Velocity model building by slope tomography

Bernard Law and Daniel Trad

Motivation

- Building long wavelength velocity model for high resolution inversion method such as full waveform inversion (FWI).

- Classical travel time tomography can estimate a macro-velocity model using reflection arrival times, but it requires horizon based travel time picking.

- Slope tomography methods use slopes and travel time picks from locally coherent events. This makes picking and potentially batch picking a much easier task.

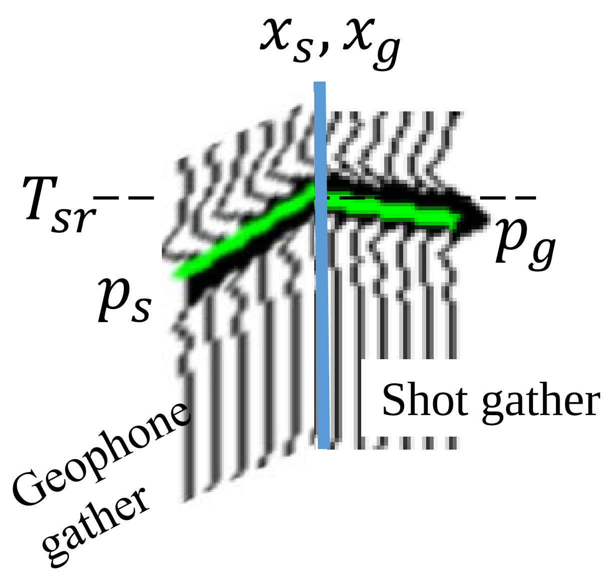
Slope tomography

- The first slope tomography method was introduced as controlled directional reception (CDR) tomography (Sword 1988).
  - named after the Russian CDR method (Riabinkin 1957)

- Stereotomography (Billette & Lambaré 1998, Chauris 2002, …)
  - has been an active research topic for many years
  - has gained momentum as a reliable velocity model building tool

- Adjoint slope tomography (Tavakoli, Riboddetti, Virieux & Operto 2017)
  - computes gradients using adjoint-state method
  - more efficient for large dataset

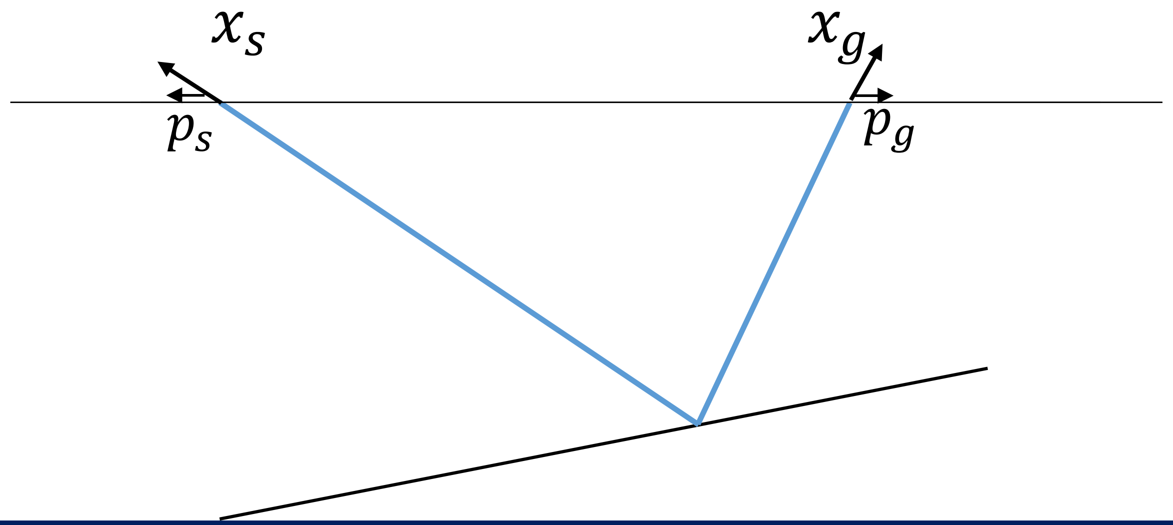
Controlled directional Reception (C.D.R.) – A Russian Pre-stack Migration Method



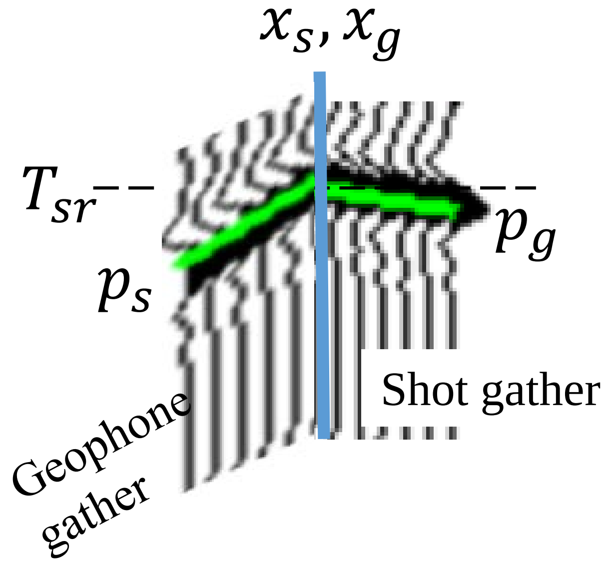
CDR method characterizes a locally coherent event with the reciprocal parameters: $x_g, x_s, P_g, P_s, T_{sr}$

Receiver ray parameter: P_g is measured on a shot gather at T_{sr}

Source ray parameter: P_s is measured on a geophone gather at T_{sr}



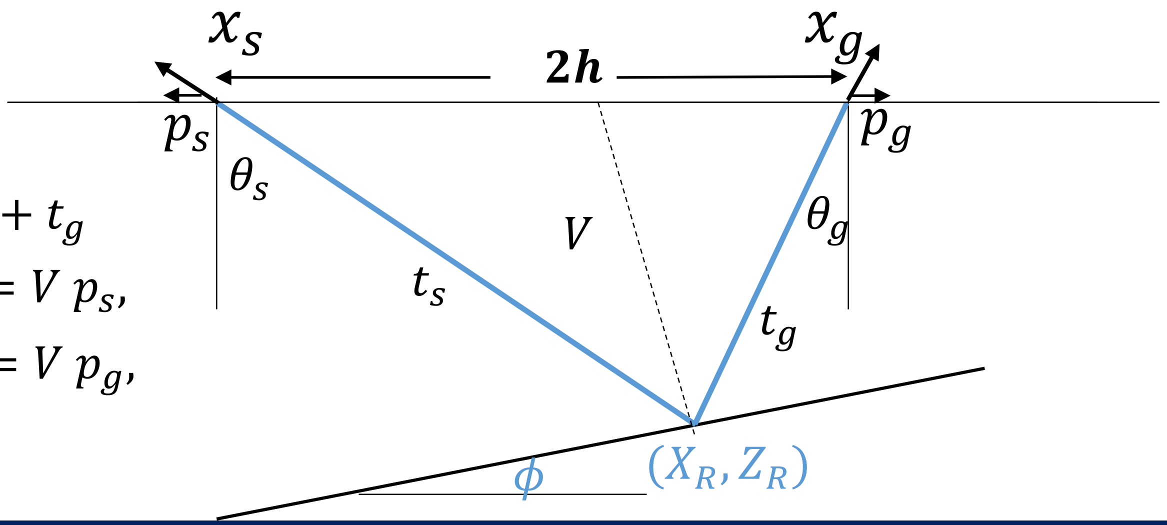
Controlled directional Reception (C.D.R.) – A Russian Pre-stack Migration Method



Using straight ray and constant velocity assumption, reciprocal parameters can be converted to reflector position (X_R, Z_R) and reflector dip angle ϕ .

