Seismic data interpolation using multichannel singular spectrum analysis

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INTRODUCTION

Why interpolation?

- It is very hard to design a seismic survey in a regular grid of sources and receivers because of economic restrictions on seismic data acquisition

- Missing traces producing sampling artifacts or noise during processing

- Many seismic processing tools for noise attenuation or imaging require high quality regularly sampled data to work properly

Methods of seismic data reconstruction

- Signal processing-based methods
  - The Fourier transform
  - The Radon transform
  - The Curvelet transform
- Wave-equation-based methods
  - Mapping and reconstruction operators
  - Finite difference offset continuation filter
- Rank reduction-based methods
  - Rank reduction of Hankel/Toeplitz matrix
  - For linear events

Introduction to MSSA

- Analysis of 1D time series
- Study of climatic records
- Signal reconstruction
- Forecasting of time series
- Filtering of digital terrain model

- In seismic data processing:
  - In frequency domain as rank reduction of the Hankel matrix
  - Random noise attenuation (Tricket, 2002. Sacchi, 2009)
  - Iterative algorithm for seismic interpolation (Oropeza and Sacchi, 2009)

THEORY

5 dimensions of a 3D seismic data:

- Vertical dimension

  Time or depth for the vertical dimension

- Spatial directions

  shot.x, shot.y, rcvr.x, rcvr.y (acquisition coordinates)

  inline, crossline, offset, azimuth (processing coordinates)

  cmp.x, cmp.y, offset.x, offset.y (processing coordinates)

Hankel matrix

Hankel Operator

$$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$$

$$A_{i,j} = A_{i+k,\, j-k}$$
$$k = 0,1,\ldots,\, j-i$$

$$A = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_2 & a_3 & a_4 & a_5 & a_6 \\ a_3 & a_4 & a_5 & a_6 & a_7 \\ a_4 & a_5 & a_6 & a_7 & a_8 \\ a_5 & a_6 & a_7 & a_8 & a_9 \end{pmatrix}$$

Hankel matrix is:

- A square matrix with constant skew-diagonals from left to right
- Is closely related to Toeplitz matrix

Toeplitz matrix is:

- Diagonal constant matrix

Toeplitz Operator

$$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$$

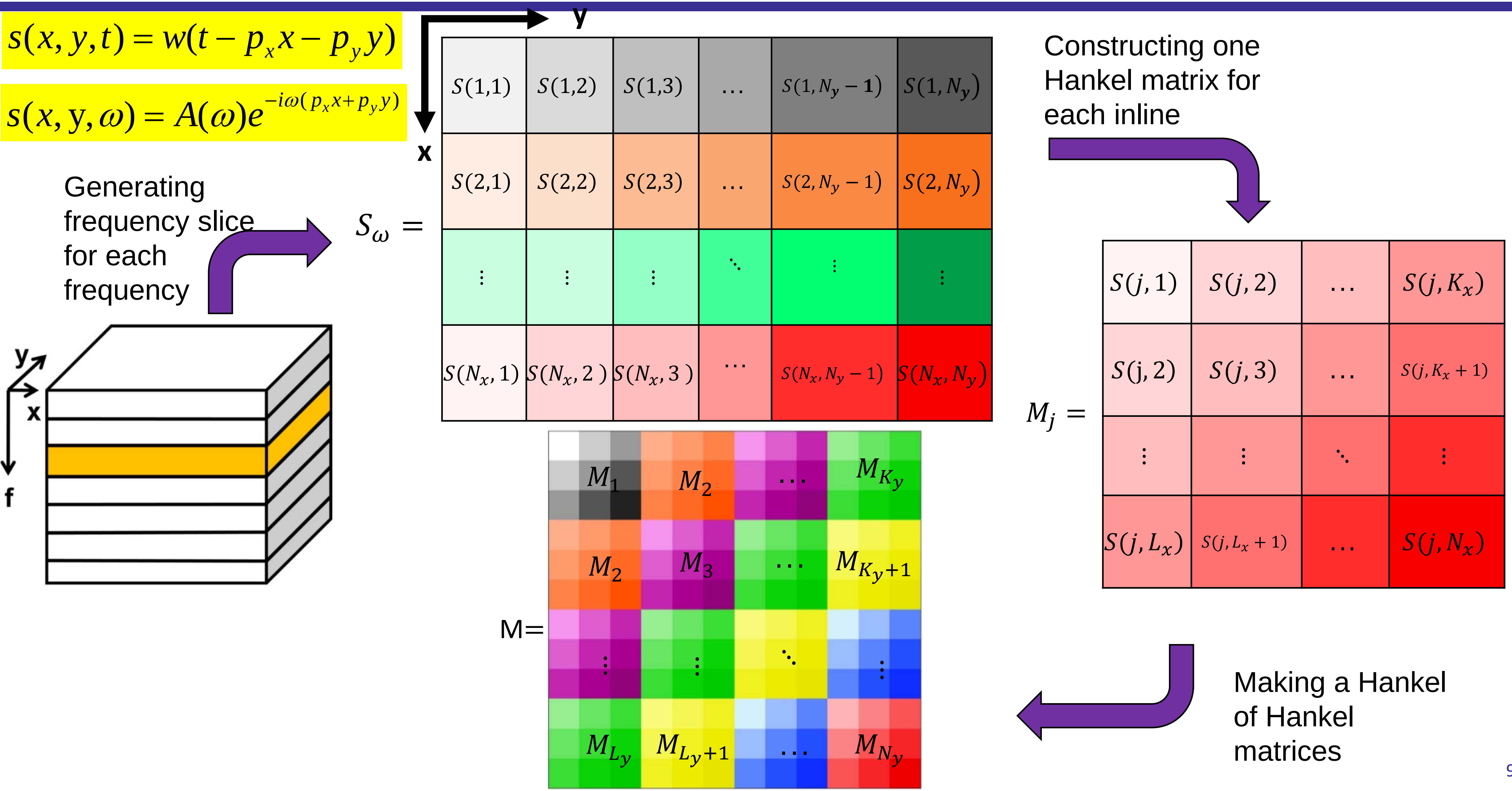
$$A_{i,j} = A_{i+1,\, j+1} = a_{i-j}$$

$$A = \begin{pmatrix} a_5 & a_4 & a_3 & a_2 & a_1 \\ a_6 & a_5 & a_4 & a_3 & a_2 \\ a_7 & a_6 & a_5 & a_4 & a_3 \\ a_8 & a_7 & a_6 & a_5 & a_4 \\ a_9 & a_8 & a_7 & a_6 & a_5 \end{pmatrix}$$

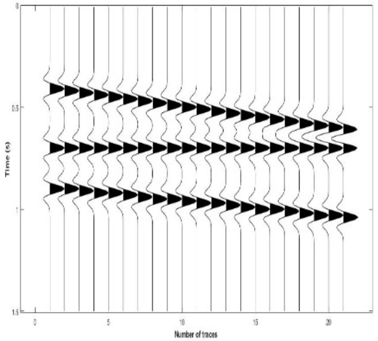
The rank reduction algorithm

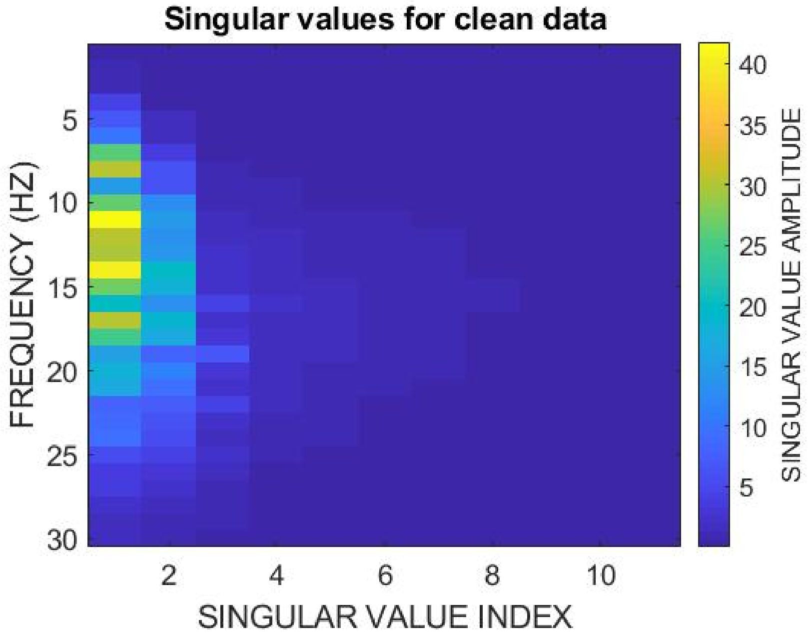
$$s(x, y, t) = w(t - p_x x - p_y y)$$

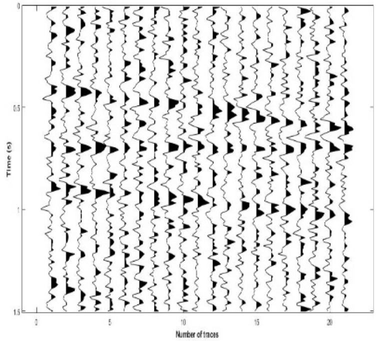
$$s(x, y, \omega) = A(\omega) e^{-i\omega(p_x x + p_y y)}$$

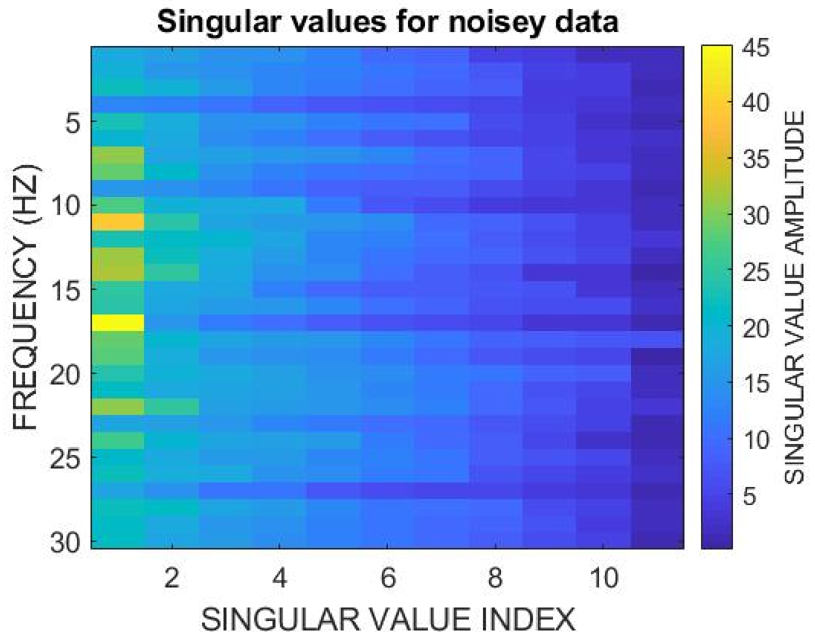


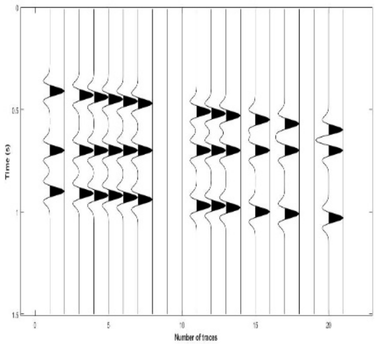
Theory of MSSA

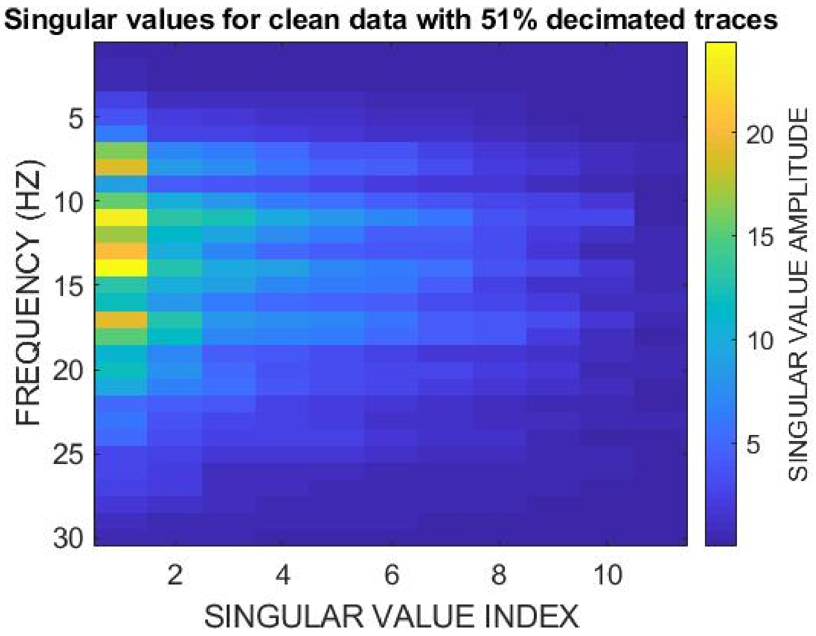


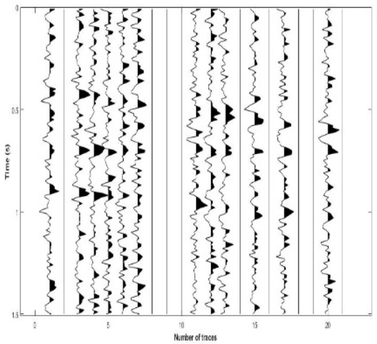


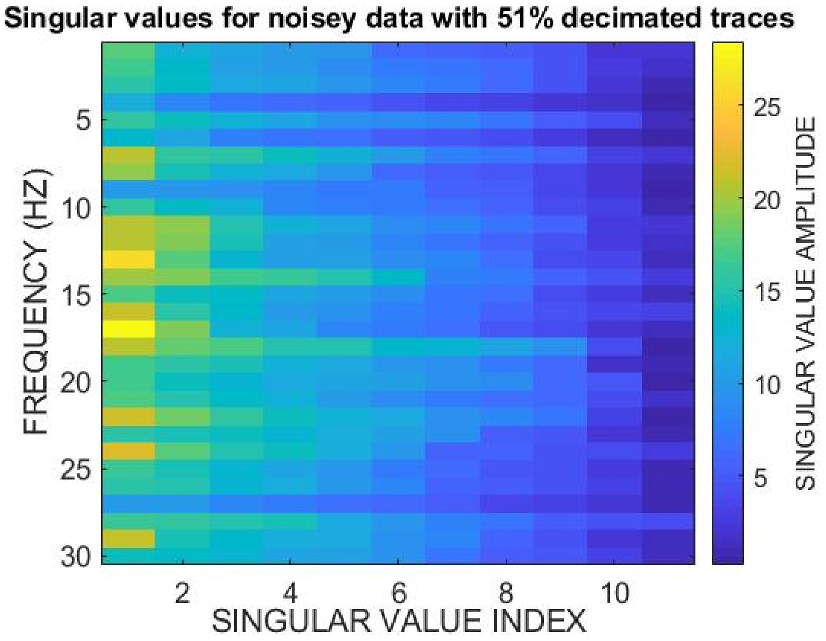












The rank reduction algorithm

SVD

$$\mathbf{M} = U\Sigma V^H$$

Rank Reduction →

$$\mathbf{M_k} = \mathbf{U_k \Sigma_k V_k}^H$$

$\mathbf{U}$ → eigenvectors of MM^H
Σ → Singular values of M (descending order)
$\mathbf{V}$ → eigen vectors of $M^H M$

