CREWES

Elastic FWI with probabilistic model constraints

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CREWES Tech Talk

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Motivation

- **Challenges of EFWI: Highly nonlinear and ill-posed**

  Uncertainty caused by insufficient data, noise, parameter crosstalk…

- **Regularization: Constraining the solution**

Objective function: $$E(\mathbf{m}) = E_d(\mathbf{m}) + \lambda E_c(\mathbf{m})$$

Data misfit: $\left\|\mathbf{d}_{obs} - \mathbf{d}(\mathbf{m})\right\|^2$

Model penalty

Regularization schemes

Smoothness: $E_c(\mathbf{m}) = \|\mathbf{D}_x\mathbf{m}\|^2 + \|\mathbf{D}_z\mathbf{m}\|^2$ Tikhonov, 1977

Prior estimate: $E_c(\mathbf{m}) = \|\mathbf{W}_m(\mathbf{m} - \mathbf{m_p})\|^2$ Asnaashari, 2013

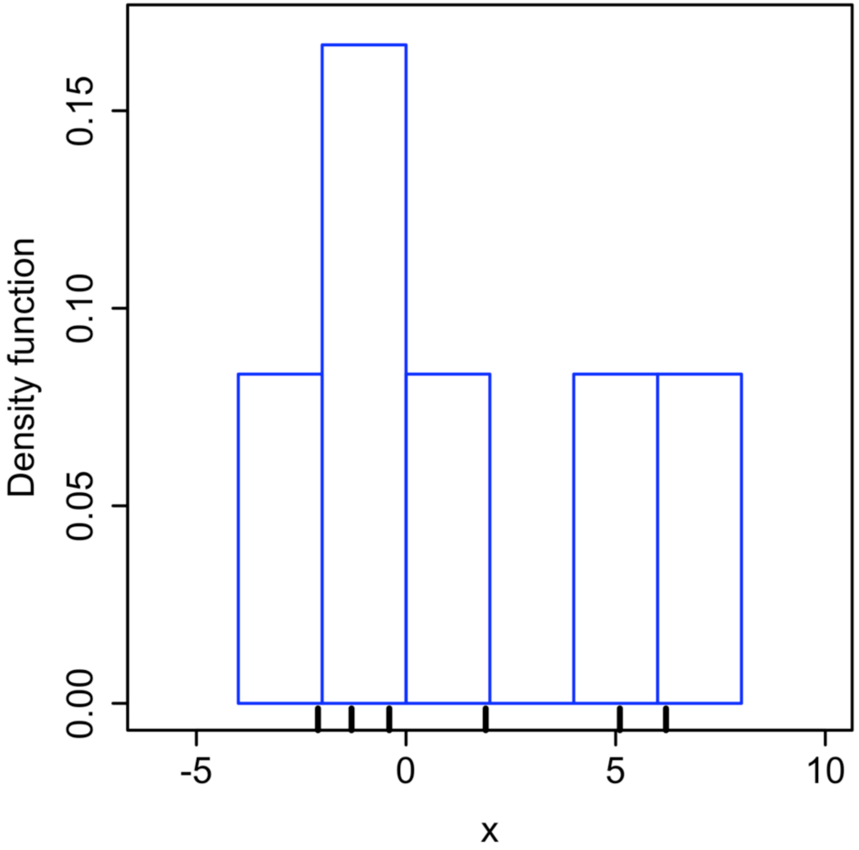
Explicit relations: $E_c(\mathbf{m}) = \|\mathbf{m}_1 - f(\mathbf{m}_2)\|^2$ Hu, 2021

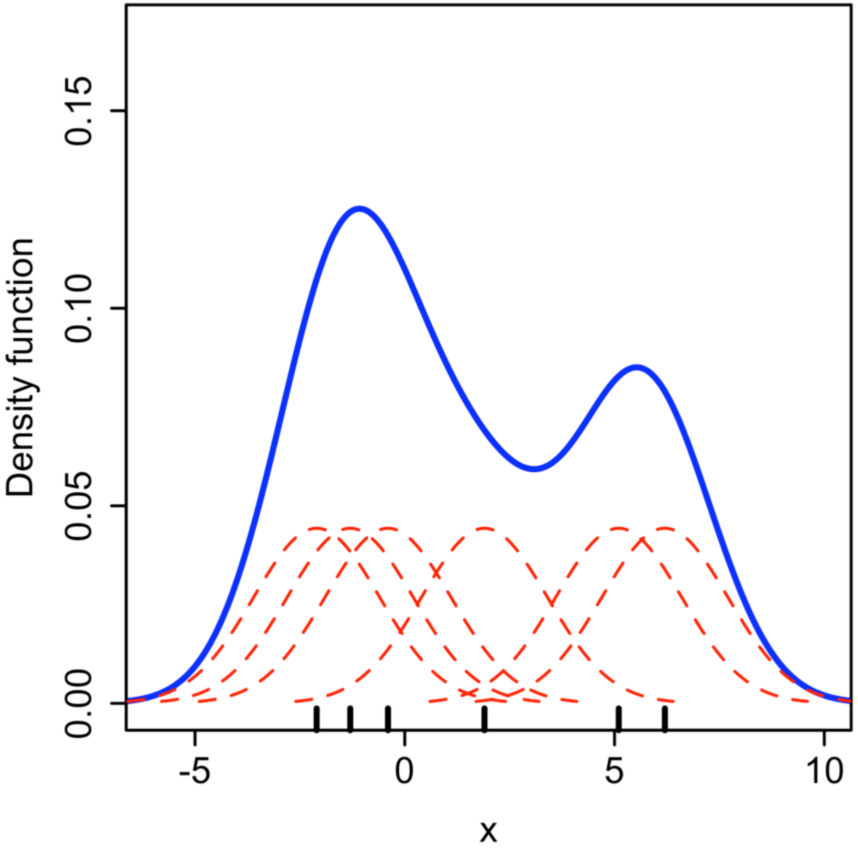
Probabilistic distribution: $E_c(\mathbf{m}) = \sum_{\mathbf{m}} D(\mathbf{m})$ Aragao, 2020

Kernel density estimation (KDE)

$$f_h(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - x_i) = \frac{1}{nh}\sum_{i=1}^{n} K(\frac{x-x_i}{h})$$

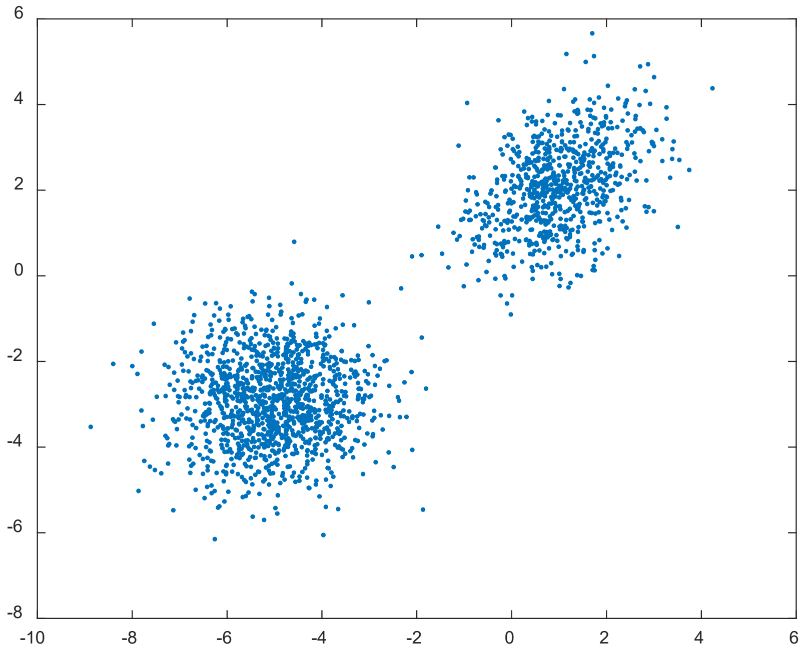
PDF



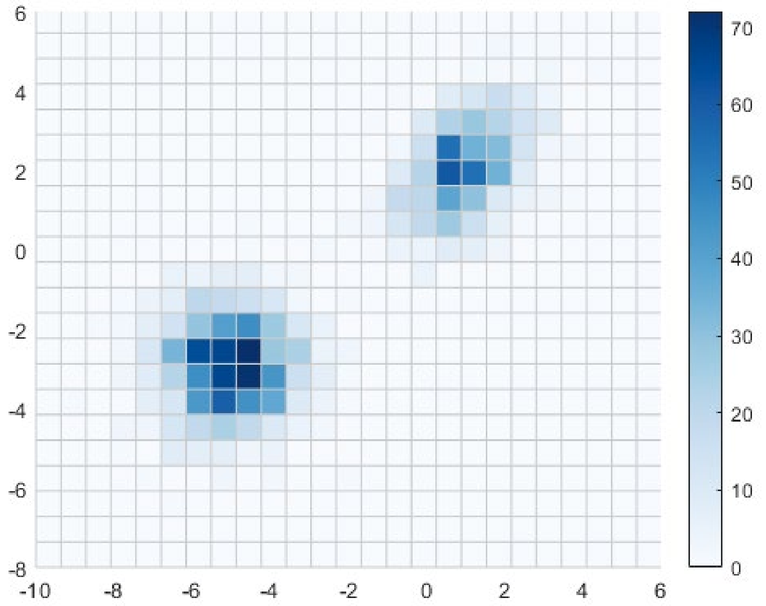


Kernel density estimation (KDE)

