The nearest approach to multiple lines in *n*-dimensional space

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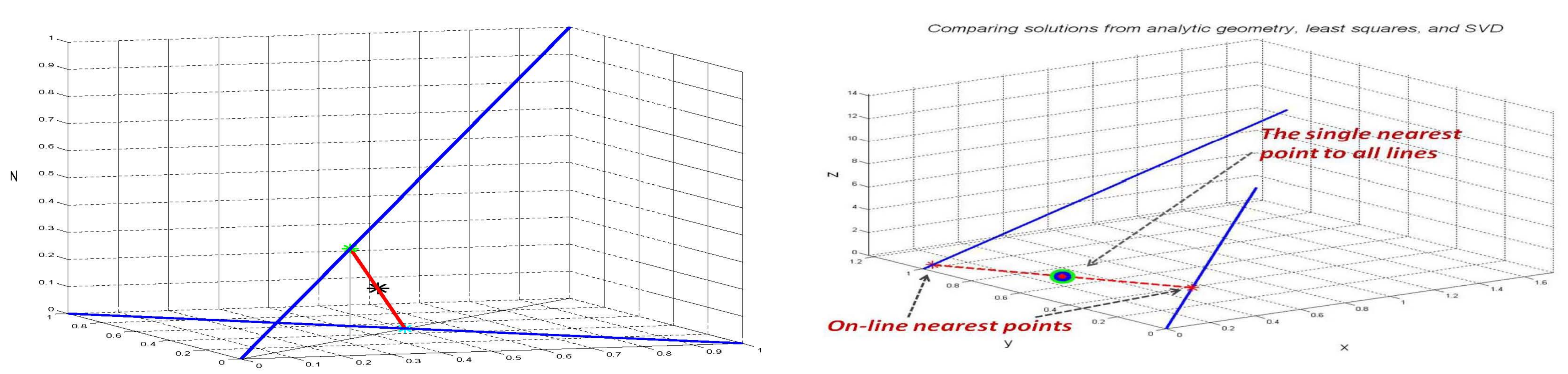
ABSTRACT

As an alternative to analytic methods, a matrix-based method is described which efficiently determines the nearest point(s) to multiple non-intersecting lines in a 3-dimensional or any *n*-dimensional space. The respective set(s) of every on-line nearest points can also be obtained simultaneously in various situations, ensuring efficiency and accuracy at the same time.

METHODOLOGY

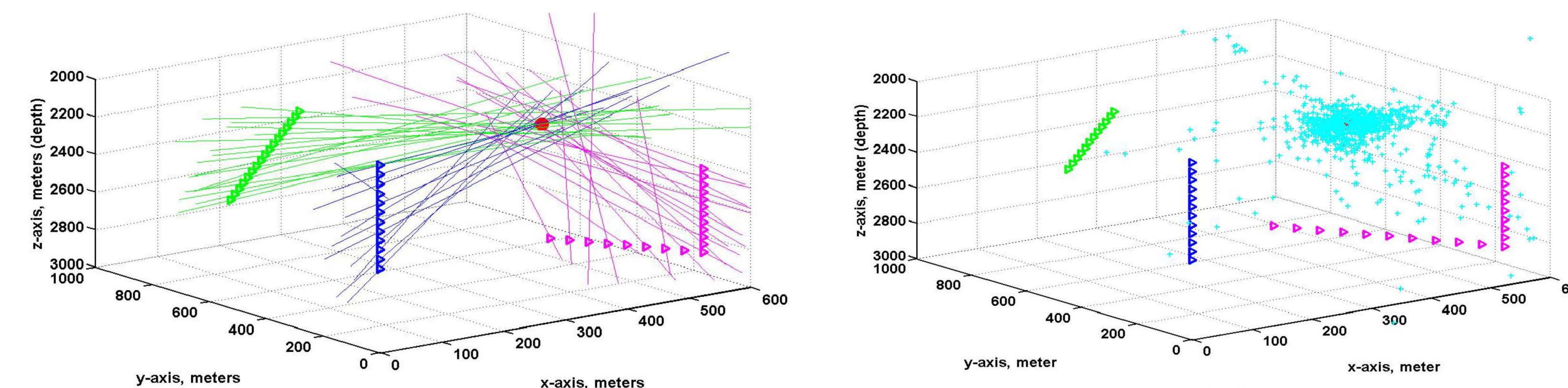
In the three approaches below, understanding the matrix-expanding pattern and then establishing the variant matrix representation of the m-line linear system respective to the number of given lines and the dimension of space is a key step.

1. **The nearest approach to two lines in 3D space:** This approach, used extensively in our seismic application, initiated other two extended approaches which are introduced below.



(a) **(b)**

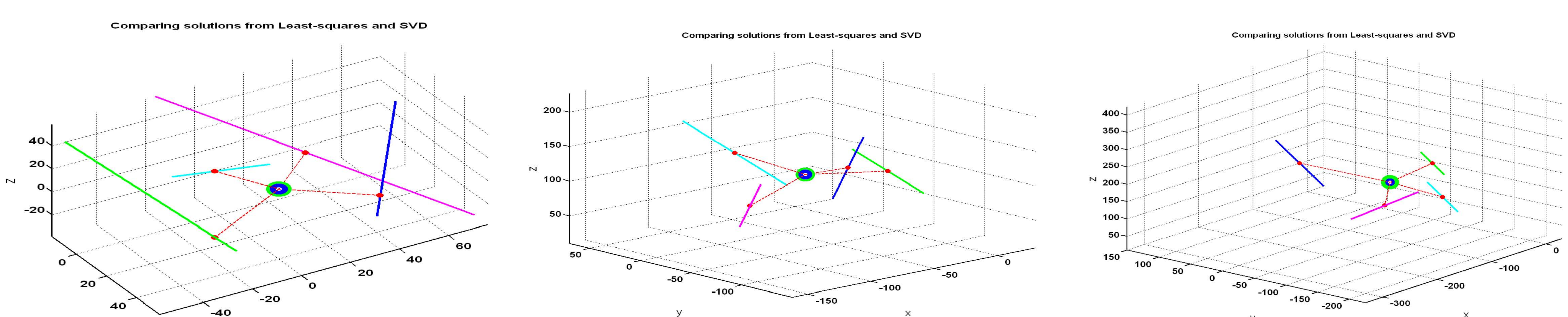
FIG.1. A perspective view of (a) the analytic geometry approach by finding the shortest line (the red dash line), then two on-line points (blue and green stars), and finally the closest point (black star), and (b) the equivalent nearest-point solutions resulting from three approaches in two methodologies: a analytic geometry method (red dot), our matrix method associated with least squares (blue circle), and our matrix method associated with SVD (green circle).



