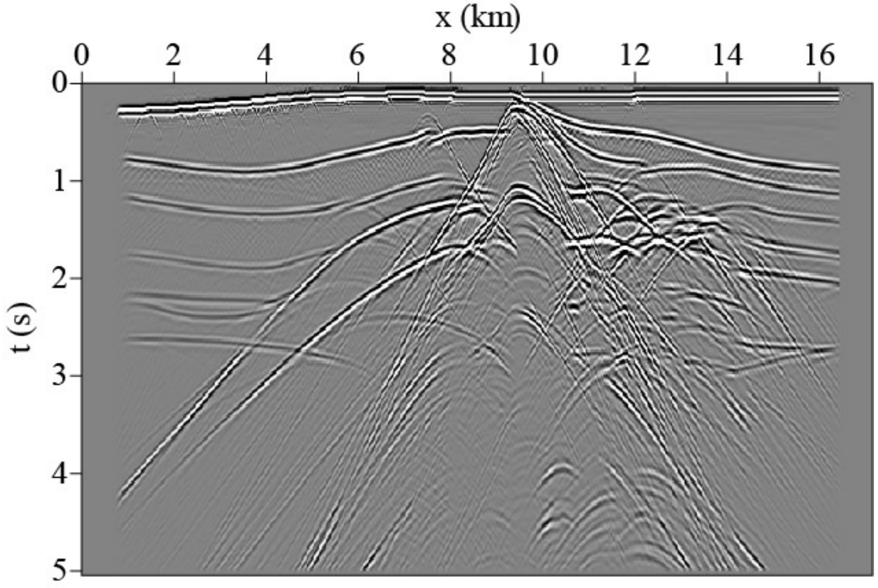


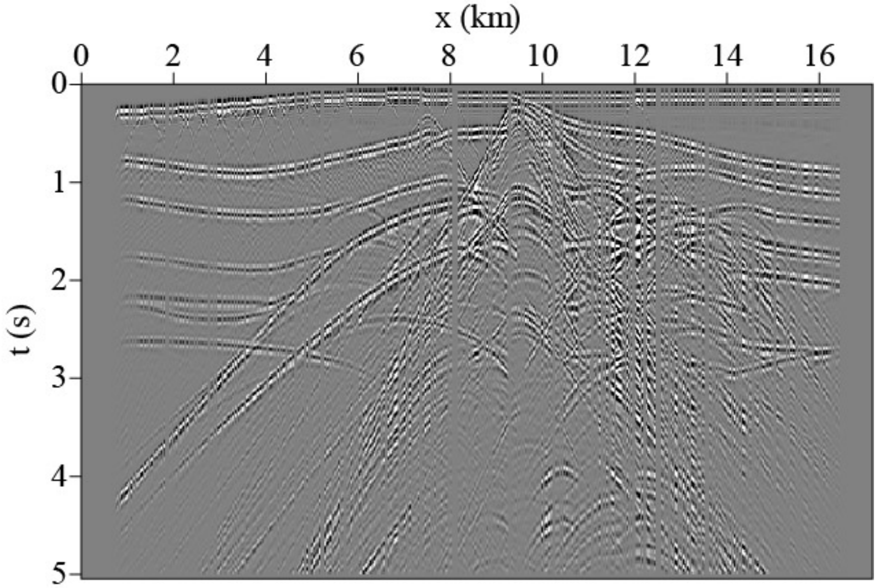
CR=30:1
Dreamlet

Threshold: **1%** of the maximal absolute value of coefficient

Data reconstruction

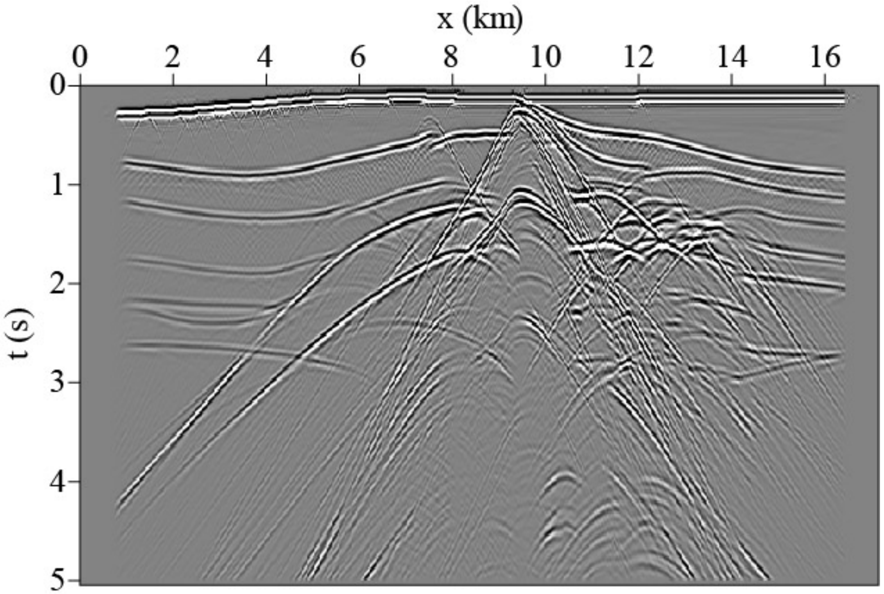
- Poststack data with 50% data missing



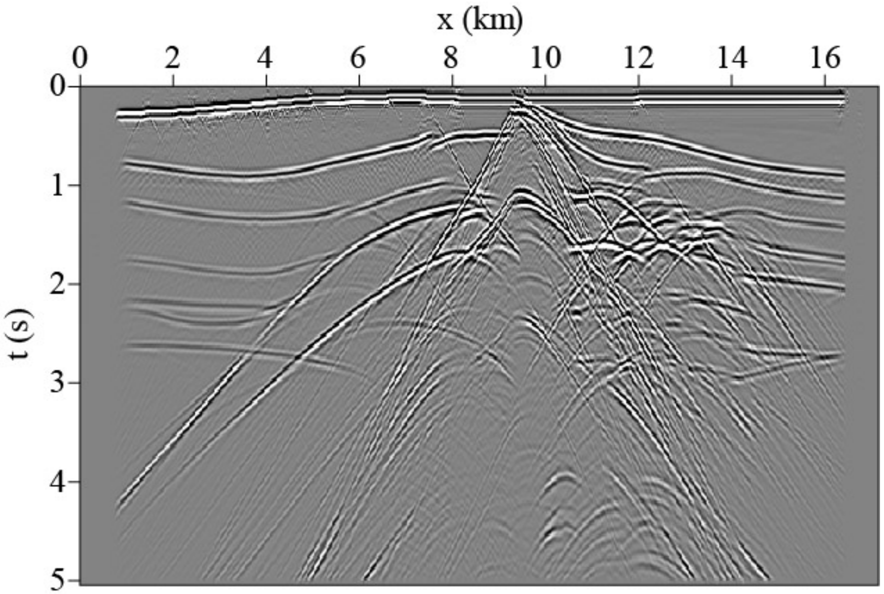


Data reconstruction

- Recovered result



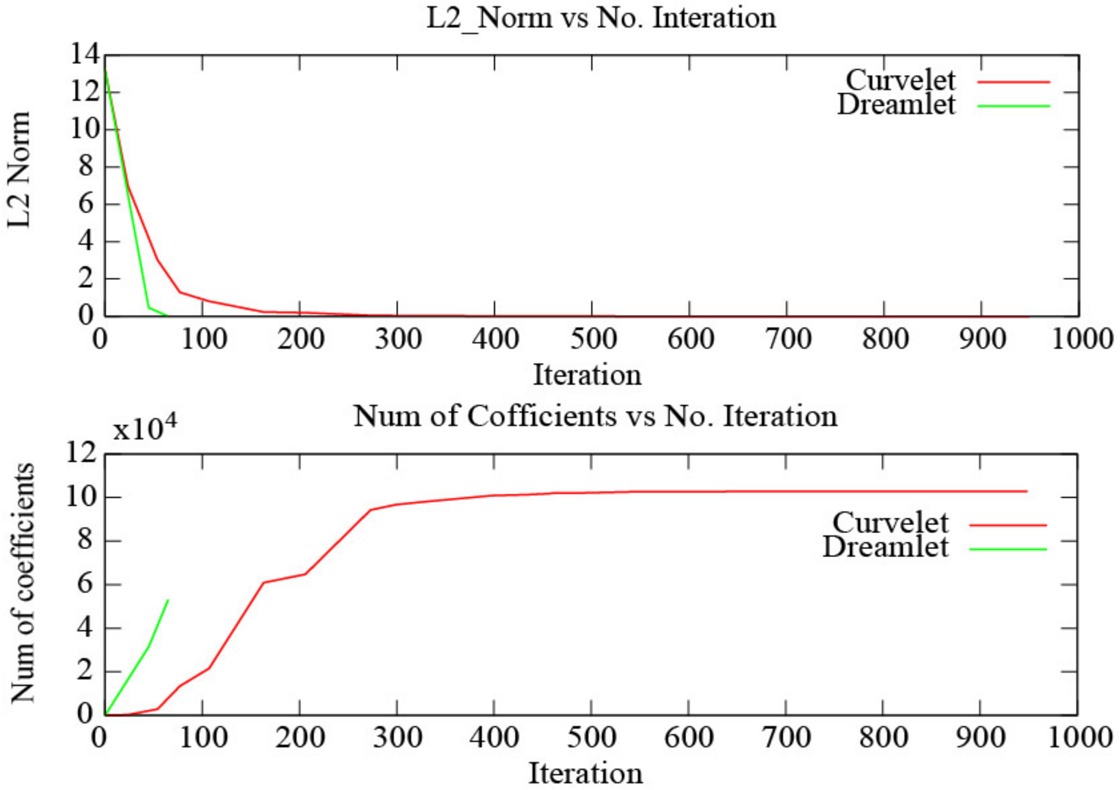
Dreamlet



Curvelet

Data reconstruction

- Comparison between Dreamlet and Curvelet



Gaussian packet

- Gaussian packets (GP), also called (space-time) Gaussian beams (Raslton 1983) or quasiphotons (Babich and Ulin 1981), are high-frequency asymptotic space-time particle-like solutions of the wave equation(Klimes 1989; Klimes 2004). They are also *waves whose envelops at a given time are nearly Gaussian functions*.
- A Gaussian packet is *concentrated to a real-valued space-time ray* (in a stationary medium, a Gaussian packet propagates along its-real-valued spatial central ray), as a Gaussian beam to a spatial ray.

Gaussian Packet

- Assume that in 3D case, wave propagation is described by the scalar wave equation:

$$\frac{1}{V^2(x_i)}\frac{\partial^2 u(x_\alpha)}{\partial x_4{}^2} - \nabla^2 u(x_\alpha) = 0$$