$$V^2 = \frac{4h}{t} \frac{t - h(p_g - p_s)}{t(p_g - p_s) + 4hp_s p_g}$$

$$tan\phi = \frac{V(p_s + p_g)}{\sqrt{1-(Vp_s)^2} + \sqrt{1-(Vp_g)^2}}$$



$t = t_s + t_g$

$sin\theta_s = V\,p_s,$

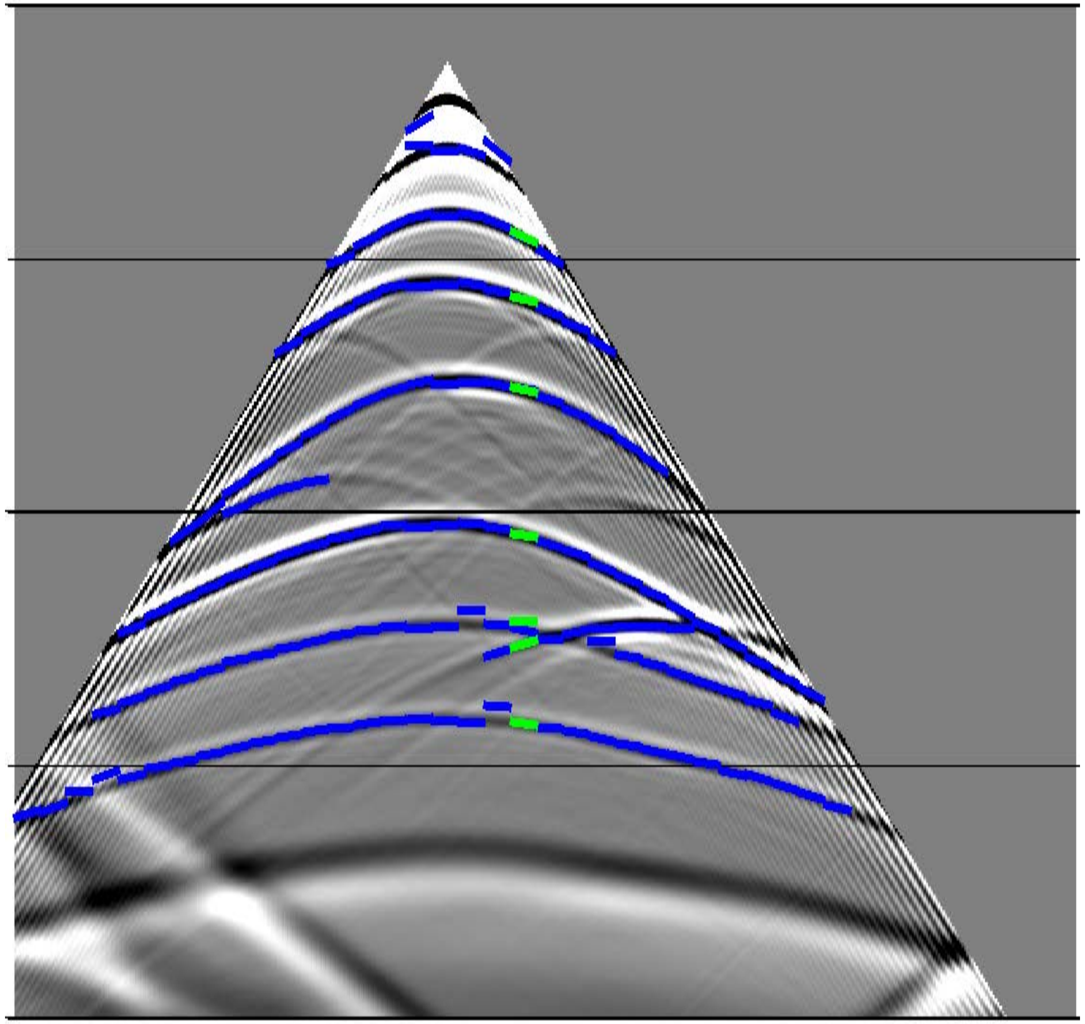
$sin\theta_g = V\,p_g,$

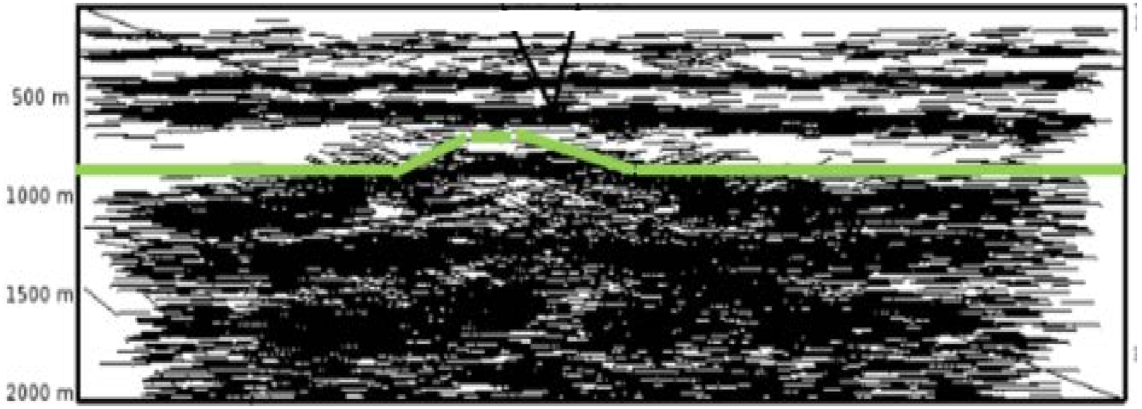
$$X_R = 0.5 * (X_s + X_g) + \frac{0.5Vt(tan\phi)}{\sqrt{1 - \frac{4h^2}{V^2t^2} + \tan^2\phi}}$$

$$Z_R = \frac{0.5Vt(1 - \frac{4h^2}{V^2t^2})}{\sqrt{1 - \frac{4h^2}{V^2t^2} + \tan^2\phi}}$$

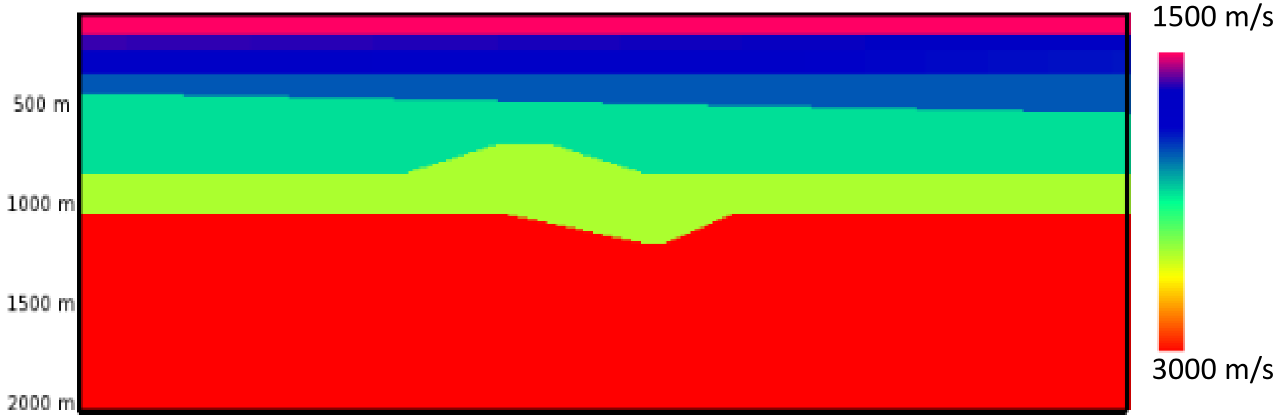
Controlled directional Reception (C.D.R.) – A Russian Pre-stack Migration Method

Finite difference shot record





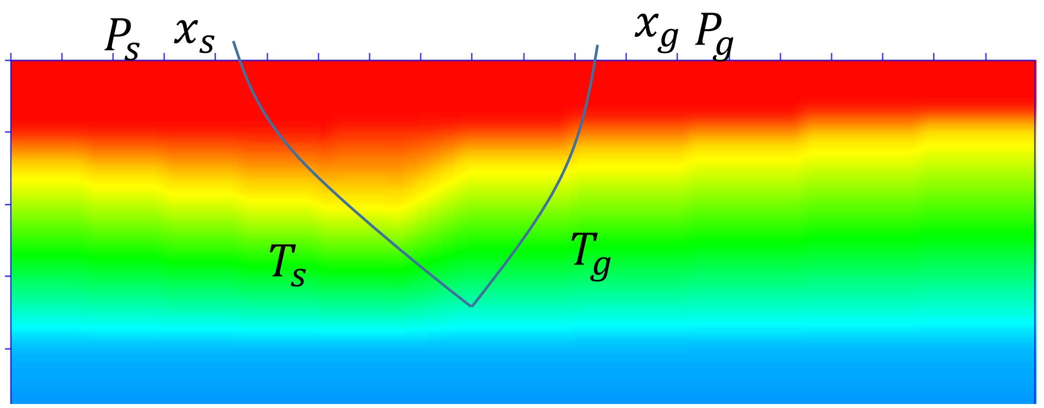
Dip bars



True model (Interval velocity)

