Seismic reflections from smooth boundaries

John C. Bancroft

CREWES

ABSTRACT

Reflections from impedance boundaries are well understood in many scientific applications including geophysics. These boundaries typically assume a step-like function that reflects the energy of all frequencies, and are consistent with geological processes. Reflections from boundaries with a smooth transition do not reflect all frequencies.

The purpose of this study is to create a numerical modelling environment that accurately propagates a waveform in a variable velocity medium, with the purpose of evaluating the reflection energy from interfaces that have various transition shapes.

INTRODUCTION

Reflection energy from a step interface between two media is defined by Zoeppritz's equations. When the incident direction is normal to the interface, the reflection and transmission coefficients are defined by the simple relationships:

$$R = \frac{Z_t - Z_i}{Z_t + Z_i}, \qquad T = \frac{2Z_i}{Z_t + Z_i},$$

where Z_i and Z_t are the acoustic impedance of the incident "*i*" and transmitted "*t*" media. The amplitude of the reflected energy is the incident waveform, scaled by $-R$, and the transmitted energy scaled by T. The reflection and transmission coefficients are only valid when the transition zone between the two media is very small relative to the wavelengths of the incident waveform. When the transmission zone is much larger than the wavelength of the incident waveform, there is no reflection. All the energy is transmitted, and the amplitude of the transmitted energy is scaled by the square-root of the acoustic impedances,

$$T_{smooth} = \sqrt{\frac{Z_i}{Z_t}}.$$

The traditional scalar wave-equation in one spatial dimension is only valid for constant impedance. For an impedance that varies with space, the following wave-equation is required:

$$\frac{\partial^2 P}{\partial t^2} = \frac{\partial}{\partial z}\left(Z^2 \frac{\partial P}{\partial z}\right) = Z^2 \frac{\partial^2 P}{\partial z^2} + \frac{\partial Z^2}{\partial z}\frac{\partial P}{\partial z}. \qquad \frac{\partial^2 P}{\partial z^2} = \frac{1}{Z^2}\frac{\partial^2 P}{\partial t^2}$$

This equation contains first and second derivatives that are approximated with finite difference equations. High order approximations are required, specifically, 6 points for the derivative, and 7 points for the second derivative.

$$\frac{dP}{dx} \approx \frac{-P_{-3} + 9P_{-2} - 45P_{-1} + 45P_1 - 9P_2 + P_3}{60\delta x}, \qquad \frac{dP}{dx} \approx \frac{P_1 - P_0}{6\delta x}$$

$$\frac{d^2P}{dx^2} \approx \frac{2P_{-3} - 27P_{-2} + 270P_{-1} - 490P_0 + 270P_1 - 27P_2 + 2P_3}{60\delta x}. \qquad \frac{d^2P}{dx^2} \approx \frac{P_1 - 2P_0 + P_{-1}}{\delta x^2}$$

EXAMPLES

I use a simple model where the velocity (impedance) changes from 1000 m/s to 2000 m/s. The time sampling is 0.25 ms, and the depth sampling is 1 m. The chirp isa linear sweep from 20 to 100 Hz over a time of 400 ms. The chirp moves spatially to the right at each time increment. The red line in the following figures represents the shape of the transition zone.

Figure 1 shows a reflected and transmitted wavelet with incorrect amplitudes and reflected polarity because the constant velocity wave equation (WE) was used. Figure 2 shows a moving chirp in (a),(b), and (c), then (d) with a better finite difference (FD) solution, (e) with correct WE but poor FD, and (f) with correct WE and FD, but a step transition.

Figure 3: good transition (3m), Figure 4: transition close to the wavelengths, Figure 5: large transition, Figure 6: modulated carrier.

FIG. 1 Wavelet:
Bad WE bad FD
Wrong trans. amp.
Wrong refl. polarity
Correct lengths

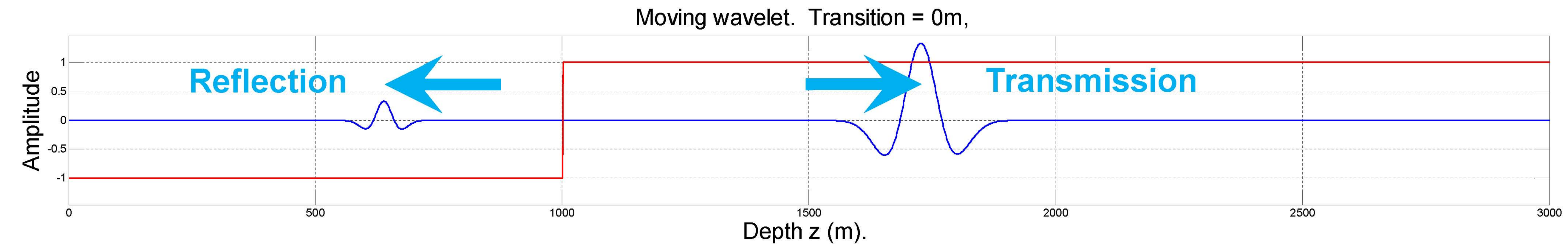

FIG. 2 Chirp:
a Chirp arriving at boundary
b Chirp exiting boundary
c Chirp passed boundary
d Bad WE, good FD
e Good WE, bad FD
f Good WE, bad FD,
good transition (3m)