Σ_k → Singular values of M_k (the first k largest singular values)

To recover the filtered data →
Making average along the anti-diagonals of the rank reduced Hankel matrix

The interpolation algorithm

$$S^{obs} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \square \begin{pmatrix} S(1,1) & S(1,2) & S(1,3) & S(1,4) \\ S(2,1) & S(2,2) & S(2,3) & S(2,4) \\ S(3,1) & S(3,2) & S(3,3) & S(3,4) \\ S(4,1) & S(4,2) & S(4,3) & S(4,4) \end{pmatrix} = \begin{pmatrix} S(1,1) & S(1,2) & S(1,3) & 0 \\ S(2,1) & 0 & 0 & S(2,4) \\ 0 & 0 & S(3,3) & S(3,4) \\ S(4,1) & S(4,2) & S(4,3) & 0 \end{pmatrix}$$

T=
- 1 if the point contains an observation
- 0 if the point contains an unobserved data

for p=1:Niter

for f=f1:fN

$$S^{p} = S^{obs} + (I - T) \square \ \mathrm{MSSA}(S^{p-1})$$

end

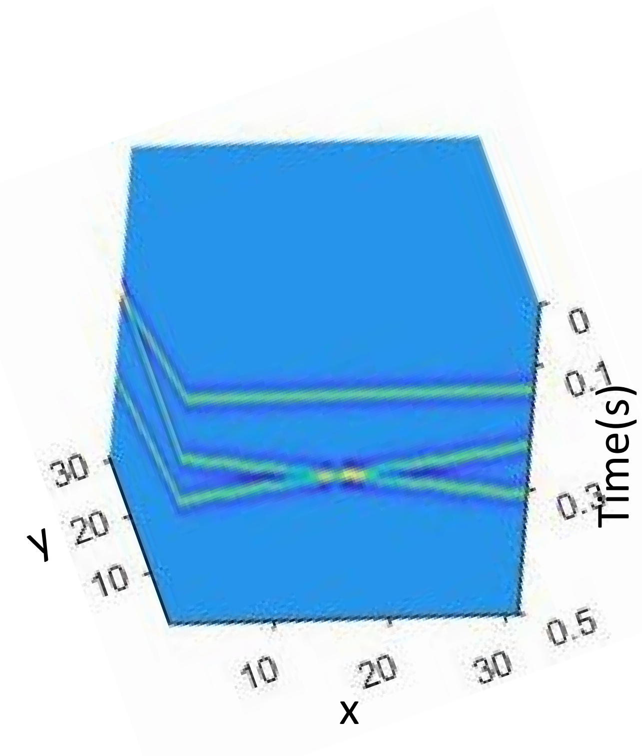
end

The algorithm stops:

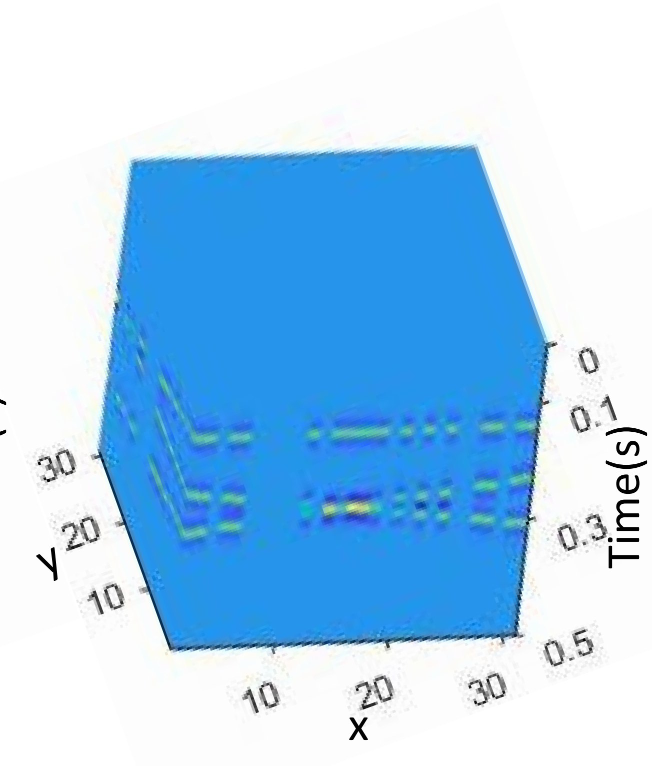
1. Maximum number of iterations is reached
2. $\left\| S^{p} - S^{p-1} \right\|_F^2 \leq \varepsilon$

