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Numerical simulation of seismic wave propagation in attenuative transversely isotropic media

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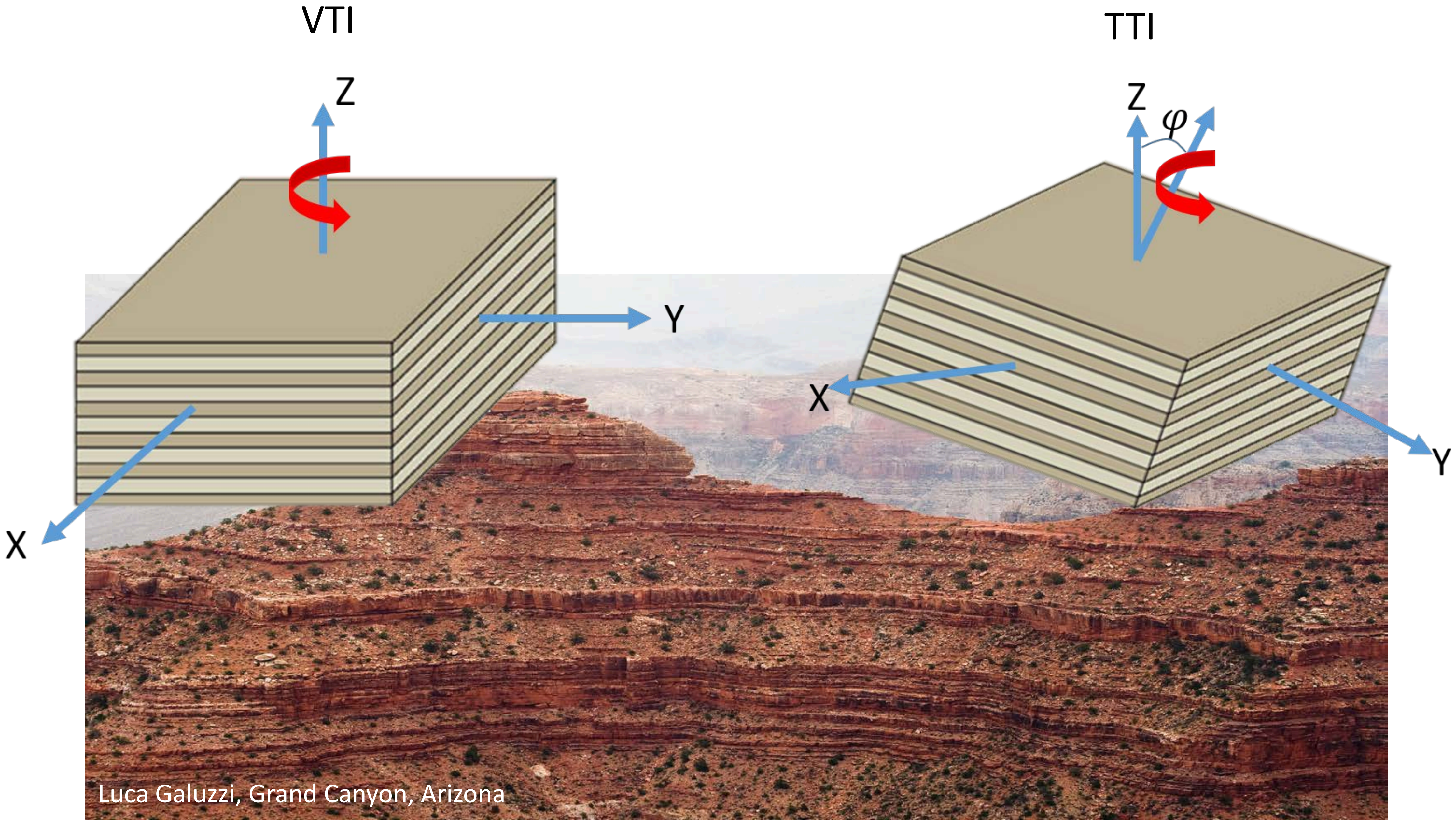
Motivation



Theory

General stress-strain relationship in attenuating media

$$\sigma_{ij}(\mathrm{t}) = C_{ijkl}(t) * \dot{\varepsilon}_{kl}(t)$$

Isotropic attenuation

Anisotropic attenuation

$$C_{ijkl}(t) = M_{ijkl}\left(1 - \frac{1}{L}\sum_{l=1}^{L}\left(1 - \frac{\tau^{\varepsilon l}}{\tau^{\sigma l}}\right)e^{-\frac{t}{\tau^{\sigma l}}}\right)H(t)$$

Anelasticity tensor for VTI media using TI approximation (Vs=0)

$$C(t) = \begin{pmatrix} C_{11} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{11} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_{11} = \rho V_P^2(1 + 2\varepsilon)$$

$$M_{13} = \rho V_P^2\sqrt{1 + 2\delta}$$

$$M_{33} = \rho V_P^2$$

(Bland, 1960, Alkhalifah, 2000, and Duveneck et al., 2008)

Theory

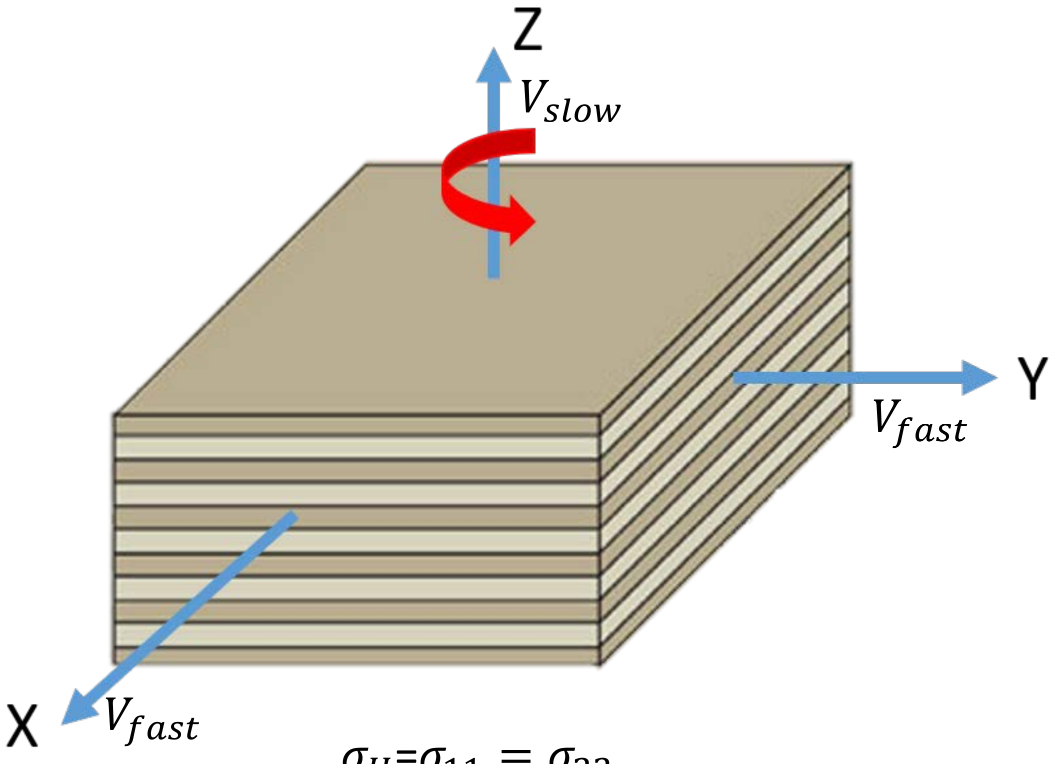
Isotropic attenuation

$$C(t) = \begin{pmatrix} M_{11} & M_{11} & M_{13} \\ M_{11} & M_{11} & M_{13} \\ M_{13} & M_{13} & M_{33} \end{pmatrix} \overbrace{\left(1 - \frac{1}{L}\sum_{l=1}^{L}\left(1 - \frac{\tau^{\varepsilon l}}{\tau^{\sigma l}}\right) e^{-\frac{t}{\tau^{\sigma l}}}\right) H(t)}^{F(t)}$$

Viscoacoustic wave equation in VTI media

$$\sigma_H = M_{11} F * \left(\partial_x u_x + \partial_y u_y\right) + M_{13} F * \partial_z u_z$$

$$\sigma_V = M_{13} F * \left(\partial_x u_x + \partial_y u_y\right) + M_{33} F * \partial_z u_z$$



$\sigma_H = \sigma_{11} = \sigma_{22}$

$\sigma_V = \sigma_{33}$

For all directions $\tau = \frac{\tau^{\varepsilon l}}{\tau^{\sigma l}} - 1$

Anisotropic attenuation

$$C(t) = \begin{pmatrix} C_{11} & C_{11} & C_{13} \\ C_{11} & C_{11} & C_{13} \\ C_{13} & C_{13} & C_{33} \end{pmatrix} \rightarrow \begin{cases} \varepsilon & \text{Associated to horizontal propagation} \\ \delta & \text{Associated to moveout} \\ V_P & \text{Relaxed vertical velocity} \end{cases}$$

Strain relaxation times

$$\begin{cases} \tau_{11}^{\varepsilon l} = \tau_h^{\varepsilon l} \rightarrow \text{Associated to } \varepsilon \\ \tau_{13}^{\varepsilon l} = \tau_n^{\varepsilon l} \rightarrow \text{Associated to } \delta \\ \tau_{33}^{\varepsilon l} = \tau_v^{\varepsilon l} \rightarrow \text{Associated to } V_P \end{cases}$$