  - the lower case indices take the values $k, l = 1, 2, 3$
  - x_α stands for general space-time coordinates $x_\alpha = (x, y, z, t)$
- A Gaussian packet is the zero-order term of the asymptotic ray series.

$$u^{gp}(x_\alpha) = A\exp(i\omega\tau)$$

  - A complex amplitude
  - τ complex phase function

Propagating Gaussian Packet

- In the two-dimensional case, a Gaussian packet can be written as

$$u^{gp}(x,z,t) = A\exp(i\omega\tau)$$

ω is arbitrary positive parameter, and it can be understood as the center frequency of the wave packet.

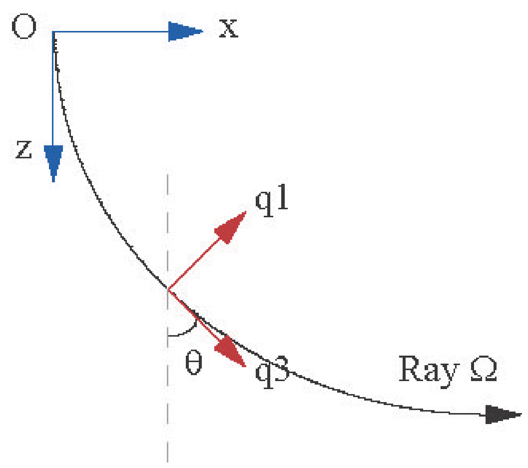
- Using Taylor expansion, the phase function can be further expanded to its second order

$$\tau \approx p_x(x-x_r) + p_z(z-z_r) + p_t(t-t_r)$$
$$+\frac{1}{2}N_{xx}(x-x_r)^2 + \frac{1}{2}N_{zz}(z-z_r)^2 + \frac{1}{2}N_{tt}(t-t_r)^2$$
$$+N_{xz}(x-x_r)(z-z_r) + N_{xt}(x-x_r)(t-t_r) + N_{zt}(z-z_r)(t-t_r)$$

Propagating Gaussian Packet

- In ray-centered coordinates system, the second space-time derivatives matrix $M_{\alpha\beta}$ of the phase function can be written as

$$M_{ij} = \frac{\partial^2 \tau}{\partial q_i \partial q_j} \quad M_{i4} = \frac{\partial^2 \tau}{\partial q_i \partial x_4} \quad M_{44} = \frac{\partial^2 \tau}{\partial x_4 \partial x_4}$$