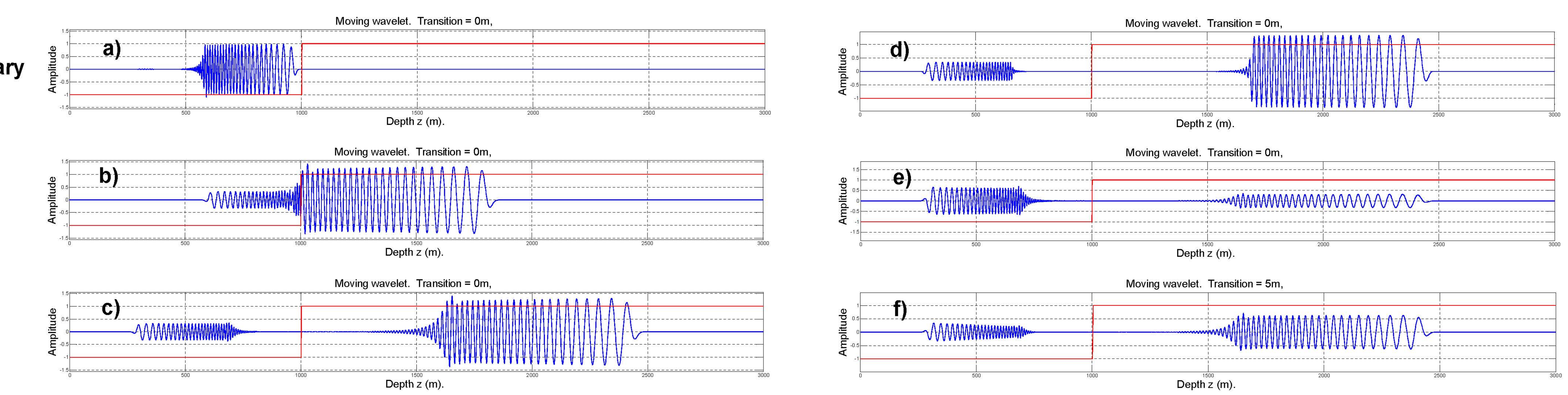

FIG. 3 Chirp:
Good WE good FD
Good transition
Correct amplitudes, polarity, shape, and lengths.

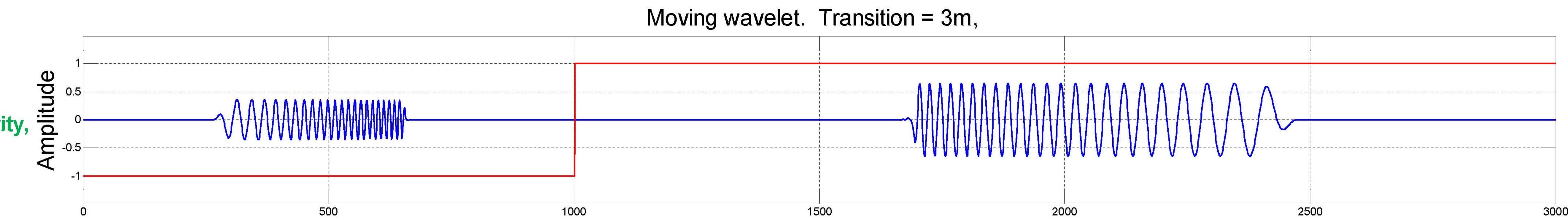

FIG. 4 Chirp:
Transition distance is close to wavelength.
Frequency dependent reflection and transmission

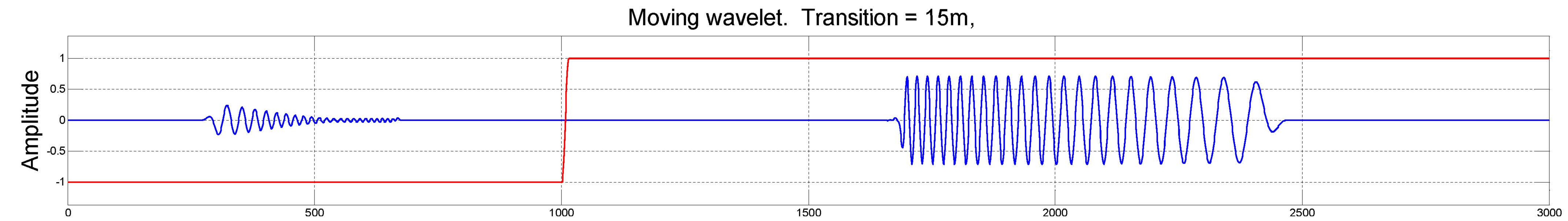

FIG. 5 Chirp:
Transition distance is much greater than wavelength.
All energy transmitted.
Amp = 0.707

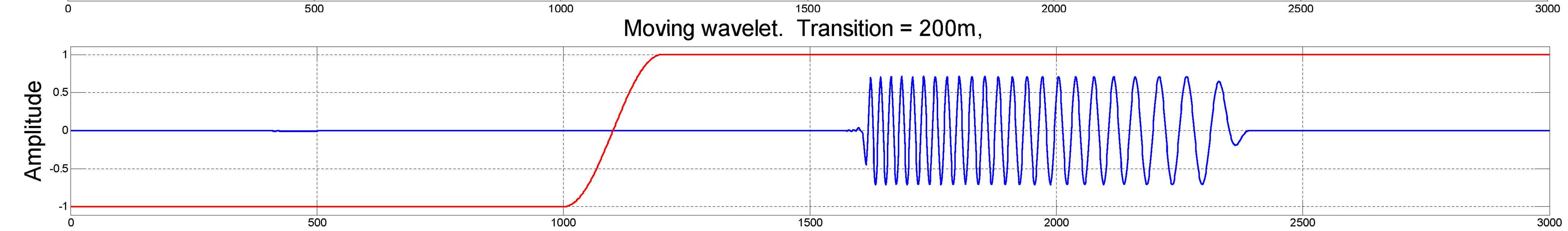

FIG. 6 Modulated carrier:
Fc = 40 Hz, Fm = 30Hz,
Linear transitions