Microseismic sensitivity for four receivers on a square grid

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ABSTRACT

There are a number of analytic solutions for estimating the location of a microseismic event from the first arrival clock-times at three or four receivers. This paper evaluates the sensitivity of four receivers on a square grid with respect to errors in the location of the receivers, jitter on the clock-times, and error in the velocity.

INTRODUCTION

The traveltime equations for raypaths between a source at (x_0, y_0, z_0) and four arbitrarily located receivers at (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), and (x_4, y_4, z_4) are

$$(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2=v^2(t_{01}-t_0)^2$$
$$(x_2-x_0)^2+(y_2-y_0)^2+(z_2-z_0)^2=v^2(t_{02}-t_0)^2$$
$$(x_3-x_0)^2+(y_3-y_0)^2+(z_3-z_0)^2=v^2(t_{03}-t_0)^2$$
$$(x_4-x_0)^2+(y_4-y_0)^2+(z_4-z_0)^2=v^2(t_{04}-t_0)^2$$

where v is the (constant) velocity, t_0 is the clock-time of the source event, and t_{01}, t_{02}, t_{03}, and t_{04}, are the clock-times of the event at the corresponding receivers. These equations are the starting point of the Apollonius solution (Bancroft and Du 2006 and 2007). If we assume the four receivers are coplanar on a square grid, and the side of the square has a dimension h, then the equations in (1) can be simplified to

$$x_0^2+y_0^2+z_0^2=v^2\left(t_{01}-t_0\right)^2$$
$$\left(x_0-h\right)^2+y_0^2+z_0^2=v^2\left(t_{02}-t_0\right)^2$$
$$x_0^2+\left(y_0-h\right)^2+z_0^2=v^2\left(t_{03}-t_0\right)^2$$
$$\left(x_0-h\right)^2+\left(y_0-h\right)^2+z_0^2=v^2\left(t_{04}-t_0\right)^2.$$

The source time t_0 can be extracted as

$$t_0=\frac{t_{01}^2-t_{02}^2-t_{03}^2+t_{04}^2}{2\left(t_{01}-t_{02}-t_{03}+t_{04}\right)},$$

which is totally independent of the geometry. The source coordinates x_0, y_0, and z_0, are computed from

$$x_0=\frac{v^2\left[2t_0\left(t_{02}-t_{01}\right)-\left(t_{02}^2-t_{01}^2\right)\right]+h^2}{2h}$$
$$y_0=\frac{v^2\left[2t_0\left(t_{03}-t_{01}\right)-\left(t_{03}^2-t_{01}^2\right)\right]+h^2}{2h}$$
$$z_0=sqrt\left[v^2\left(t_{01}-t_0\right)^2-\left(x_0^2+y_0^2\right)\right],$$

where the sign of z_0 can be chosen to suit the dimensions of the project.

SENSITIVITY ANALYSIS

Four receivers are on the corners of a square with a side h of 10 m, the velocity is 2000 m/s, and the source is located at (30, 20, -50). The receiver locations are perturbed with random noise, jitter is added to the receiver times, and the velocity is varied. The source location is then estimated from the traveltimes and h. This procedure is repeated for 100 trials with 10% of the outliers removed. The source is a blue "**+**", the receiver a green "x", and the estimated location s"**o**".