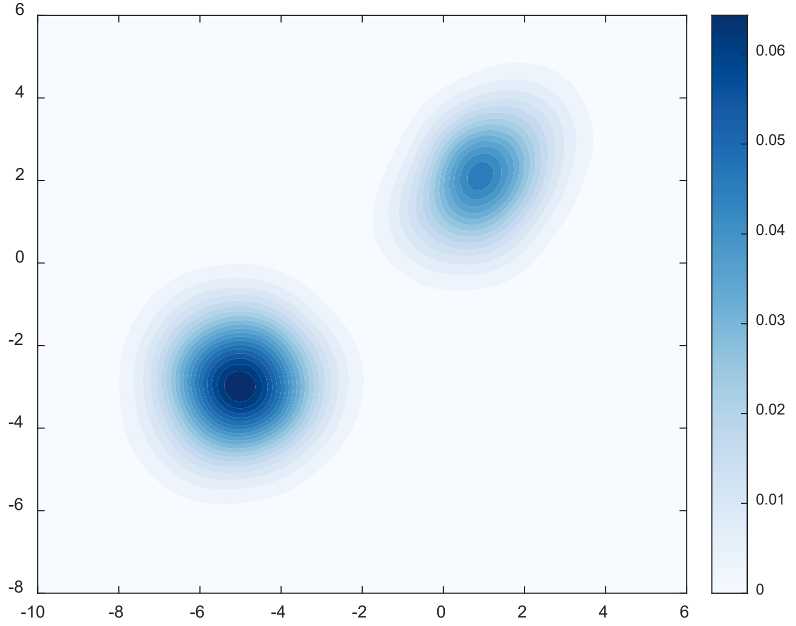
Data



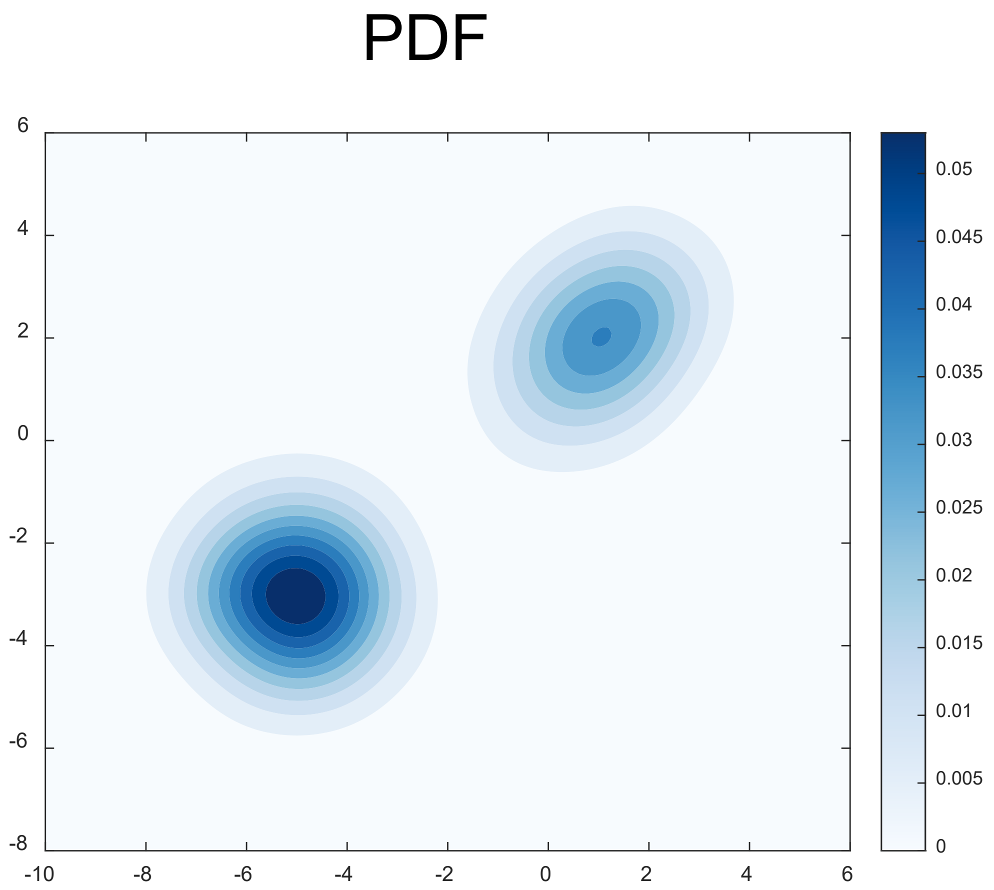
2D Histogram

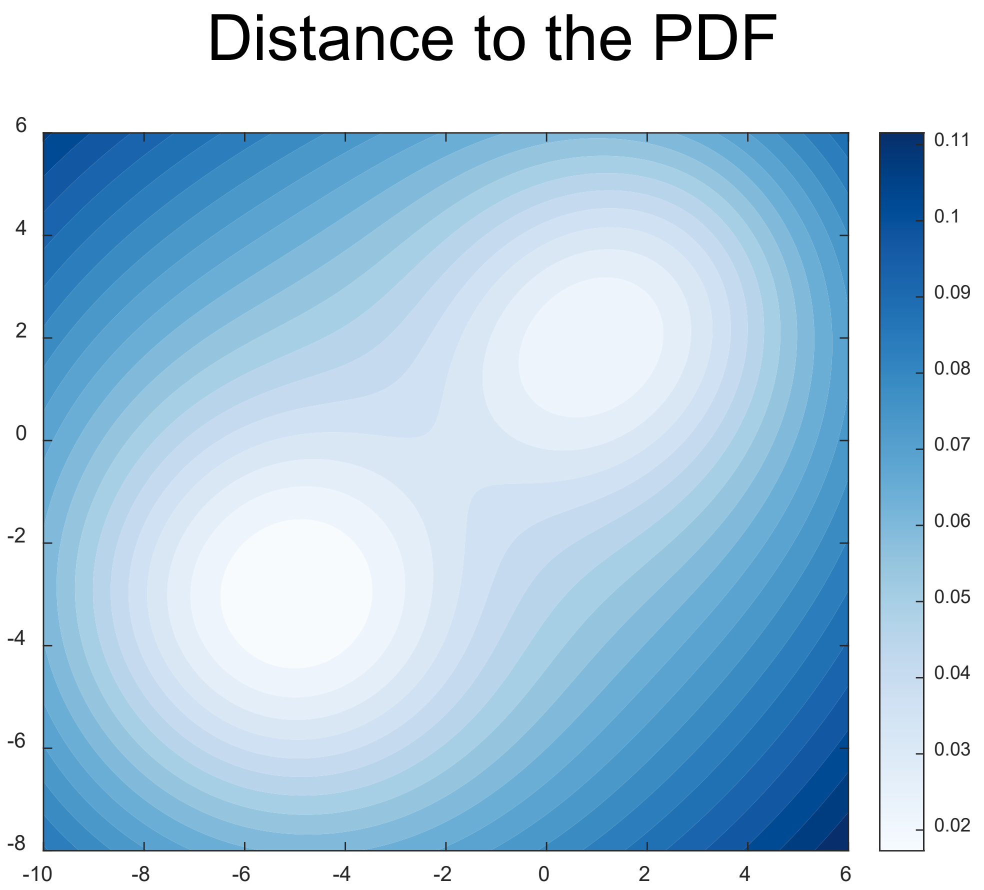


KDE



Probabilistic constraint

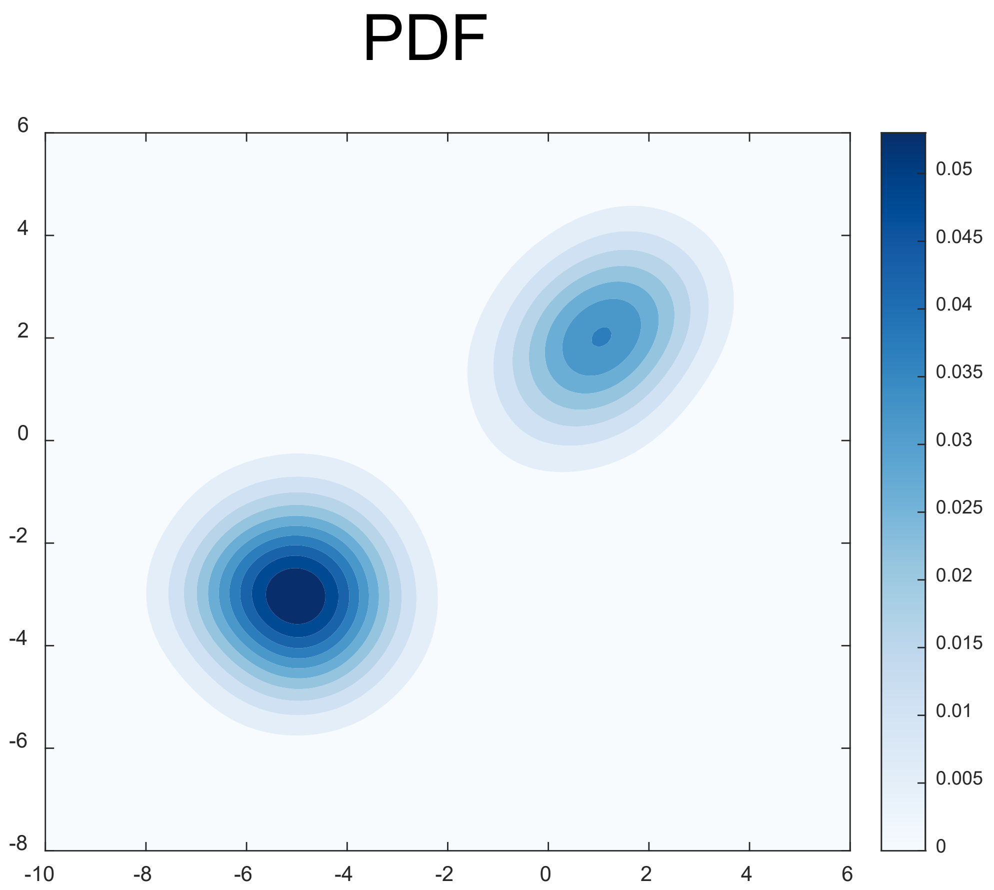


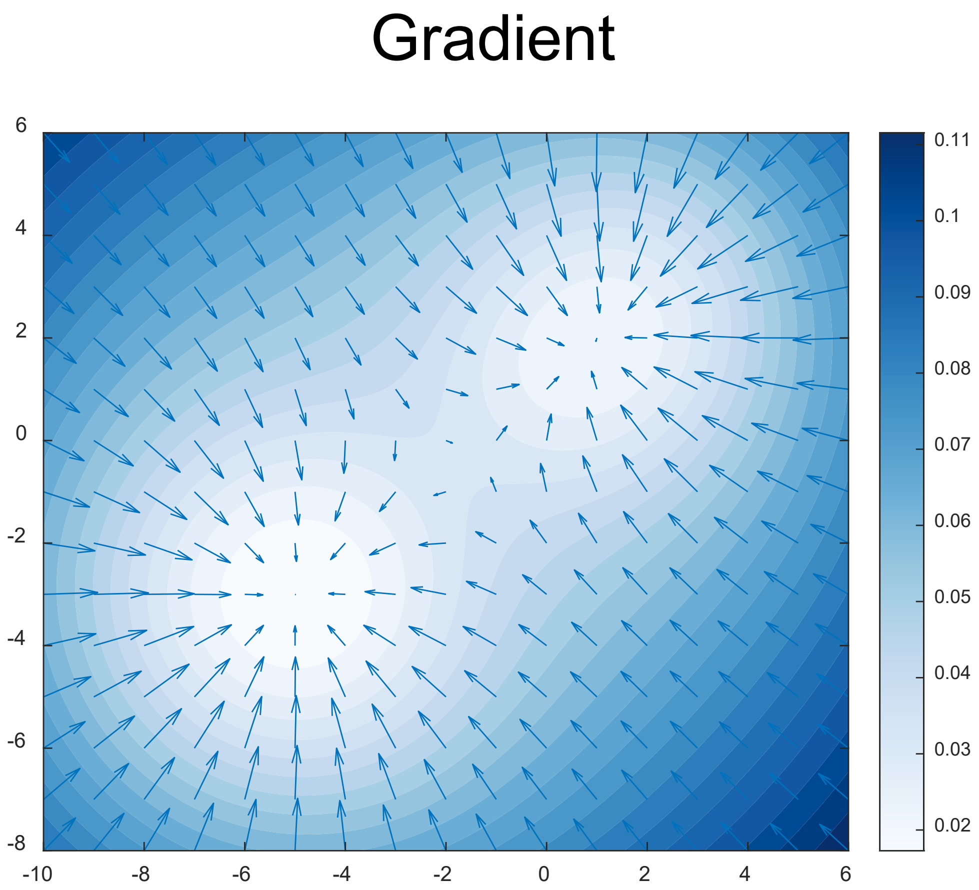


$$\frac{1}{D(\mathbf{m})} = \sum_{\mathbf{p}} \frac{f(\mathbf{p})}{d(\mathbf{m},\mathbf{p})}$$