EXAMPLES

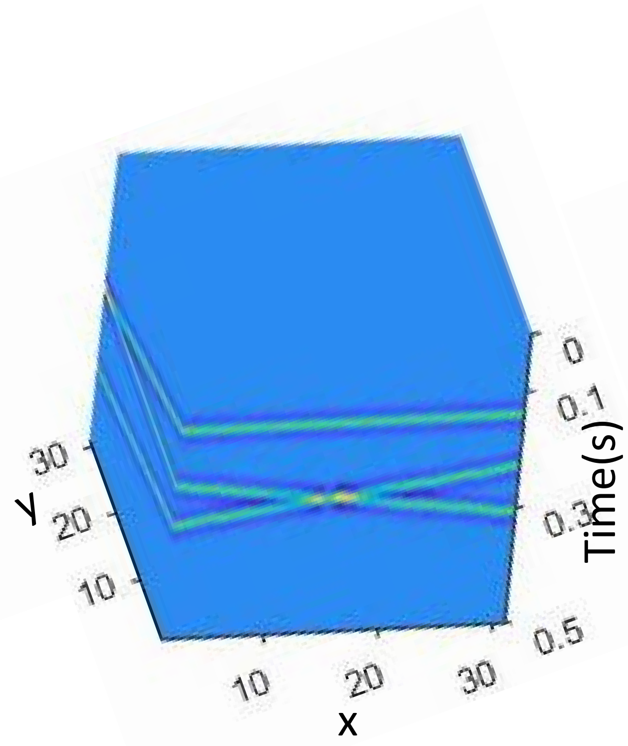
3D poststack



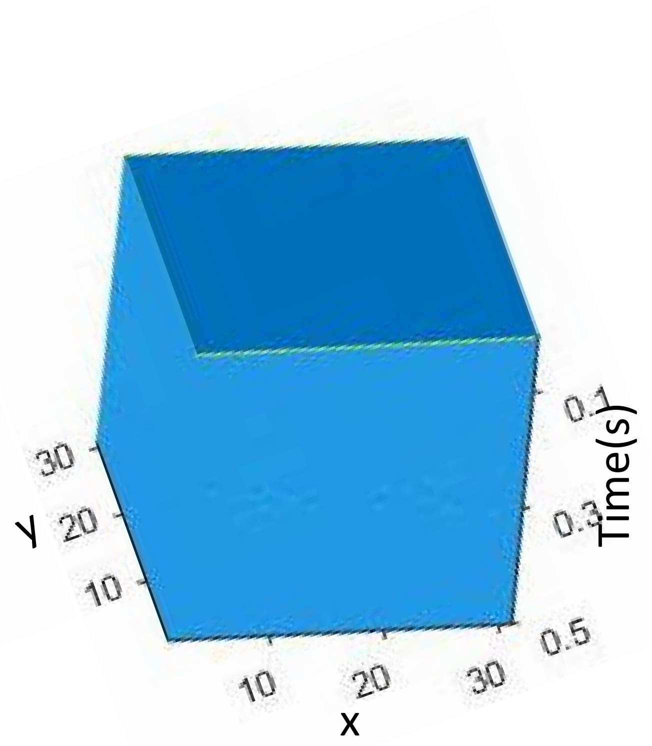
Initial data



Input data with 50% randomly decimated traces

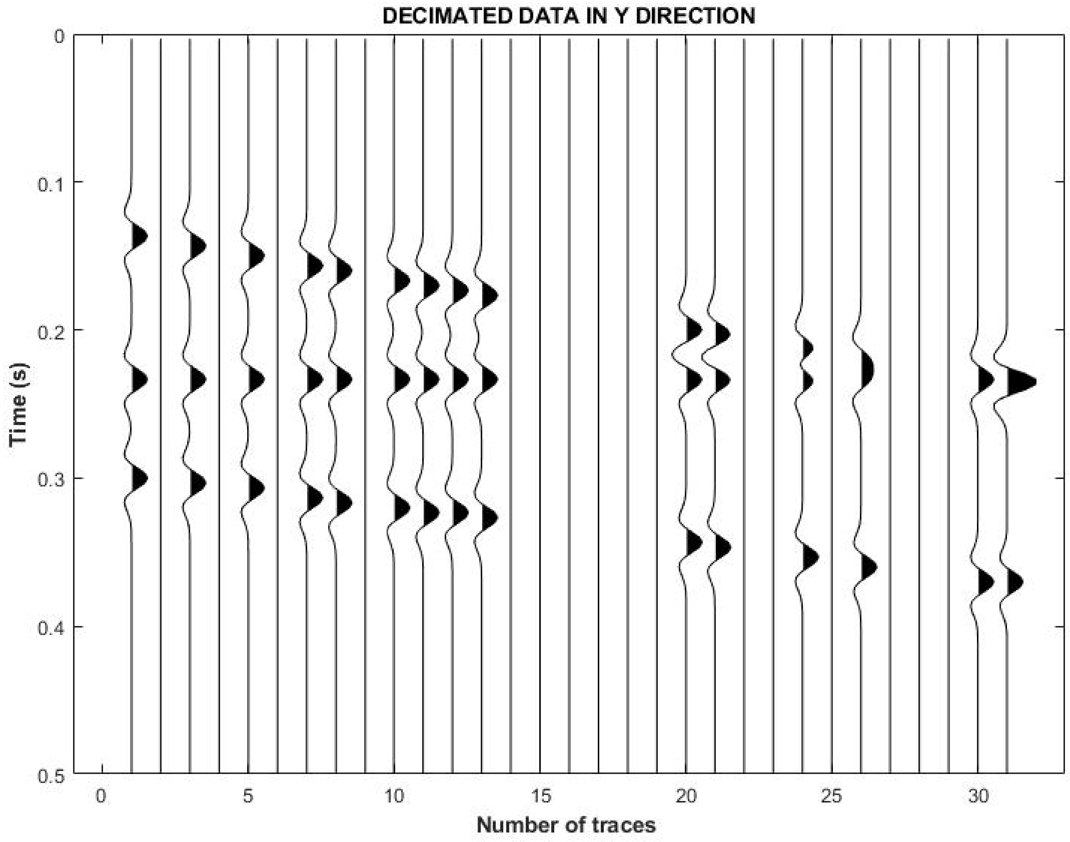


Interpolated data with MSSA

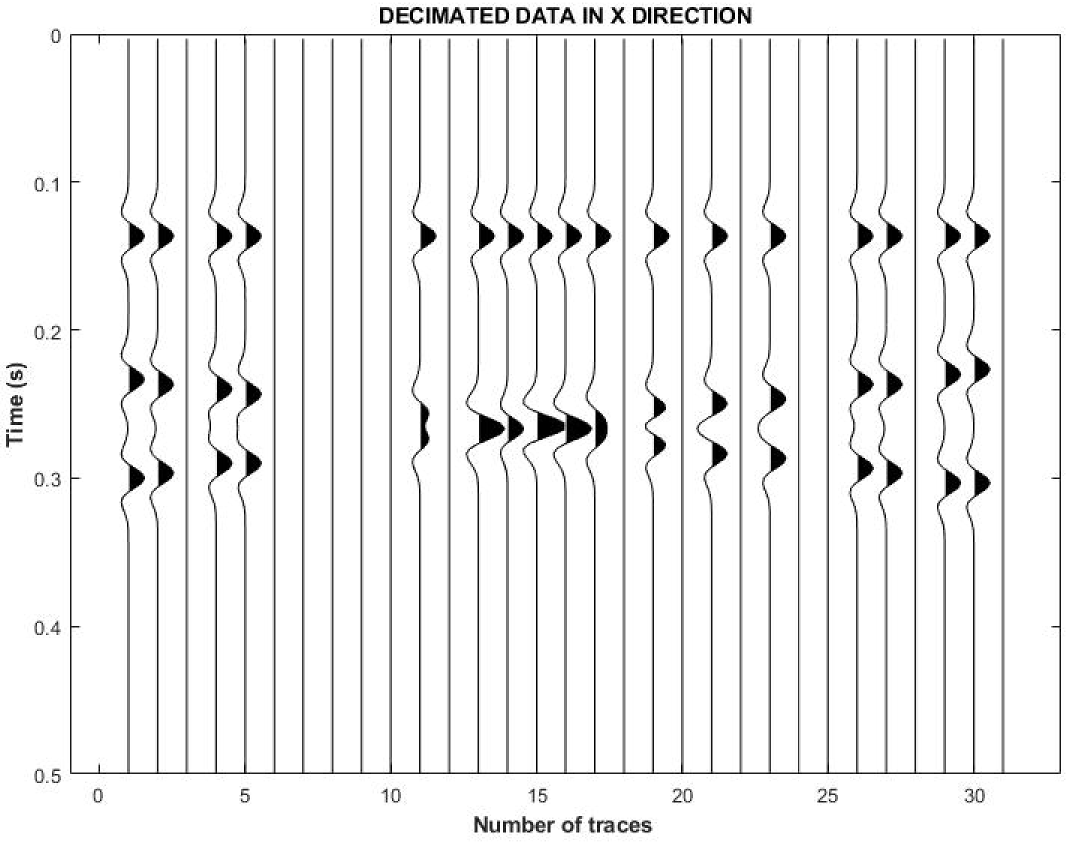


Difference with the initial data

INPUT DATA

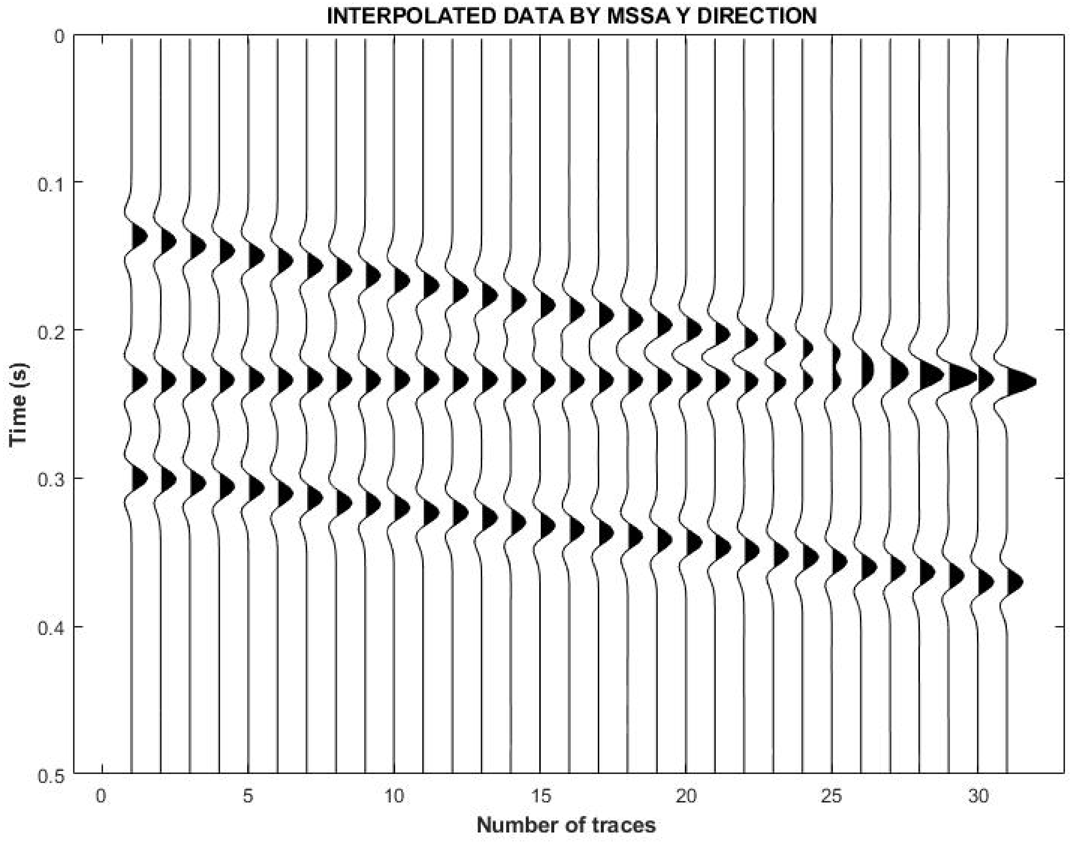


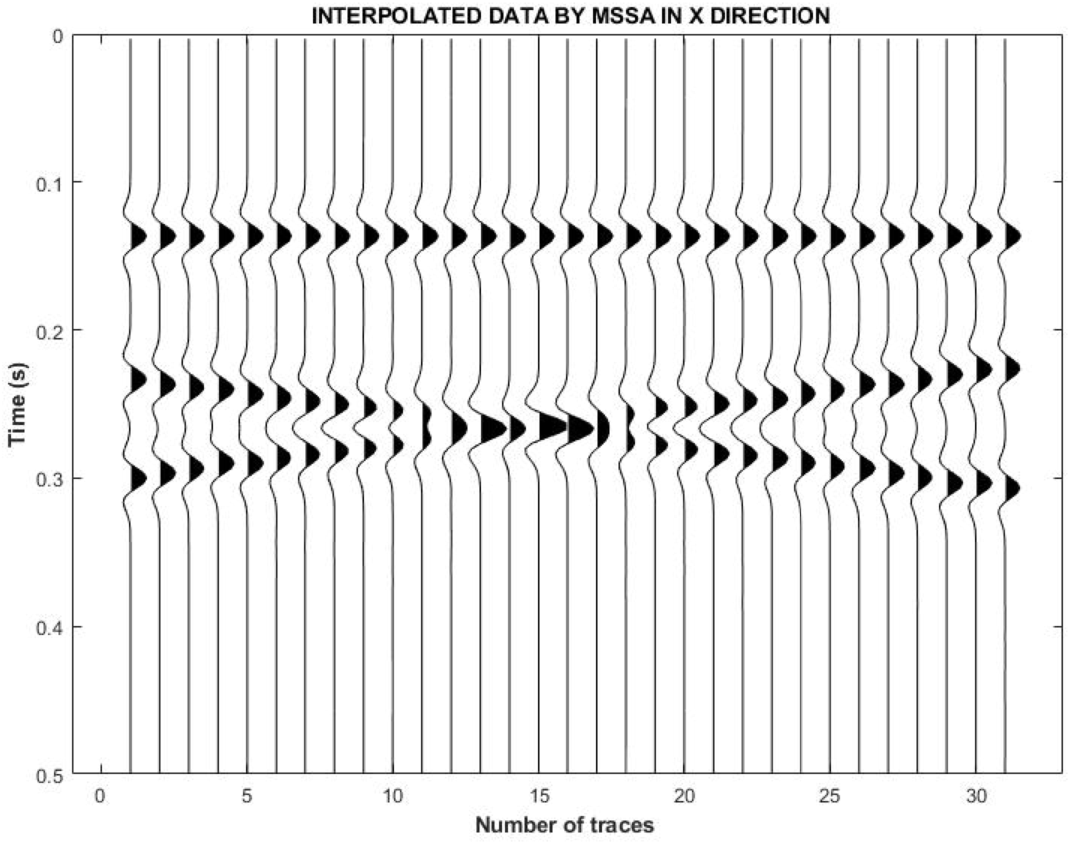
DECIMATED DATA IN Y DIRECTION
Time (s)
Number of traces



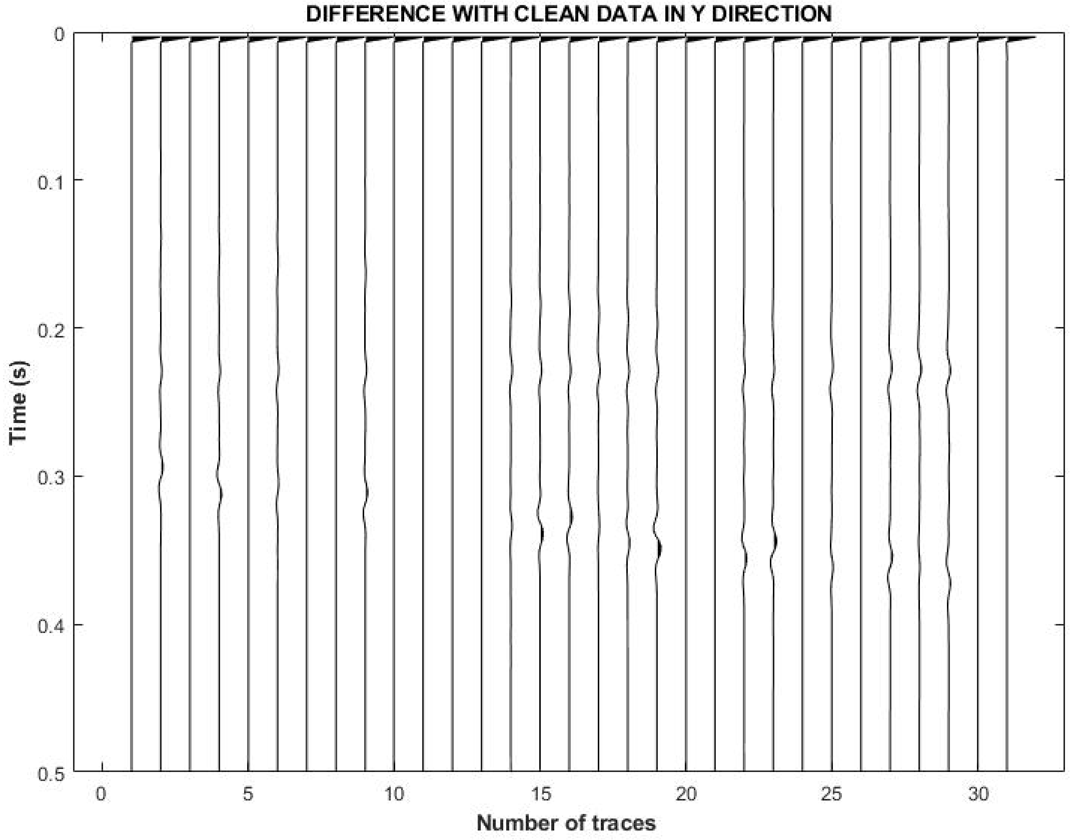
DECIMATED DATA IN X DIRECTION
Time (s)
Number of traces