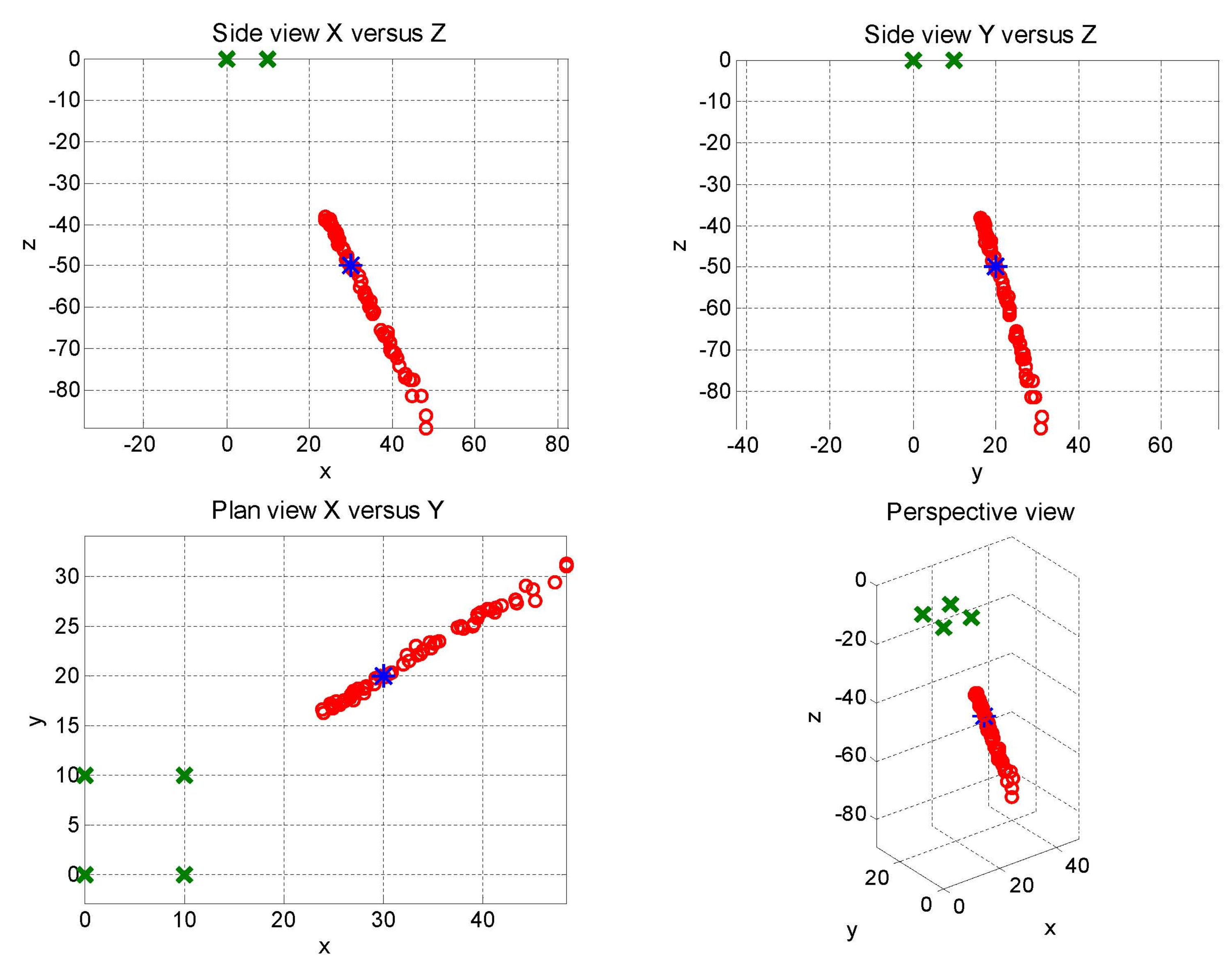

FIG. 1 Source sensitivity when the *x* and *y* coordinates of the receivers are perturbed by 1% or a SD = 0.1 m.

