Anelastic scattering and AVF/AVA inversion of absorptive reflectivity

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ABSTRACT

This year we have lain some of the groundwork for a scattering theoretic description of anelastic wave propagation. The aim is to create a framework for (1) describing the diffraction and conversion of anelastic waves in heterogeneous media, and (2) directly inverting P, S, and converted wave data taken over dissipative media. This includes expressing reference and perturbed anelastic wave equations in diagonalized forms, which are then prepared for inclusion in an appropriate Scattering, or Lippmann-Schwinger equation. We have also extended the 2009 discussion on AVF/AVA inversion of anelastic reflectivity to incorporate attenuating reference media, and more general attenuation laws. In this poster we summarize some of this work.

Anelastic scattering & reflectivity

Our scattering problem at its most general involves an appropriate casting of reference and perturbed anelastic wave equations, and a scattering potential in the form of a tensor whose elements describe interaction and conversion strengths (FIG 1).

The geophysics literature contains numerous reports of one type of anelastic scattering: frequency-dependent seismic data anomalies associated with attenuating targets. Some researchers have attributed these to the presence of a strong absorptive reflection coefficient, which, indeed, according to wave theory, places a characteristic imprint on the data. This represents a potentially important source of information of direct relevance to, e.g., reservoir characterization. One of the objectives of the work presented here is to develop theoretical insight into the problem of extracting this information.

An absorptive reflection coefficient can be analyzed mathematically and ultimately inverted by considering either its frequency variations (i.e., AVF), or its angle variations (i.e., AVA). We emphasize in particular the issue of separability -- if, and how, it is possible to determine variations in multiple an-acoustic or anelastic medium properties, including Q, occurring simultaneously at a reflecting boundary.