Aragao, 2020

Probabilistic constraint





Probabilistic constraint

Proposed method:

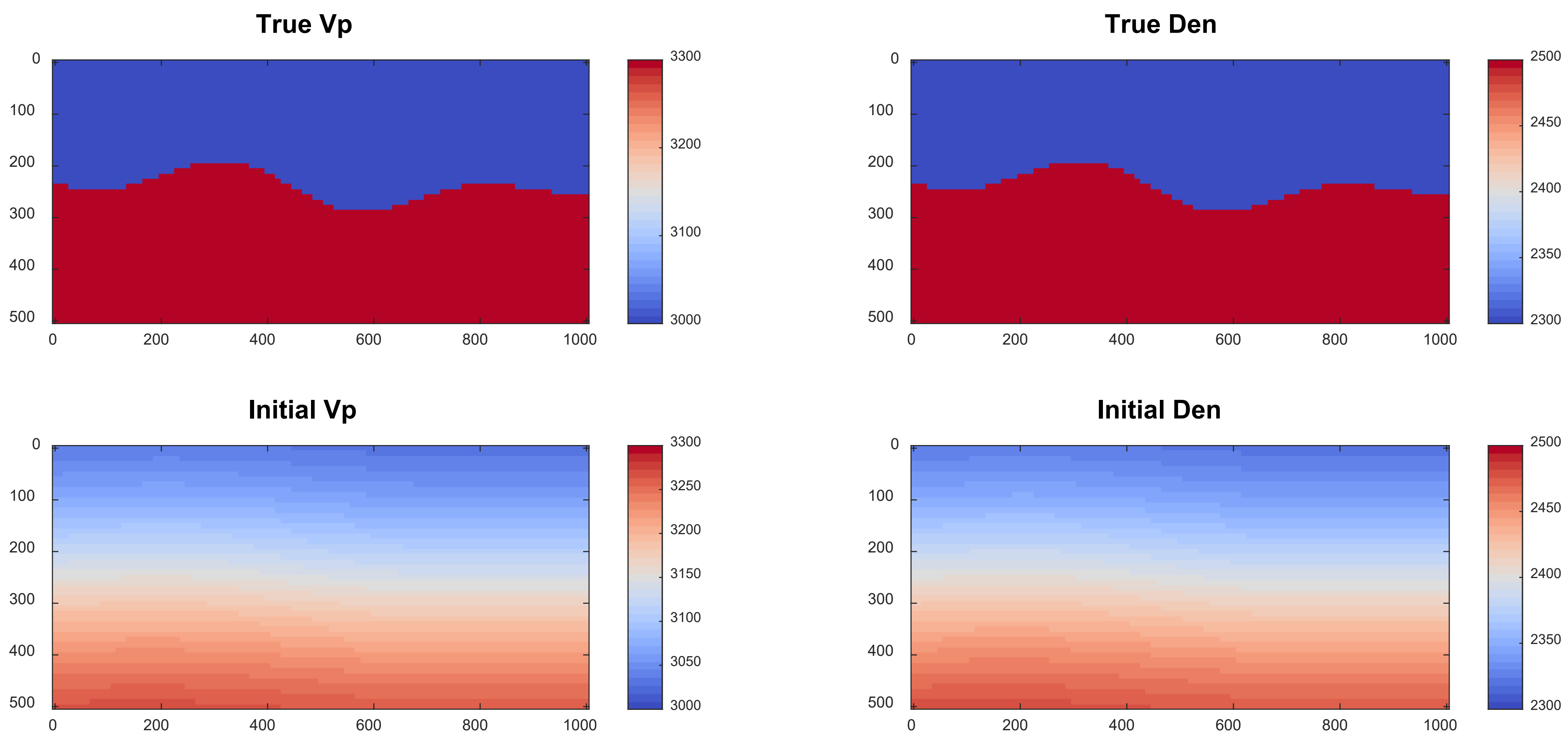
$$E_c(\mathbf{m}) = \left\| f(\mathbf{m}) - f_{\mathrm{p}} \right\|^2$$

Model Penalty PDF of current model Prior PDF

$$f(m^*) = \frac{1}{n} \sum_{i=1}^{n} K(m^*, m_i)$$

Gaussian Kernel: $$K(m^*, m_i) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(m^* - m_i)^2}{2\sigma^2}\right)$$

Example one: layer model



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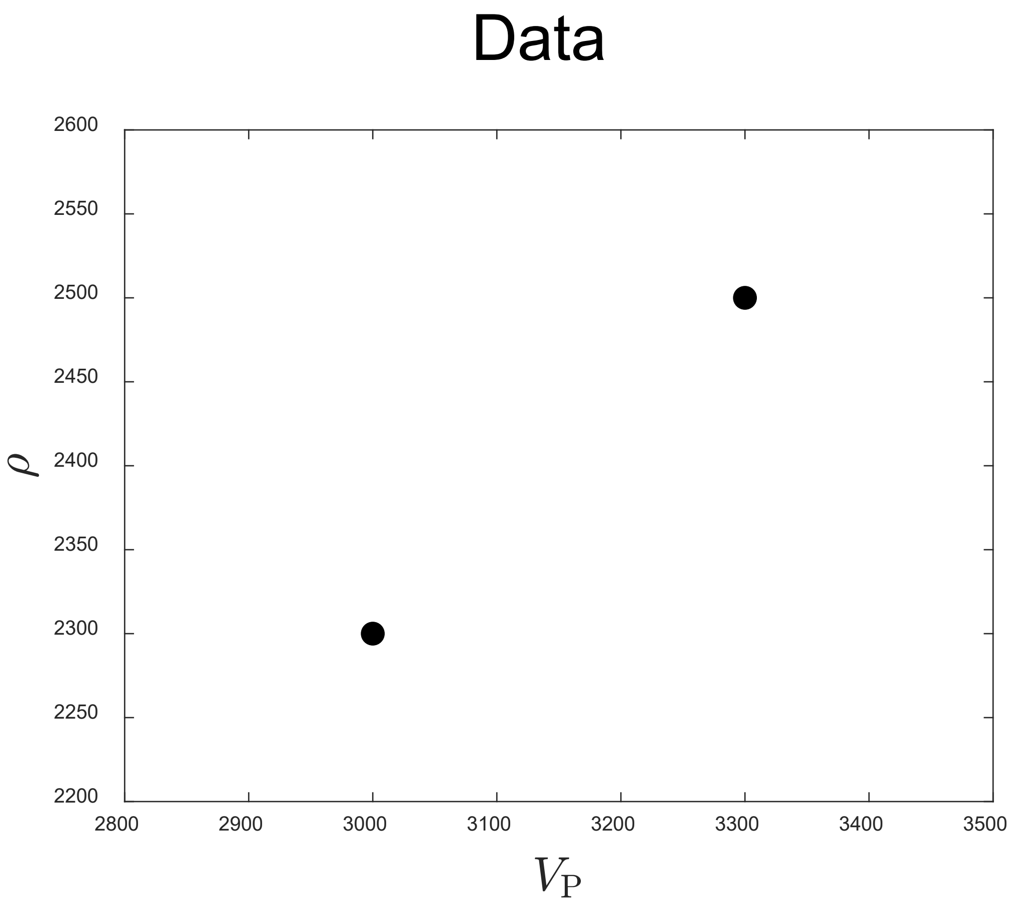