Ray-Centered coordinate and Cartesian coordinate along a ray in 2D case

- Let H_{jm} be the transformation matrices from ray-centered coordinate system to Cartesian coordinates,

$$H_{im} = \frac{\partial x_i}{\partial q_m}$$

- Then

$$N_{jk} = H_{jm} H_{kn} M_{mn}$$
$$N_{j4} = H_{jm} M_{m4}$$
$$N_{44} = M_{44}$$

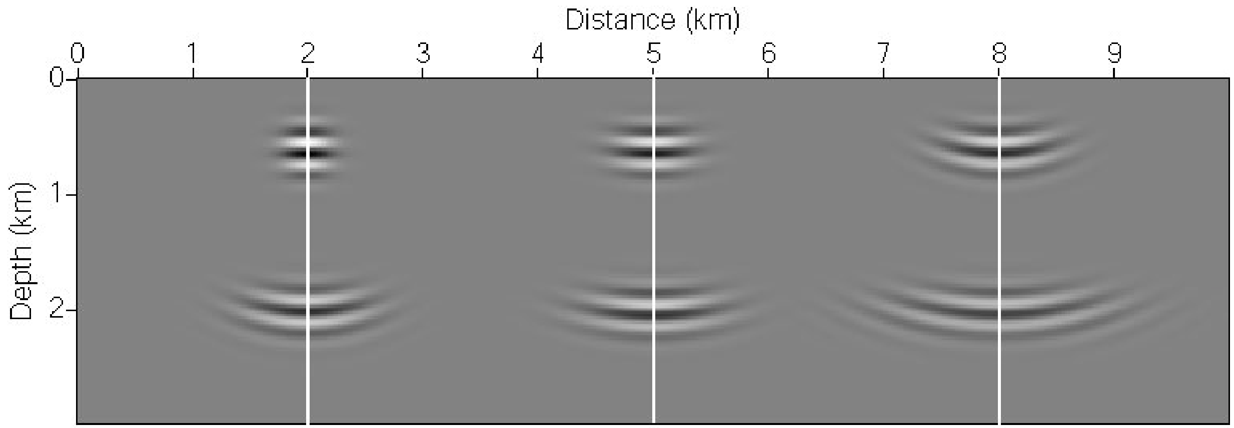
Propagating Gaussian Packet

- Before propagating a Gaussian packet, we need to know
  - Beam center x_r, z_r, t_r
  - Central frequency, ray parameter ω, p_x, p_z, p_t=-1
  - Amplitude A
  - Parameter related to dynamic ray tracing
    - M_{33}, M_{34} depend on M_{11}, M_{14} and M_{44}
    - M_{11} functions similar as in Gaussian beam. Real part: curvature, imaginary part: width perpendicular to the central ray

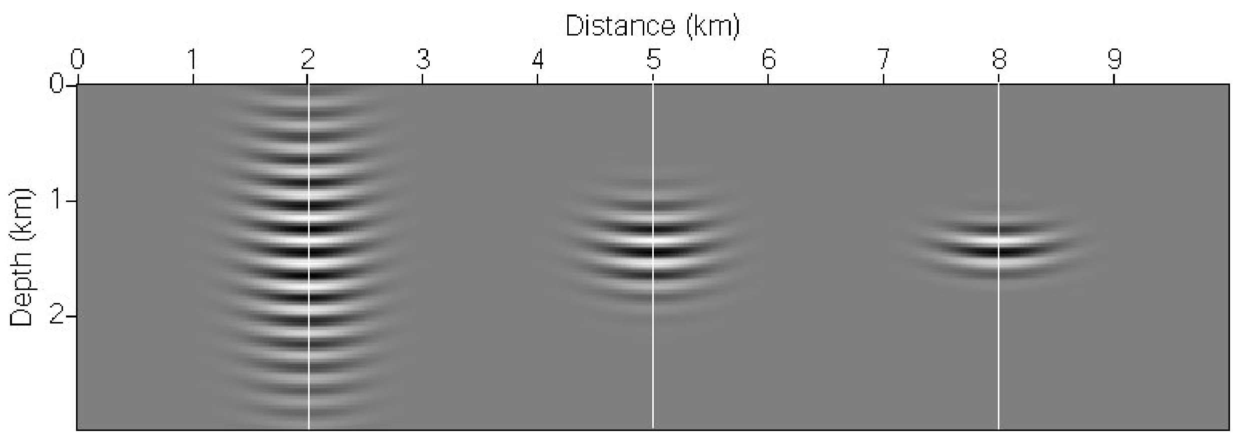
$$M_{11}^{0} = i/\omega_r w_0^2$$

    - M_{14} and $\mathrm{Re}(M_{44})$ control Gaussian packets` symmetry. Symmetric Gaussian packet when they equal to 0.
    - $\mathrm{Im}(M_{44})$ controls pulse duration along the central ray.