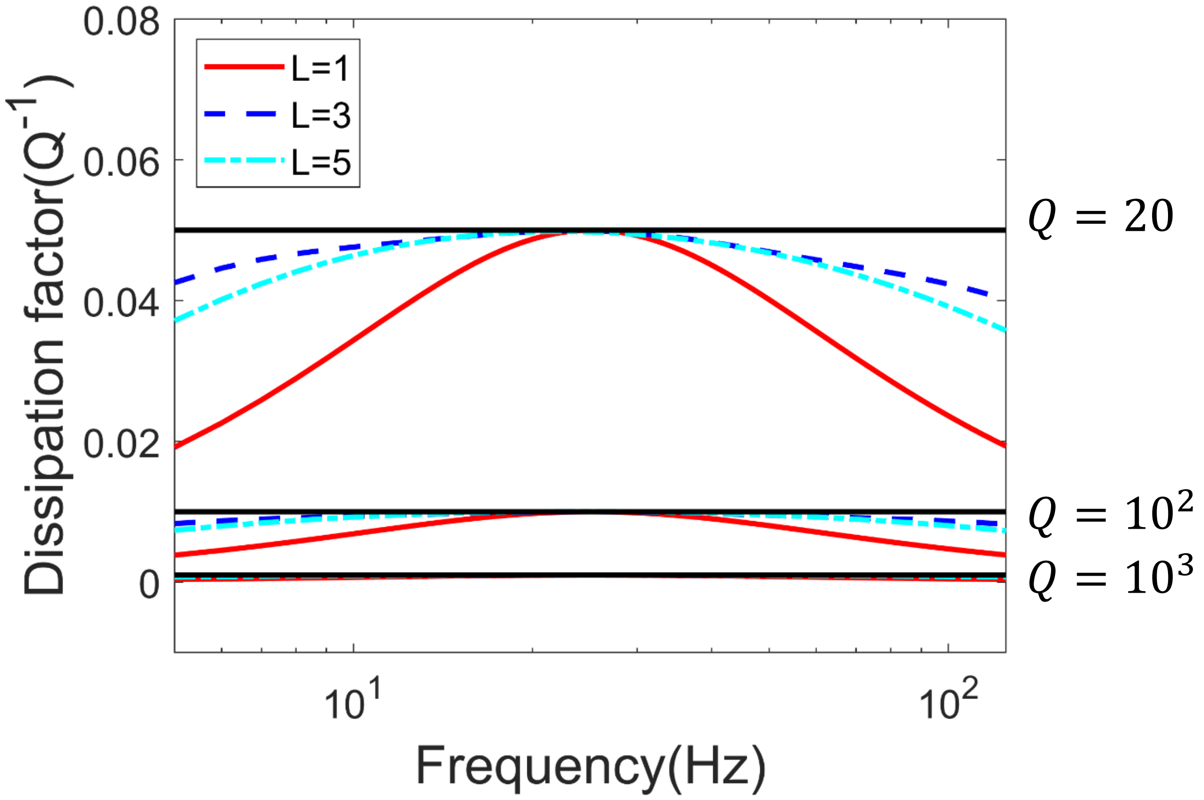
$$\tau_m = \frac{\tau_m^{\varepsilon l}}{\tau^{\sigma l}} - 1\,, \qquad m = h, n, v.$$

Application of constant-Q method

$$\tau_m = \frac{\tau_m^{\varepsilon l}}{\tau^{\sigma l}}, \qquad m = v, h, n.$$

Each one of the direction-depending strain relaxation times is then associated to a direction-depending quality factor as

$$Q^{-1} \approx \frac{1}{L}\sum_{l=1}^{L}\frac{\omega\left(\tau_m^{\varepsilon l} - \tau^{\sigma l}\right)}{1 + \omega^2 \tau_m^{\varepsilon l}\tau^{\sigma l}}$$



VTI wave equation in anisotropy attenuation media

Anisotropic attenuation

$$C(t)=\begin{pmatrix} M_{11}F_h(t) & M_{11}F_h(t) & M_{13}F_n(t)\\ M_{11}F_h(t) & M_{11}F_h(t) & M_{13}F_n(t)\\ M_{13}F_n(t) & M_{13}F_n(t) & M_{33}F_v(t)\end{pmatrix}, \qquad F_m(t)=\left(1-\frac{1}{L}\sum_{l=1}^{L}\left(1-\frac{\tau_m^{\varepsilon l}}{\tau^{\sigma l}}\right)e^{-\frac{t}{\tau^{\sigma l}}}\right)H(t);\quad m=h,n,v$$

Viscoacoustic wave equation in VTI media:

$$\sigma_H=M_{11}F_h*\left(\partial_x u_x+\partial_y u_y\right)+M_{13}F_n*\partial_z u_z$$

$$\sigma_V=M_{13}F_n*\left(\partial_x u_x+\partial_y u_y\right)+M_{33}F_v*\partial_z u_z$$

$$\partial_t\sigma_H=M_{11}\left[1-\frac{1}{L}\sum_{l=1}^{L}\left(1-\frac{\tau_h^{\varepsilon l}}{\tau^{\sigma l}}\right)\right]\left(\partial_x u_x+\partial_y u_y\right)+M_{13}\left[1-\frac{1}{L}\sum_{l=1}^{L}\left(1-\frac{\tau_n^{\varepsilon l}}{\tau^{\sigma l}}\right)\right]\left(\partial_z u_z\right)+\frac{1}{L}\sum_{l=1}^{L}\left(M_{11}r_H+M_{13}r_N\right)$$

$$\partial_t\sigma_V=M_{13}\left[1-\frac{1}{L}\sum_{l=1}^{L}\left(1-\frac{\tau_n^{\varepsilon l}}{\tau^{\sigma l}}\right)\right]\left(\partial_x u_x+\partial_y u_y\right)+M_{33}\left[1-\frac{1}{L}\sum_{l=1}^{L}\left(1-\frac{\tau_v^{\varepsilon l}}{\tau^{\sigma l}}\right)\right]\left(\partial_z u_z\right)+\frac{1}{L}\sum_{l=1}^{L}\left(M_{13}r_N+M_{13}r_V\right)$$

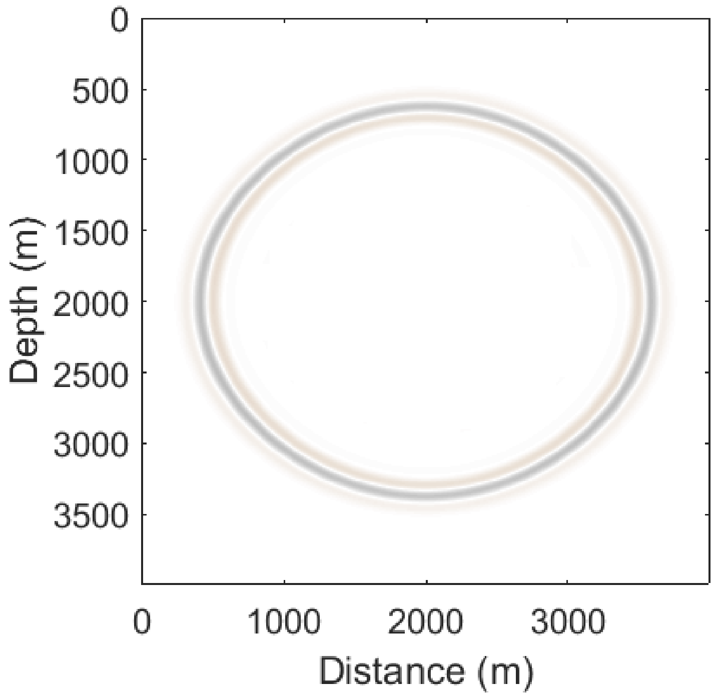
Dependency of attenuation with angle and stability

VTI media

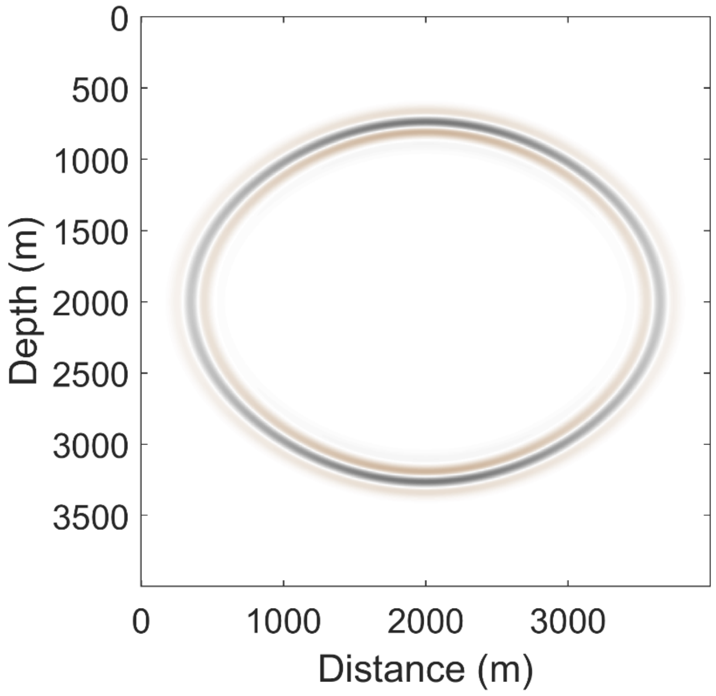
$$\theta = 0^{\circ}, \varphi = 0^{\circ}$$