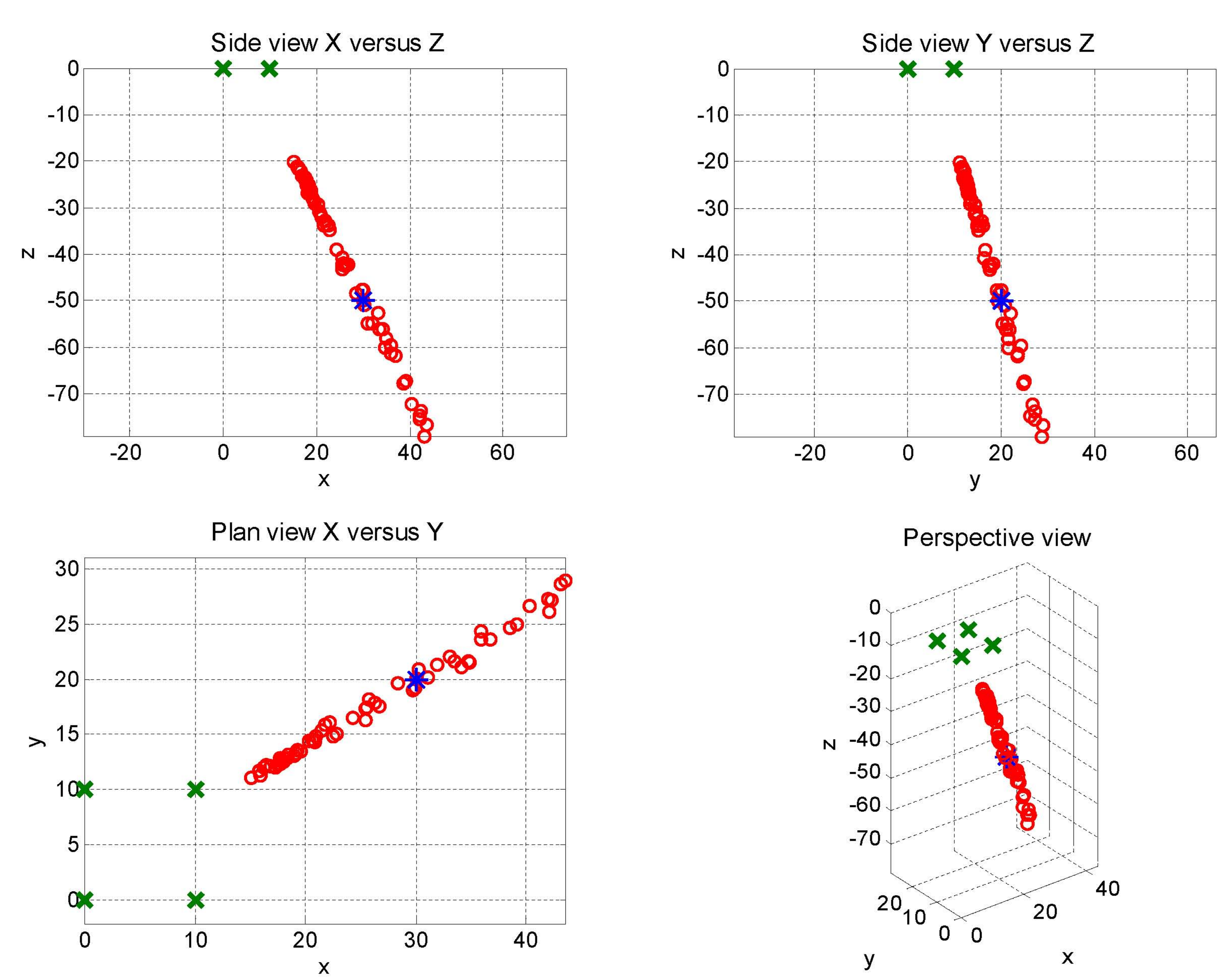

FIG. 2 Plot of source sensitivity when the *z* coordinate of the receivers is perturbed by 1% or a SD = 0.1 m.