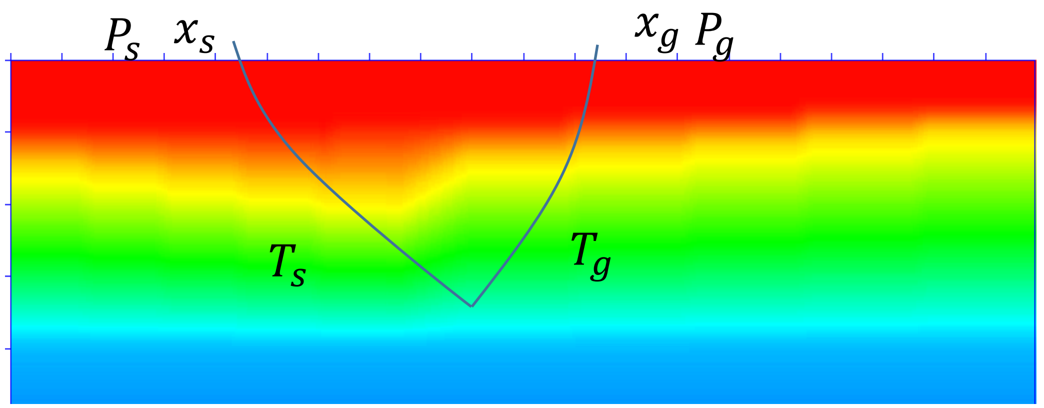
CDR tomography (Sword,1988)

- The principle behind CDR tomography is that ray paths can be reconstructed with source and receiver ray parameters, and correct velocity

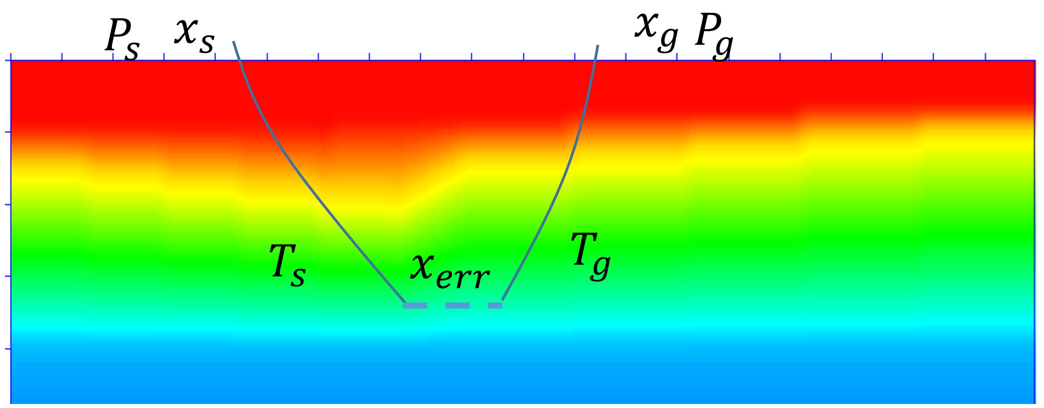


Correct velocity model

- The principle behind CDR tomography is that ray paths can be reconstructed with source and receiver ray parameters, and correct velocity

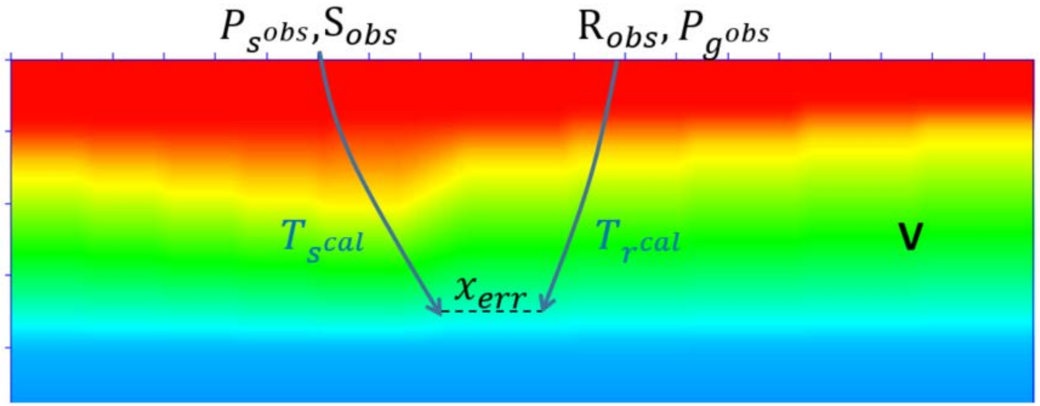


Correct velocity model



Incorrect velocity model

Slope tomography



CDR tomography

Model space : $[V]_{i=1,M}$

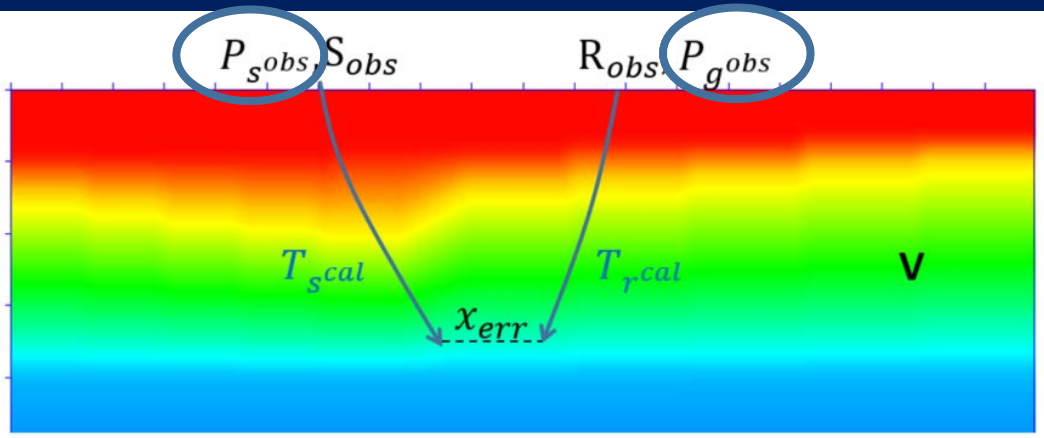
Data space : $[X_{err}]_{j=1,N}$

Fréchet derivative: $A_{ij} = \frac{\partial X_{err\,j}}{\partial V_i}$

Inversion: $\mathbf{A}\,\mathbf{\Delta V} = \mathbf{\Delta X_{err}}$

Sword 1988

Slope tomography



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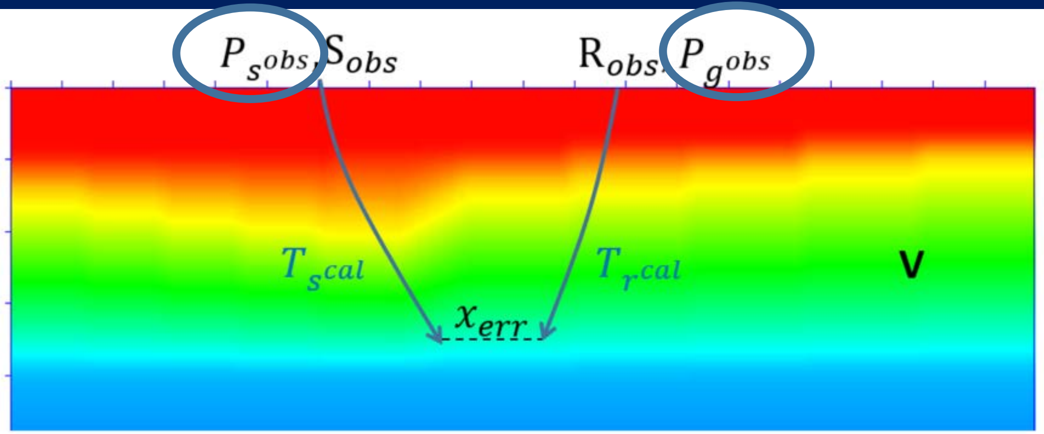
Sword 1988

Stereotomography (Billette and Lambare,1998)



CDR tomography

Model space : $[V]_{i=1,M}$

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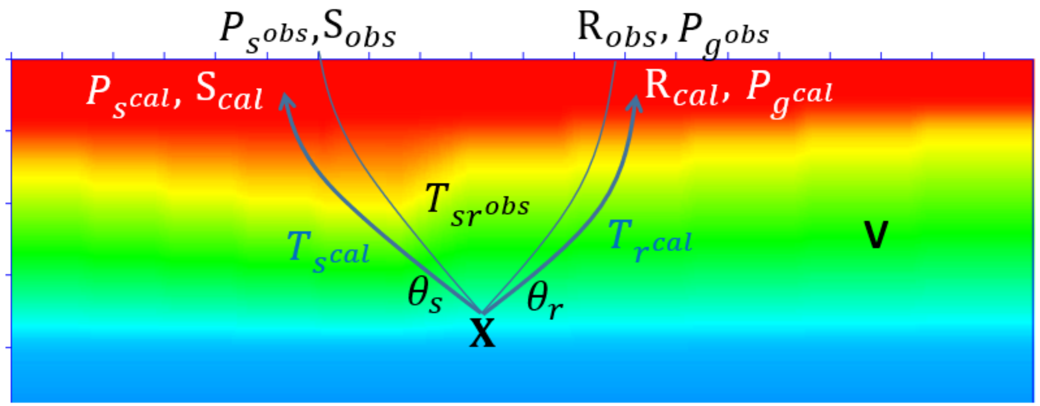