$$\varepsilon = 0.2, \delta = 0.1, V_P = 2\ km/s$$

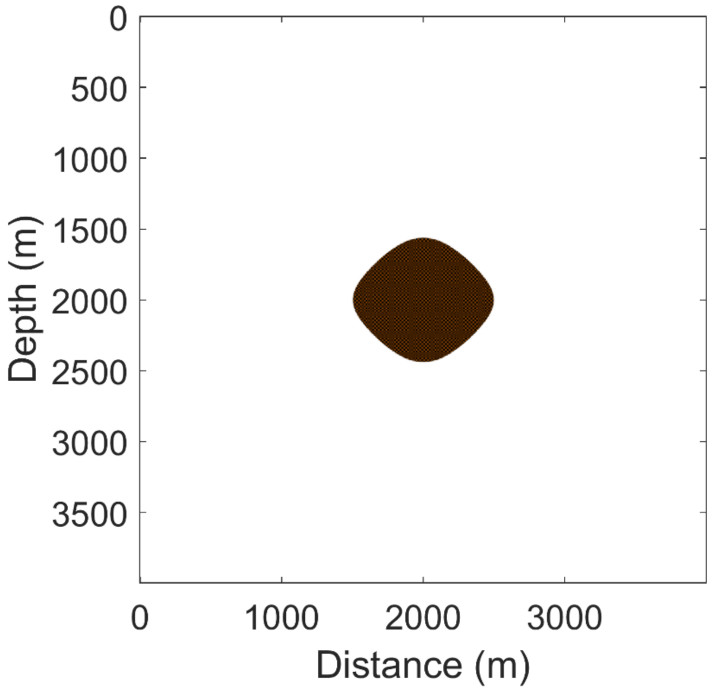
Isotropic Q, $\Delta t = 1\ ms$



Anisotropic Q, $\Delta t = 1\ ms$



$\Delta t = 1.01\ ms$

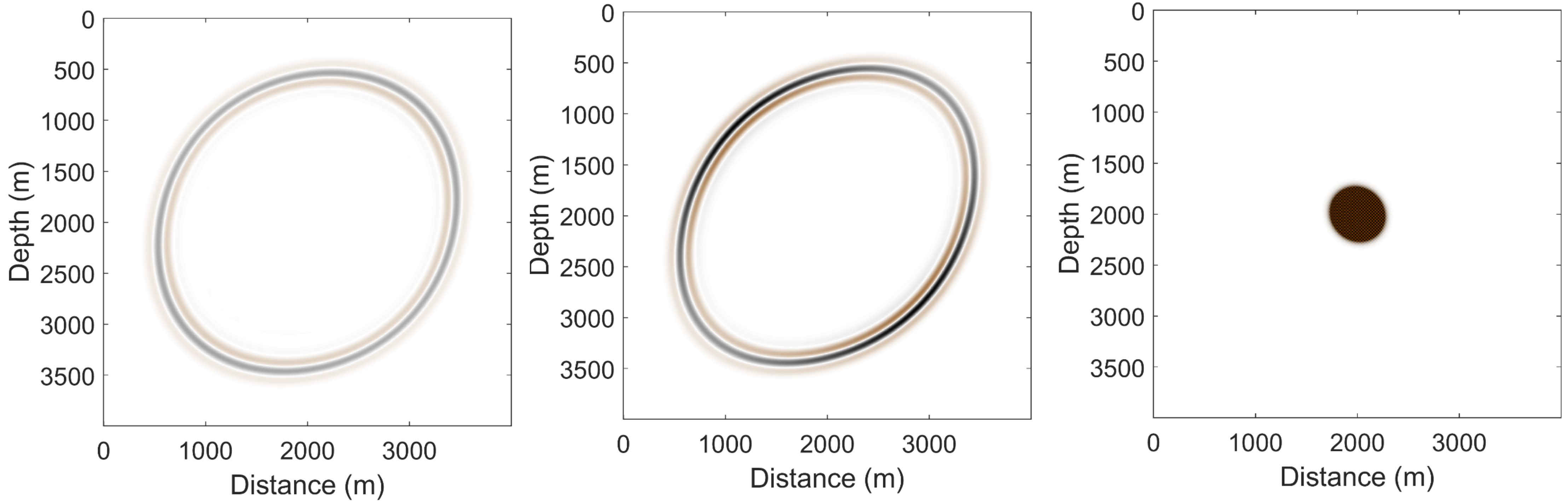


Dependency of attenuation with angle and stability

TTI media

$$\theta = 45^{\circ}, \varphi = 0^{\circ}$$

Isotropic Q, $\Delta t = 1.09\, ms$ | Anisotropic Q, $\Delta t = 1.09\, ms$ | $\Delta t = 1.1\, ms$



Dependency of attenuation with angle and stability

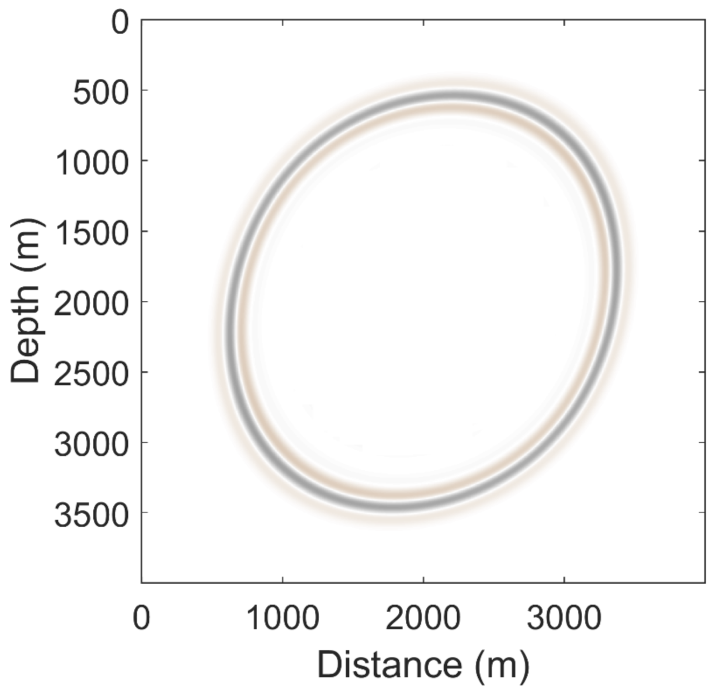
TTI media

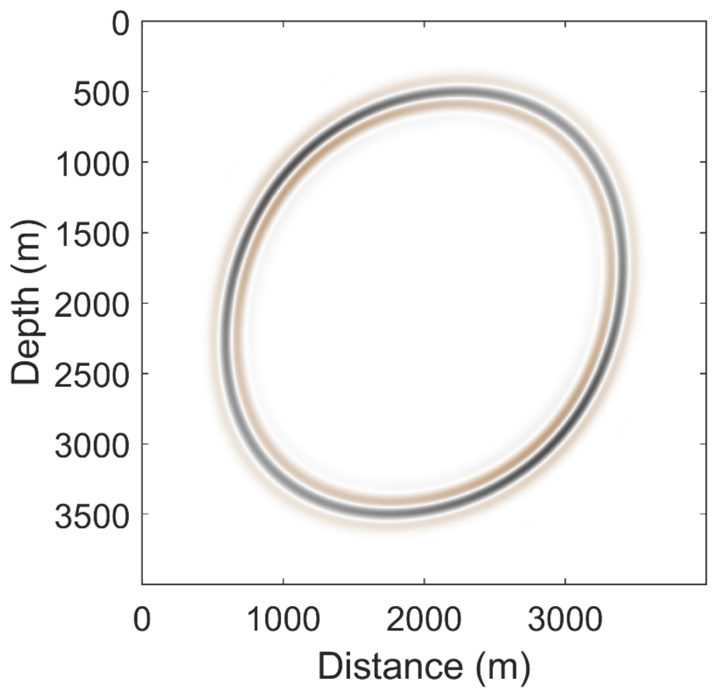
$$\theta = 45^{\circ}, \varphi = 20^{\circ}$$