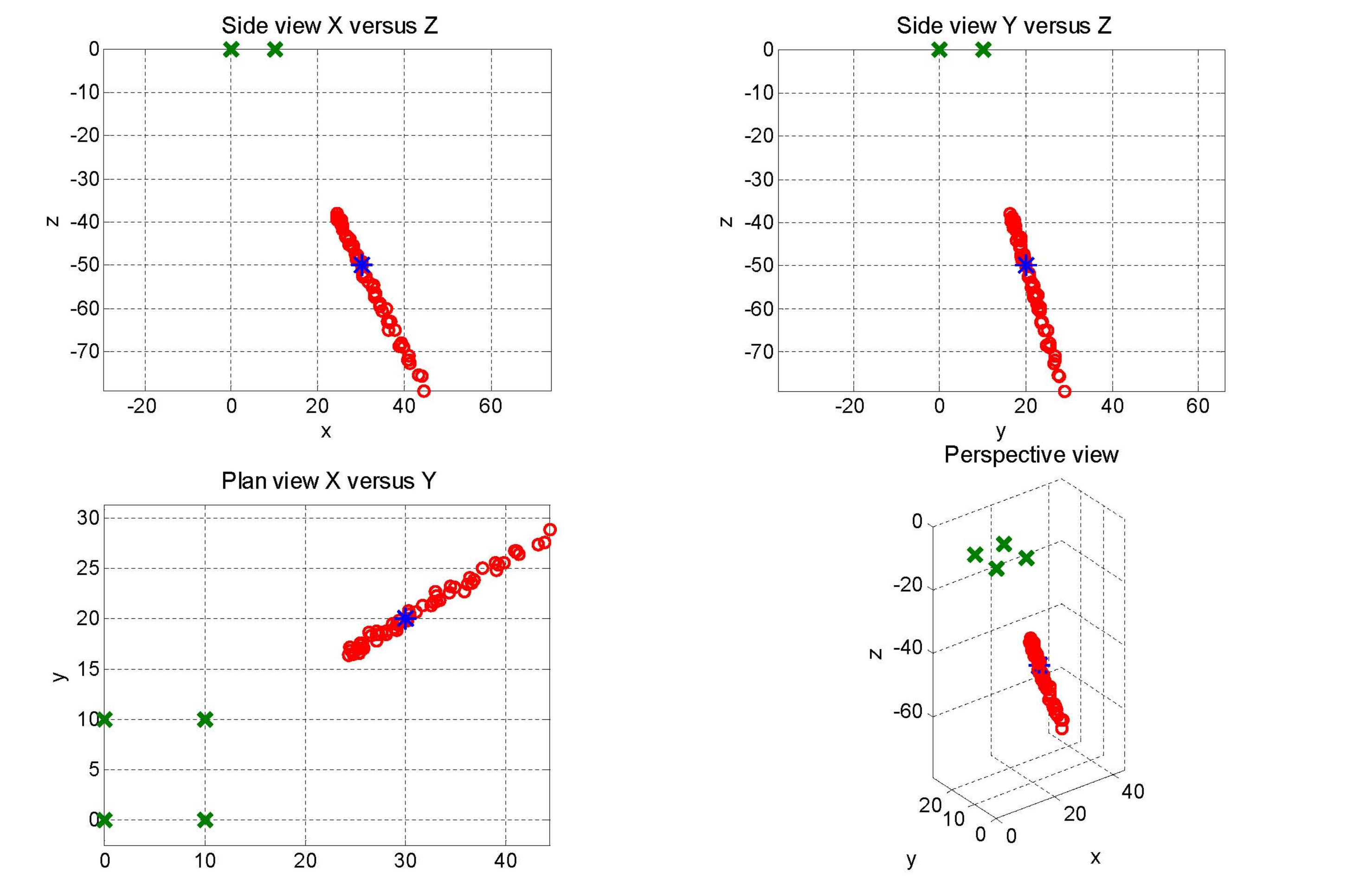

FIG. 3 Plot of source sensitivity when the traveltime from the source to the receivers is perturbed with a jitter that has a SD = 0.025 ms.