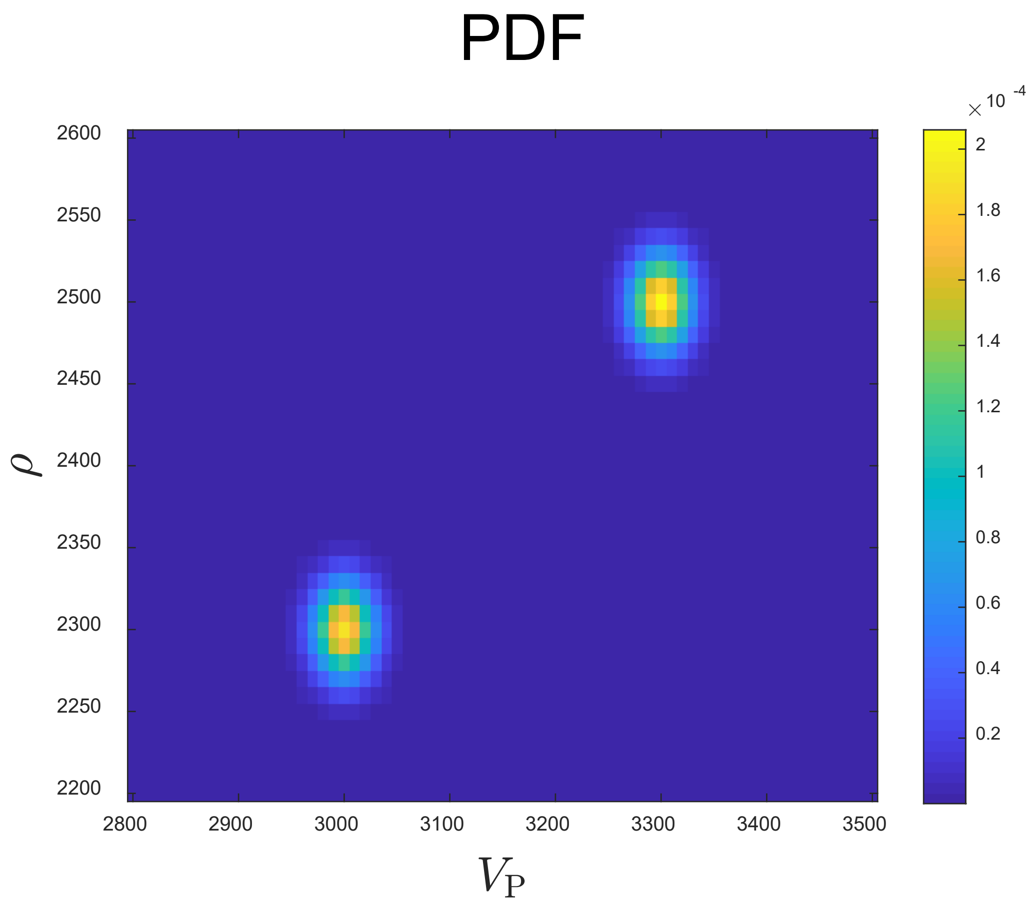

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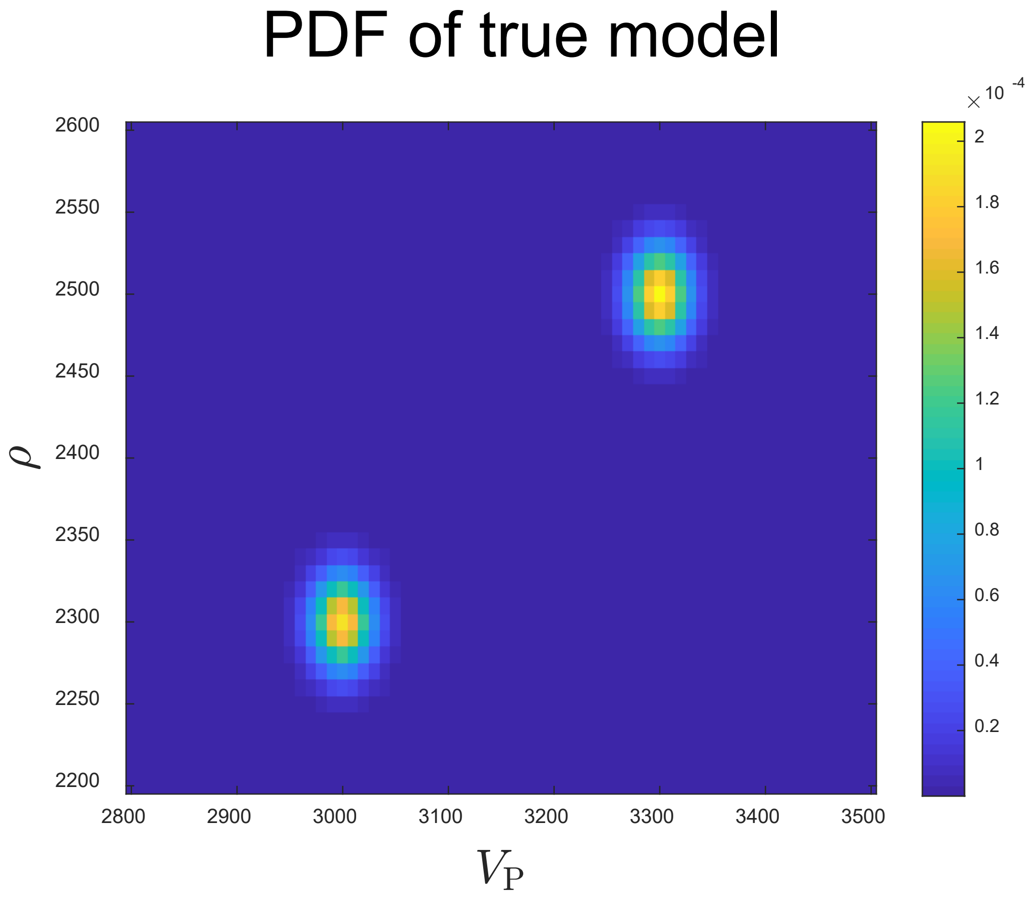
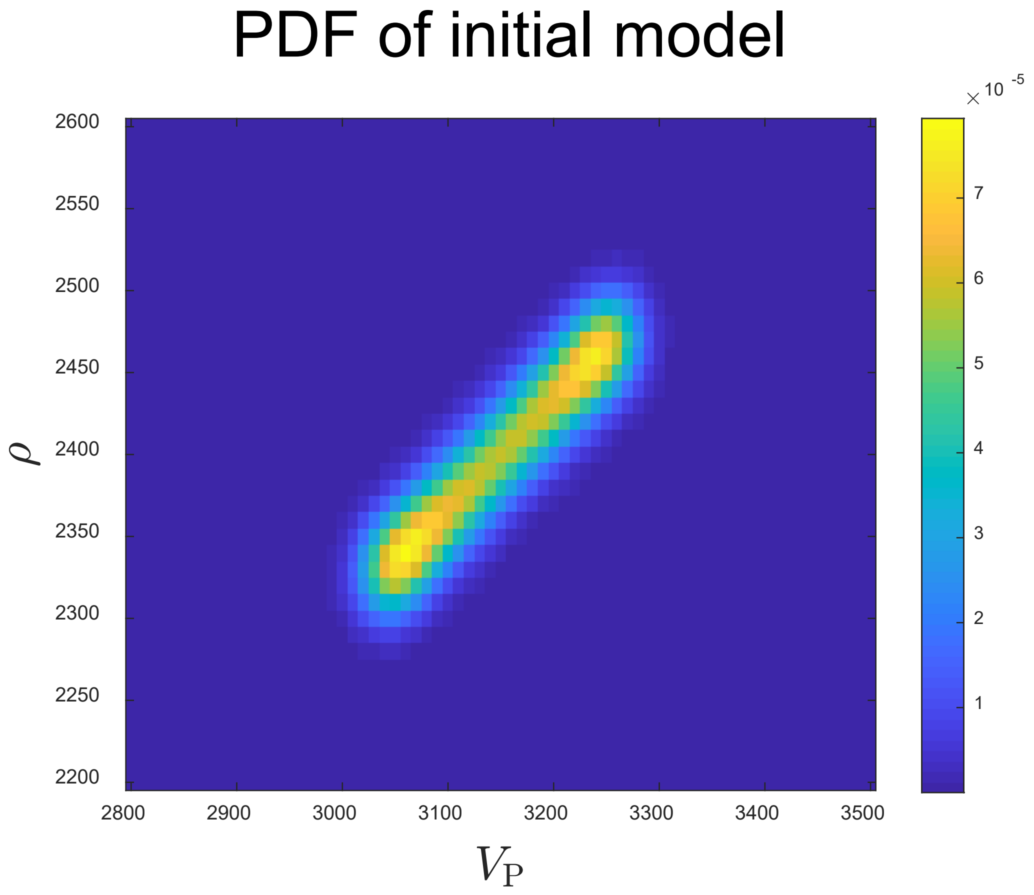
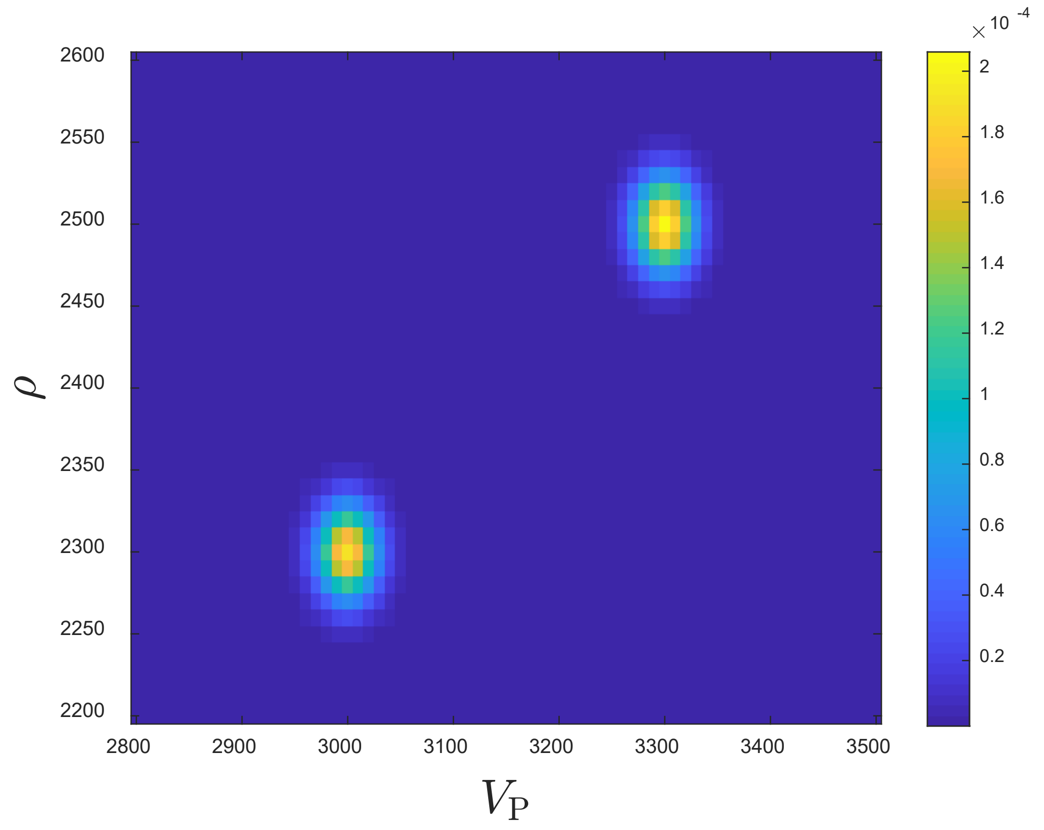