Propagating Gaussian Packet

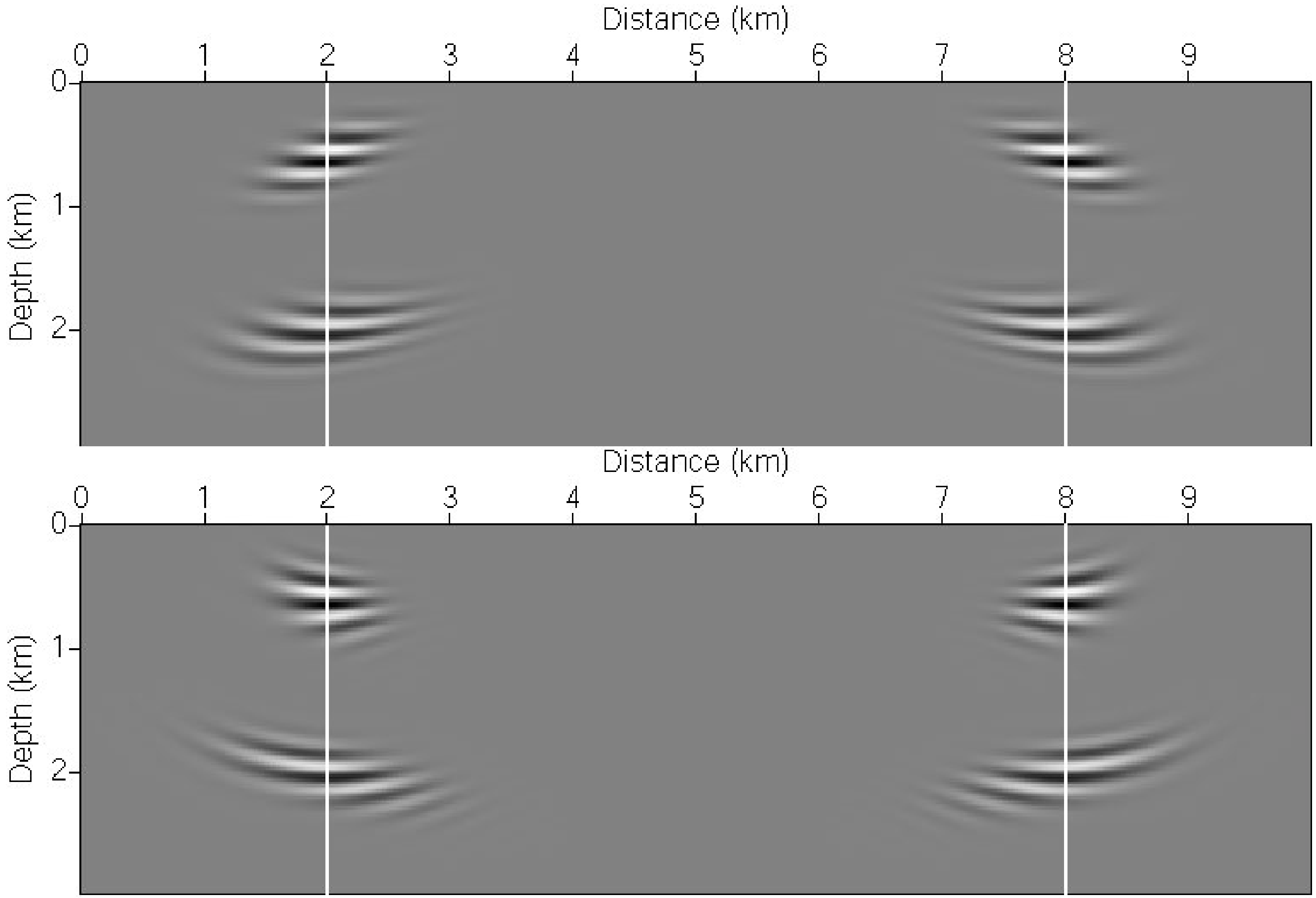


Snapshot at $t = 0.3s,\ 1.0\mathrm{s}$ for Gaussian packets with different initial curvature. White lines stand for corresponding central ray.



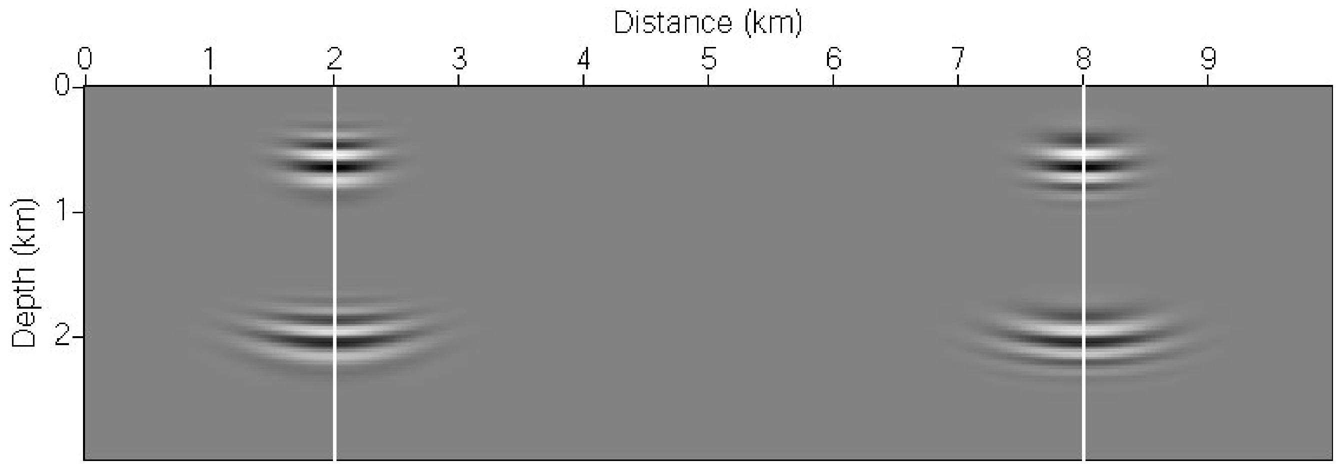
Snapshot for Gaussian packets at $t = 0.7s$ with different pulse duration

Propagating Gaussian Packet



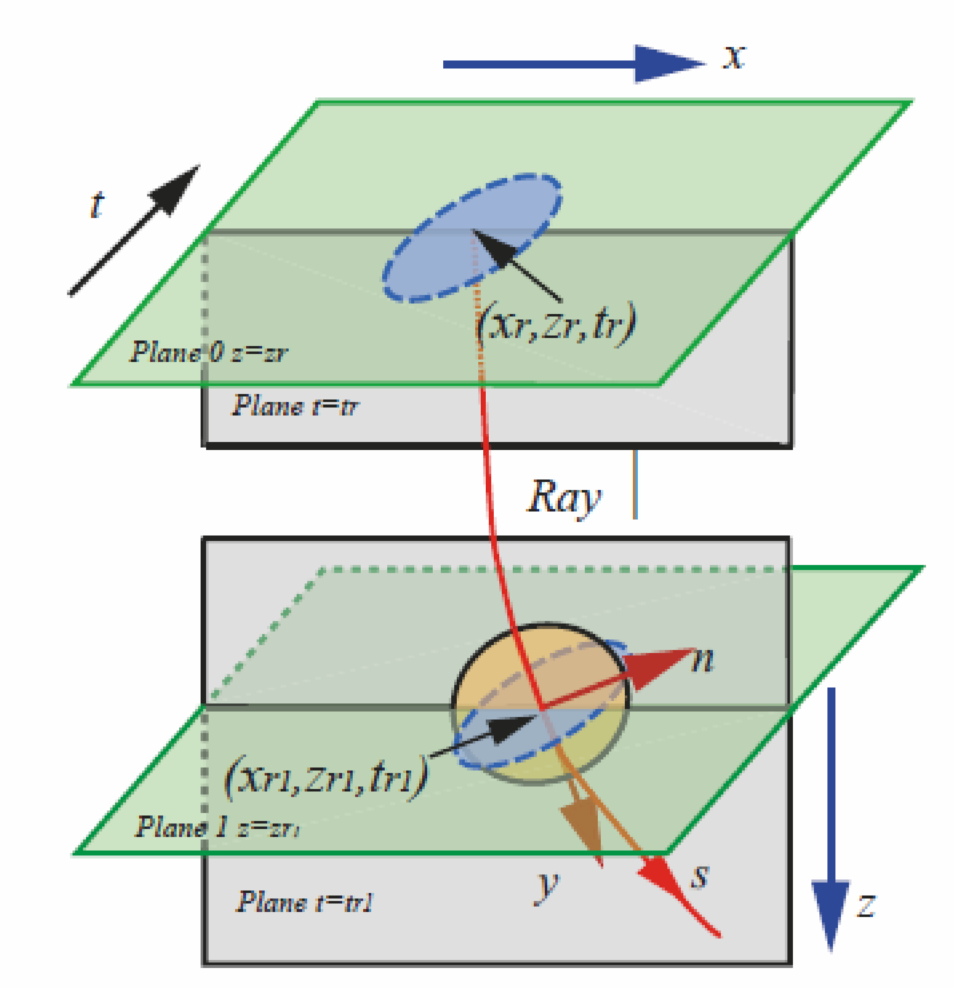
Snapshot for asymmetric Gaussian packet on $t = 0.3s,\ 1.0\mathrm{s}$