Stereotomography

Model space: $\mathbf{m} = [(X, \Theta_s, \Theta_r, T_s, T_r)_{i1=1,N}], [V]_{i2=1,M}\,]$

Data space : $\mathbf{d} = \left[S, R, P_s, P_g, T_{sr}\right]_{j=1,N}$

Fréchet derivative: $A_{ij} = \frac{\partial(S,R,P_s,P_r,T_{sr})}{\partial(X,\Theta_s,\Theta_r,T_s,T_r,V)}$

Inversion: $\mathbf{A\,\Delta m = \Delta d}$

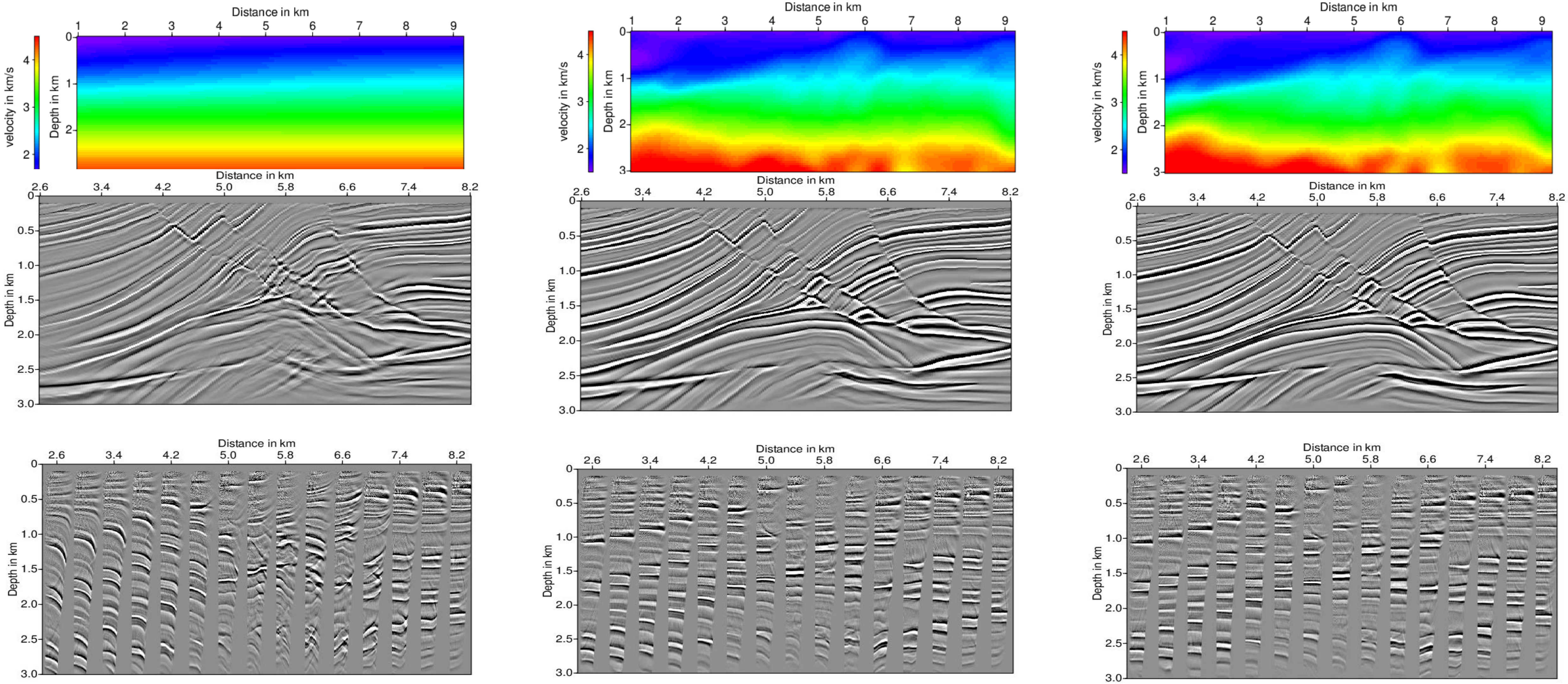
Billette and Lambaré 1998

Stereotomography

Input : Vertical Gradient 400m x 4000m
Output: Model "A"

Input : Model "A" Interpolated to 200m x 250m
Output : Model "B"

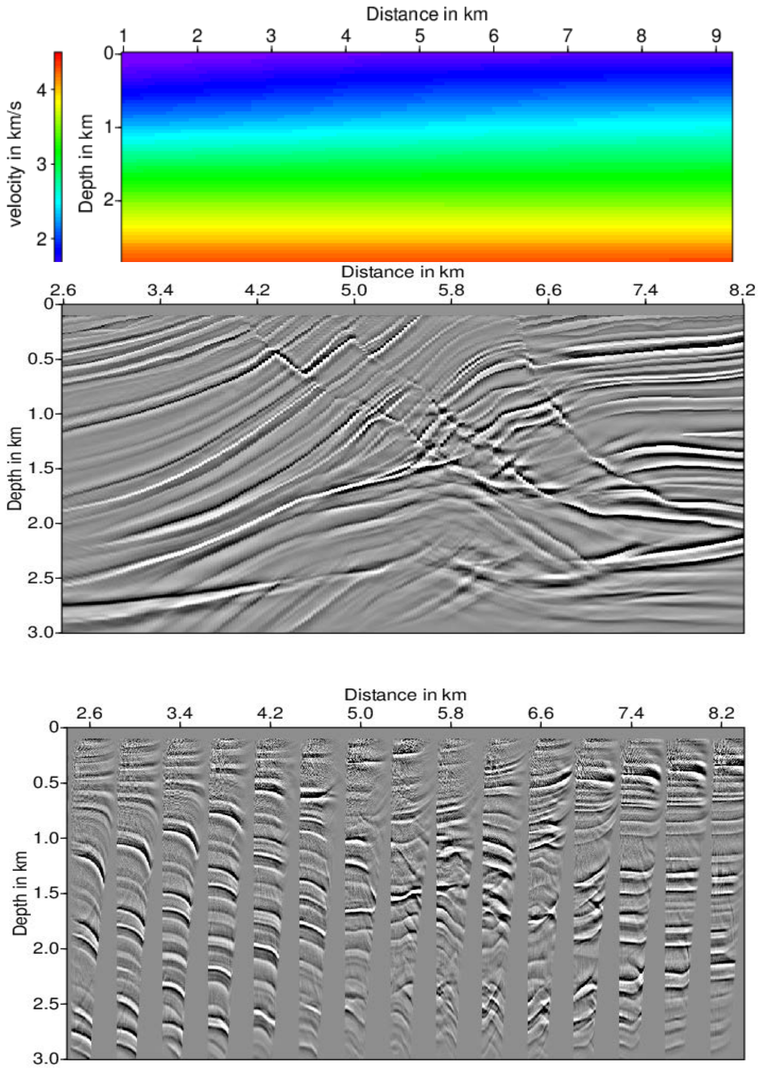
Input : Model "B" and edited picks
Output: Model "C"