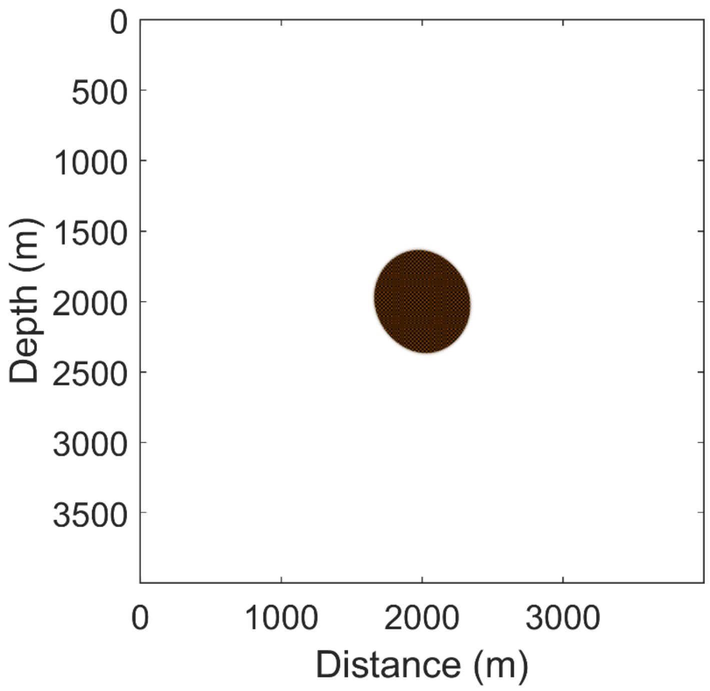
Isotropic $Q, \Delta t = 1.13\ ms$

Anisotropic $Q, \Delta t = 1.13\ ms$

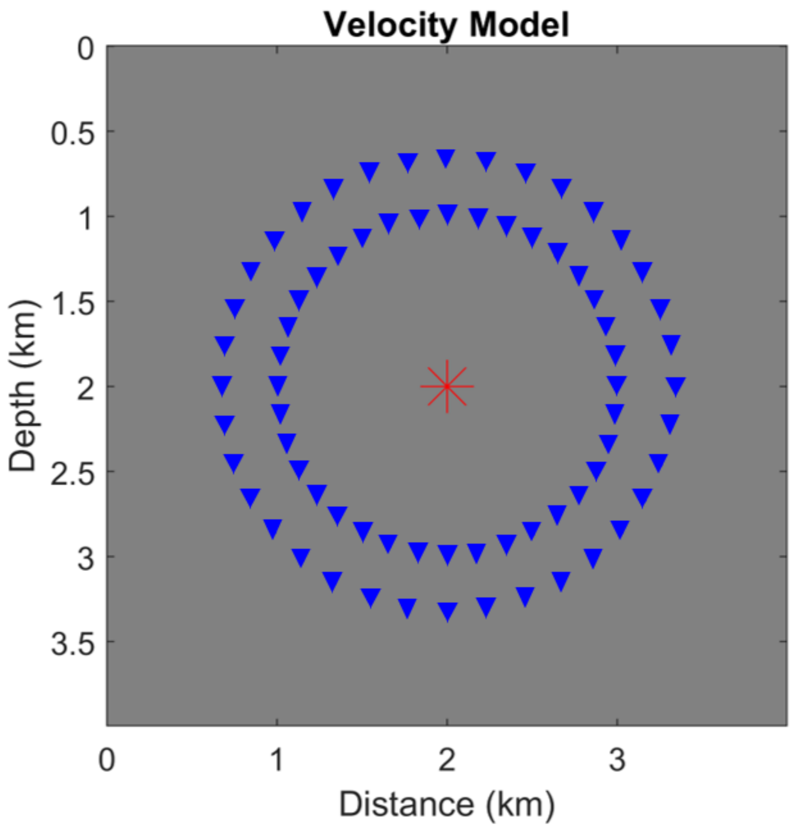
$\Delta t = 1.14\ ms$



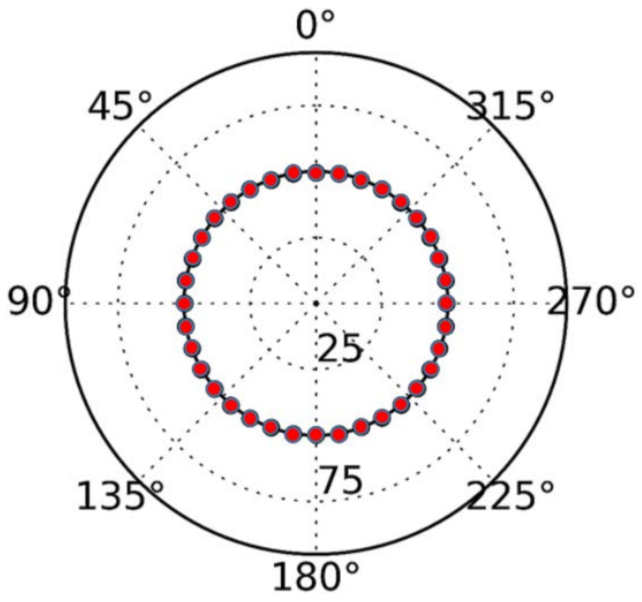




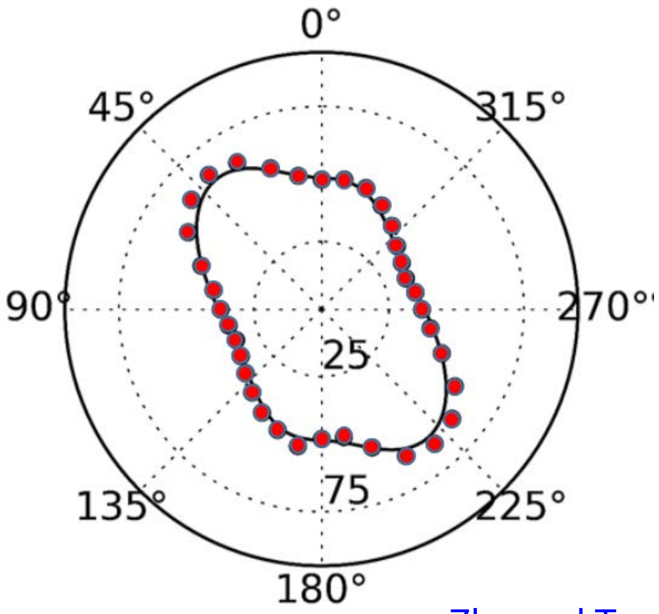
Dependency of the estimated Q with azimuth and tilt angles



Isotropic attenuation (VTI)
$Q = 50$

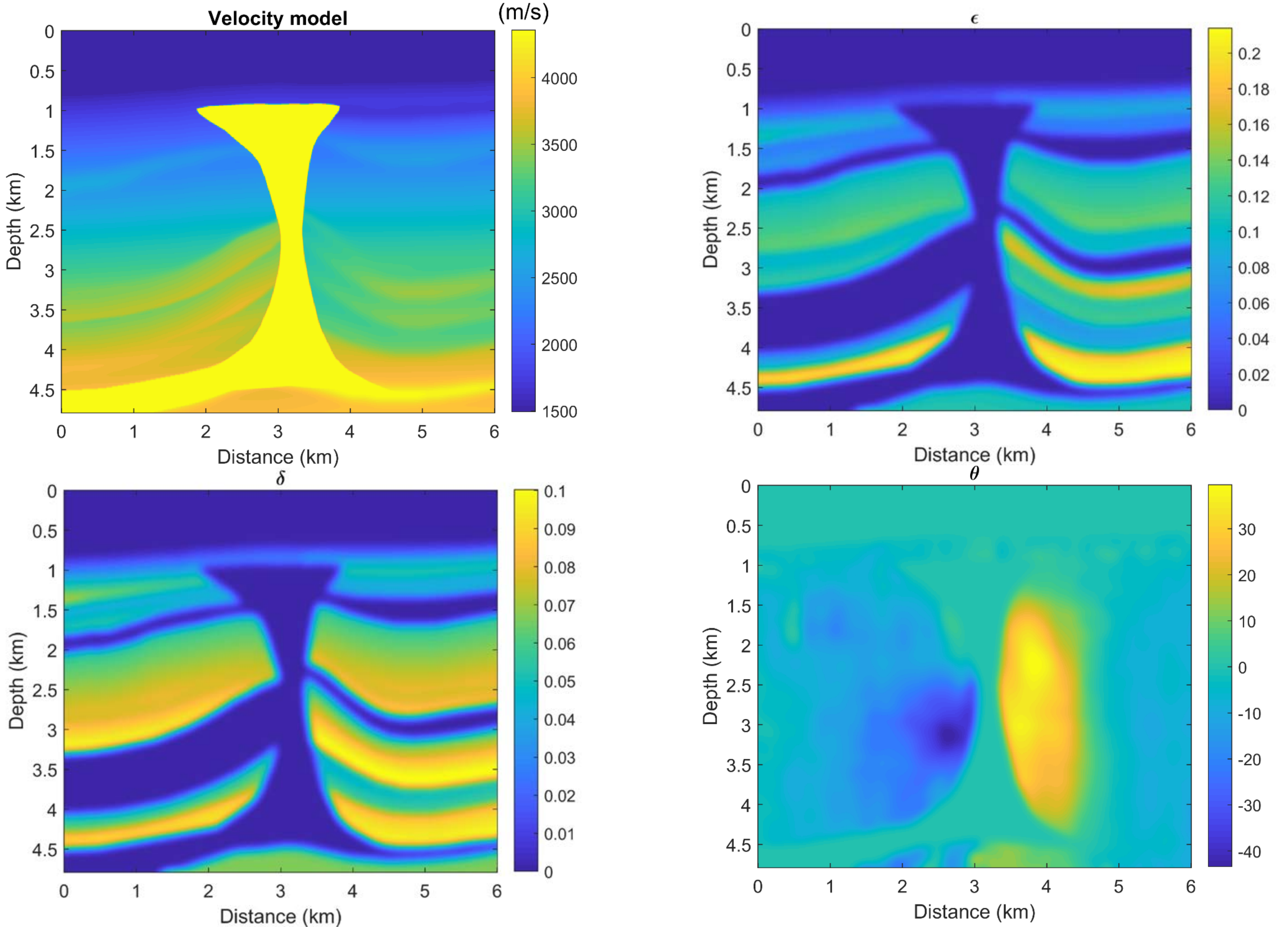


Anisotropic attenuation (TTI)
$Q_h = 80, Q_v = 50, Q_n = 40$
$\theta = 35^{\circ}, \varphi = 50^{\circ}$