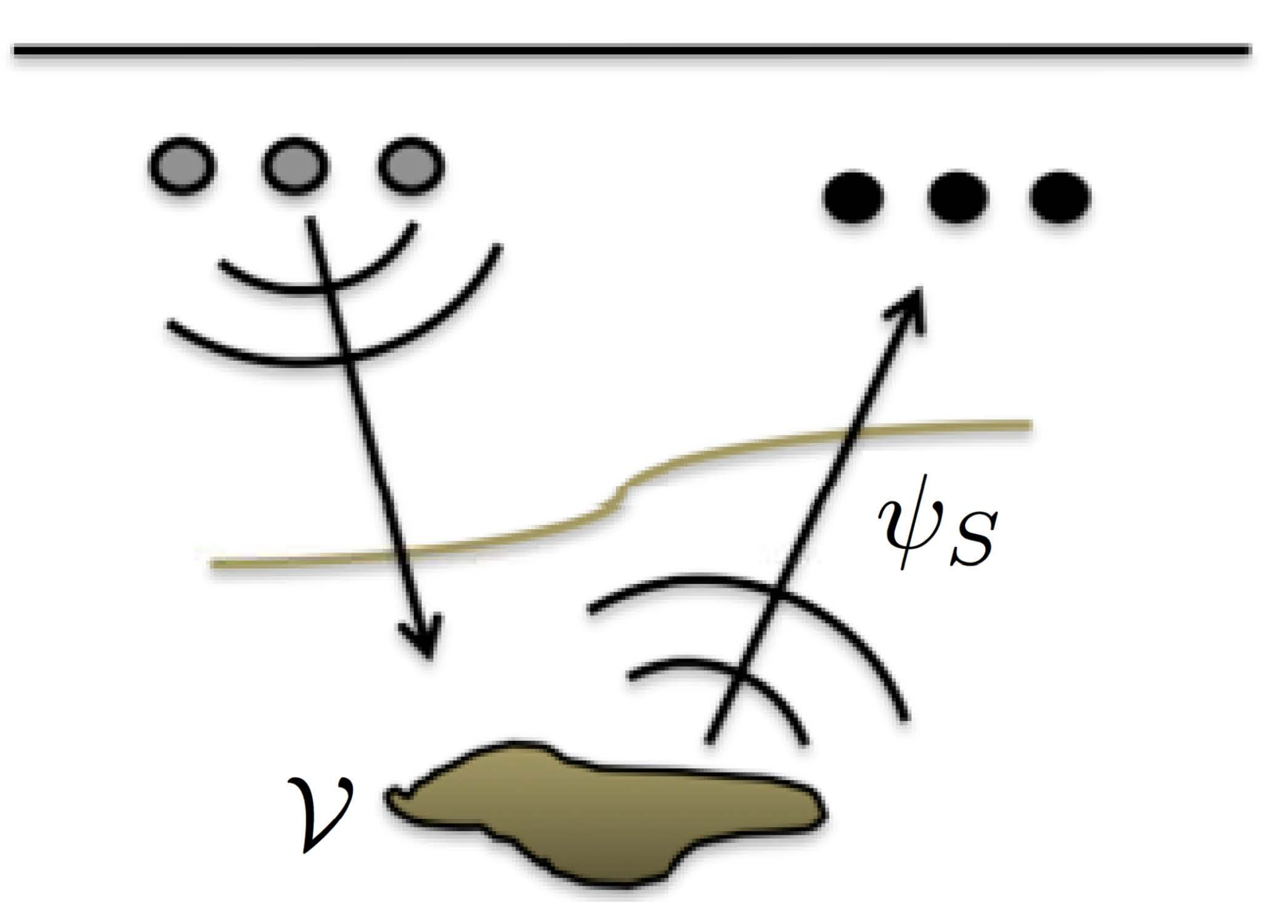

FIG. 1. Schematic diagram containing the general elements of the anelastic scattering problem. This includes wave equations describing propagation in a smooth or homogeneous background medium (white) and a discontinuous perturbed medium (green), and perturbation operators V describing the difference between the two. The perturbed field ψ_S is the result of a nonlinear combination of the reference field and V.

AVF/AVA modeling and inversion

We have made progress by considering the scattering sub-problem of modeling and inverting anelastic reflectivity. The scattering problem reduces to an expansion of the reflection coefficient about P-wave velocity and Q perturbations a_c and a_Q:

$$R(k_z,\theta) = \left[\left(\frac{1}{4}a_c - \frac{1}{2}F_{kz}(\theta)a_Q\right) + \left(\frac{1}{8}a_c^2 + \frac{1}{4}F_{kz}^2(\theta)a_Q^2\right) + ...\right](\sin^2\theta)^0$$
$$+ \left[\left(\frac{1}{4}a_c - \frac{1}{2}F_{kz}(\theta)a_Q\right) + \left(\frac{1}{4}a_c^2 - \frac{1}{2}F_{kz}(\theta)a_c a_Q + \frac{3}{4}F_{kz}^2(\theta)a_Q^2\right) + ...\right](\sin^2\theta)^1$$
$$+ \left[\left(\frac{1}{4}a_c - \frac{1}{2}F_{kz}(\theta)a_Q\right) + \left(\frac{3}{8}a_c^2 - F_{kz}(\theta)a_c a_Q + \frac{5}{4}F_{kz}^2(\theta)a_Q^2\right) + ...\right](\sin^2\theta)^2$$
$$+ ... \; .$$

This and other forms may be truncated at various orders and examined as approximations (FIG 2).