Propagating Gaussian Packet



Snapshot for asymmetric Gaussian packet on t=0.3s, 1.0s with complex pulse duration parameter

Data representation



Profiles of Gaussian Packets

Initial parameters for decomposing seismic data	
x_r	Space center location
t_r	**Time center location**
ω	Central frequency
p_x	Ray parameter
w_0	Gaussian window width along space direction (beam width)
N_{tt}^{0}	**Gaussian window width along time direction (pulse duration)**

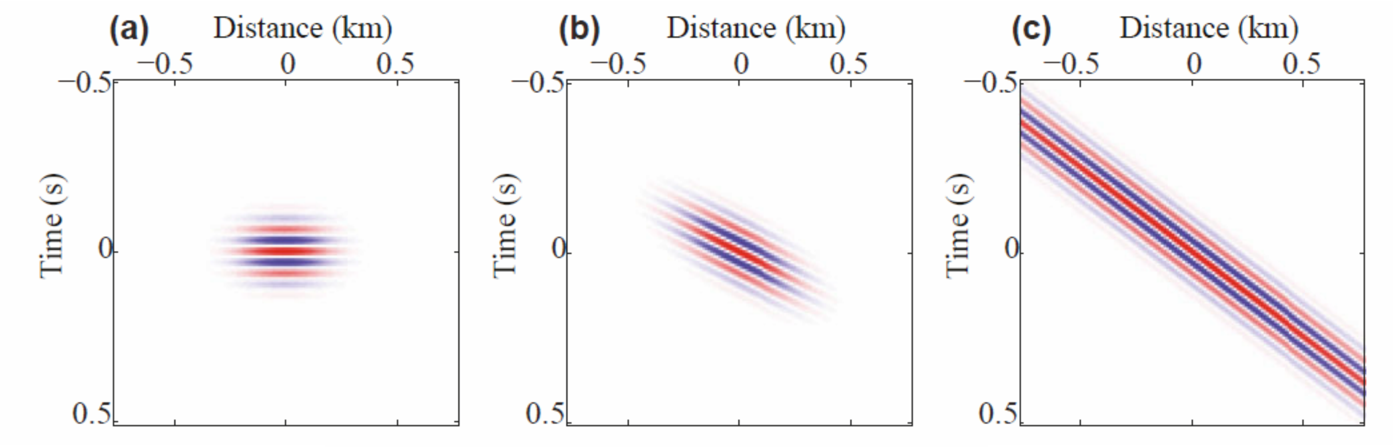
Data representation by Gaussian packet

- Gaussian packet profile at the surface

$$u_s^{gp}(x,t) = A\exp\{i\omega[p_x(x-x_r)-(t-t_r)$$
$$+\frac{1}{2}N_{xx}(x-x_r)^2+\frac{1}{2}N_{tt}(t-t_r)^2+N_{xt}(x-x_r)(t-t_r)]\}$$

  - Cross term between time and space

$$N_{xt} = -\frac{\sin\theta}{V}N_{tt} = -p_x N_{tt}^0$$

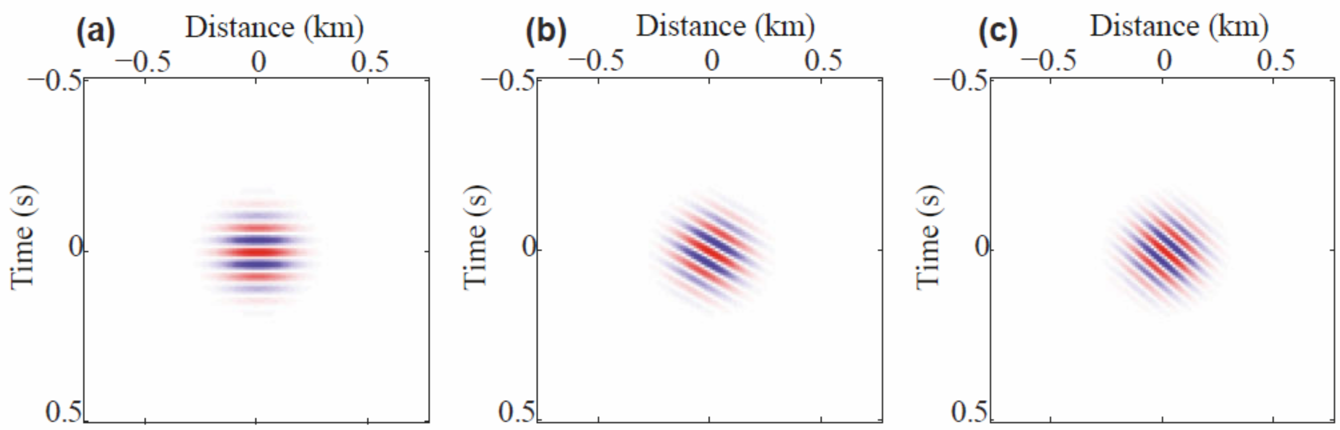


(a) $\theta = 0°$. (b) $\theta = 41°$. (c) $\theta = 82°$.