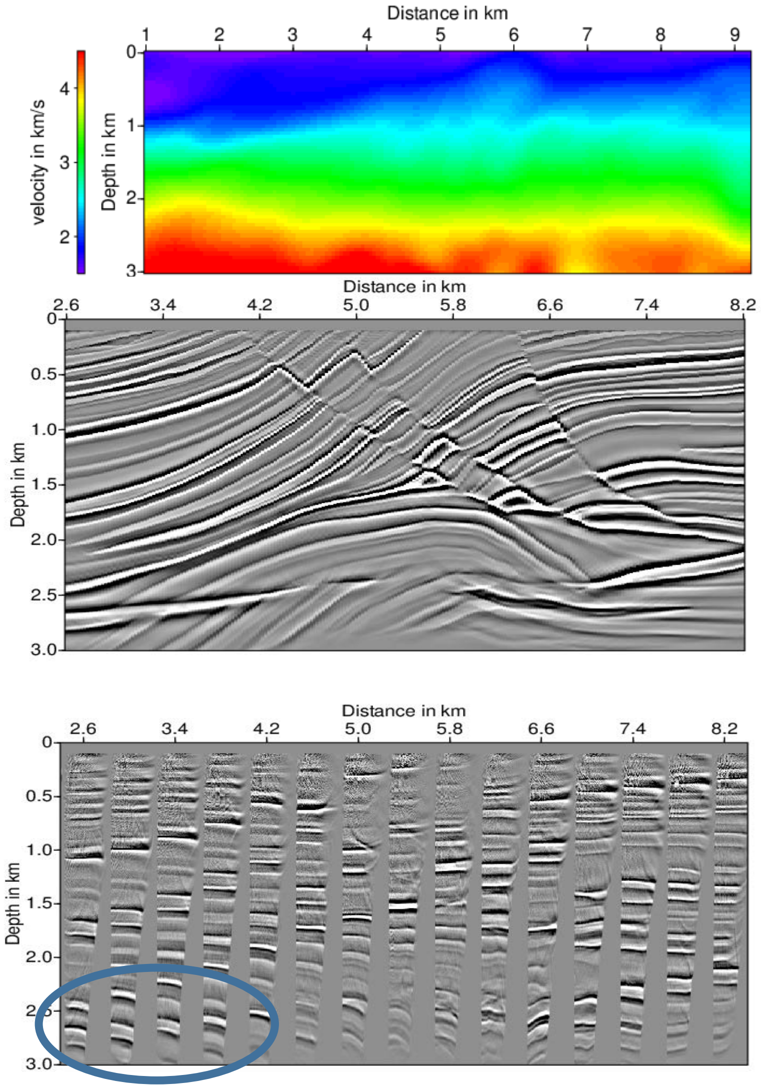
Stereotomography

MINES ParisTech

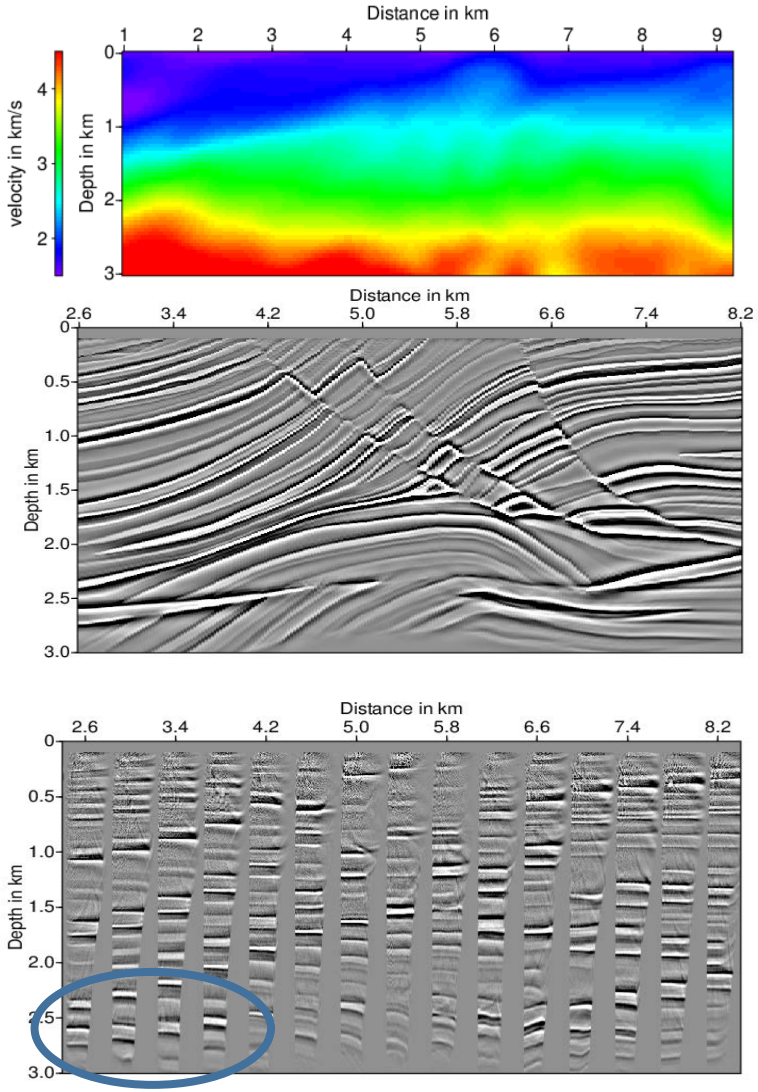
Input : Vertical Gradient 400m x 4000m
Output: Model “A”



Input : Model “A” Interpolated to 200m x 250m
Output : Model “B”

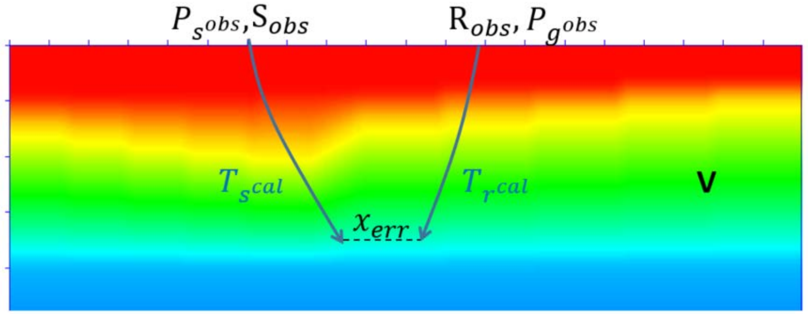


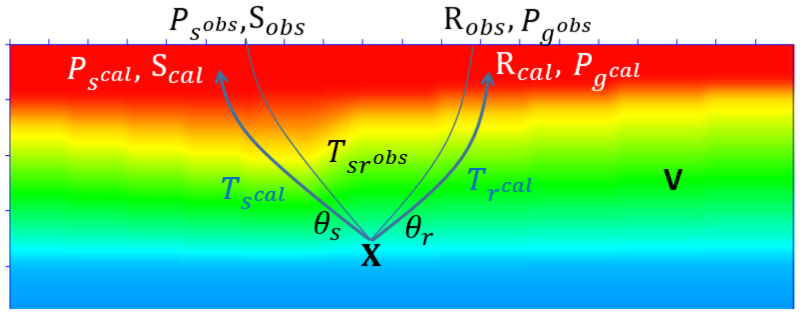
Input : Model “B” and edited picks
Output: Model “C”



Slope tomography

	CDR tomography	Stereotomography
Model space :	$[V]_{i=1,M}$	$[(X,\Theta_s,\Theta_r,T_s,T_r)_{j=1,N}],[V]_{i=1,M}]$
Data space :	$[X_{err}]_{j=1,N}$	$[T_{sr},P_s,P_g,S,R]_{j=1,N}$
Inversion:	$\mathbf{A\,\Delta V = \Delta X_{err}}$	$\mathbf{A\,\Delta m = \Delta d}$
Memory Requirement:	$\mathbf{A}[\mathrm{N,M}]$	$\mathbf{A}[\mathrm{5N, 5N+M}]$