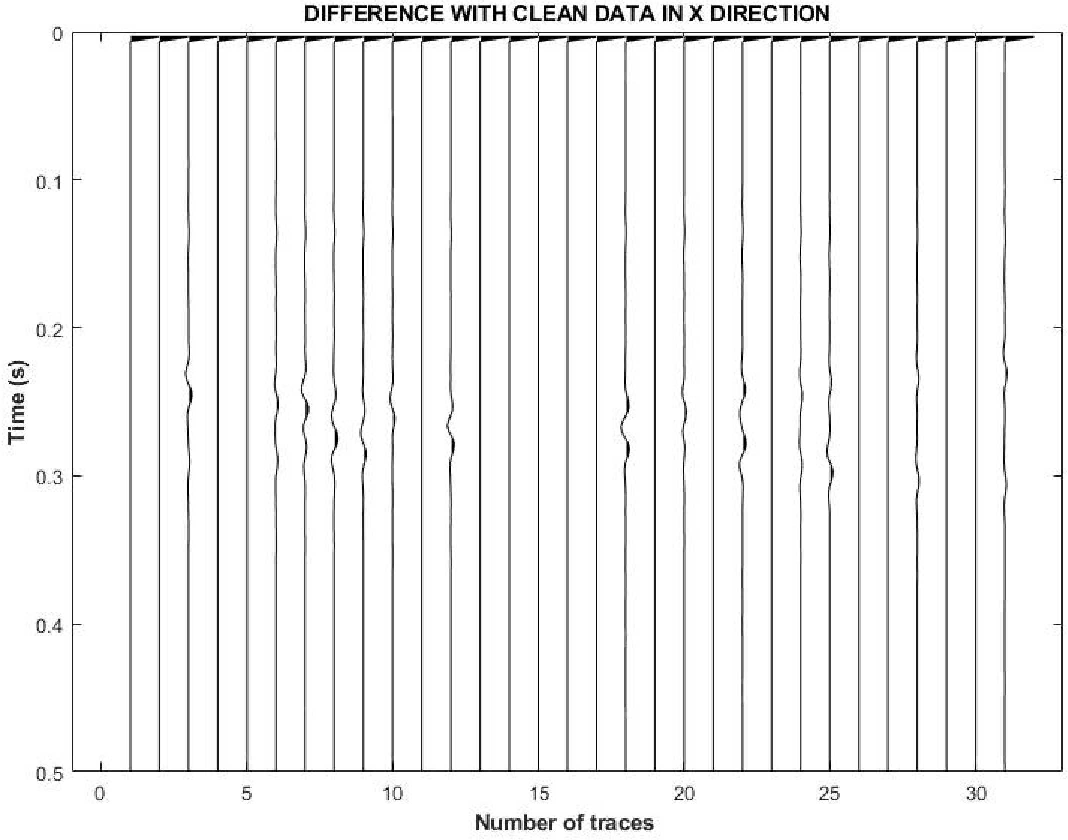
INTERPOLATED DATA



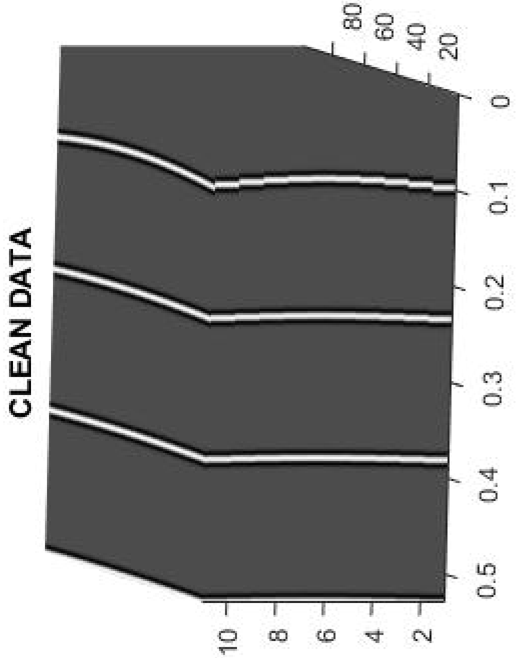


THE DIFFERENCES

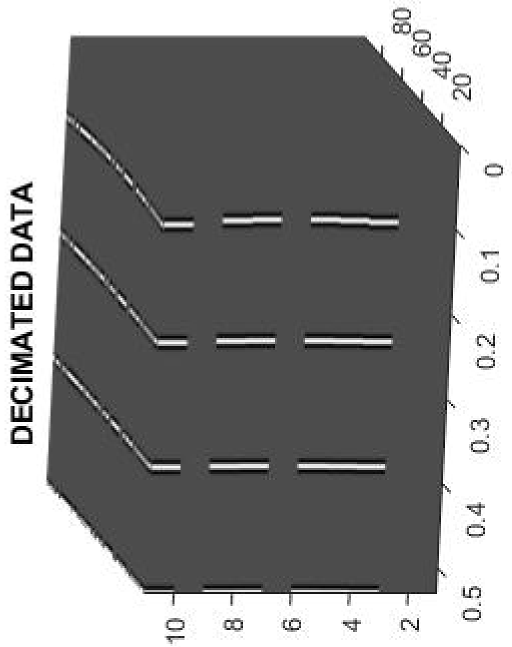




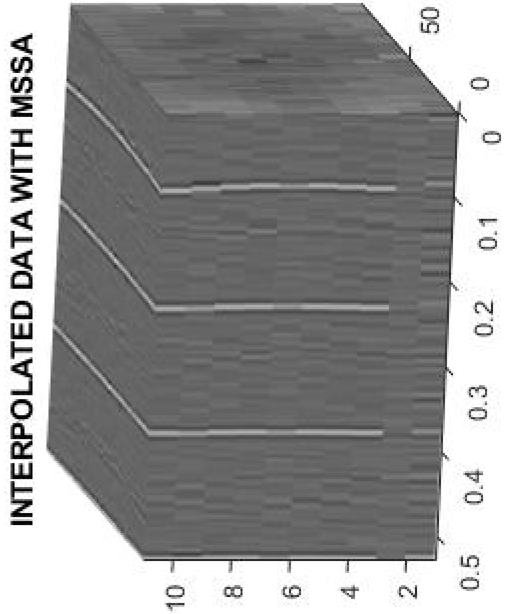
3D prestack data



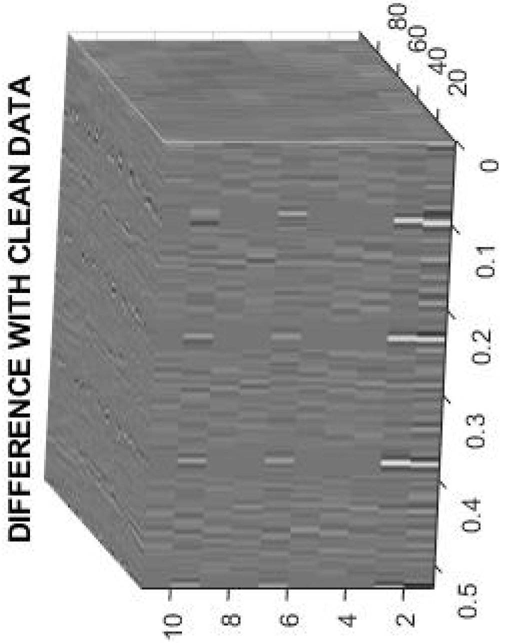
Initial data



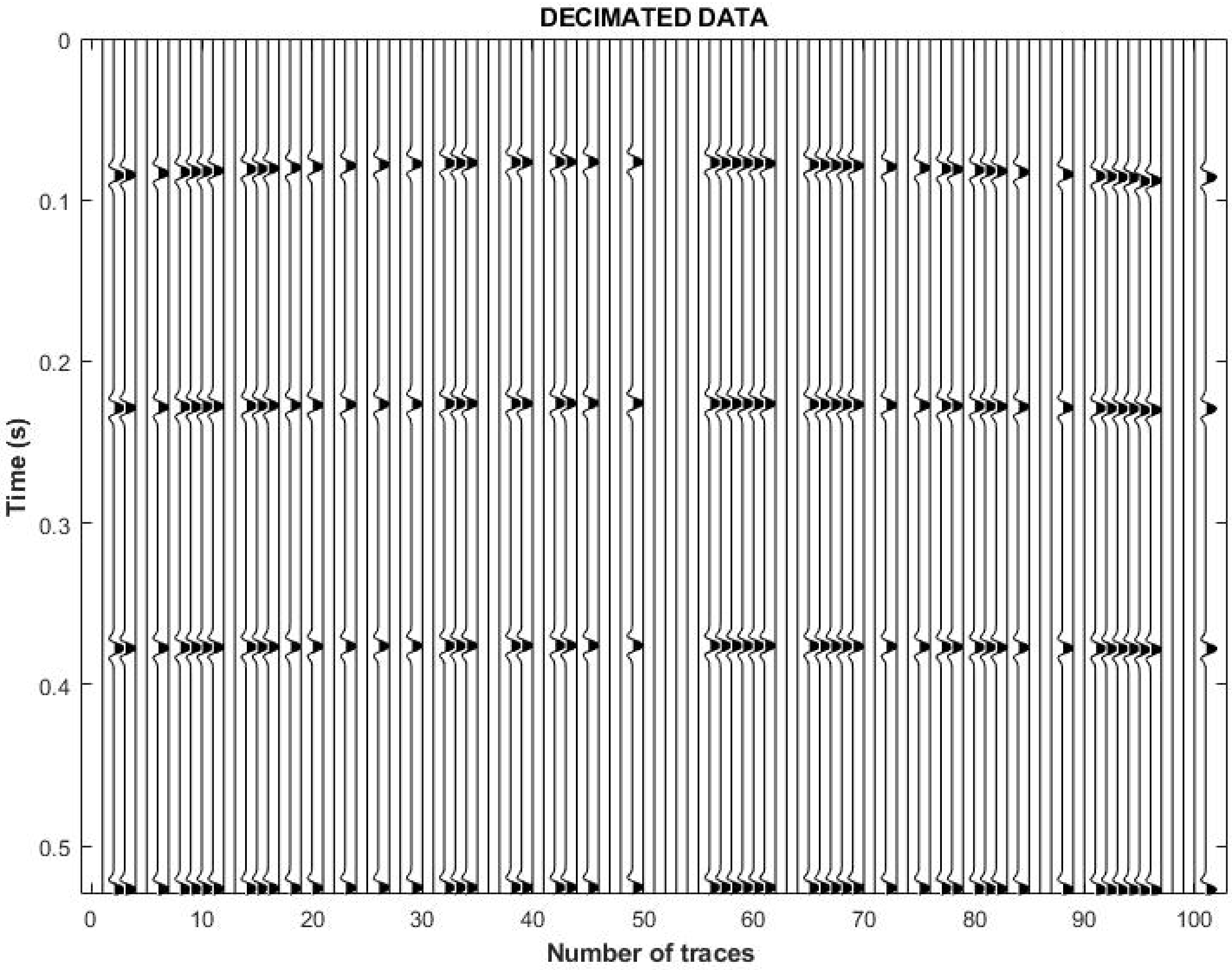
Input data with
50% decimated traces

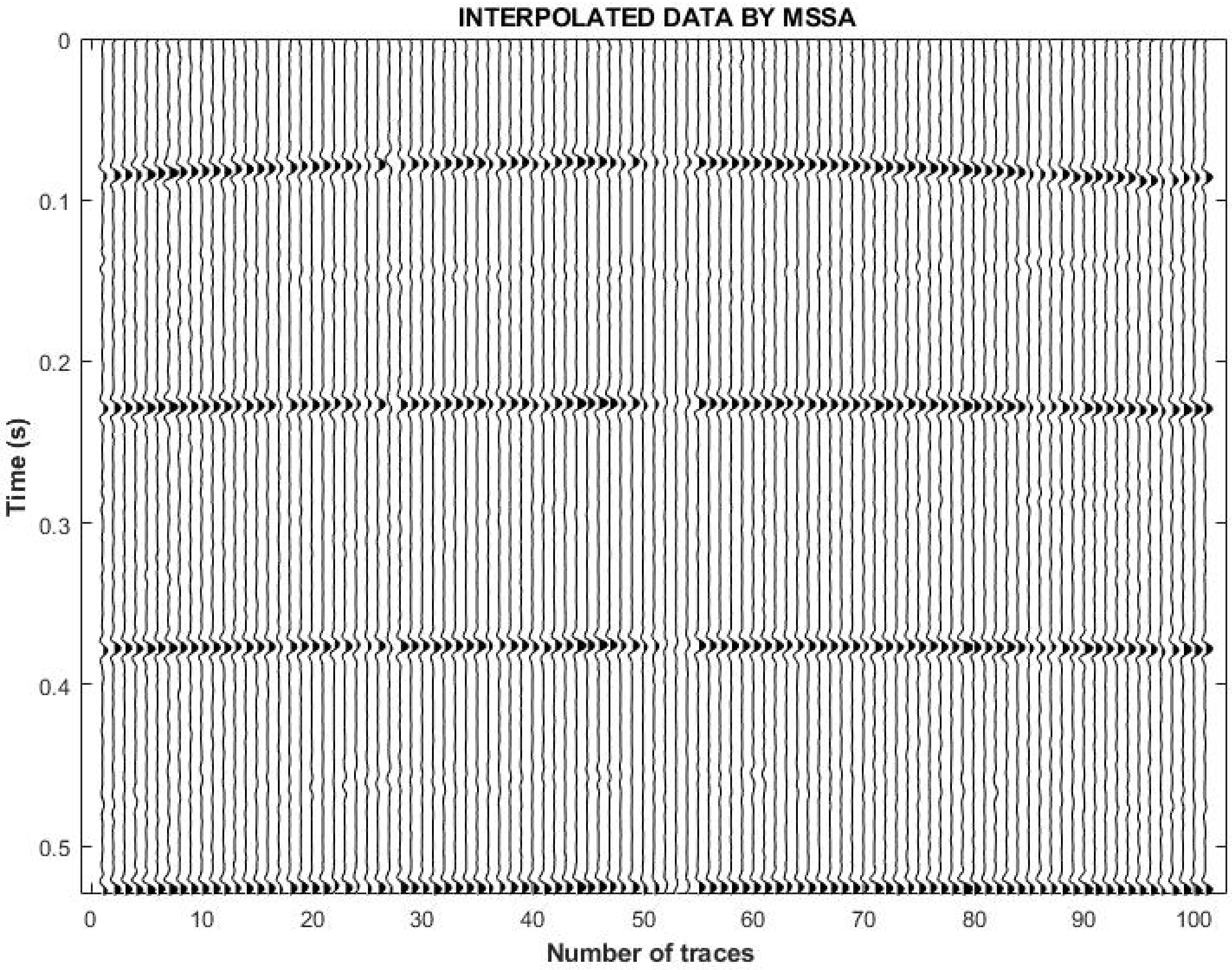


Interpolated traces
by MSSA

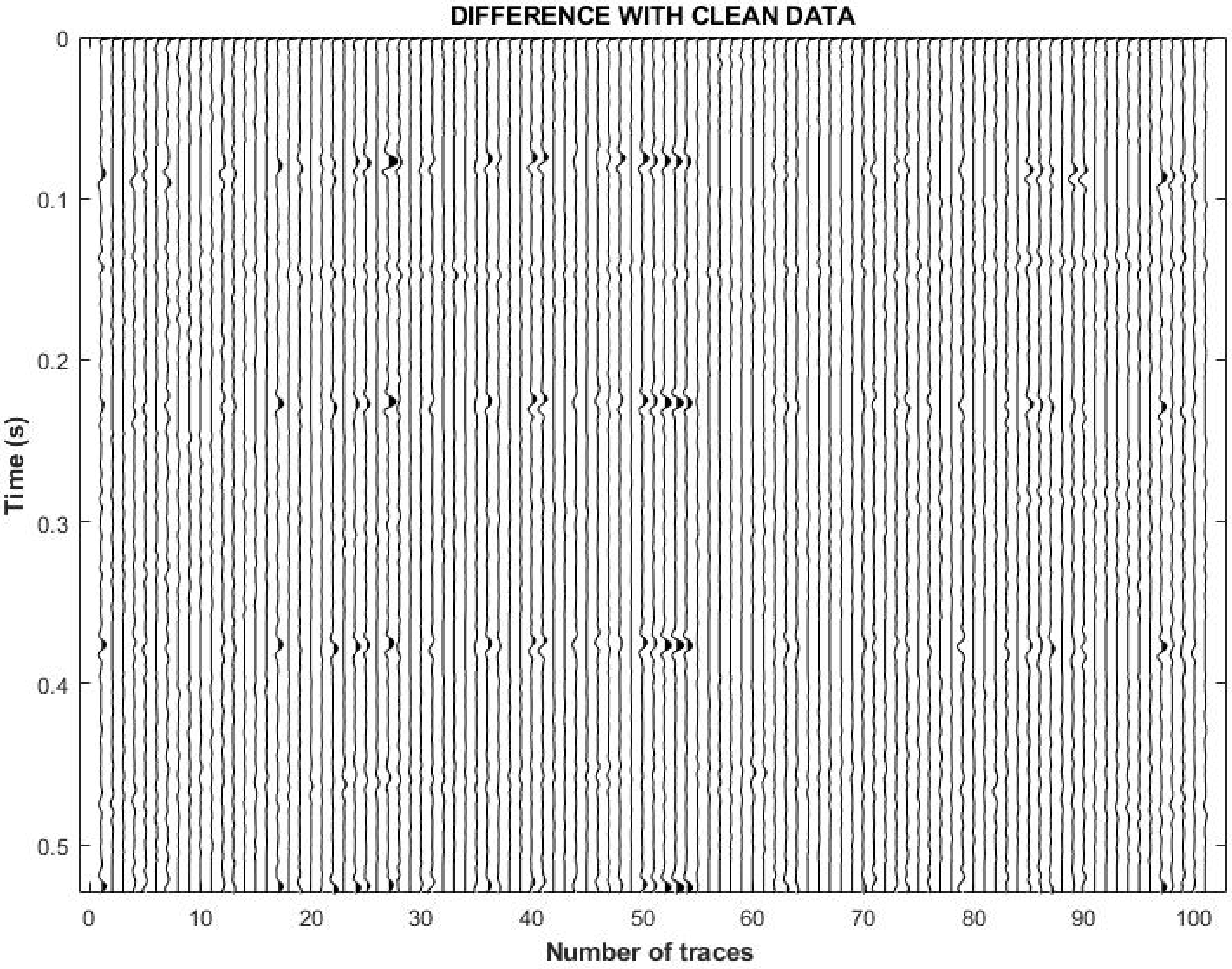