Data representation by Gaussian packet

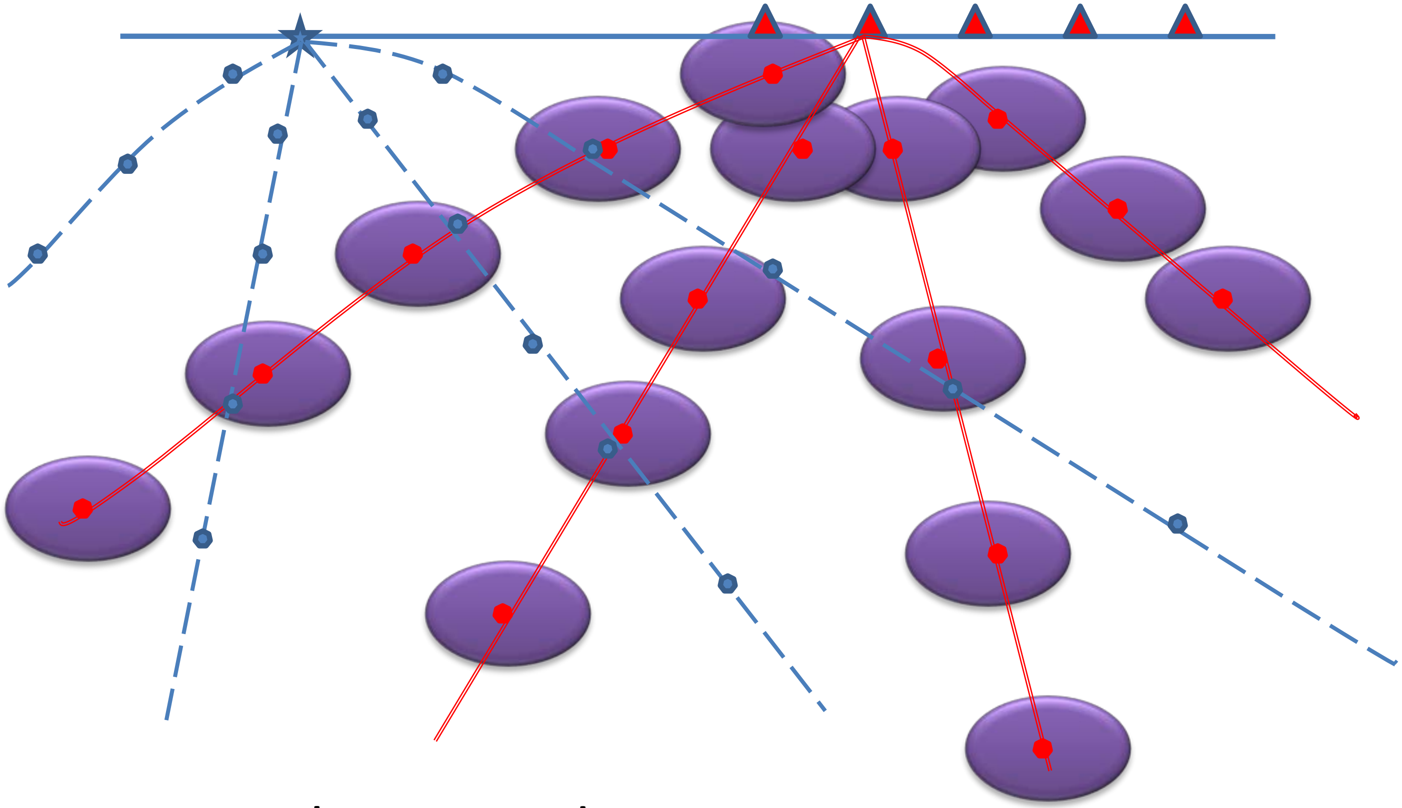
- When N_{xt} is small enough, the cross term can be ignored. Thus, the Gaussian packets are reduced into Gabor frame based Dreamlet with phase shift.

$$u_s^{gp}(x,t) \approx \tilde{d}_{m,n,p,q}(x,t)\exp\left(-i\bar{\xi}_m \bar{x}_n + i\bar{\varpi}_p \bar{t}_q\right)$$



(a) $\theta = 0^\circ$. (b) $\theta = 41^\circ$. (c) $\theta = 82^\circ$.

Imaging of GP



For poststack seismic data

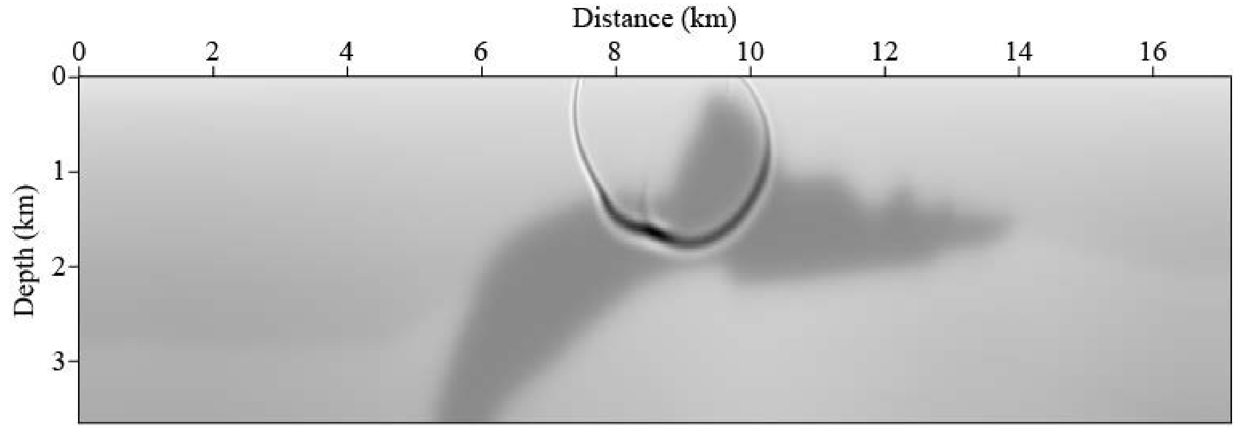
$$u(x,z,0) \approx \mathrm{Re}\left(\sum_{m,n,p,q} C^{gp}_{m,n,p,q} u^{gp}(x,z,t=t_r) \right)$$