- Sensitive to picking errors

- Much larger model and data space than CDR tomography
- Produce stable solution

Slope tomography

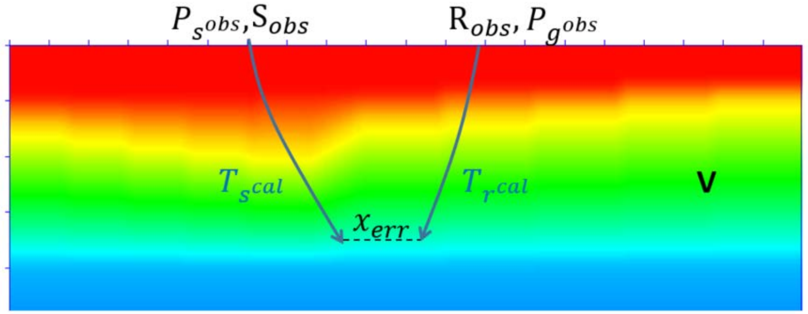
CDR tomography

Model space : $[V]_{i=1,M}$

Data space : $\left[X_{err}\right]_{j=1,N}$

Inversion: $\mathbf{A}\,\mathbf{\Delta V} = \mathbf{\Delta X_{err}}$

Memory Requirement: $\mathbf{A}[\mathrm{N},\mathrm{M}]$



- Sensitive to picking errors

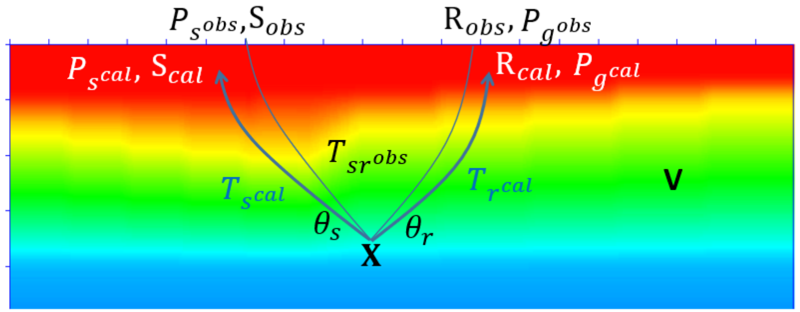
Stereotomography

Model space: $[(X, \Theta_s, \Theta_r, T_s, T_r)_{j=1,N}],[V]_{i=1,M}\ ]$

Data space: $\left[T_{sr}, P_s, P_g, S, R\ \right]_{j=1,N}$

Inversion: $\mathbf{A}\,\mathbf{\Delta m} = \mathbf{\Delta d}$

Memory Requirement: $\mathbf{A}[5\mathrm{N}, 5\mathrm{N}+\mathrm{M}]$



- Much larger model and data space than CDR tomography
- Produce stable solution

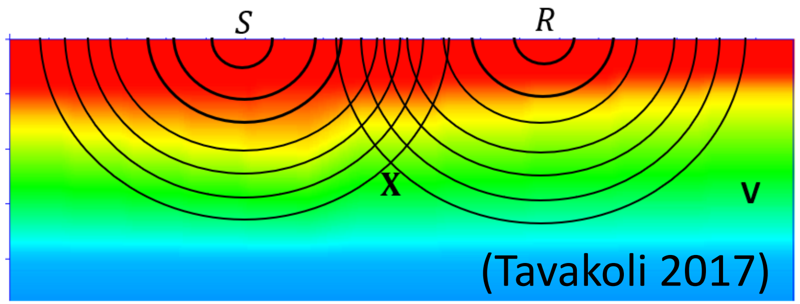
Adjoint slope tomography

Model space: $[X_{j=1,N}],[V]_{i=1,M}\ ]$

Data space: $\left[T_{sr}, P_s, P_g\ \right]_{j=1,N}$

Inversion: $\mathbf{m}_{\ k+1} = \mathbf{m}_{\ k} + \alpha_k\ \frac{\partial J}{\partial \mathbf{m}}$

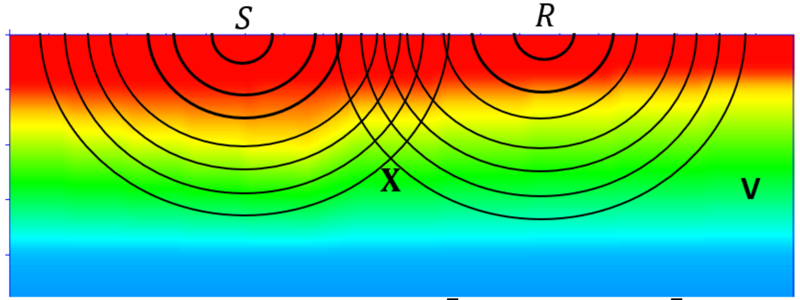
Memory Requirement: $\frac{\partial J}{\partial \mathbf{m}}$ [N+M]



- Matrix free solution
- Gradients are computed using adjoint-state method
- Forward modeling uses eikonal solver
- Efficient for large dataset

Adjoint slope tomography

(Tavakoli 2017)



Data space : $\mathbf{d} = \left[T_{sr}, P_s, P_g\right]_{j=1,N}$

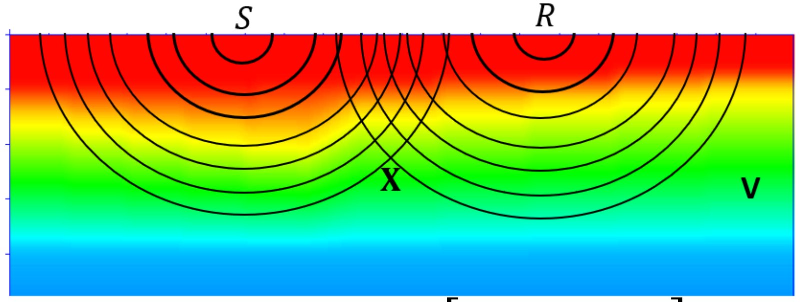
Model space : $\mathbf{m} = [X_{j=1,N}],[V]_{i=1,M}]$

Model update : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{m}}$

Adjoint slope tomography

(Tavakoli 2017)

Cost function: $J = \sum \left(T_{sr} - T_{sr}^{obs}\right)^2 + \sum \left(P_s - P_s^{obs}\right)^2 + \sum \left(P_r - P_r^{obs}\right)^2$



Data space : $\mathbf{d} = \left[T_{sr}, P_s, P_g\right]_{j=1,N}$

Model space : $\mathbf{m} = \left[X_{j=1,N}\right], \left[V\right]_{i=1,M}$

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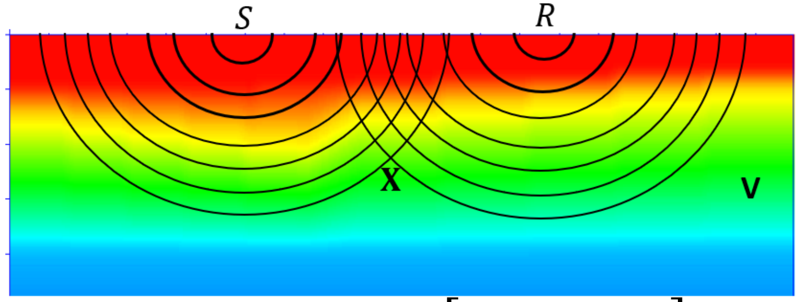
Adjoint slope tomography

(Tavakoli 2017)

Cost function: $J = \sum \left(T_{sr} - T_{sr}^{obs}\right)^2 + \sum \left(P_s - P_s^{obs}\right)^2 + \sum \left(P_r - P_r^{obs}\right)^2$

State variables	State equations
T_{sr}	$T_{sr} - T_s + T_r = 0$
P_s	$P_s - (T_{s+1} - T_{s-1})/2\Delta s = 0$
P_r	$P_r - (T_{r+1} - T_{r-1})/2\Delta r = 0$
T_s	$\lvert\nabla T_s(x)\rvert^2 - \frac{1}{v(x)^2} = 0$
T_r	$\lvert\nabla T_r(x)\rvert^2 - \frac{1}{v(x)^2} = 0$



Data space : $\mathbf{d} = \left[T_{sr}, P_s, P_g\right]_{j=1,N}$

Model space : $\mathbf{m} = \left[X_{j=1,N}\right],\left[V\right]_{i=1,M}$]

Model update : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial J}{\partial \mathbf{m}}$

Adjoint slope tomography

(Tavakoli 2017)

Cost function: $J = \sum \left(T_{sr} - T_{sr}^{obs}\right)^2 + \sum \left(P_s - P_s^{obs}\right)^2 + \sum \left(P_r - P_r^{obs}\right)^2$

State variables	**State equations**
T_{sr}	$T_{sr} - T_s + T_r = 0$
P_s	$P_s - (T_{s+1} - T_{s-1})/2\Delta s = 0$
P_r	$P_r - (T_{r+1} - T_{r-1})/2\Delta r = 0$
T_s	$\lvert\nabla T_s(x)\rvert^2 - \frac{1}{v(x)^2} = 0$
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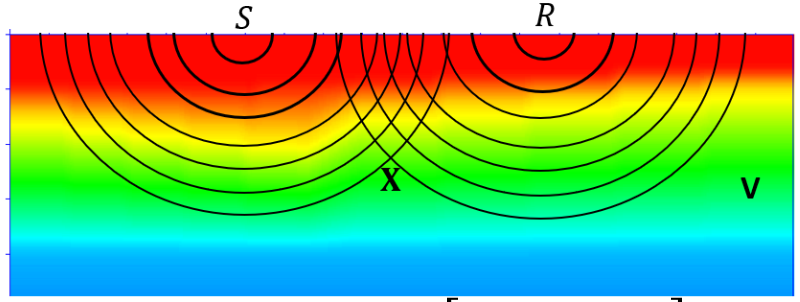


Data space : $\mathbf{d} = \left[T_{sr}, P_s, P_g\right]_{j=1,N}$

Model space : $\mathbf{m} = \left[[X_{j=1,N}],[V]_{i=1,M}\right]$

Model update : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial J}{\partial \mathbf{m}}$