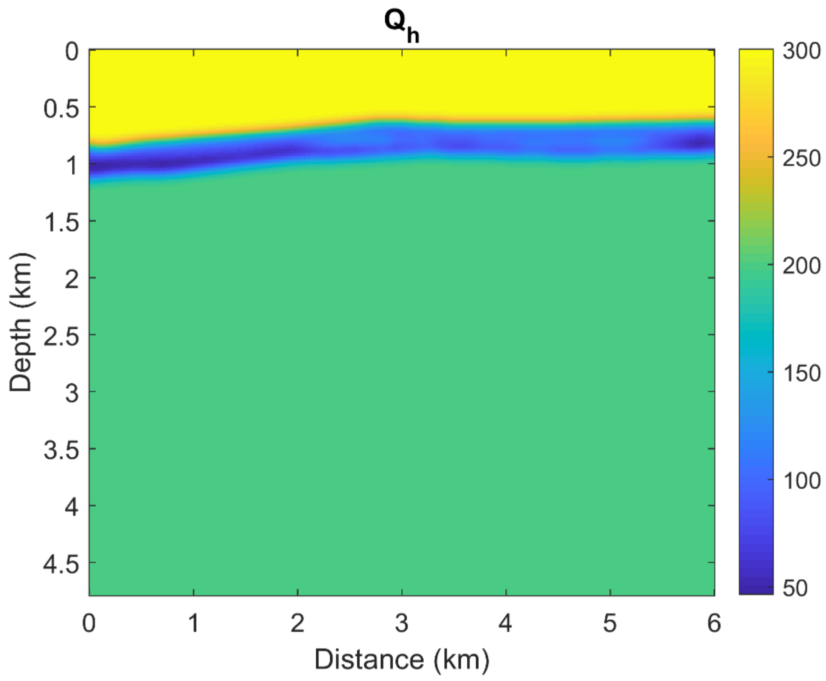
Zhu and Tsvankin, 2006

Anisotropic viscoacoustic BP model

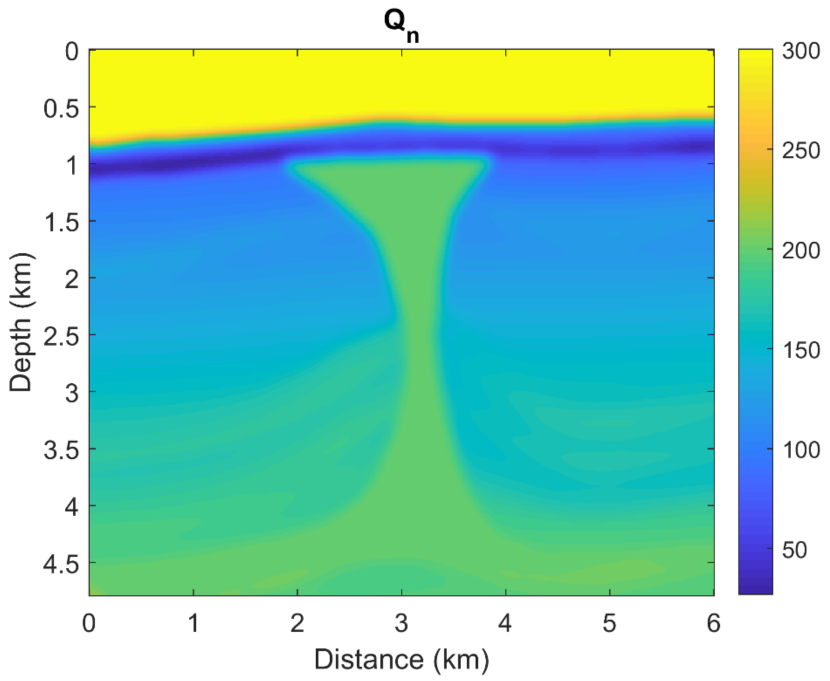


Anisotropic viscoacoustic BP model

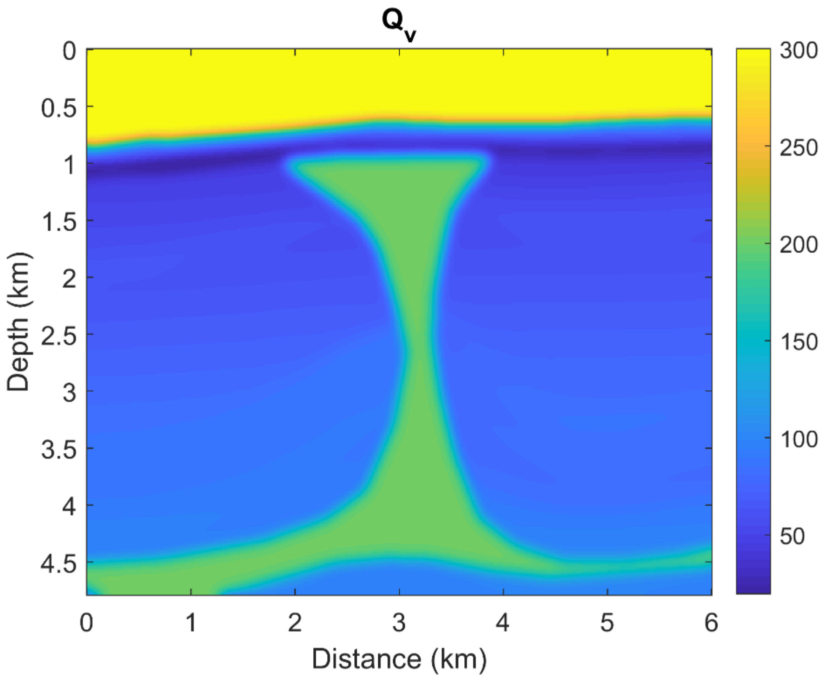
Horizontal attenuation



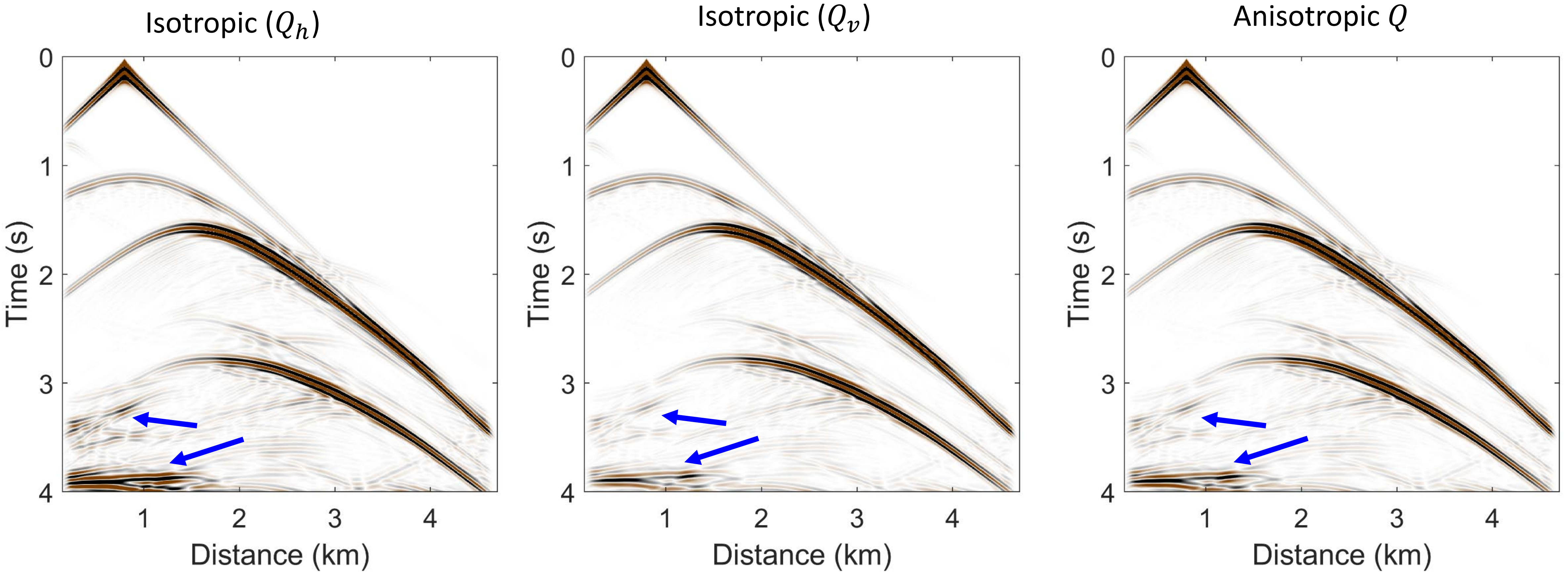
Normal attenuation



Vertical attenuation

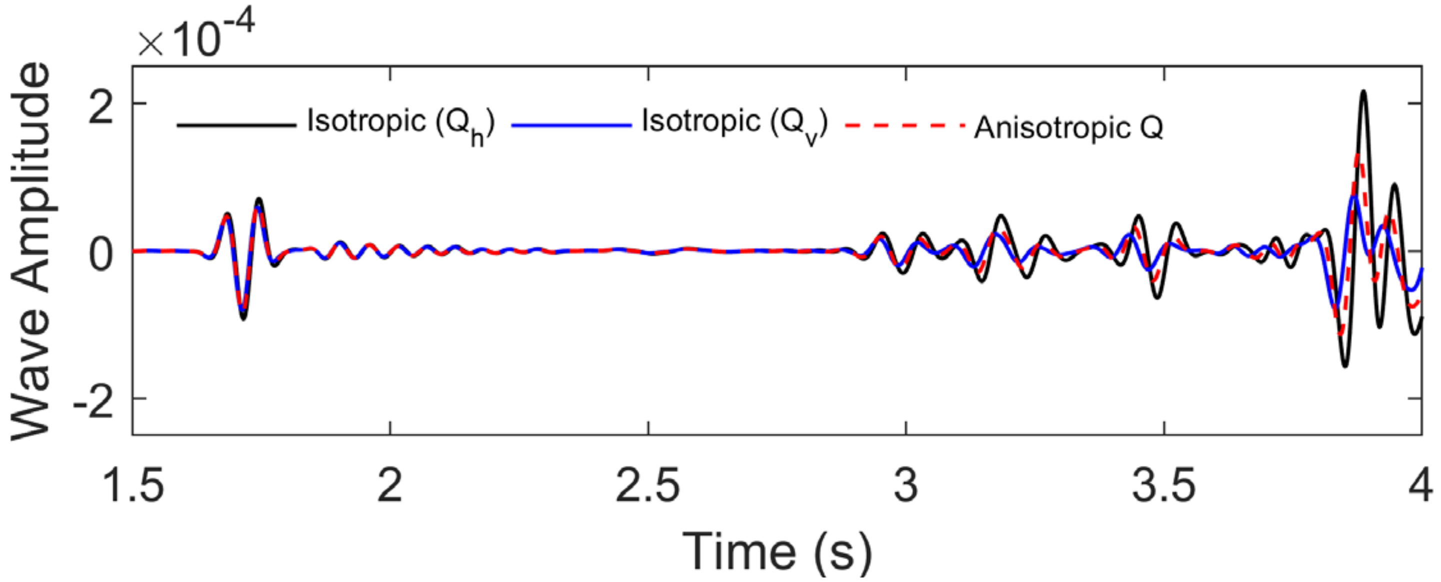


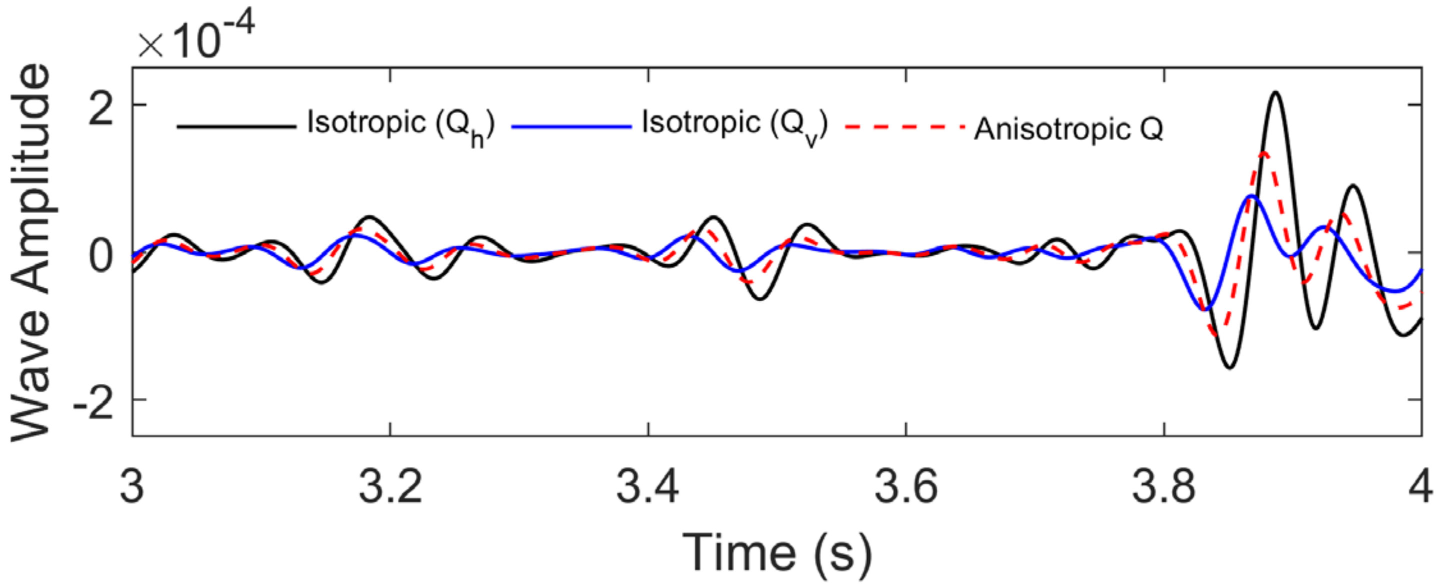
Anisotropic viscoacoustic BP model