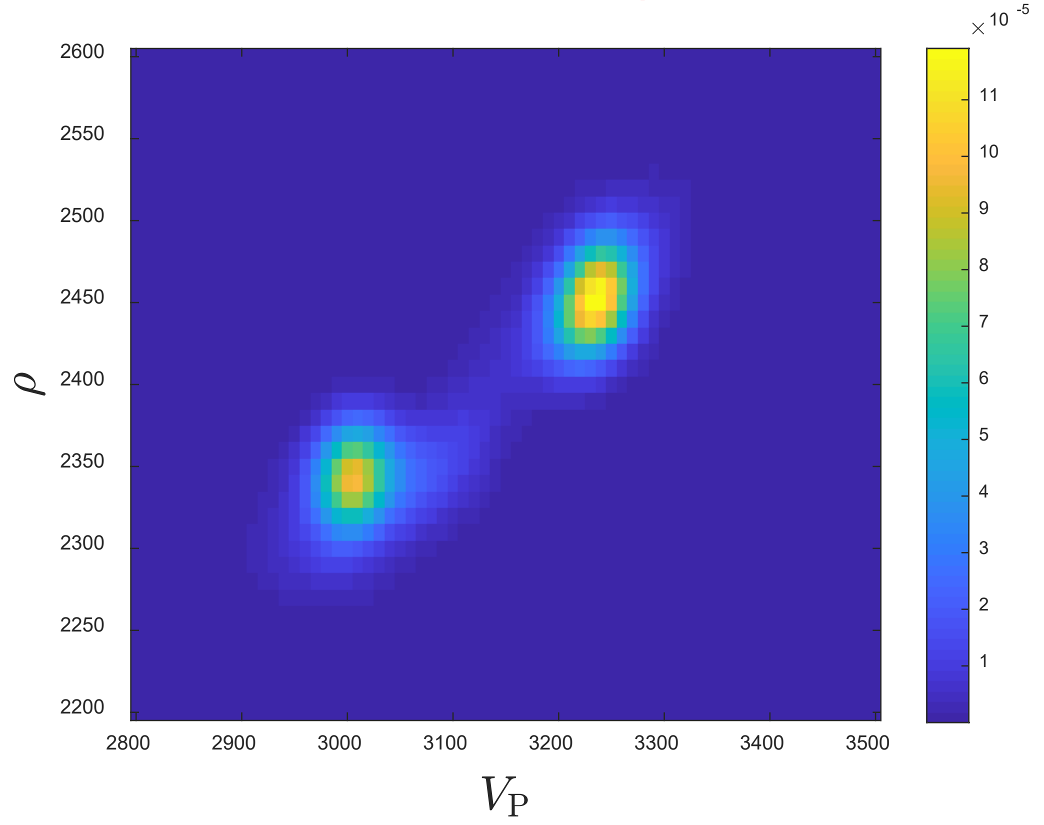


Example one: layer model

PDF of true model

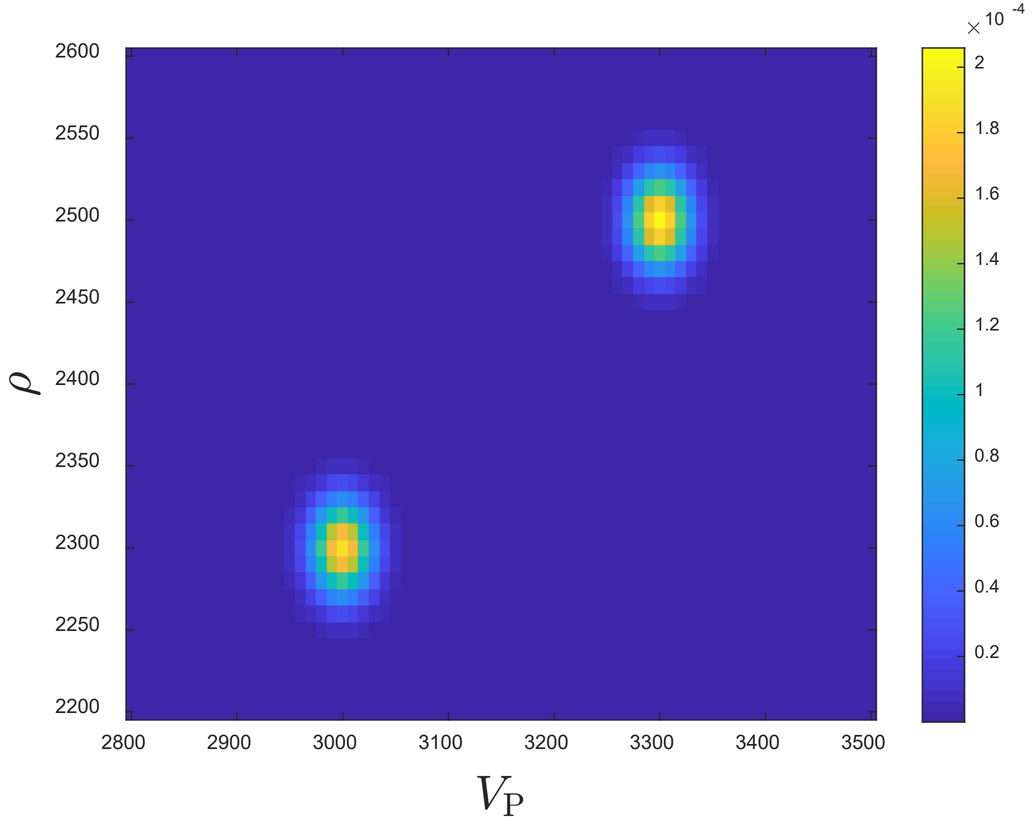


PDF of inverted model (Unconstrained)