Implementation procedure

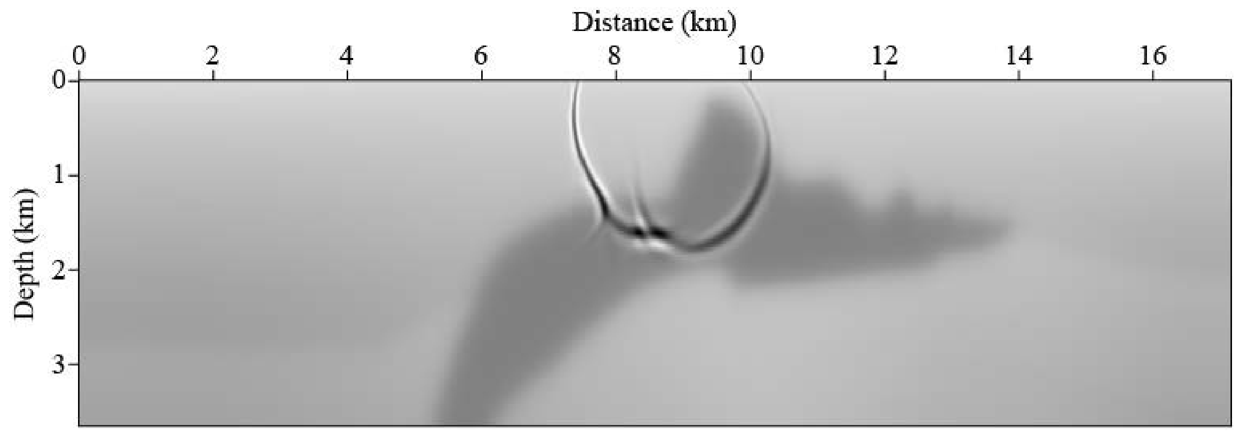
- Decompose seismic data into Dreamlets, determine related Gaussian packets` initial value.
  - threshold
- Propagate Gaussian packets into subsurface
- Apply imaging condition