Anisotropic viscoacoustic BP model

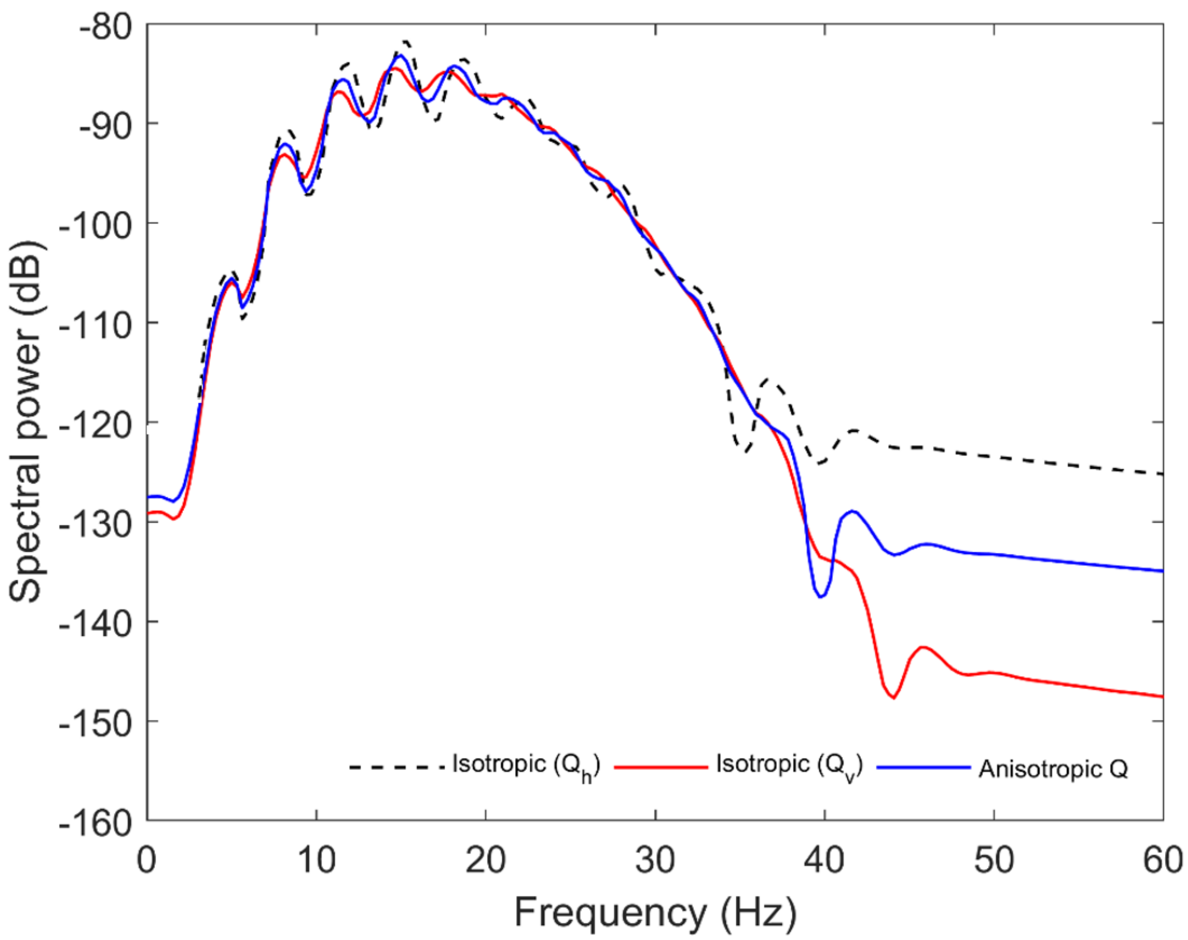
Traces at an offset of 1.2 km





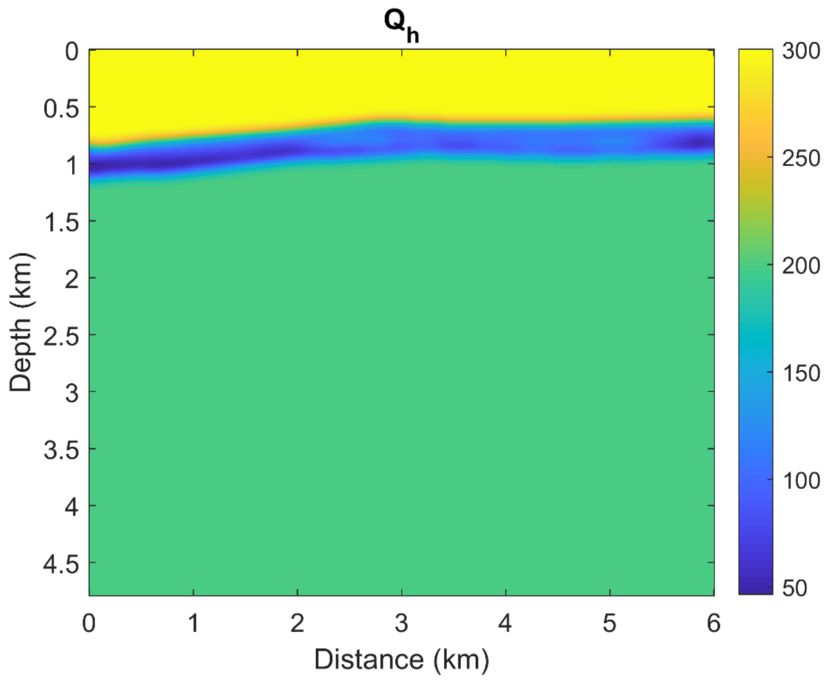
Comparison of the amplitude spectrum

1.2 km

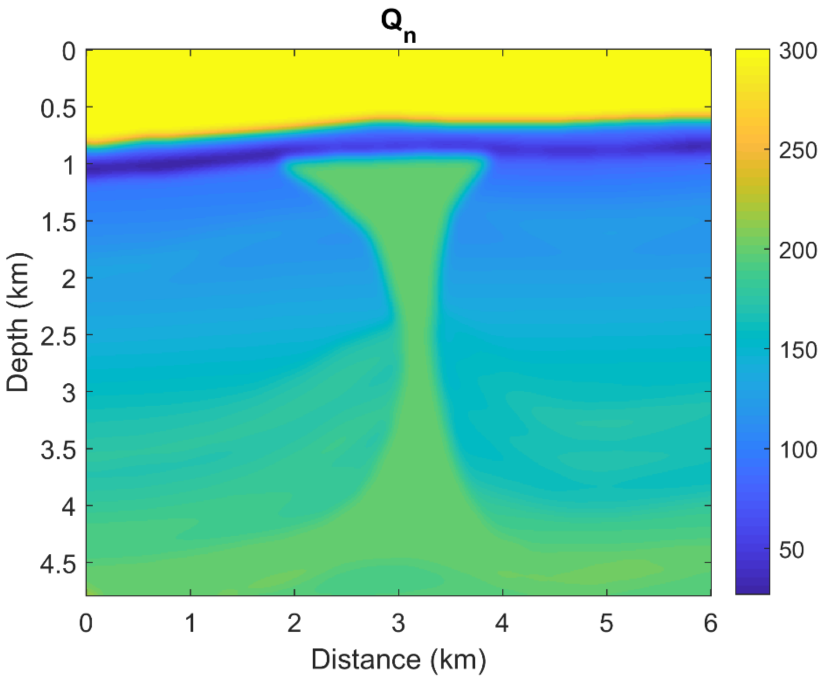


Anisotropic viscoacoustic BP model

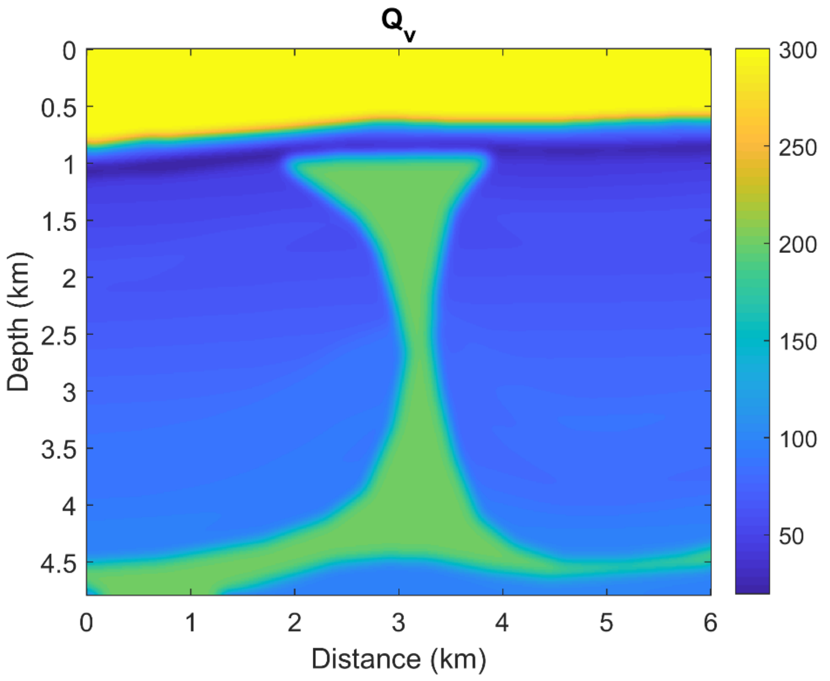
Horizontal attenuation



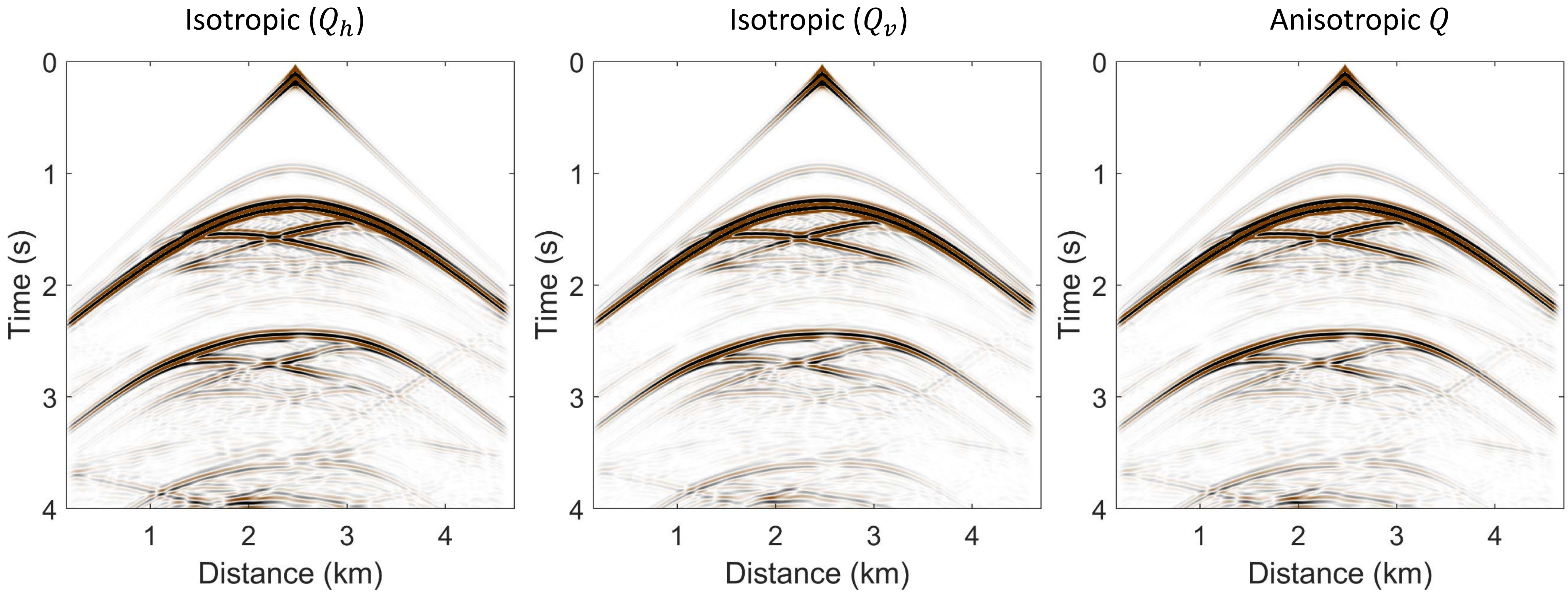
Normal attenuation



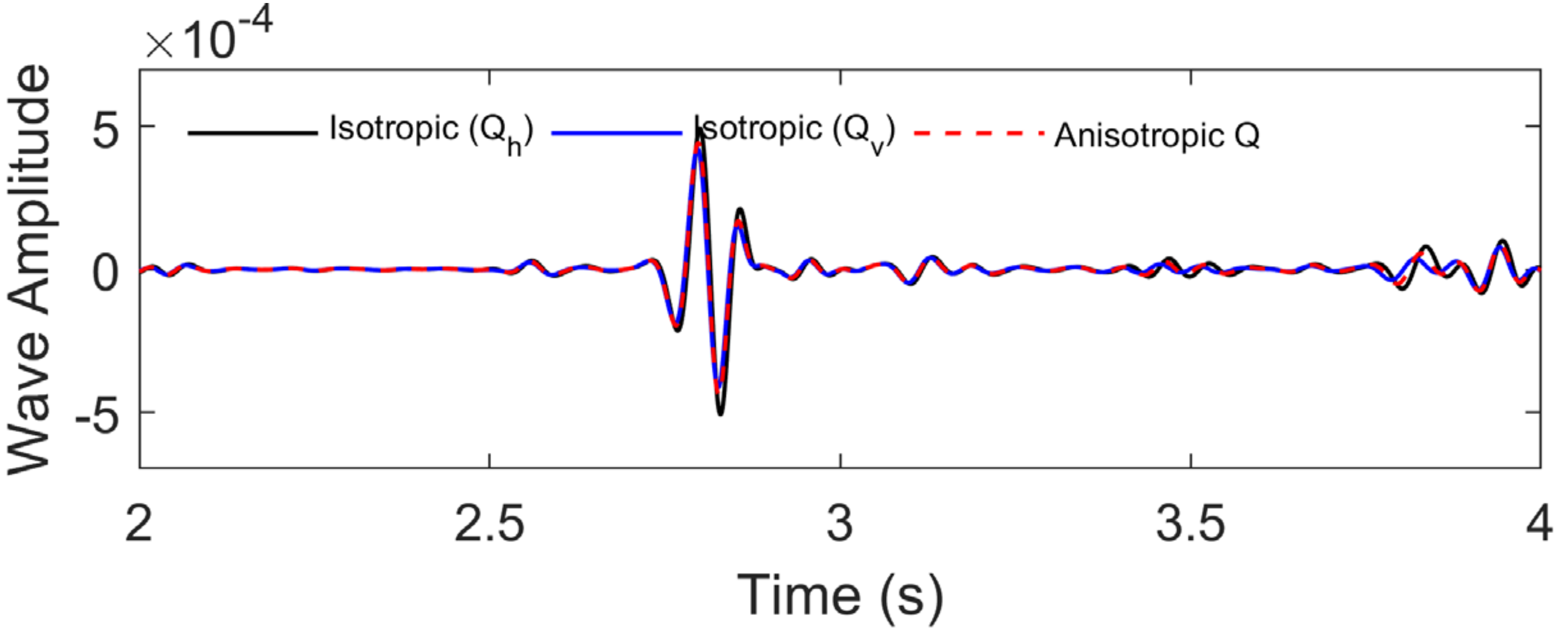