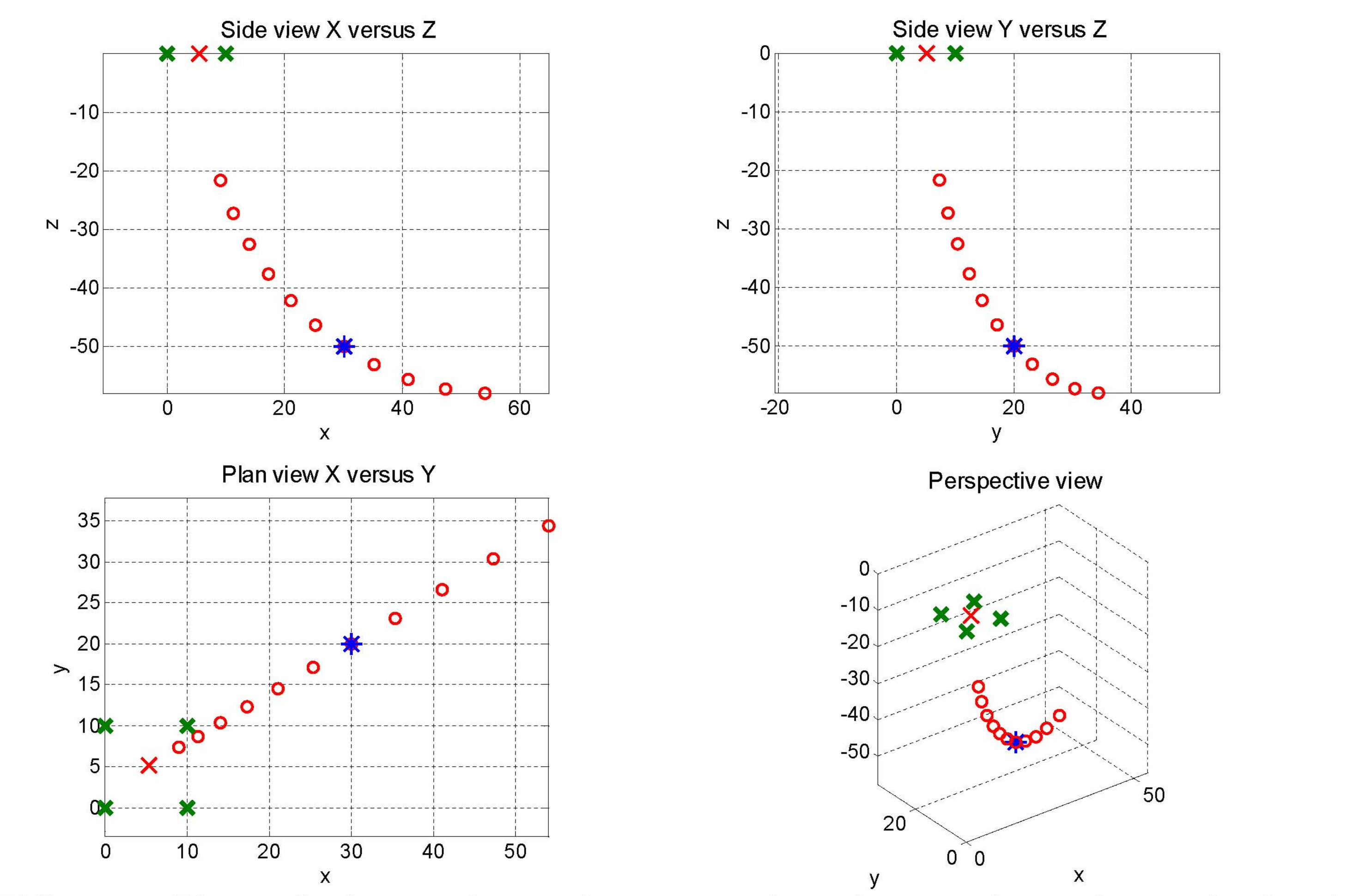

FIG. 4 Plot of the estimated source locations when the velocity is incremented from 0.4v to 1.4v.