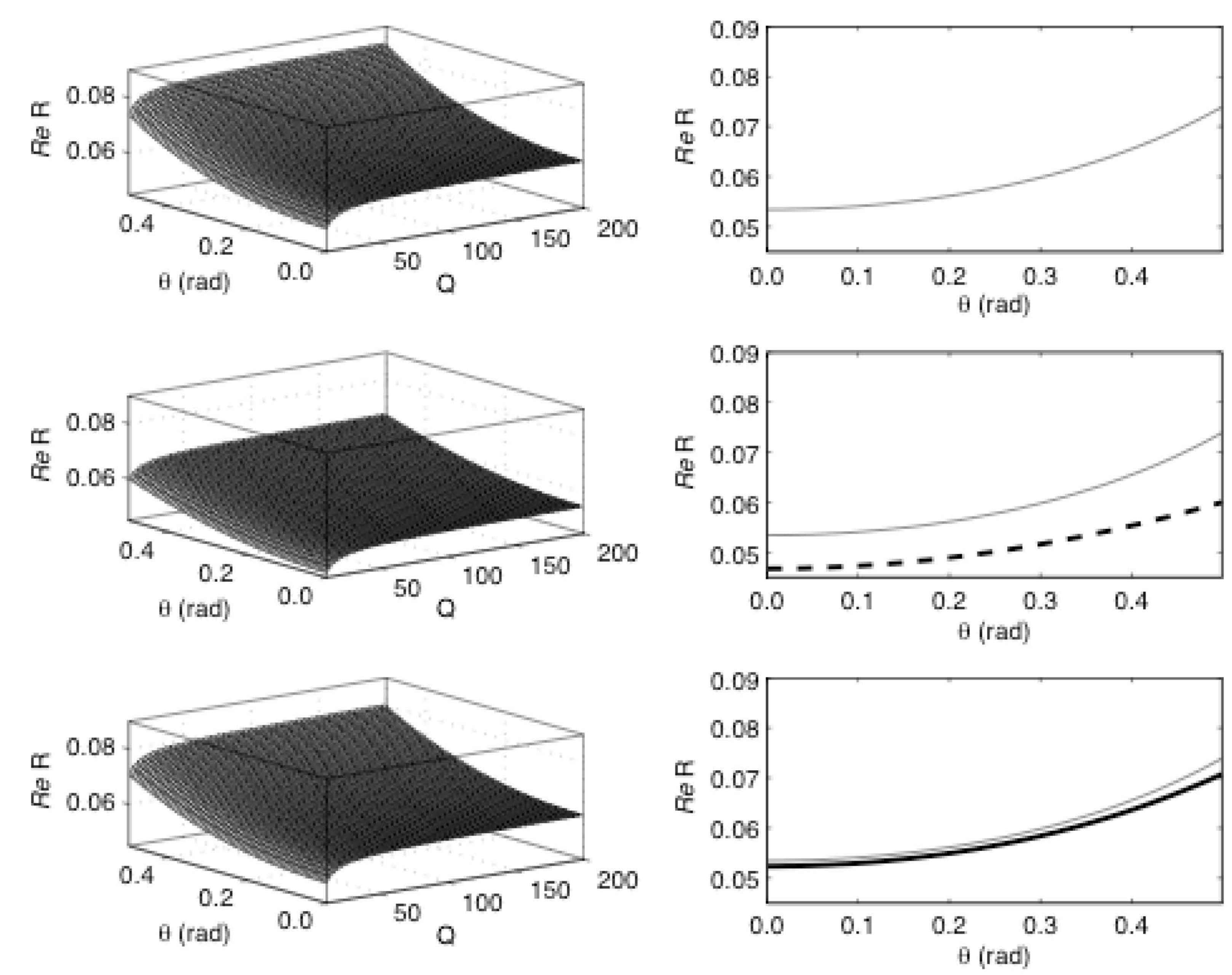

FIG. 2. Various AVA plots of attenuative R. Top row: exact R for a range of angles and target Q values (left), and a range of angles for low Q=10 (right). Middle row: linear approximation (on the right the approximations is dashed, compared to exact in black). Bottom row: second order approximation vs. exact. We conclude nonlinear R(Q) relationship will play an important role, with second order corrections likely sufficient.

The 'anacoustic' problem generalizes to the anelastic problem by segmenting the determinants of the Zoeppritz equation based on order in any of five parameter contrasts (V_P, V_S, ρ, Q_P and Q_S; for details, please see report):

$$R_P = \frac{\hat{\det}\mathcal{A}_P}{\hat{\det}\mathcal{A}} = \hat{\det}\mathcal{A}_P^{(1)} + \left(\hat{\det}\mathcal{A}_P^{(2)} - \hat{\det}\mathcal{A}_P^{(1)}\hat{\det}\mathcal{A}^{(1)}\right) + ...,$$

$$R_S = \hat{\det}\mathcal{A}_S^{(1)} + \left(\hat{\det}\mathcal{A}_S^{(2)} - \hat{\det}\mathcal{A}_S^{(1)}\hat{\det}\mathcal{A}^{(1)}\right) + ...,$$

Which may be used for modeling (FIG 3) and direct inversion (FIG 4).