Vertical attenuation

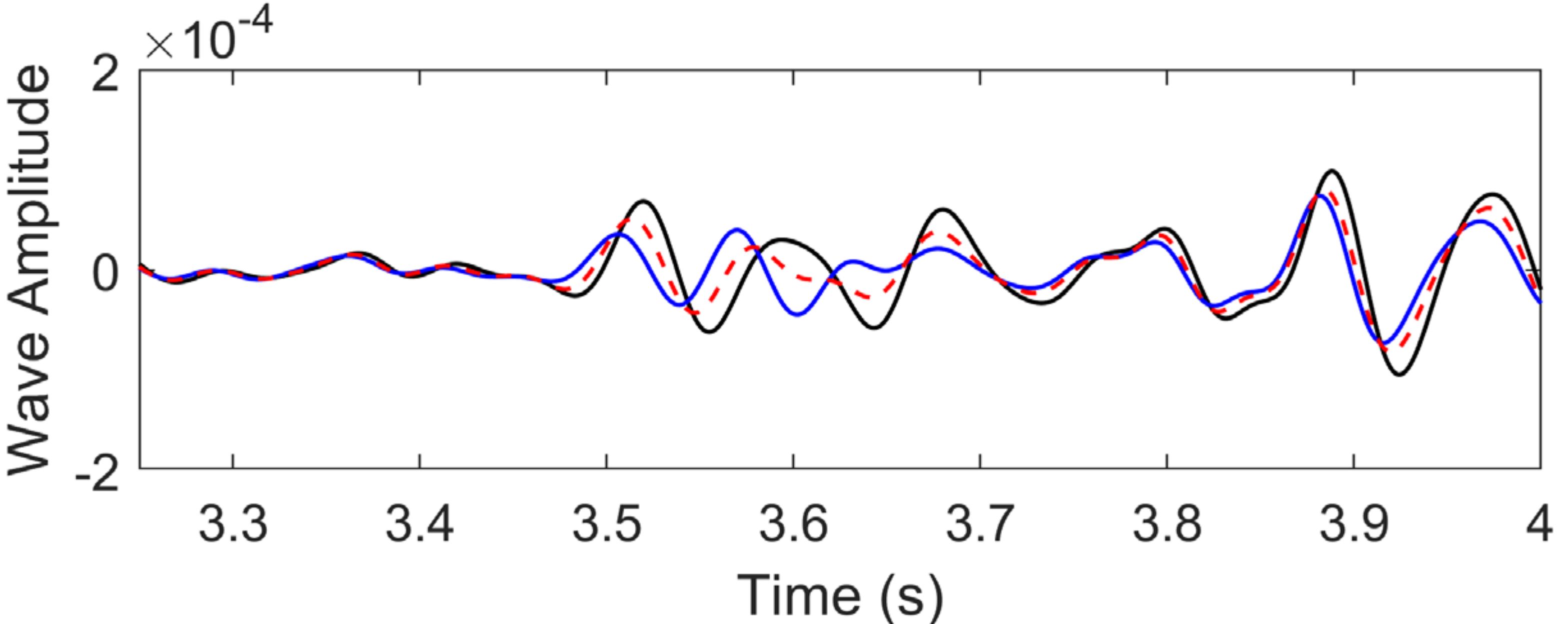


Anisotropic viscoacoustic BP model



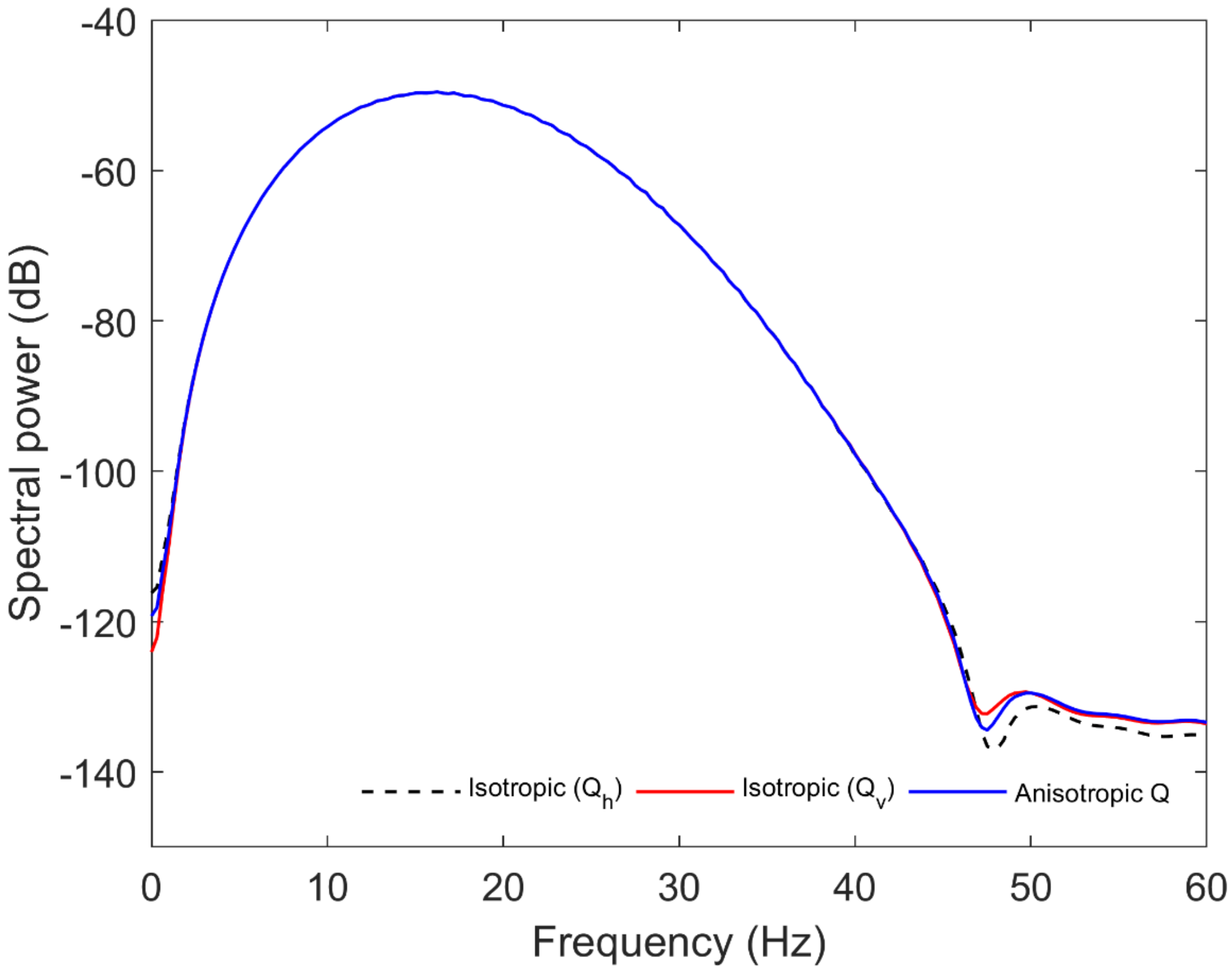
Anisotropic viscoacoustic BP model





Comparison of the amplitude spectrum

Traces at an offset of 3.1 km



Conclusions

- We have developed a derivation of a system of equations for acoustic waves in a medium with transverse isotropy (TI) in velocity and attenuation.

- Attenuation anisotropy is introduced in the wave equation based on the constant-Q model.

- Comparison with analytical solutions, and modeling examples, demonstrates that our modeling approach is capable of capturing TI effects in intrinsic attenuation.

- The proposed method is useful for seismic modeling, imaging, and inversion, and our future research aims toward its application on the analysis of real seismic data.

Acknowledgments

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