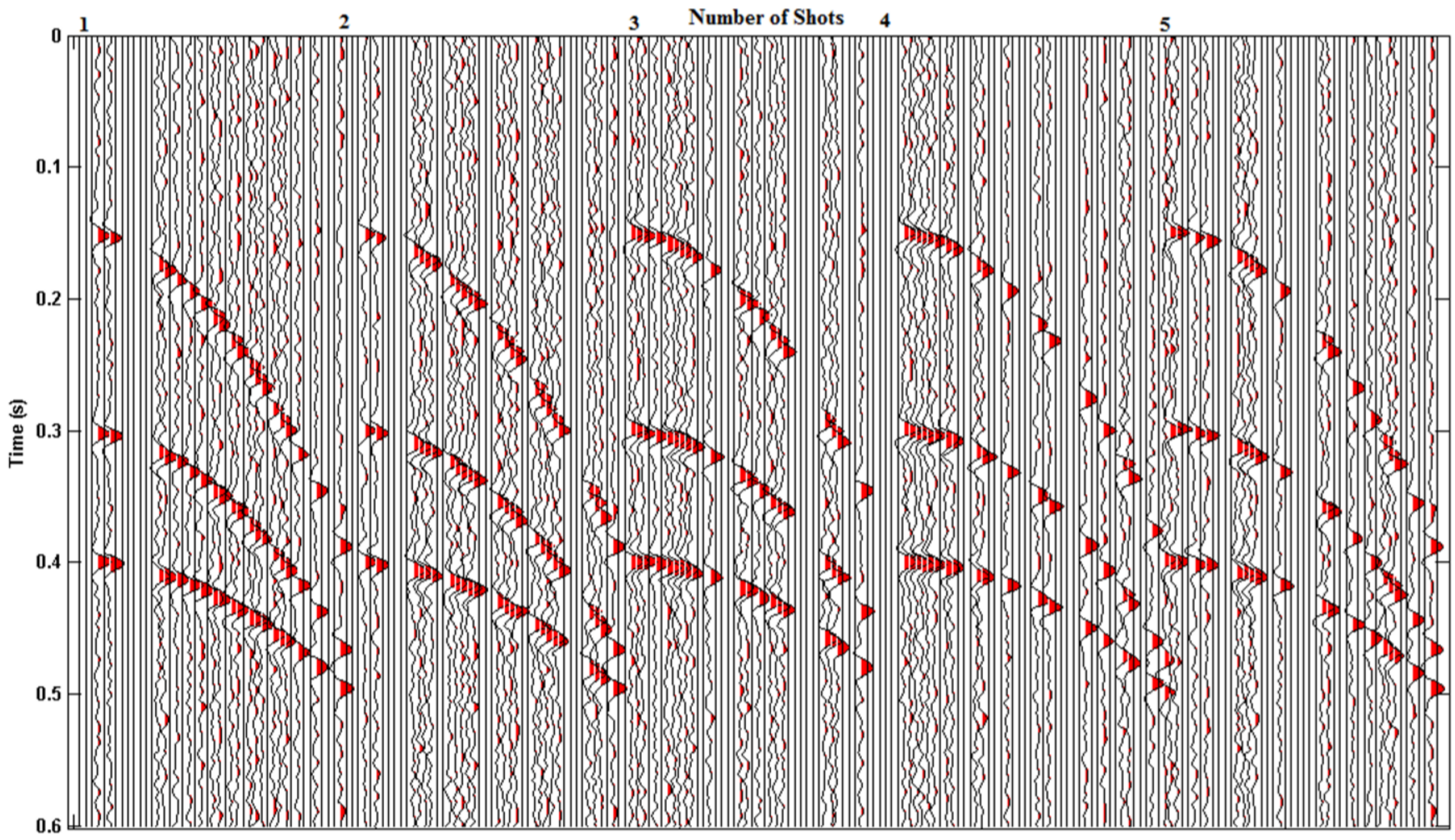
Residuals

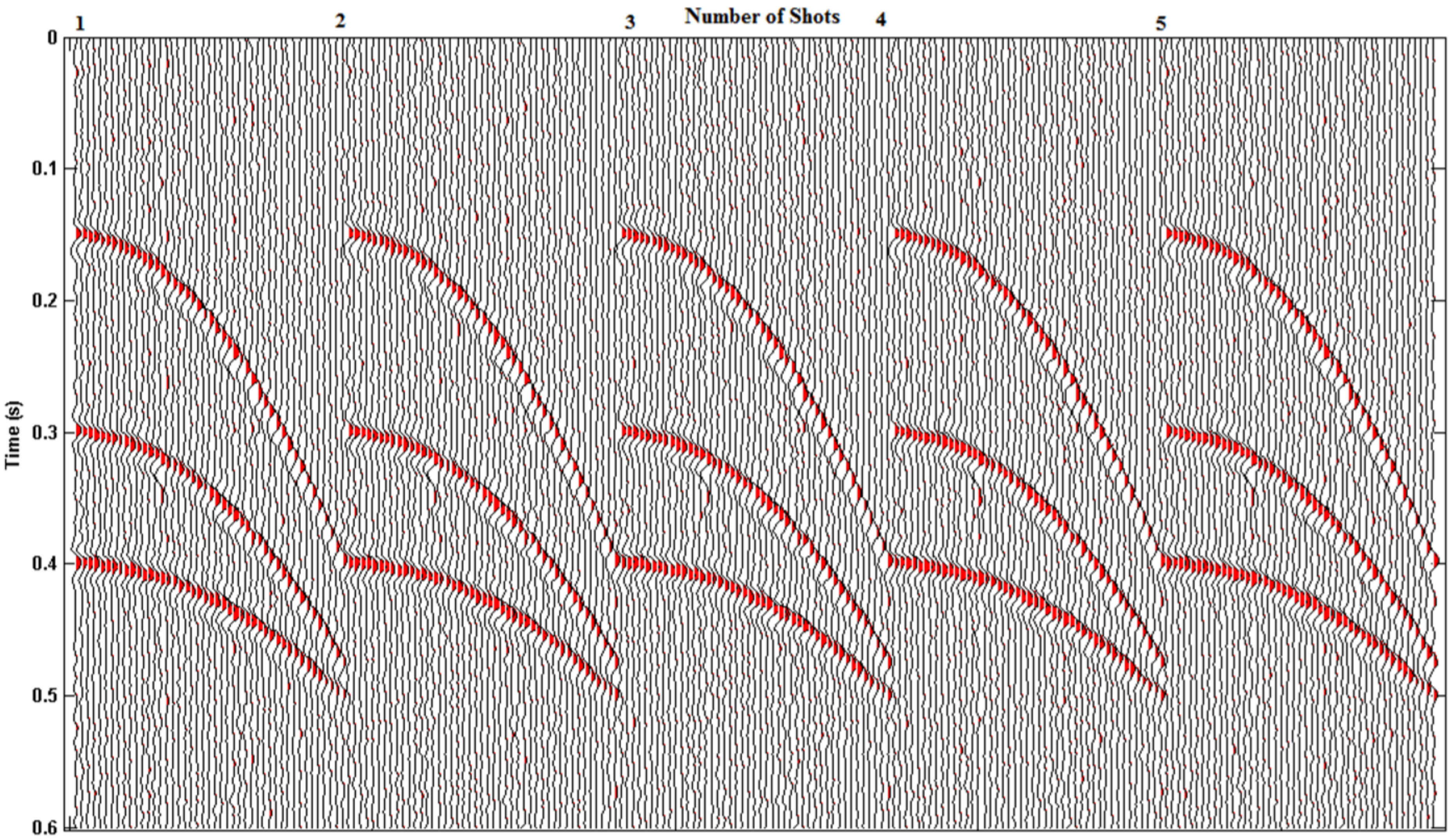

DECIMATED DATA
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
50
60
70
80
90
100
Number of traces



INTERPOLATED DATA BY MSSA
Time (s)
Number of traces
0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
50
60
70
80
90
100

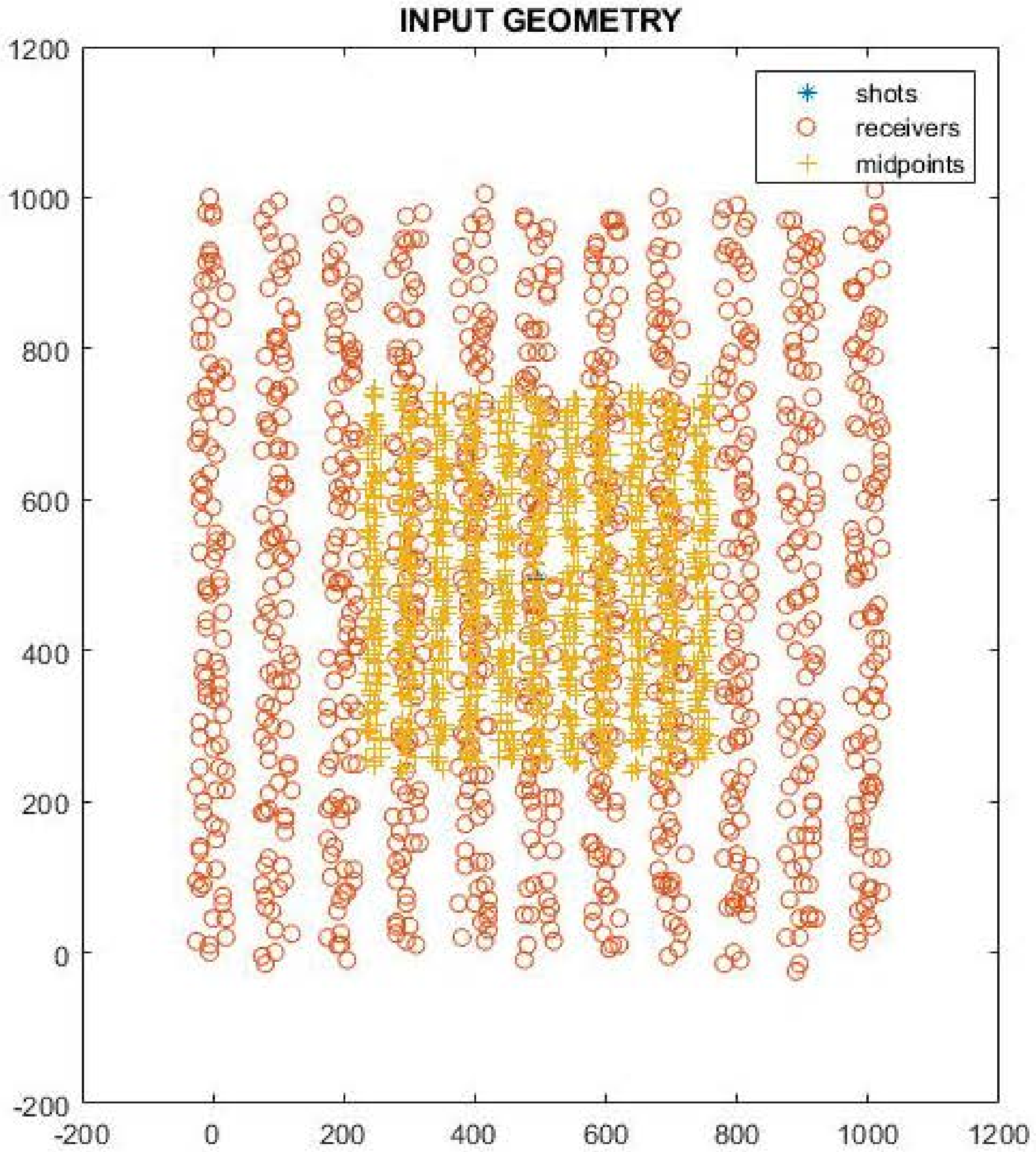

DIFFERENCE WITH CLEAN DATA
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
50
60
70
80
90
100
Number of traces