(a) **(b)**

FIG.2. An application of our matrix approach in a seismic project. By simulating a 3-well (color-coded triangles) monitoring system for a single microseism (red dot), we obtain (a) seismic propagations incident at three wells (color-coded raypaths) and (b) mutual nearest points (cyan crosses) by our matrix approach, from raypaths obtained in all three wells.

2. **The nearest approach to multiple lines in 3D space:** In the context here, by "multiple", we mean three or more. The solution at this situation can be extracted from the general solution model as described below.



(a) **(b)** **(c)**

FIG.3. Case illustration of a single, unique solution to the nearest point with four non-intersecting lines (a) none are parallel (b) two parallel pairs (blue and pink; green and cyan) (c) 3 parallel lines (blue, green, and cyan)

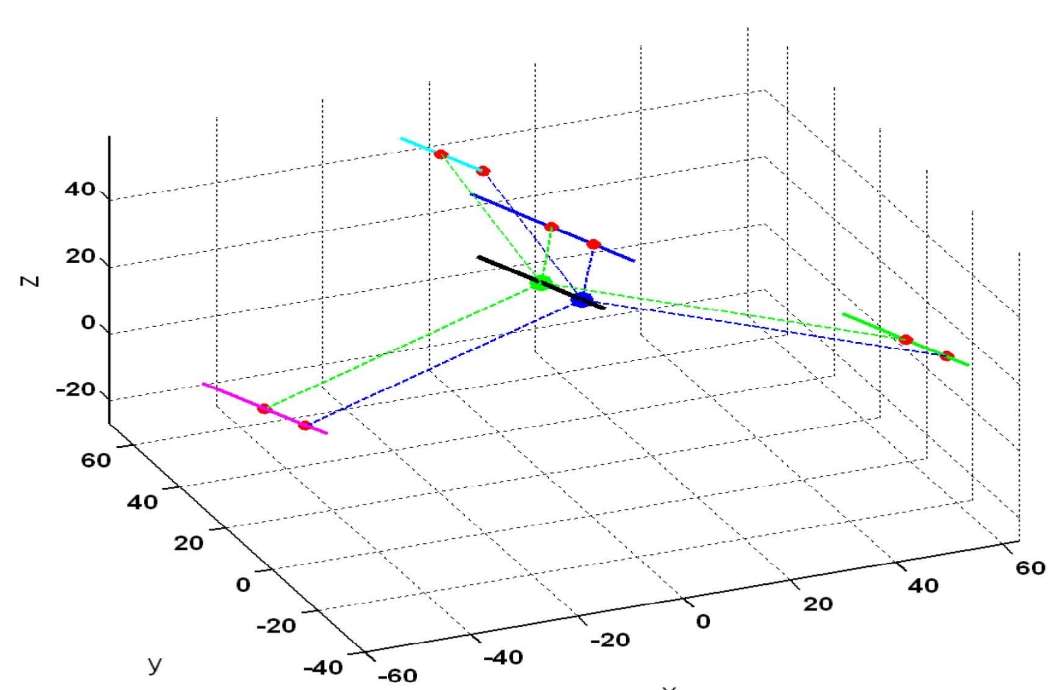


FIG.4. A case of an infinite number of nearest points due to all lines being parallel.

3. **The nearest approach to multiple lines in *n*-dimensional space:** The nearest approach to *m* lines (or vectors) given in a *n*-dimensional space is determined by our variant matrix representation, *Gm=d*, of the *m*-line linear system. The SVD solution to that matrix system provides all information to determine nearest approaches, i.e., point(s) or vector(s) nearest to all lines, and the respective set(s) of on-line points. The different situations are classified as follows:

A. If there is any non-parallel pair, as illustrated in Figure 3, the solution set includes the following:

- ***The single nearest point (or vector) to all given points (or vectors)*:** $nrst_{\_toAll} = m_{svd}[1:n]$, where m_{svd} is the SVD solution of Gm=d.
- ***The m nearest on-line points (or vectors)*:** $\{nrst\_l_1, nrst\_l_2, nrst\_l_3, nrst\_l_4, \ldots\ldots, nrst\_l_m\} = \{P1+U1*m_{svd}[n+1], P2+U2*m_{svd}[n+2], P3+U3*m_{svd}[n+3], P4+U4*m_{svd}[n+4], \ldots\ldots, Pm+Um*m_{svd}[n+m]\}$, where the Pi and Ui pair represents the known point and direction cosines to a set of basis for each line or vector.

B. If all given lines are parallel, as illustrated in Figure 4, the solution set is defined by

- $nrst_{\_toAll} = m_{svd}[1:n] + h * V_0$ where V_0 is a p-element column vector spanning the null space of G^T, and h is a single scaling parameter scaling V_0.

ACKNOWLEDGEMENT

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