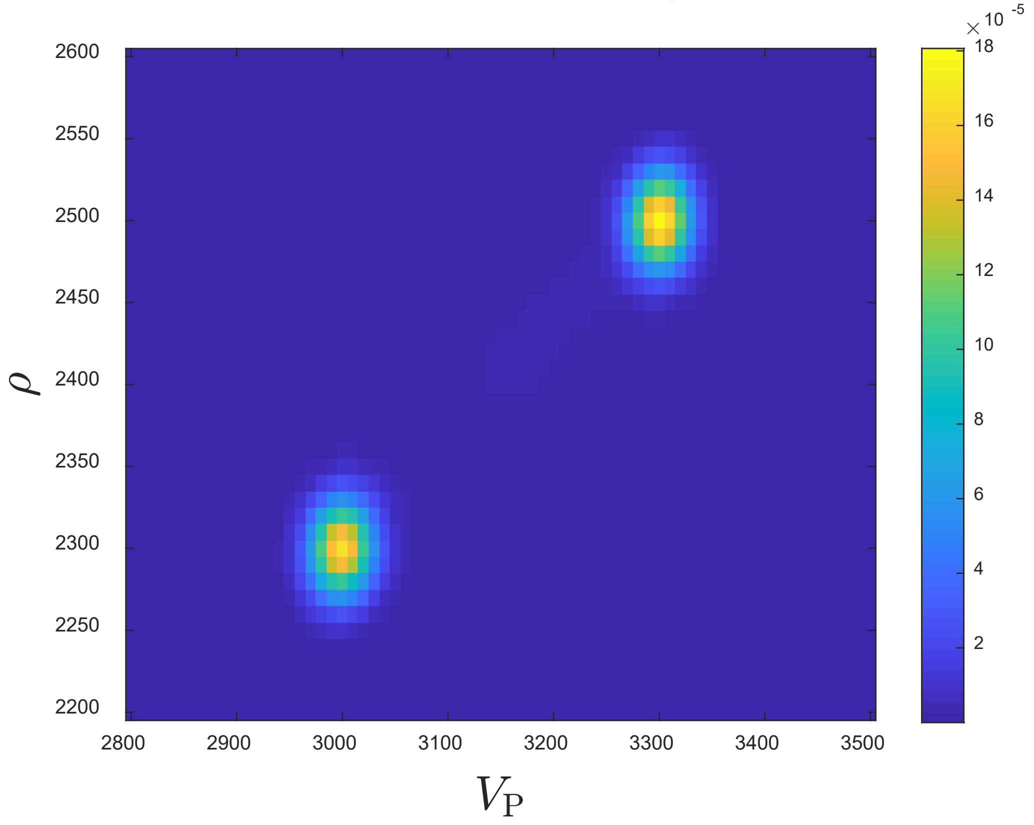


Example one: layer model

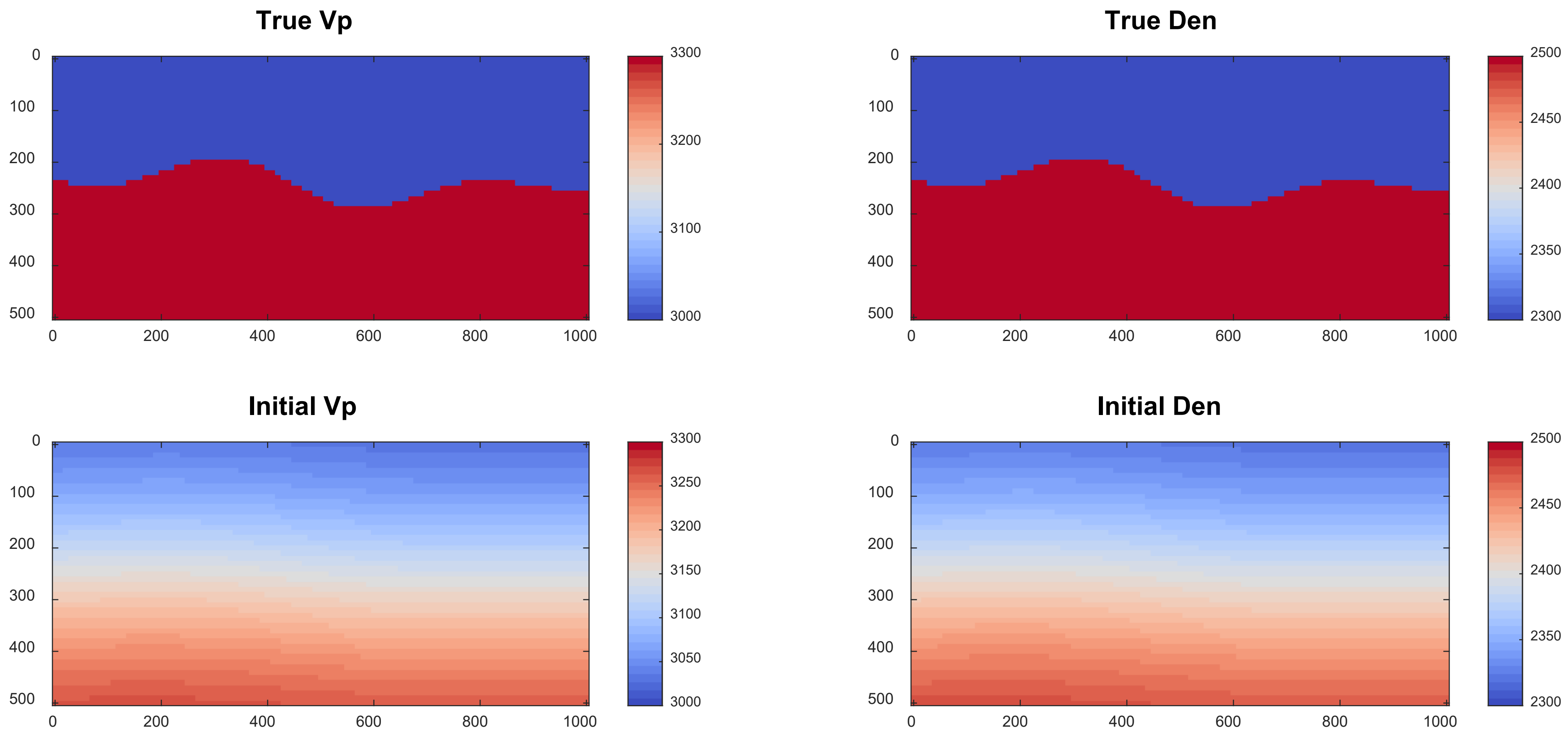
PDF of true model



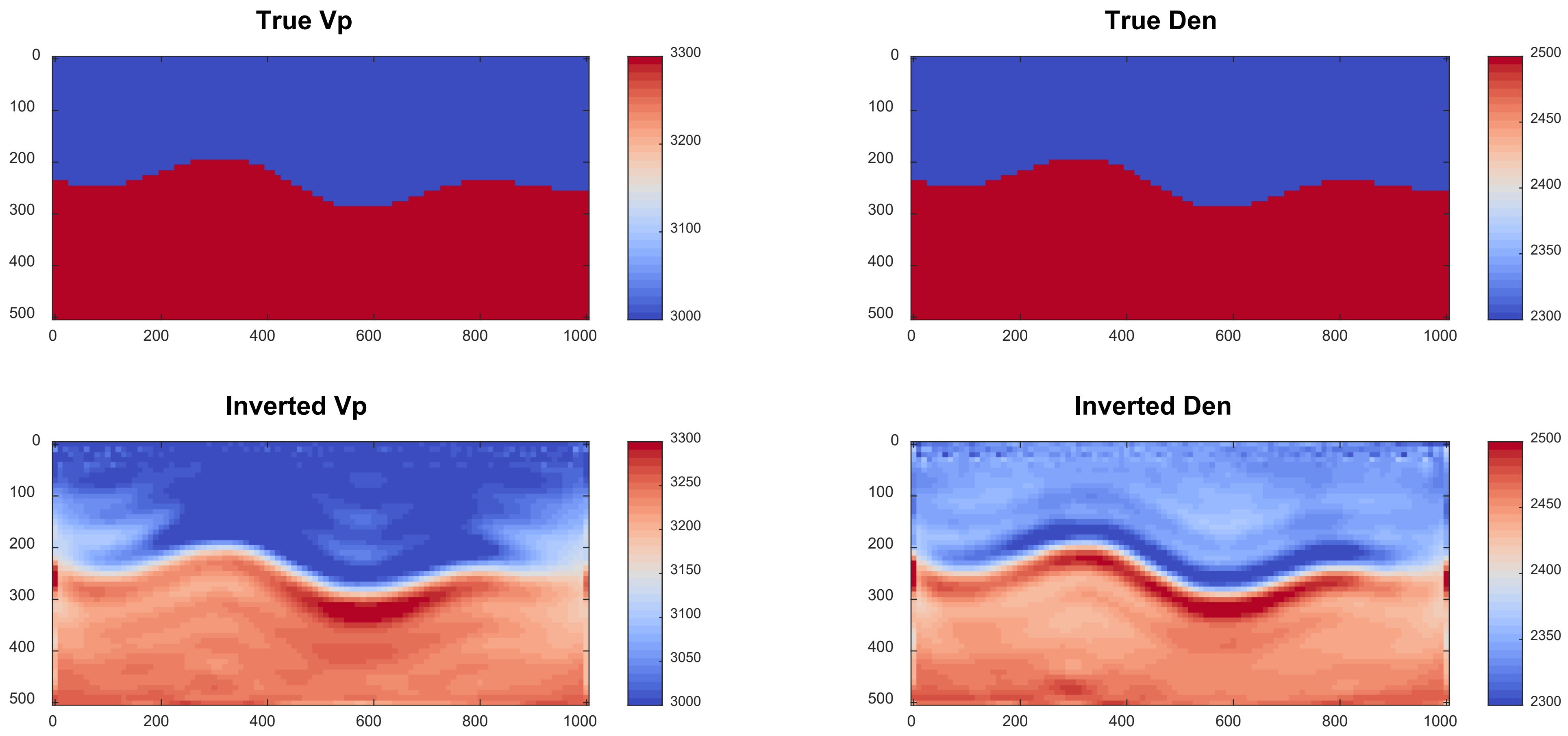
PDF of inverted model (Constrained)



Example one: layer model

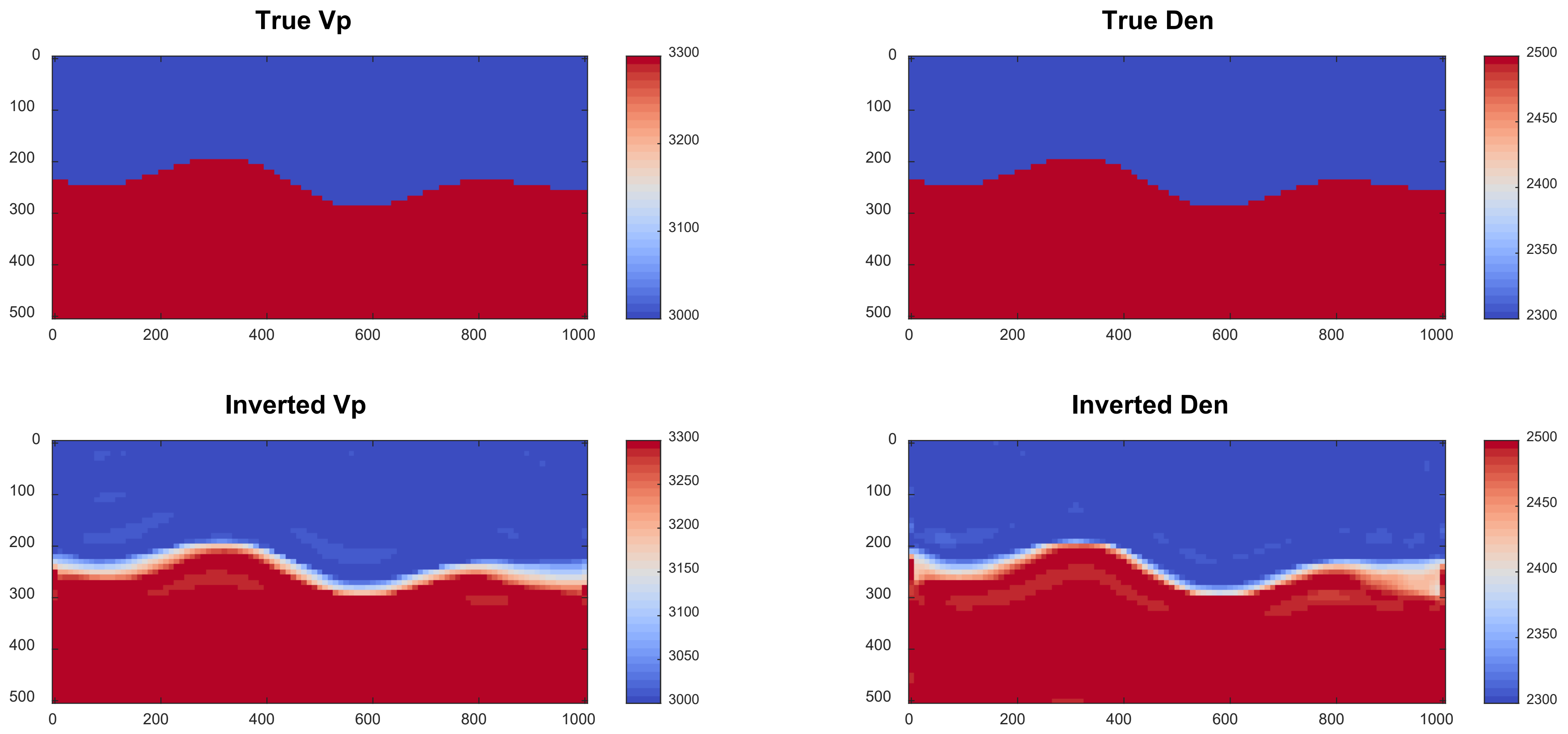


Example one: layer model



Unconstrained

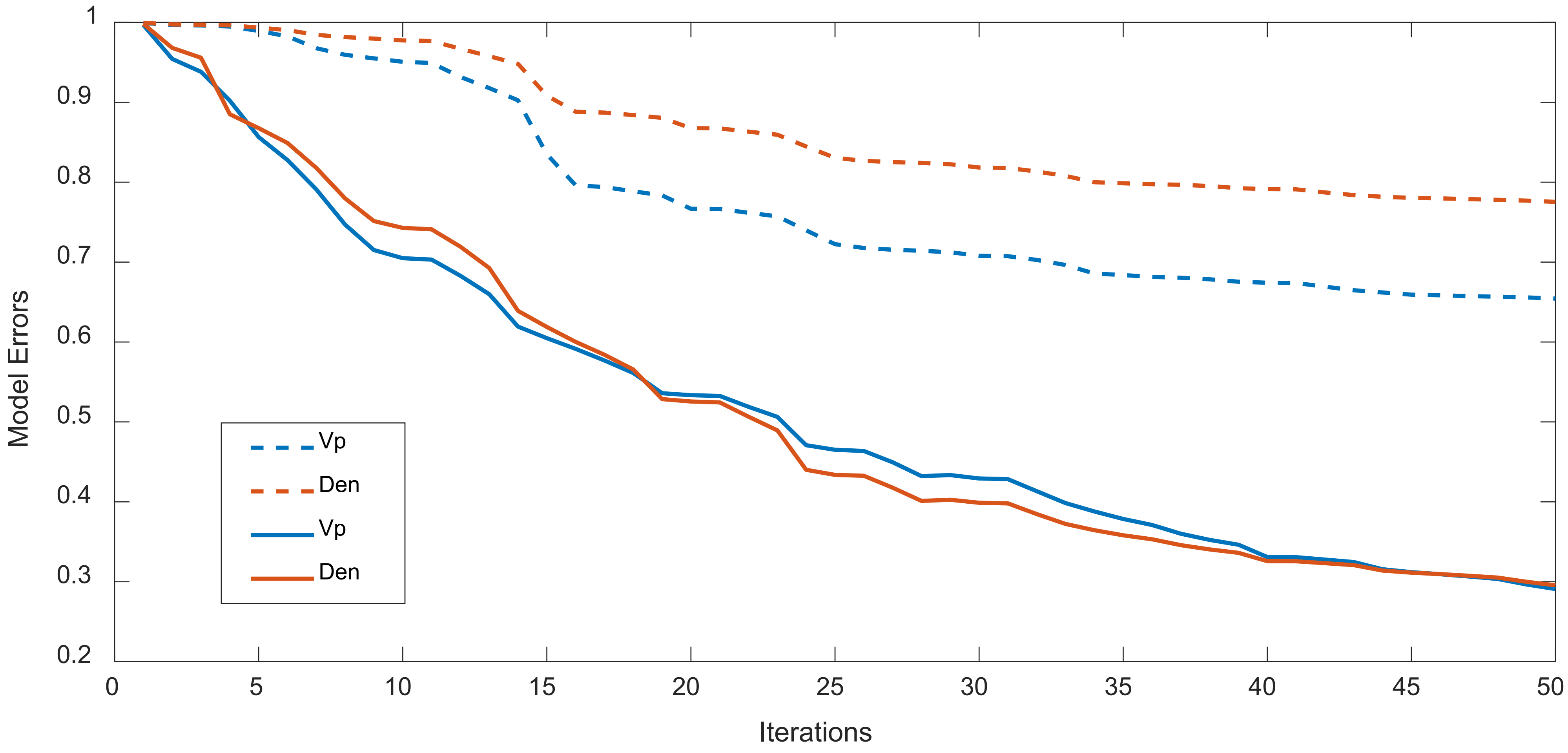
Example one: layer model



Constrained

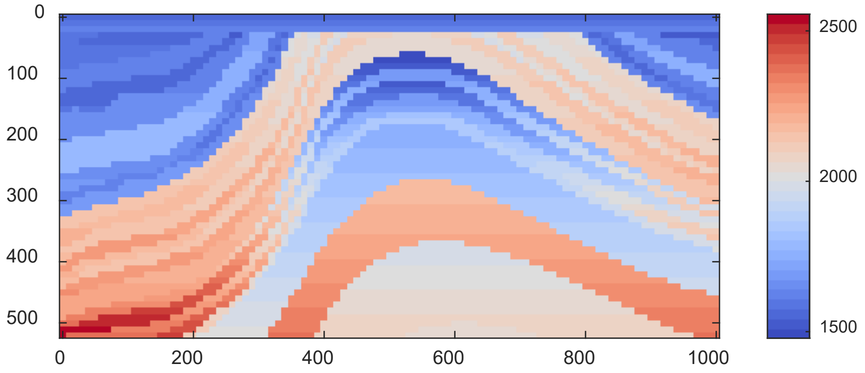
Example one: layer model

Convergence:

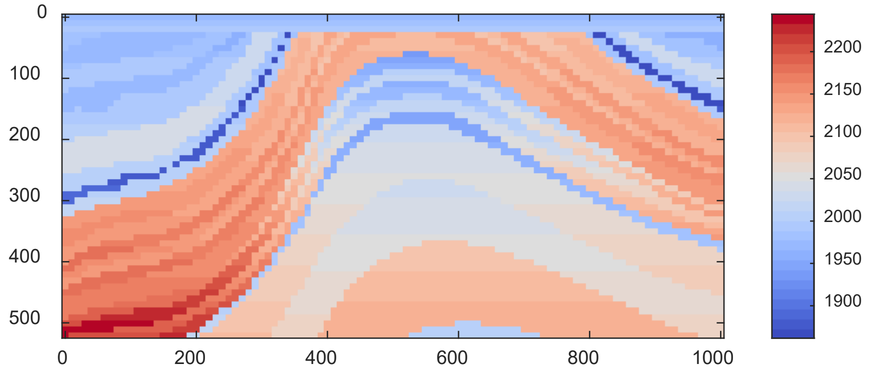


Example two: Marmousi model

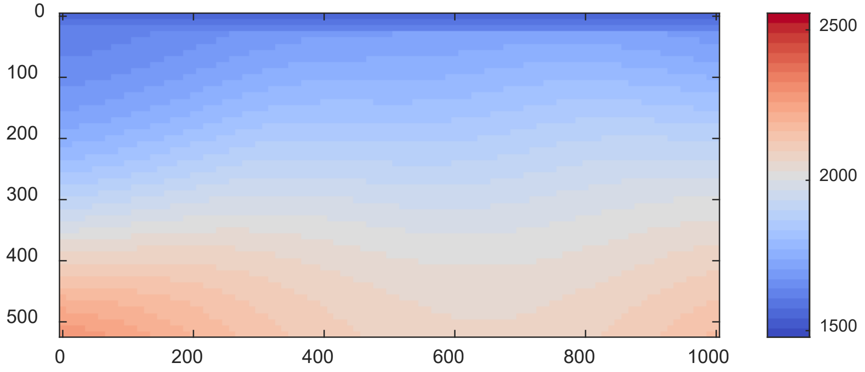
True Vp



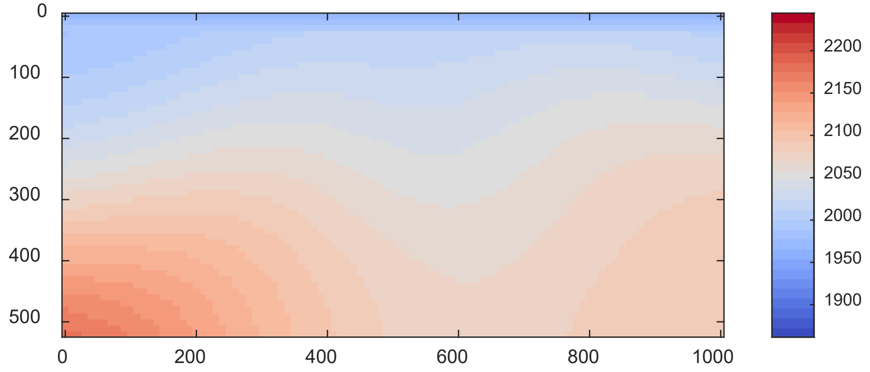
True Den



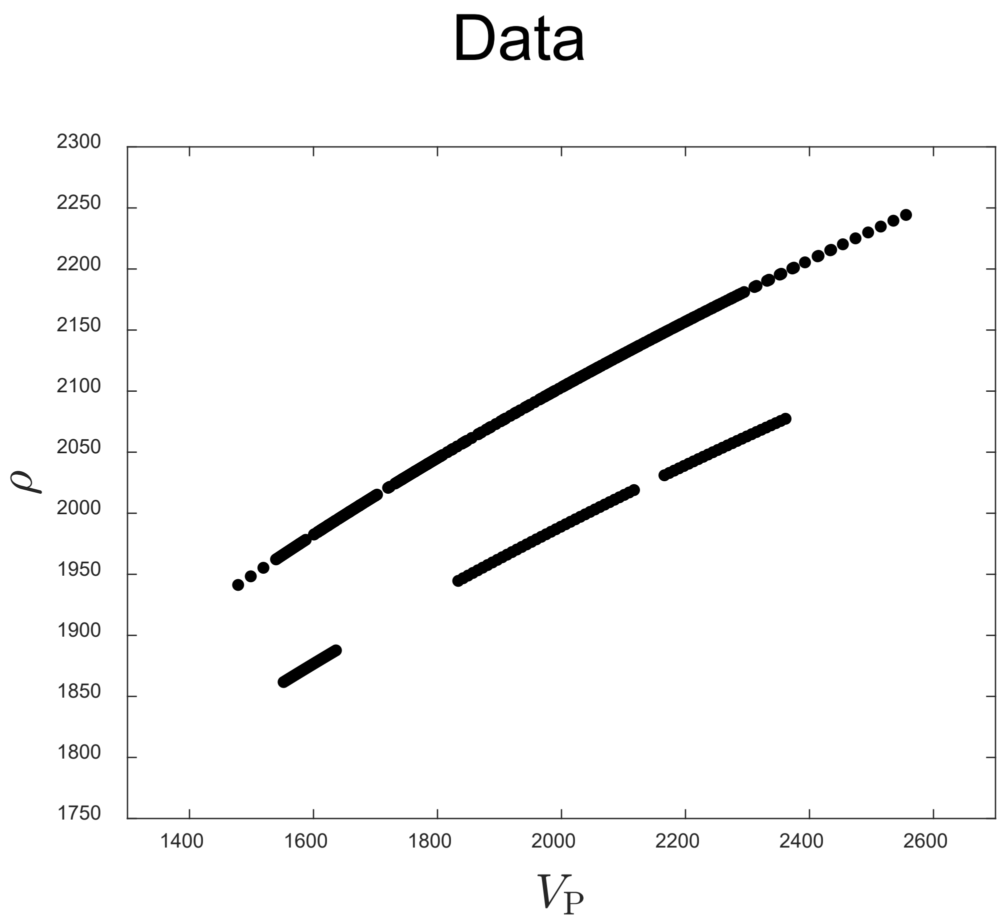
Initial Vp

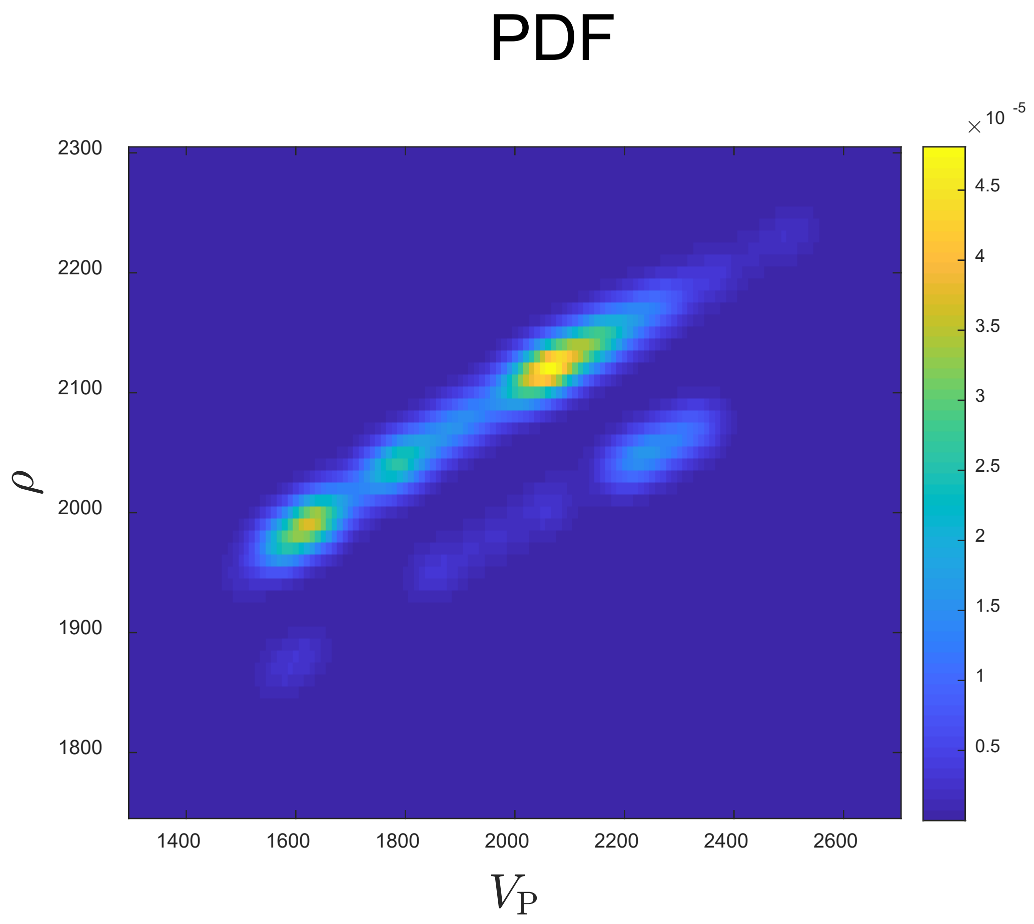


Initial Den



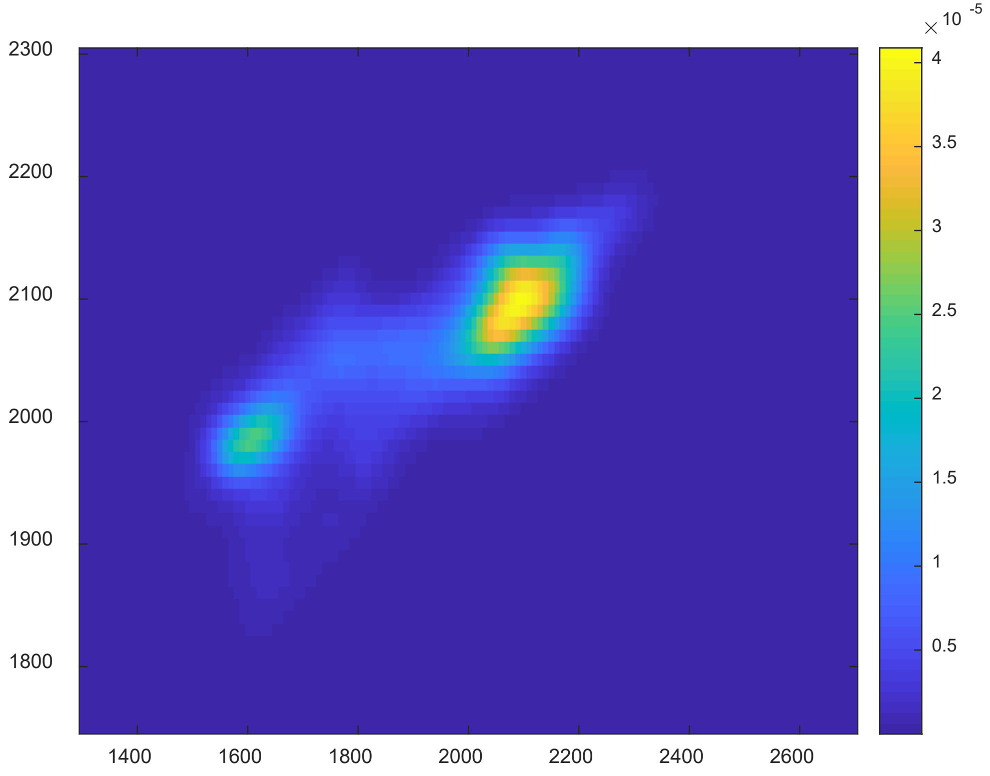
Example two: Marmousi model



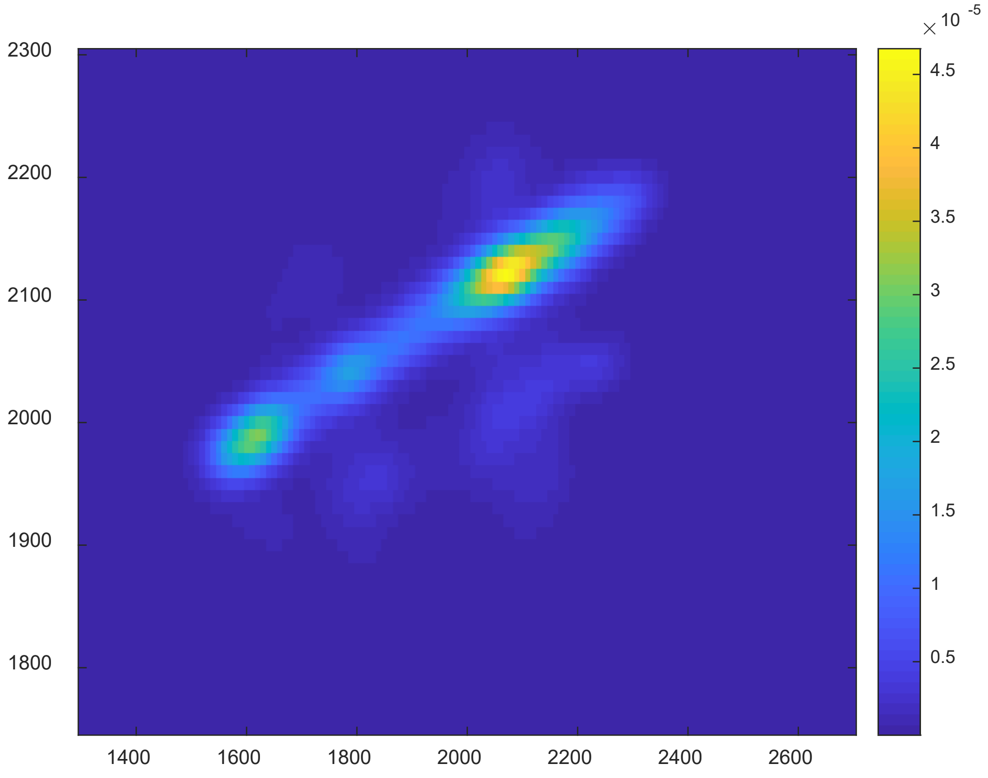


Example two: Marmousi model

Inverted (unconstrained)



Inverted (constrained)