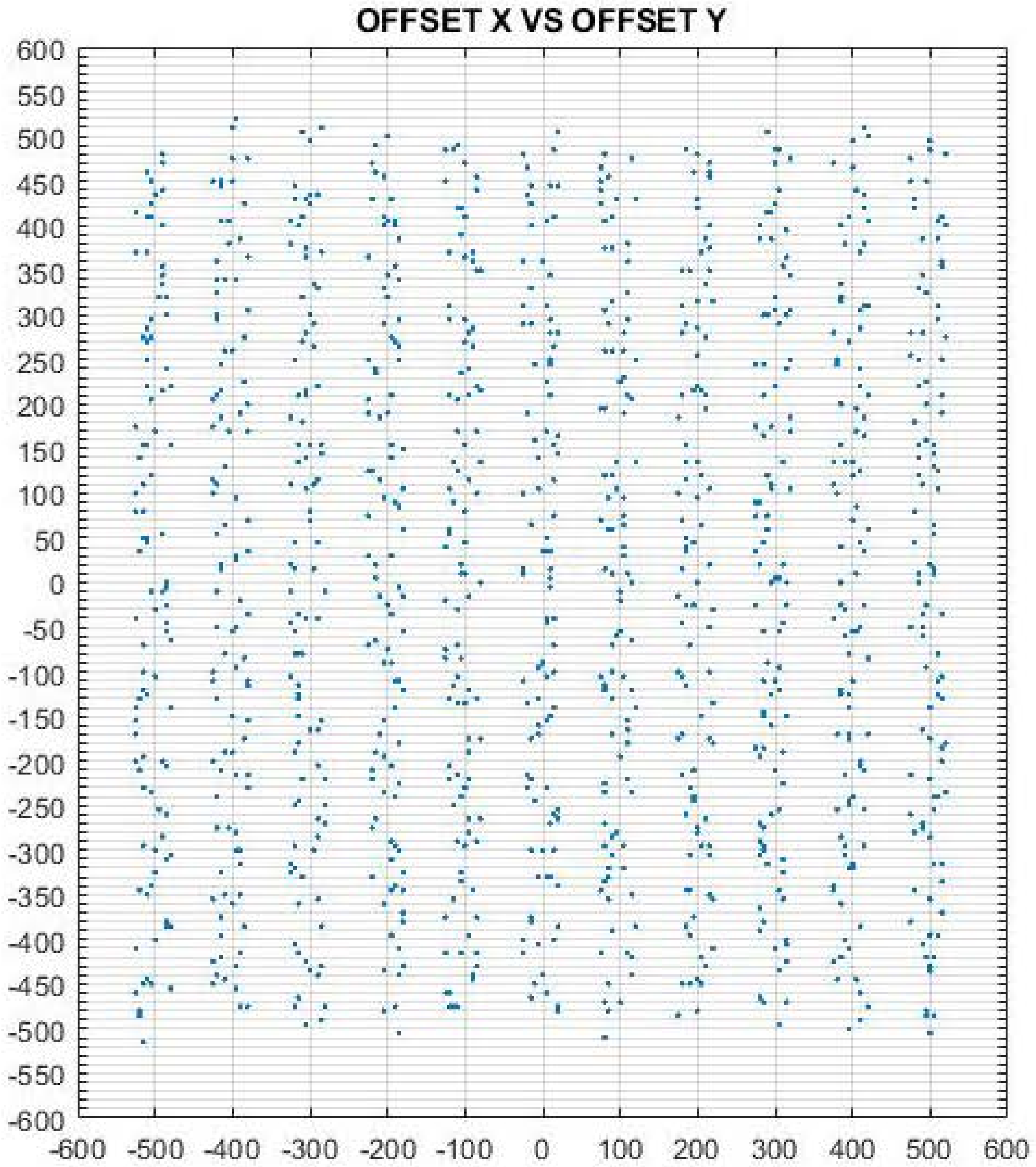

Number of Shots
1
2
3
4
5
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0.6

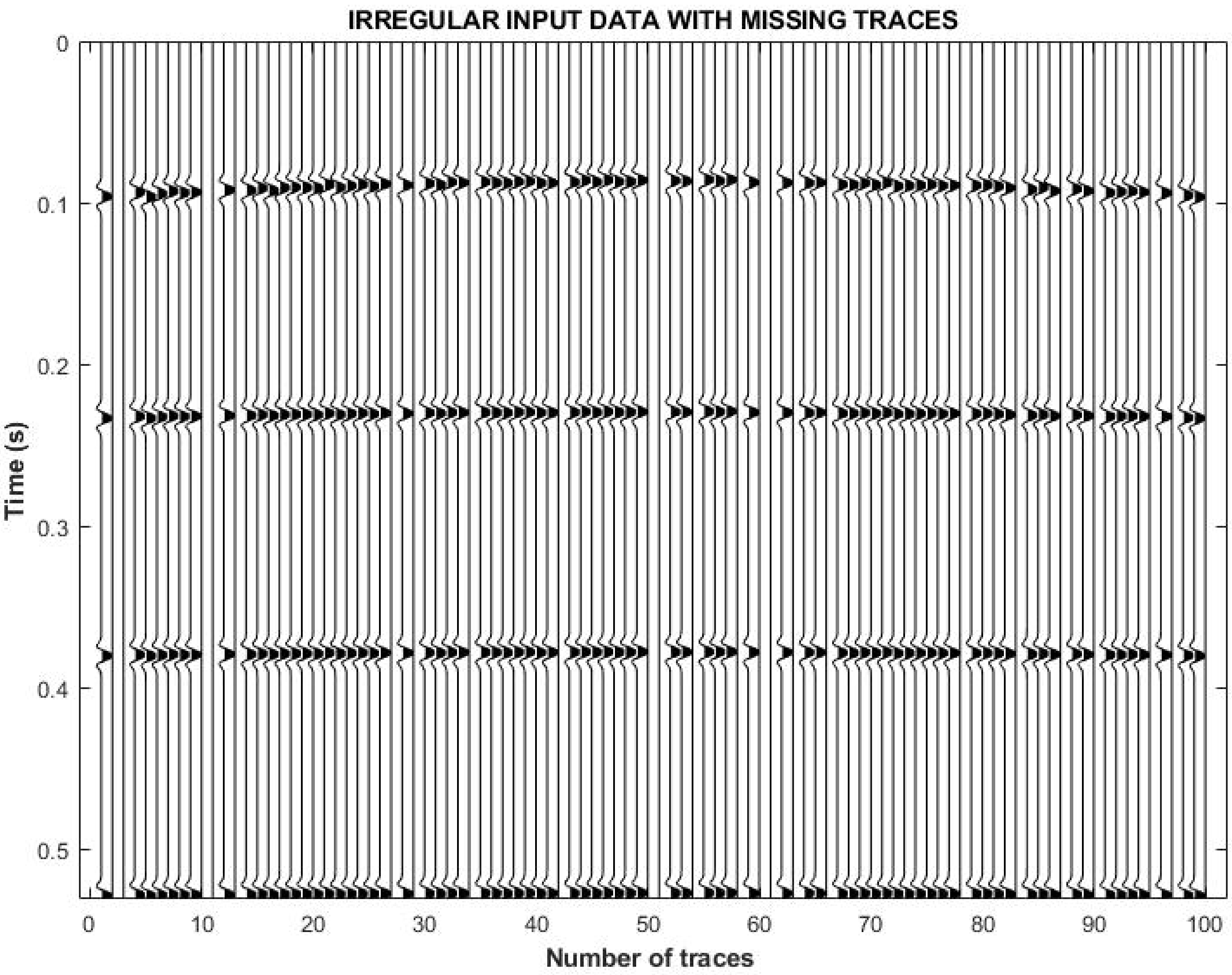

Number of Shots
1
2
3
4
5
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0.6