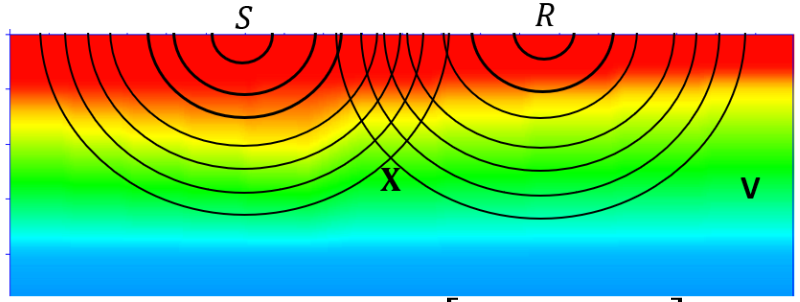
Augmented cost function with equality constraints from the state equations :

$$\mathcal{L} = J - \sum \mu_{sr}\left(T_{sr}^{obs} - T_s - T_r\right) - \sum \xi_s\left(P_s^{obs} - (T_{s+1} - T_{s-1})/2\Delta s\right) - \sum \xi_r\left(P_r^{obs} - (T_{r+1} - T_{r-1})/2\Delta r\right) - \frac{1}{2}\sum\left\langle \lambda_s \middle| \lvert\nabla T_s(x)\rvert^2 - \frac{1}{v(x)^2}\right\rangle - \frac{1}{2}\sum\left\langle \lambda_r \middle| \lvert\nabla T_r(x)\rvert^2 - \frac{1}{v(x)^2}\right\rangle$$

Adjoint slope tomography

(Tavakoli 2017)

$$\mathcal{L} = \sum \left(T_{sr} - T_{sr}^{obs}\right)^2 + \sum \left(P_s - P_s^{obs}\right)^2 + \sum \left(P_r - P_r^{obs}\right)^2 - \sum \mu_{sr}\left(T_{sr}^{obs} - T_s - T_r\right) - \sum \xi_s\left(P_s^{obs} - (T_{s+1} - T_{s-1})/2\Delta s\right) - \sum \xi_r\left(P_r^{obs} - (T_{r+1} - T_{r-1})/2\Delta r\right) - \frac{1}{2}\sum\left\langle \lambda_s \middle| |\nabla T_s(x)|^2 - \frac{1}{v(x)^2}\right\rangle - \frac{1}{2}\sum\left\langle \lambda_r \middle| |\nabla T_r(x)|^2 - \frac{1}{v(x)^2}\right\rangle$$



Gradient of cost function:

$$\frac{\partial J}{\partial v(x)} = \frac{\partial \mathcal{L}}{\partial v(x)} = -\frac{1}{v(x)^3}\sum\left(\lambda_s(x) + \lambda_r(x)\right)$$

$$\frac{\partial J}{\partial X} = \frac{\partial \mathcal{L}}{\partial X} = \mu_{sr}\left(\frac{\partial T_s}{\partial X} + \frac{\partial T_r}{\partial X}\right) + \frac{\xi_s}{2\Delta s}\left(\frac{\partial T_{s+1}}{\partial X} - \frac{\partial T_{s-1}}{\partial X}\right) + \frac{\xi_r}{2\Delta r}\left(\frac{\partial T_{r+1}}{\partial X} - \frac{\partial T_{r-1}}{\partial X}\right)$$

Data space : $\mathbf{d} = \left[T_{sr}, P_s, P_g\right]_{j=1,N}$

Model space : $\mathbf{m} = [X_{j=1,N}],[V]_{i=1,M}]$

Model update : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial \boldsymbol{J}}{\partial \mathbf{m}}$

Adjoint slope tomography

(Tavakoli 2017)

$$\mathcal{L} = \sum \left(T_{sr} - T_{sr}^{obs}\right)^2 + \sum \left(P_s - P_s^{obs}\right)^2 + \sum \left(P_r - P_r^{obs}\right)^2 - \sum \mu_{sr}\left(T_{sr}^{obs} - T_s - T_r\right) - \sum \xi_s\left(P_s^{obs} - (T_{s+1} - T_{s-1})/2\Delta s\right) -$$

$$\sum \xi_r\left(P_r^{obs} - (T_{r+1} - T_{r-1})/2\Delta r\right) - \frac{1}{2}\sum\left\langle \lambda_s \middle| |\nabla T_s(x)|^2 - \frac{1}{v(x)^2}\right\rangle - \frac{1}{2}\sum\left\langle \lambda_r \middle| |\nabla T_r(x)|^2 - \frac{1}{v(x)^2}\right\rangle$$

Gradient of cost function:

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State variables | **Adjoint State variables and equations**

T_{sr} : $\frac{\partial \mathcal{L}}{\partial T_{sr}} = 0 \rightarrow \quad \mu_{sr} = (T_{sr} - T_{sr}^{obs})$

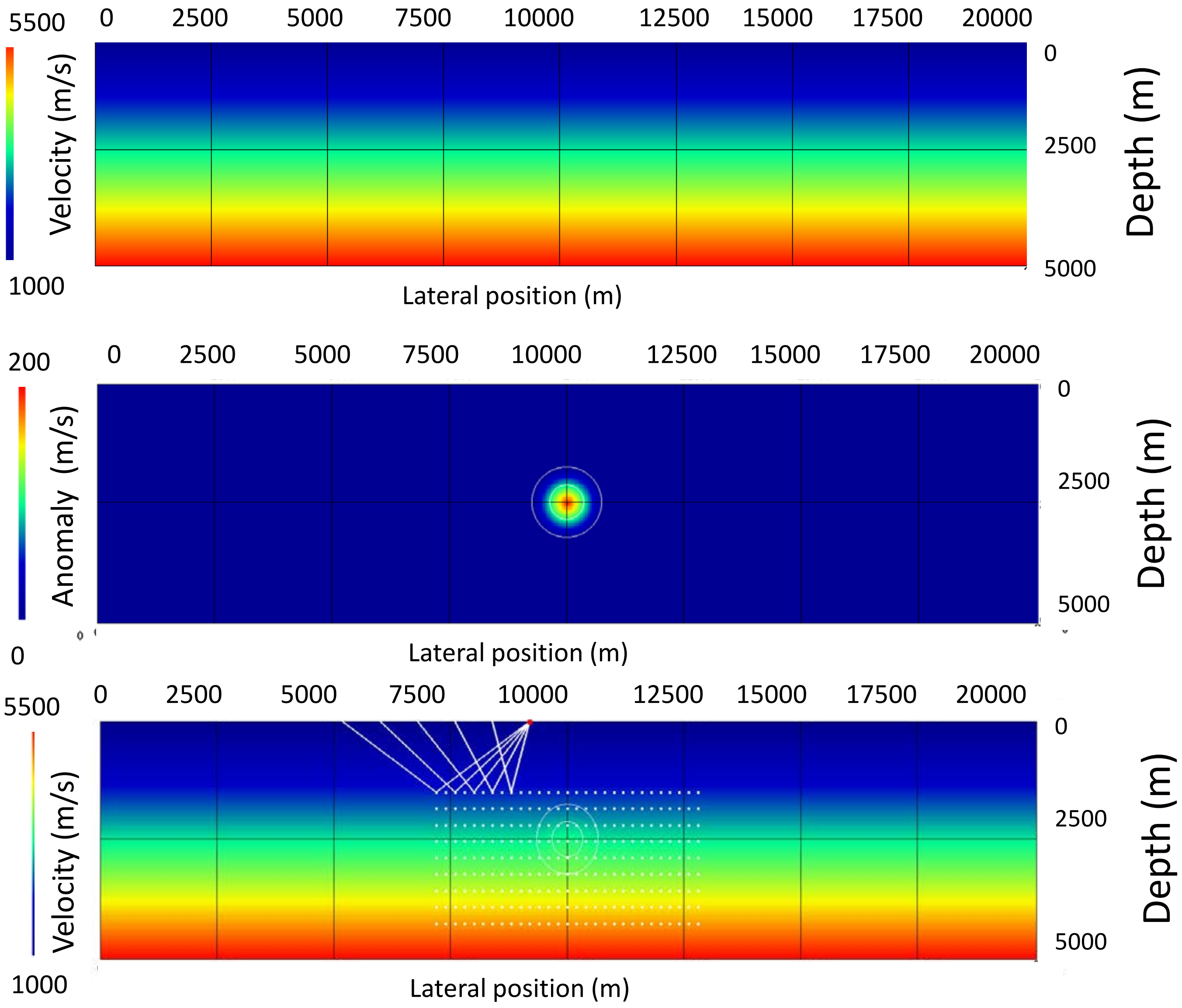
P_s : $\frac{\partial \mathcal{L}}{\partial P_s} = 0 \rightarrow \quad \xi_s = (P_s - P_s^{obs})$

P_r : $\frac{\partial \mathcal{L}}{\partial P_r} = 0 \rightarrow \quad \xi_r = (P_g - P_g^{obs})$

T_s : $\frac{\partial \mathcal{L}}{\partial T_s} = 0 \rightarrow \quad \frac{\partial}{\partial x}\left(\frac{\partial T_s}{\partial x}\lambda_s\right) + \frac{\partial}{\partial z}\left(\frac{\partial T_s}{\partial z}\lambda_s\right) = -\sum u_{sr} + \frac{1}{2\Delta s}\left(\sum \xi_{s+1} - \sum \xi_{s-1}\right)$

T_r : $\frac{\partial \mathcal{L}}{\partial T_r} = 0 \rightarrow \quad \frac{\partial}{\partial x}\left(\frac{\partial T_r}{\partial x}\lambda_r\right) + \frac{\partial}{\partial z}\left(\frac{\partial T_r}{\partial z}\lambda_r\right) = -\sum u_{sr} + \frac{1}{2\Delta r}\left(\sum \xi_{r+1} - \sum \xi_{r-1}\right)$

Adjoint slope tomography Numerical test



37 shots with shot spacing of 200 m and maximum offset of 4000 m.