Impulse response in complex media

Finite Difference

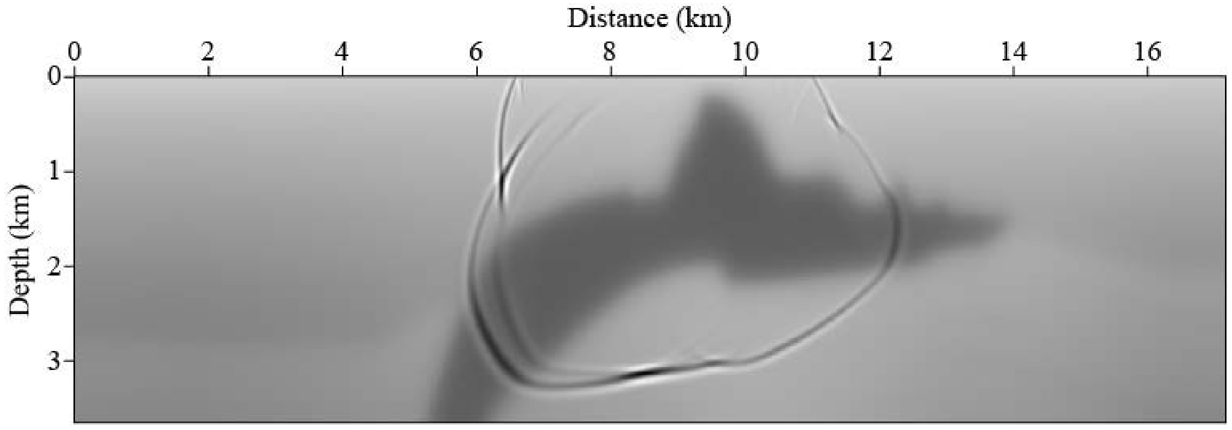


Gaussian Packet

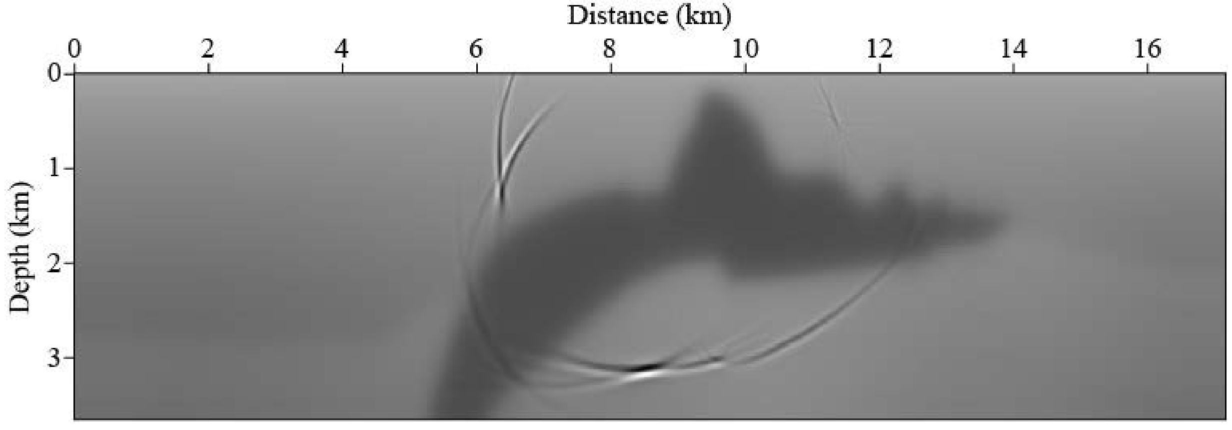


Impulse response in complex media

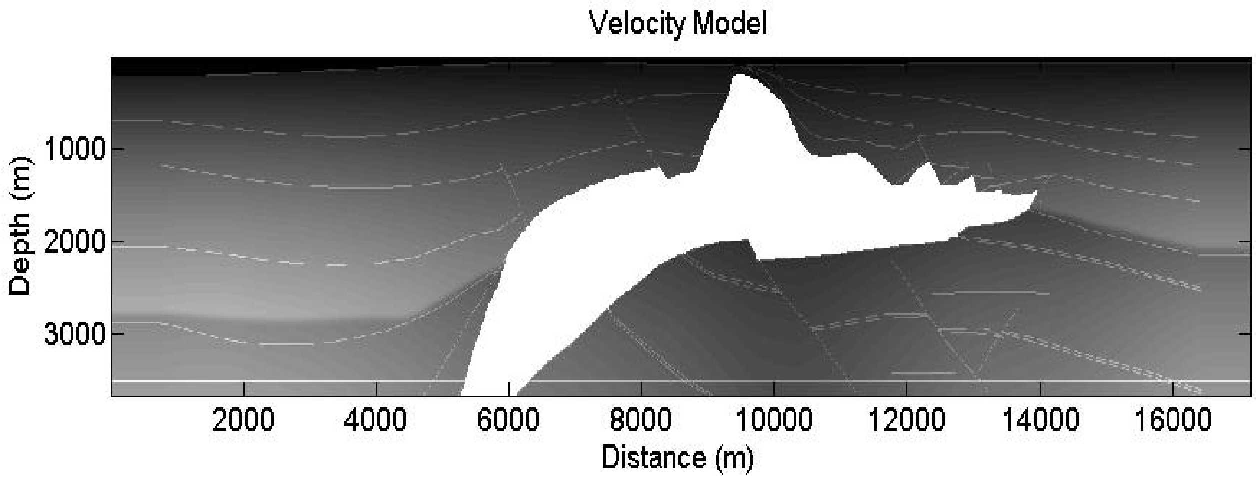
Finite Difference

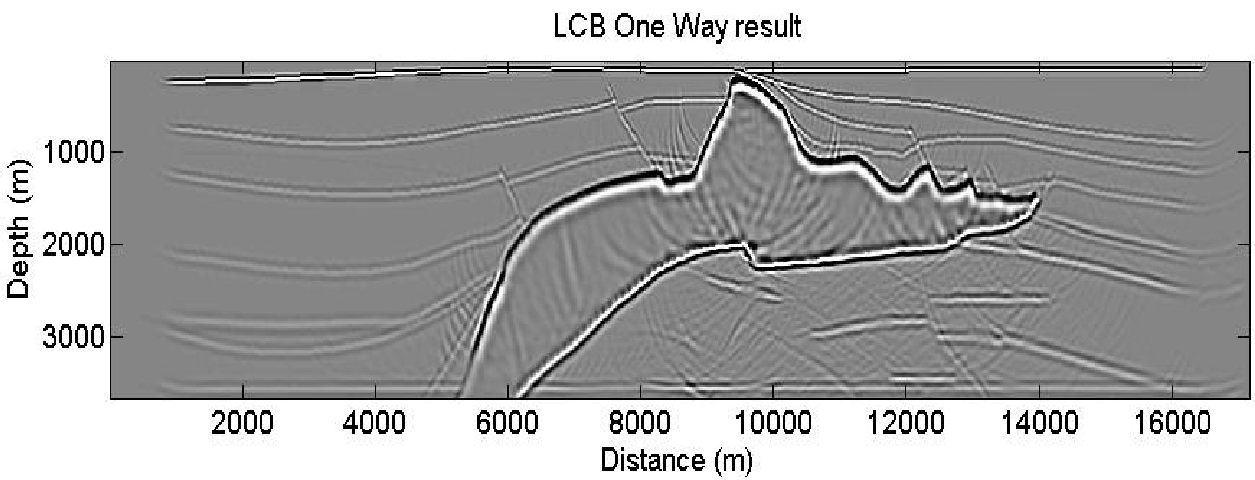


Gaussian Packet



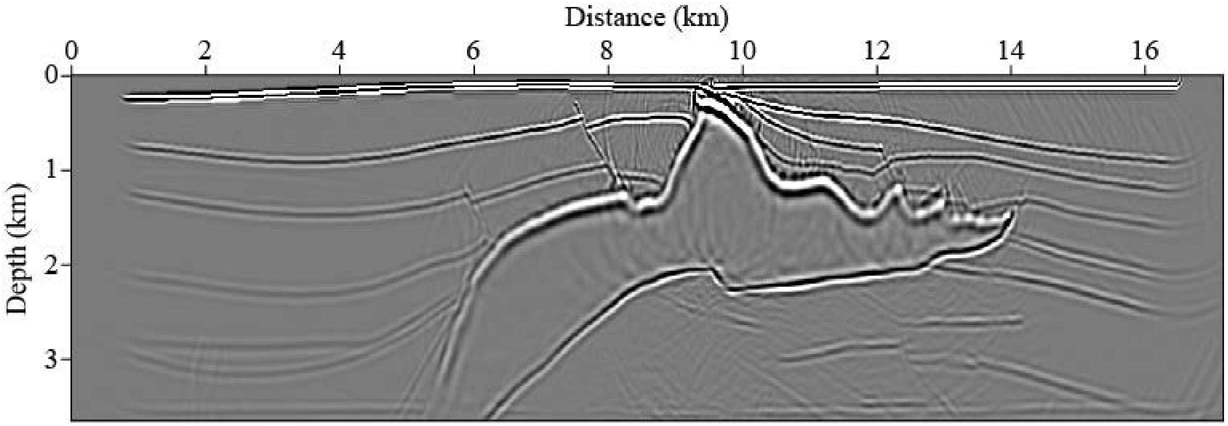
Imaging Example



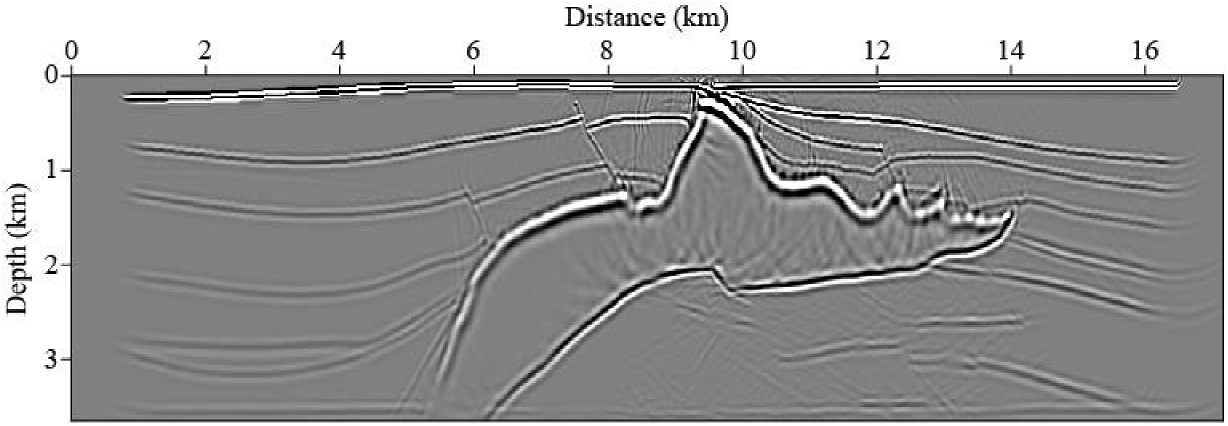


local cosine beamlet migration method (Wu, Wang, and Luo 2008)

Imaging Example



Dreamlet with Gabor frames' redundancy of 8
$N\omega$=32 $N\xi$= 64
1% largest coefficients are used



t: Gabor frame redundancy of 8
$N\omega$=32
x: Local slant stack

Conclusions

- Dreamlet decomposition
  - defined in a local time–frequency space–wavenumber domain
  - represent seismic data sparsely and efficiently
- Dreamlet based Gaussian packets migration
  - Efficient decomposition
  - Efficient propagation
    - E.g., With Gabor frame's redundancy 8, only 32 local frequencies are used for summation.
    - The relationship between each packet's time position and the travel time from the surface to the imaging point makes the imaging using Gaussian packets simple and easy to perform.
  - Accurate imaging result

Acknowledgments

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