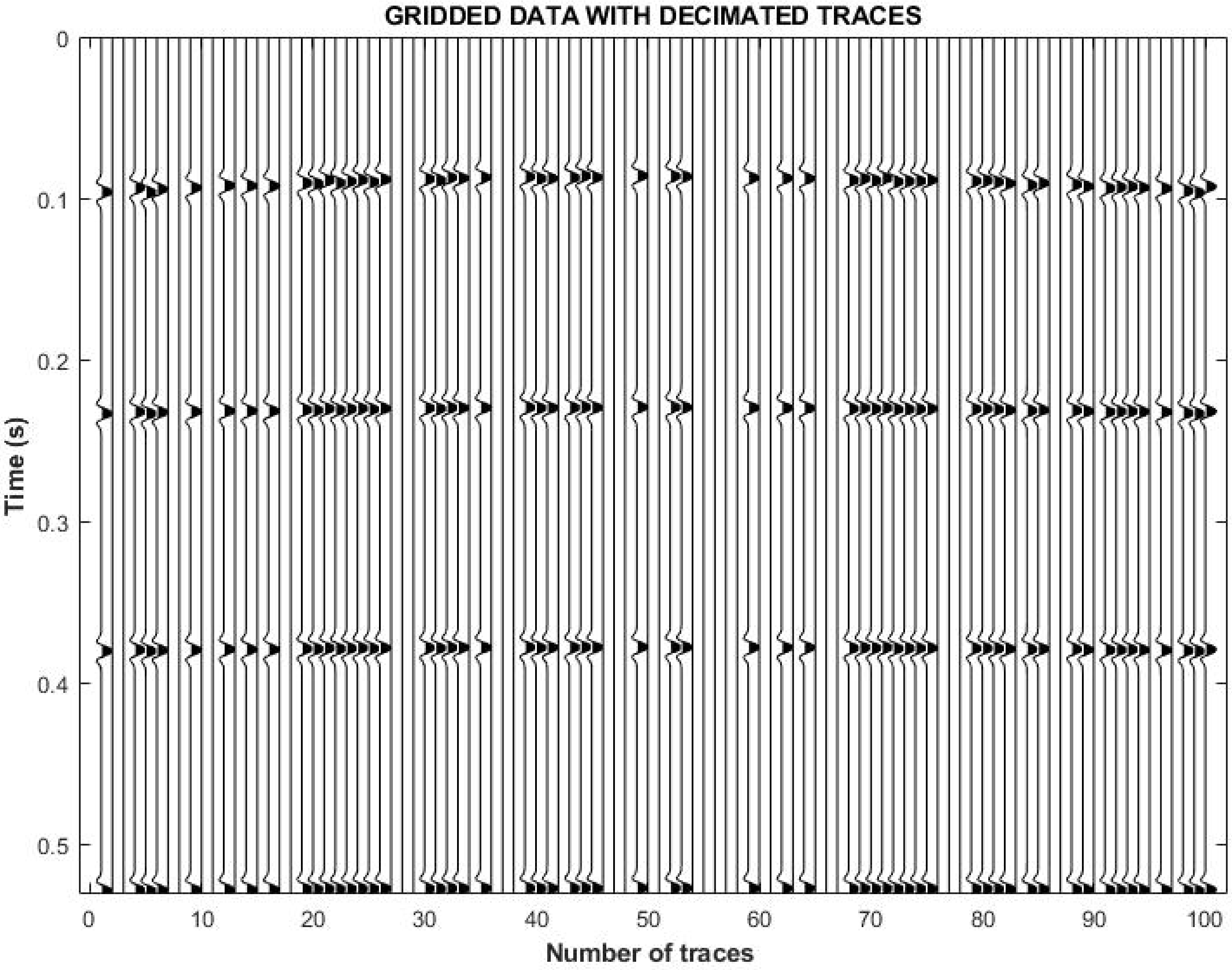
Future work

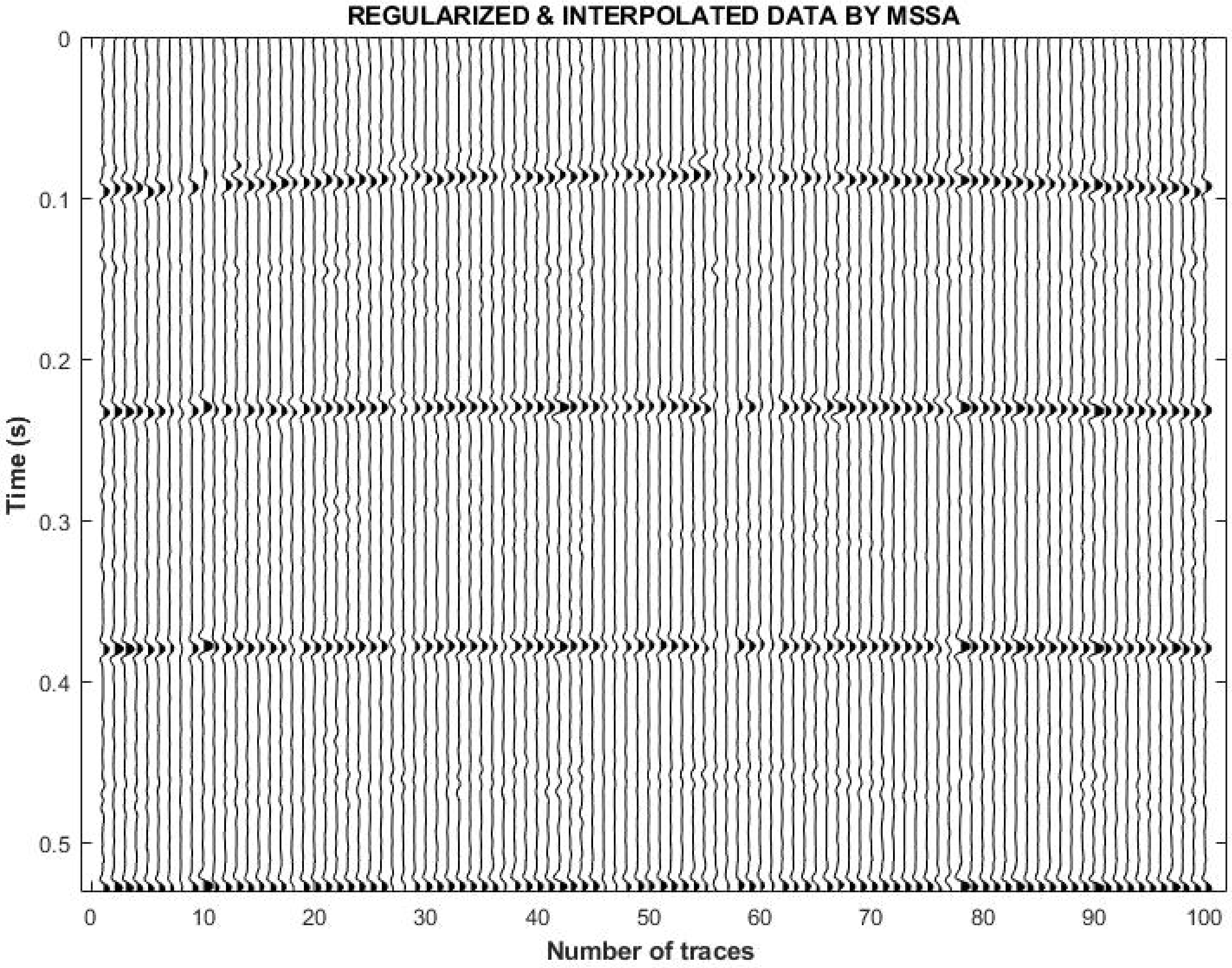
Irregular sampling



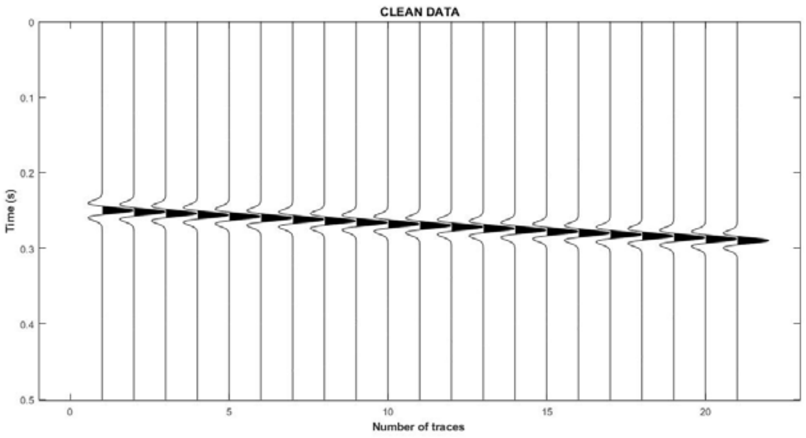


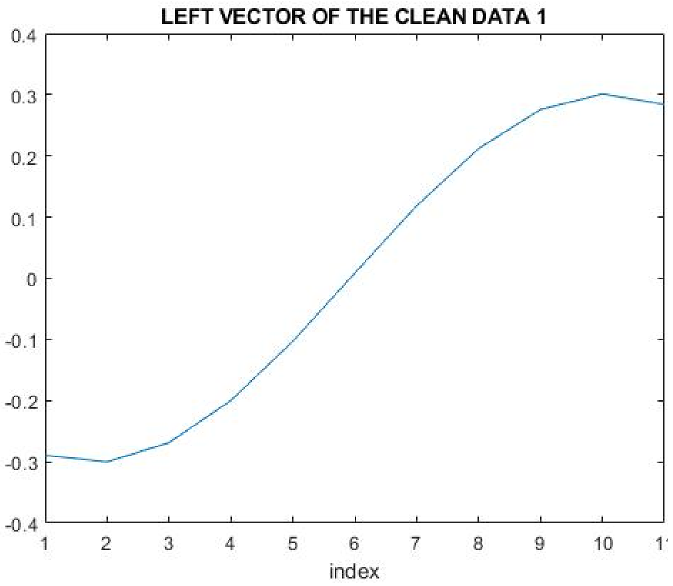

IRREGULAR INPUT DATA WITH MISSING TRACES
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
50
60
70
80
90
100
Number of traces

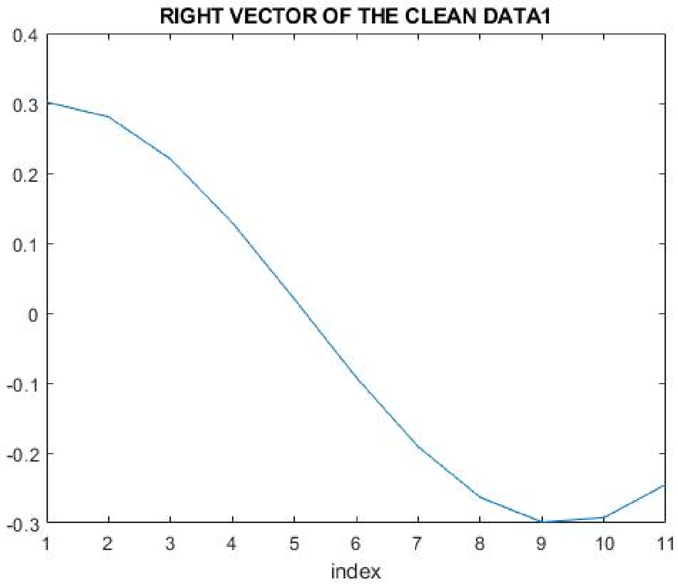

GRIDDED DATA WITH DECIMATED TRACES
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
50
60
70
80
90
100
Number of traces

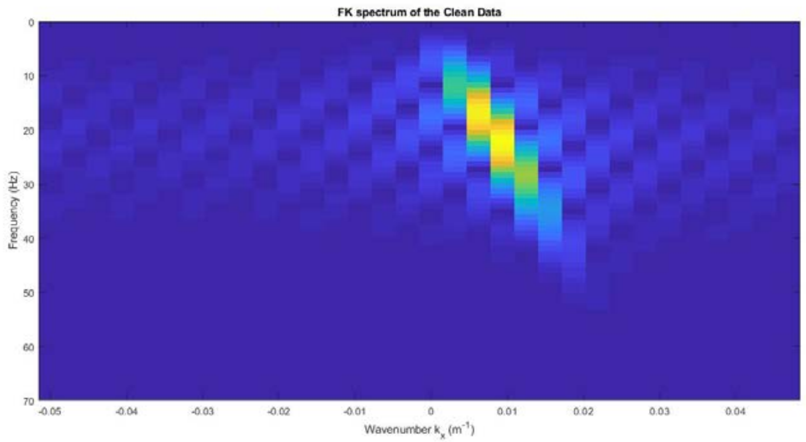

REGULARIZED & INTERPOLATED DATA BY MSSA
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
50
60
70
80
90
100
Number of traces