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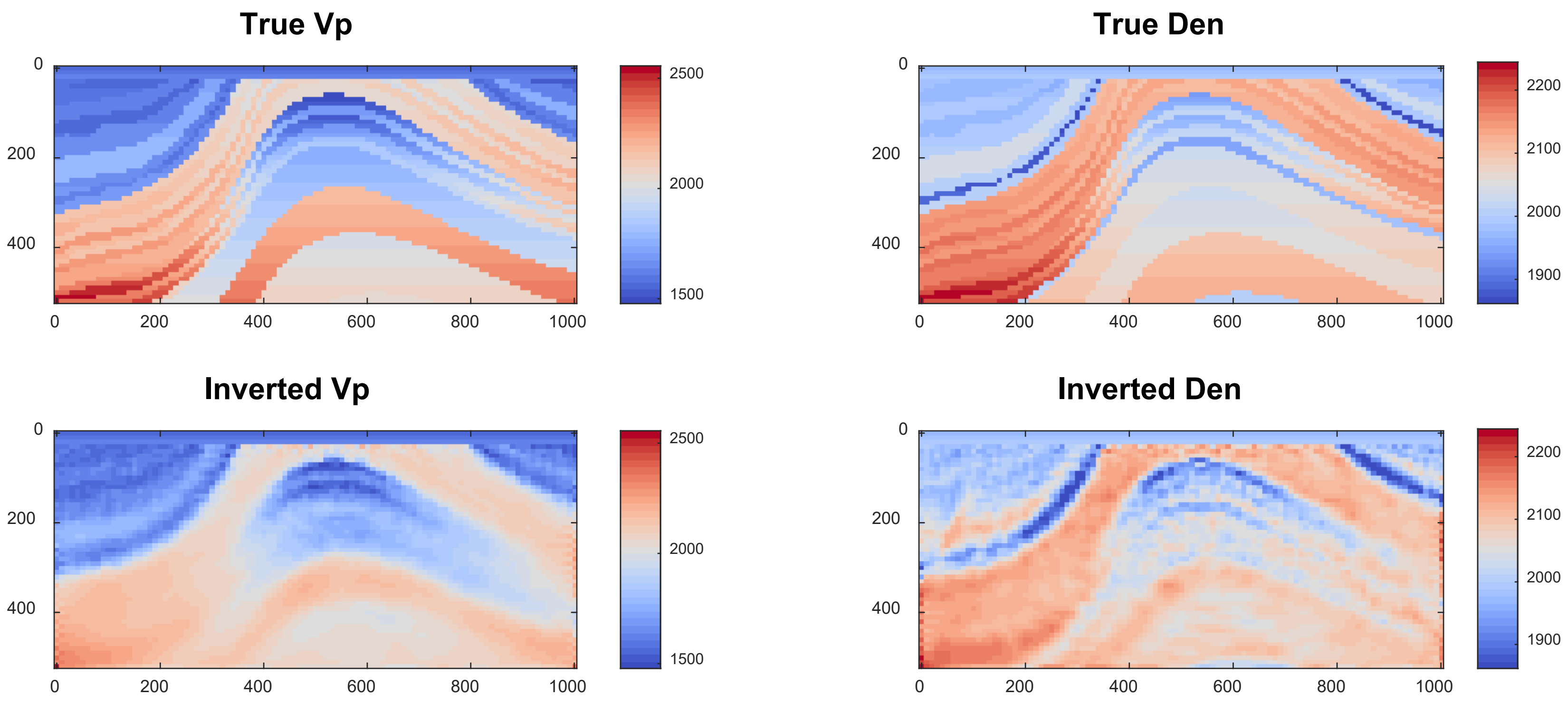
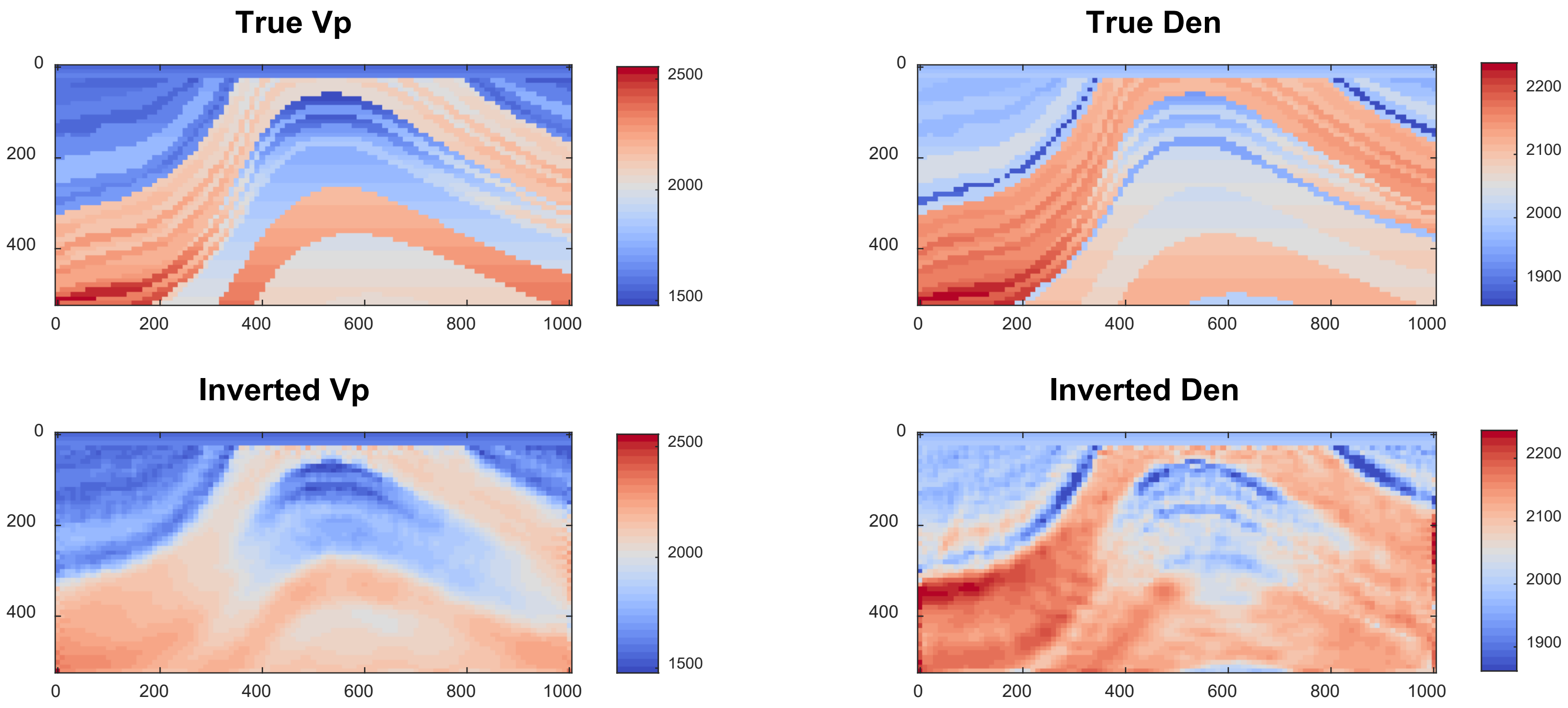


Unconstrained

Example two: Marmousi model



Constrained

Conclusions

- We propose a new FWI scheme to constrain the probability density distribution of the solution.
- We need more analysis to figure out the adaptability of the approach.
- The method can be extended to single-parameter acoustic inversion and three-parameter elastic inversion.

Acknowledgments

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