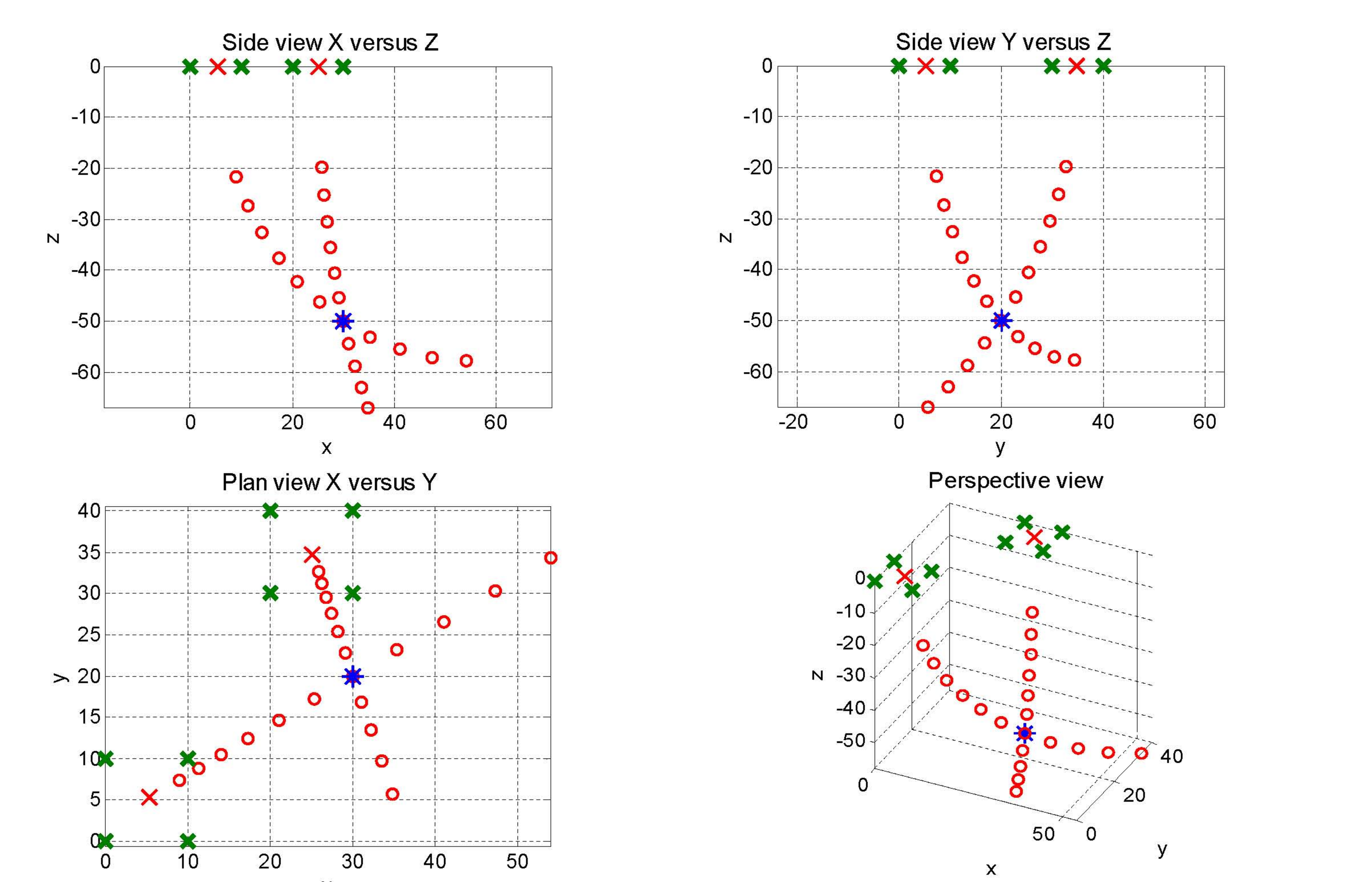

FIG. 5 Two sets of four receivers illustrating intersection of variable velocity curves to define a more accurate velocity and location.