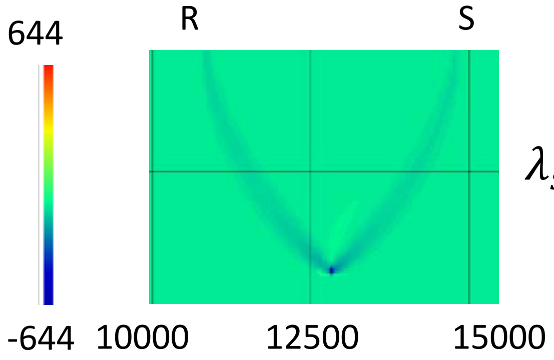
Compute reciprocal parameters every 800 m within each shot. (equivalent to picking 5 traces per shot)

1305 reciprocal parameter picks were created

Numerical test

$$\frac{\partial}{\partial x}\left(\frac{\partial T_s}{\partial x}\lambda_s\right) + \frac{\partial}{\partial z}\left(\frac{\partial T_s}{\partial z}\lambda_s\right) = -\sum u_{sr} + \frac{1}{2\Delta s}\left(\sum \xi_{s+1} - \sum \xi_{s-1}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T_r}{\partial x}\lambda_r\right) + \frac{\partial}{\partial z}\left(\frac{\partial T_r}{\partial z}\lambda_r\right) = -\sum u_{sr} + \frac{1}{2\Delta r}\left(\sum \xi_{r+1} - \sum \xi_{r-1}\right)$$



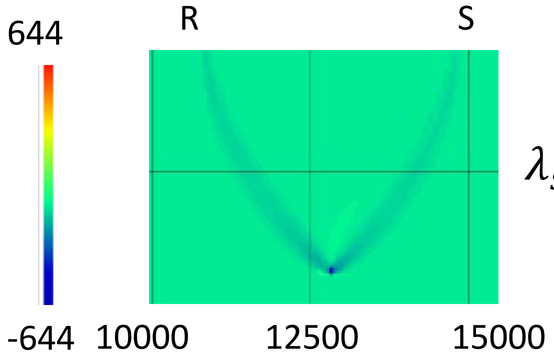
λ_s and λ_r for single T_{sr}, P_s and P_g pick

Adjoint slope tomography

Numerical test

$$\frac{\partial}{\partial x}\left(\frac{\partial T_s}{\partial x}\lambda_s\right)+\frac{\partial}{\partial z}\left(\frac{\partial T_s}{\partial z}\lambda_s\right)=-\sum u_{sr}+\frac{1}{2\Delta s}\left(\sum\xi_{s+1}-\sum\xi_{s-1}\right)$$

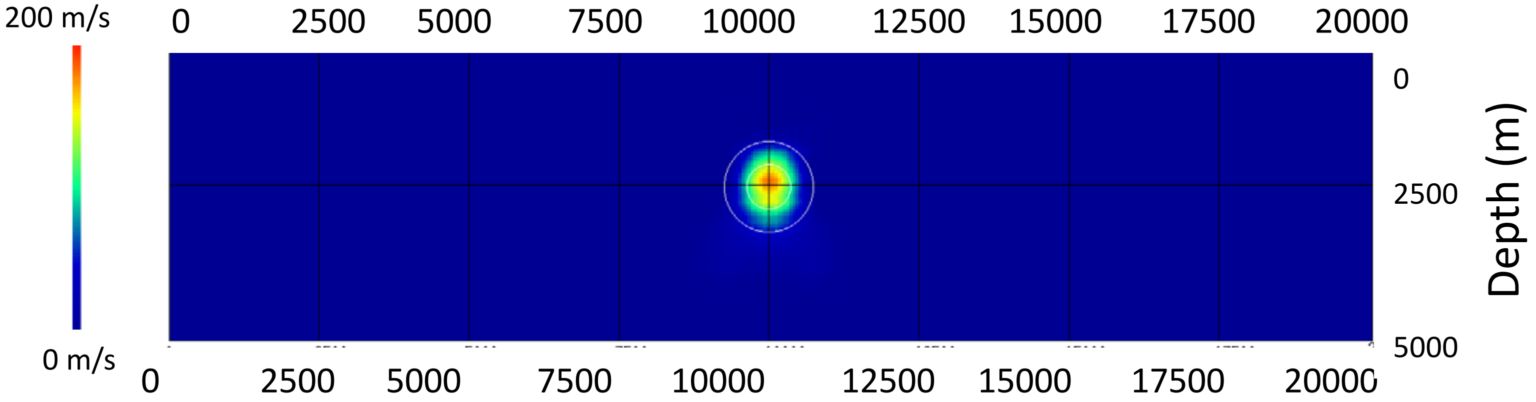
$$\frac{\partial}{\partial x}\left(\frac{\partial T_r}{\partial x}\lambda_r\right)+\frac{\partial}{\partial z}\left(\frac{\partial T_r}{\partial z}\lambda_r\right)=-\sum u_{sr}+\frac{1}{2\Delta r}\left(\sum\xi_{r+1}-\sum\xi_{r-1}\right)$$



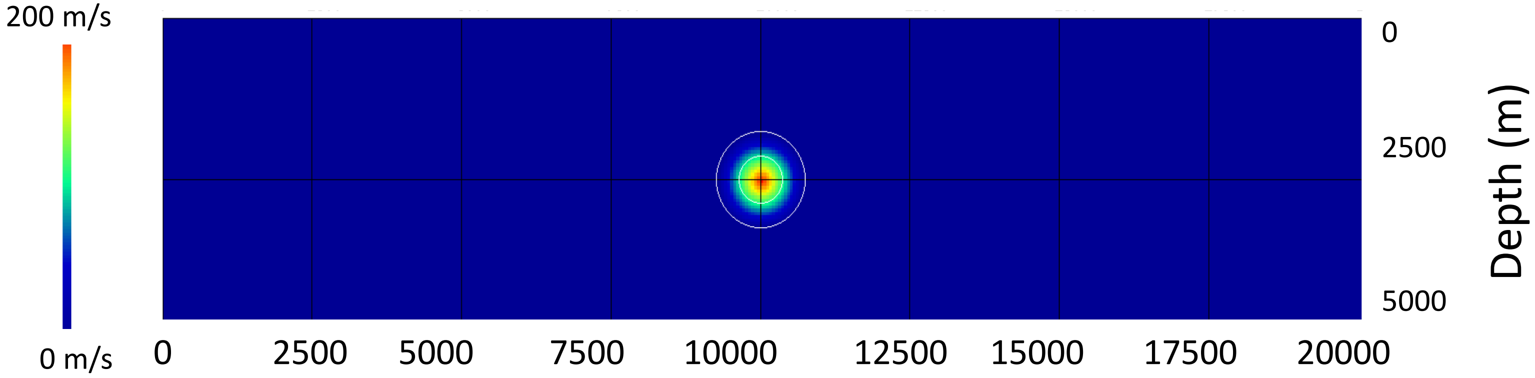
λ_s and λ_r for single T_{sr}, P_s and P_g pick

Velocity update computed from 37 shots and 1305 picks

$$\alpha\,\frac{\partial J}{\partial v(x)}$$



Actual velocity anomaly



Conclusions

- CDR tomography is sensitive to picking errors because of the inherent limitation in the algorithm

- Stereotomography can produce stable solution, but can be expensive for large dataset.

- Adjoint slope tomography is computationally more efficient for large dataset. Will continue to investigate further in this method.

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