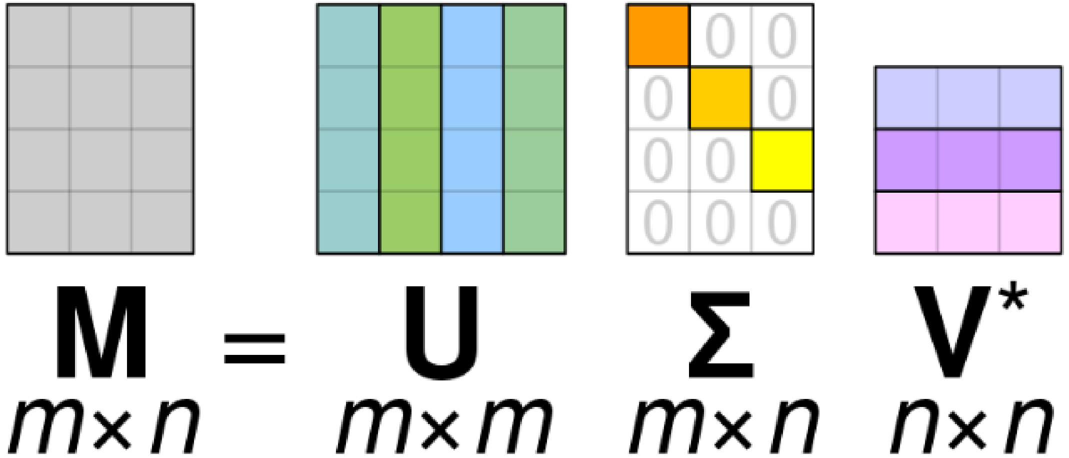
Regularly decimated traces

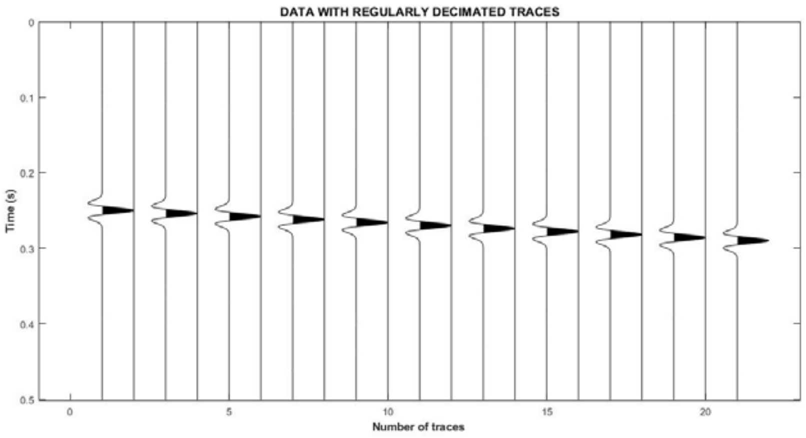


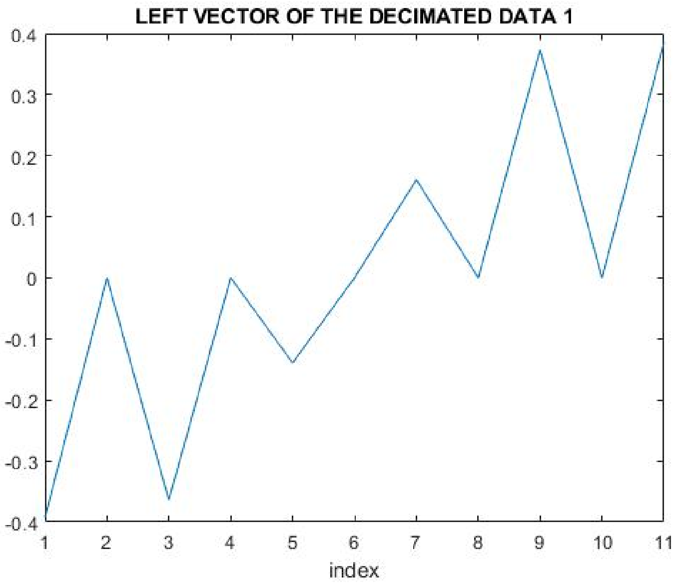


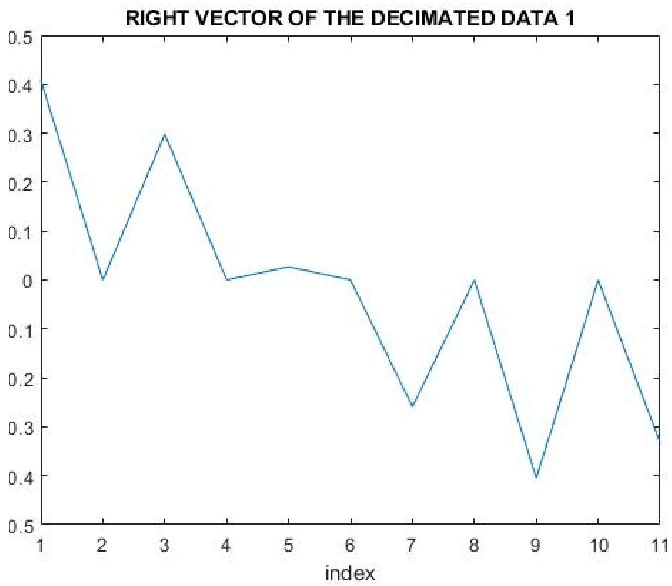


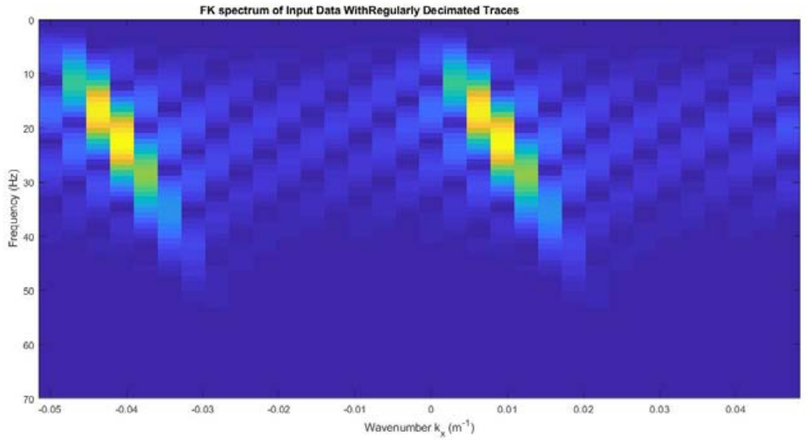


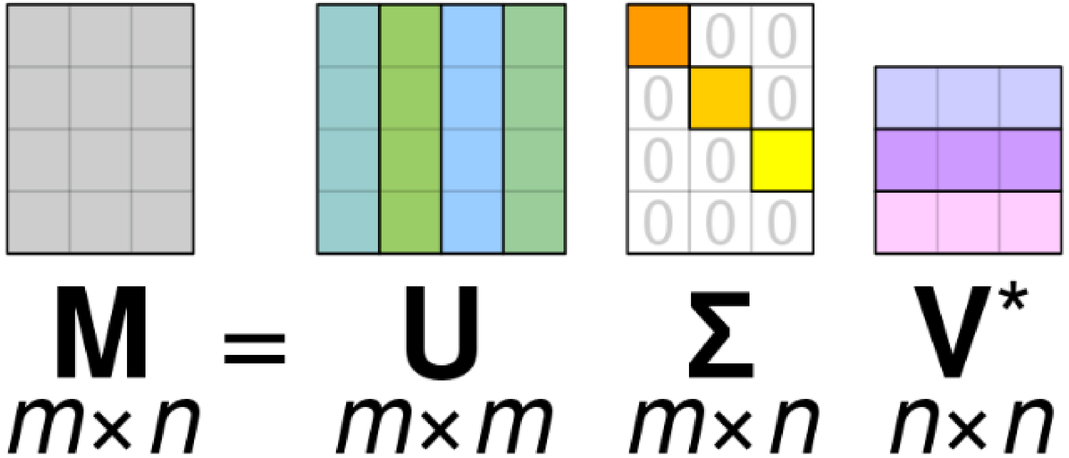


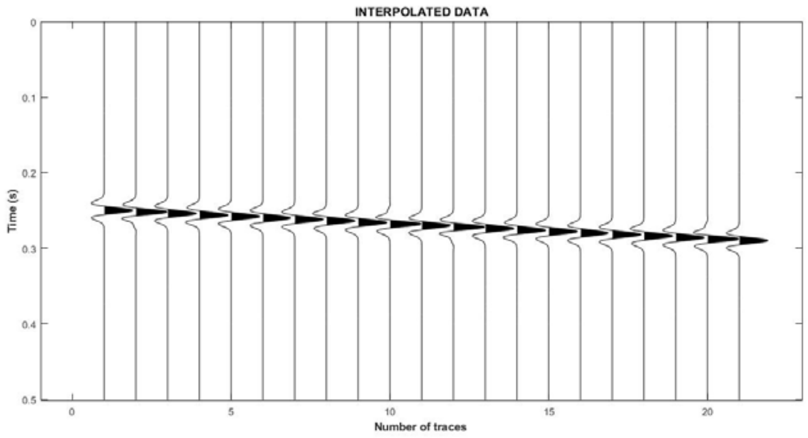

DATA WITH REGULARLY DECIMATED TRACES
Time (s)
Number of traces

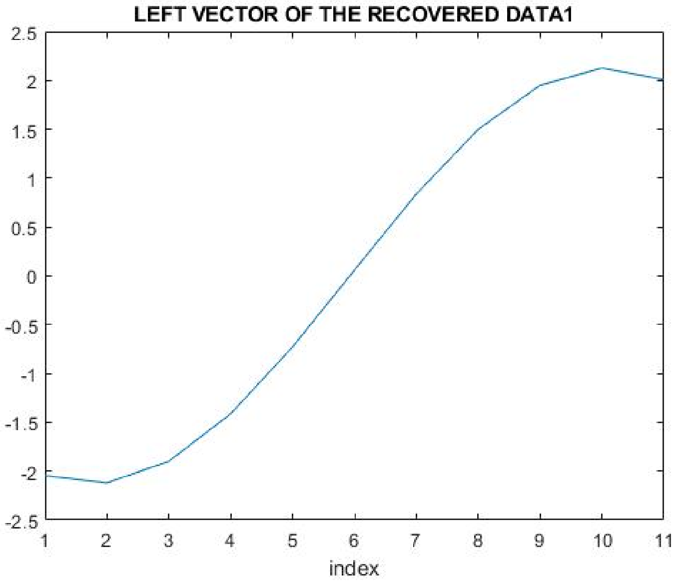

LEFT VECTOR OF THE DECIMATED DATA 1
index

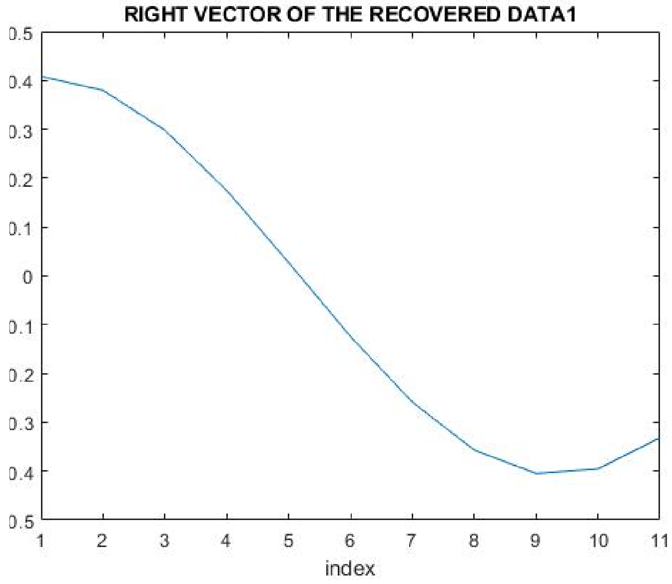

RIGHT VECTOR OF THE DECIMATED DATA 1
index

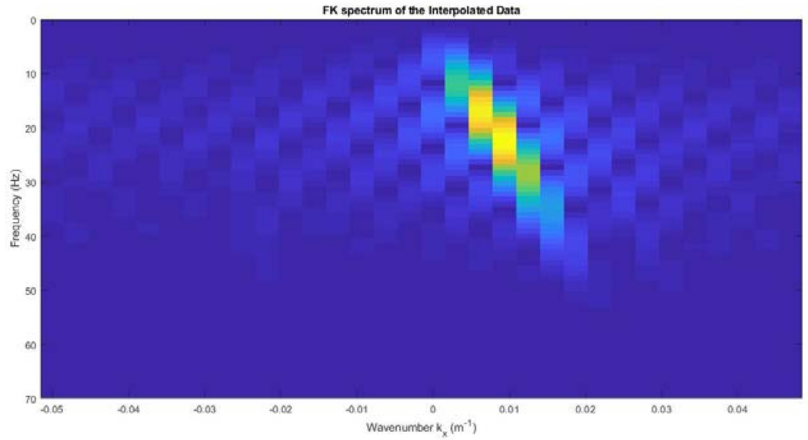

FK spectrum of Input Data WithRegularly Decimated Traces
Frequency (Hz)
Wavenumber k_x (m^-1)

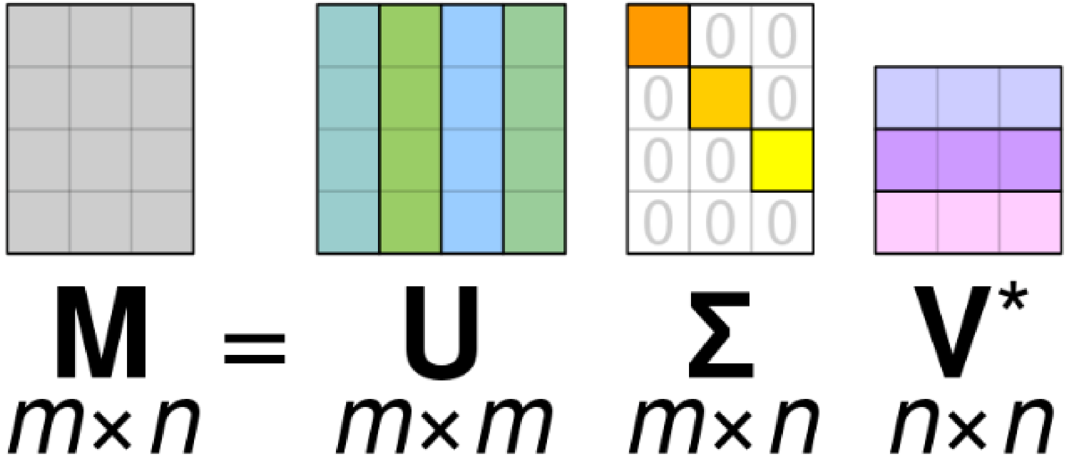

M = U Σ V*
m×n m×m m×n n×n

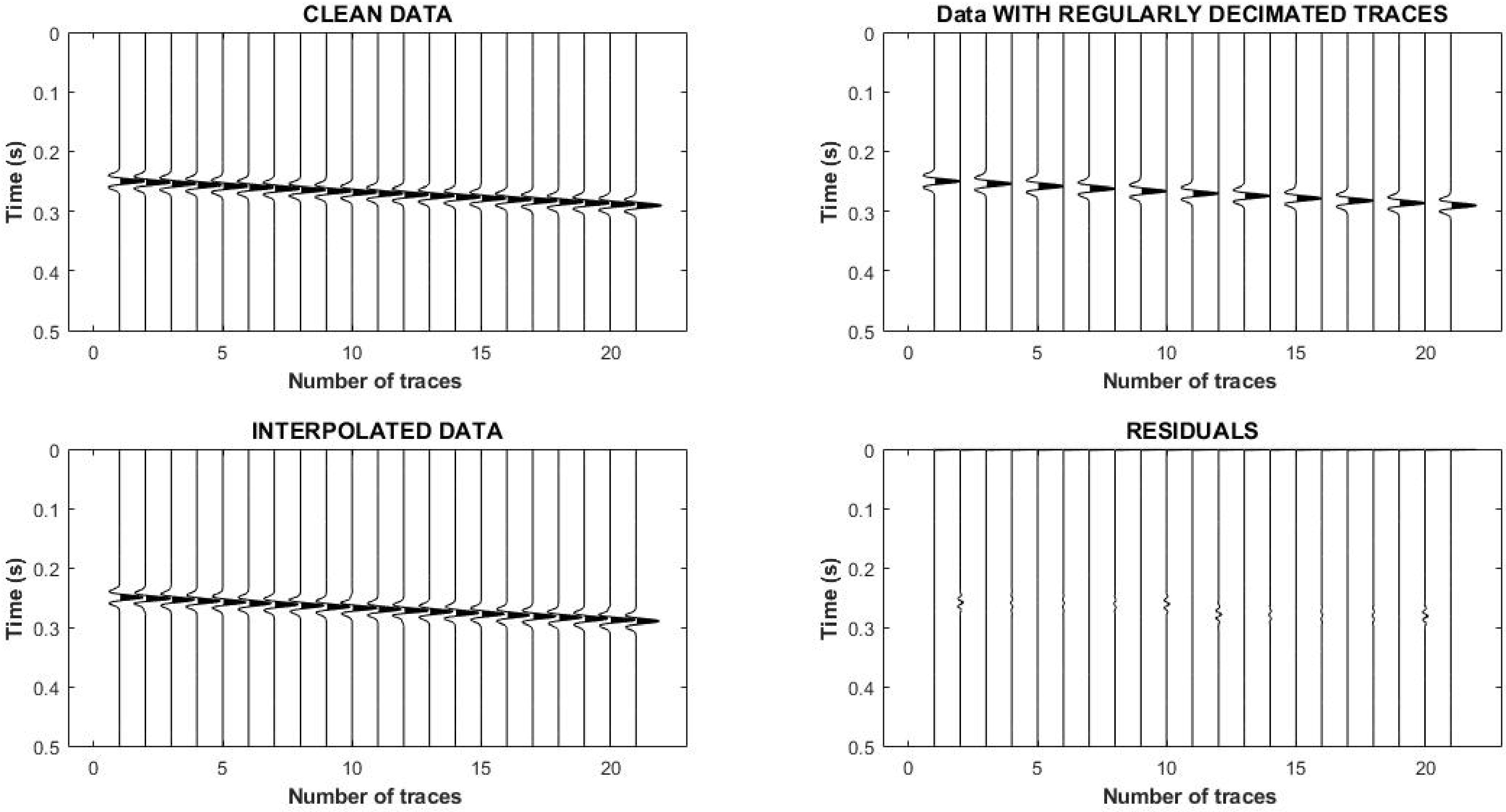

INTERPOLATED DATA
Time (s)
Number of traces


LEFT VECTOR OF THE RECOVERED DATA1
index


RIGHT VECTOR OF THE RECOVERED DATA1
index


FK spectrum of the Interpolated Data
Frequency (Hz)
Wavenumber k_x (m^{-1})


$M = U \Sigma V^*$
$m \times n$
$m \times m$
$m \times n$
$n \times n$


CLEAN DATA
Data WITH REGULARLY DECIMATED TRACES
INTERPOLATED DATA
RESIDUALS
Time (s)
Number of traces
0
0.1
0.2
0.3
0.4
0.5
5
10
15
20

Conclusions

- Low rank reduction method
- Works in the frequency-space domain
- Forms spatial data into a block of Hankel matrices
- *k* linear events rank the Hankel matrix of the data *k*
- Missing traces and additive random noise increase the rank of the Hankel matrix
- Interpolates plane waves, however for the curvature

  it needs to set small windows to assume the data linear or

  NMO correction before the interpolation to minimize the effect of curvature
- For irregular data it can work with coarse binning leads to jittering
- Doesn’t work for regularly decimated data it needs dealiasing eigen-decomposition of the Hankel matrix

Thank you!