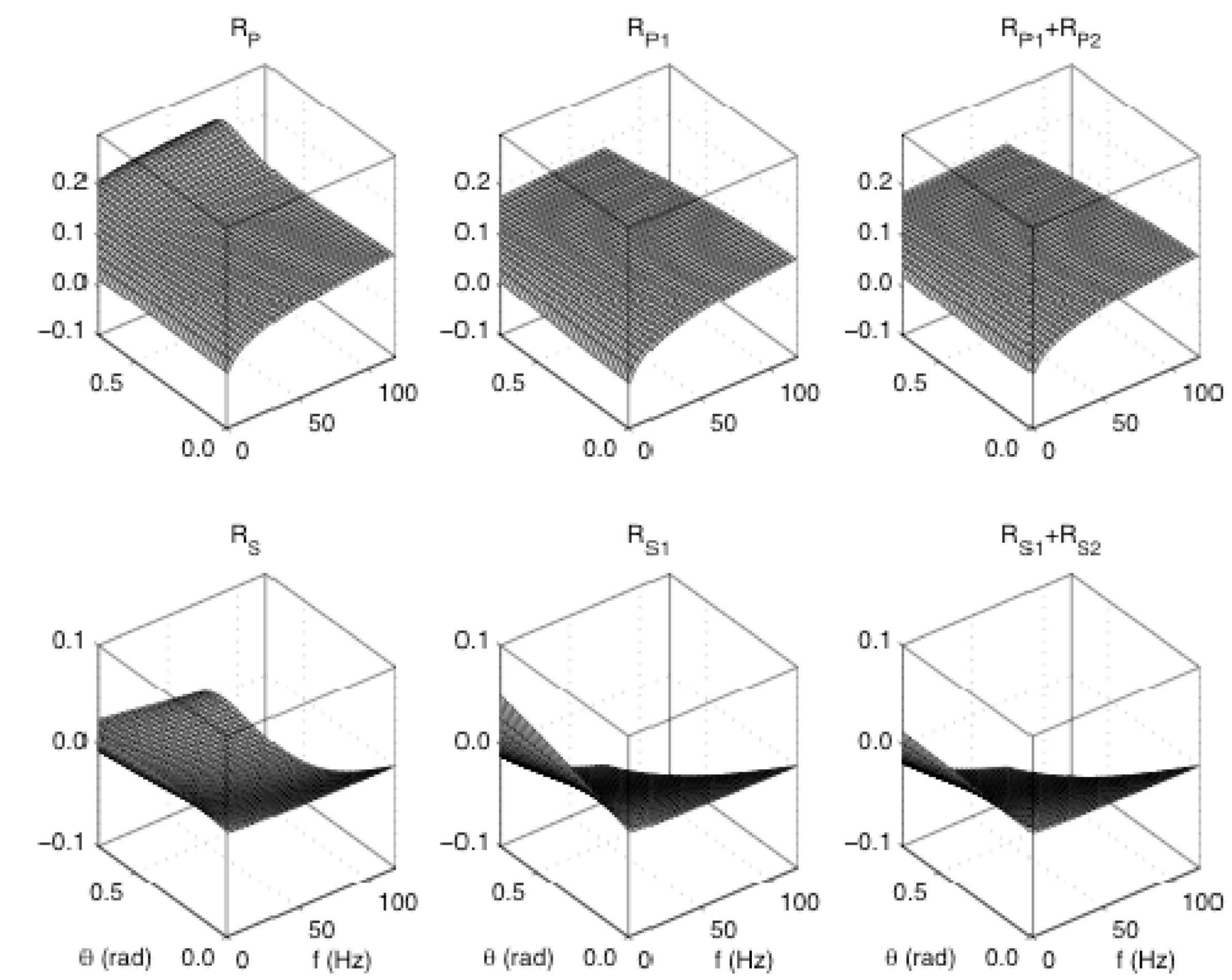

FIG. 3. R_P and R_S over a strongly anelastic target. Left column: exact coefficients; middle column: first order approximate; right column: second order approximate.

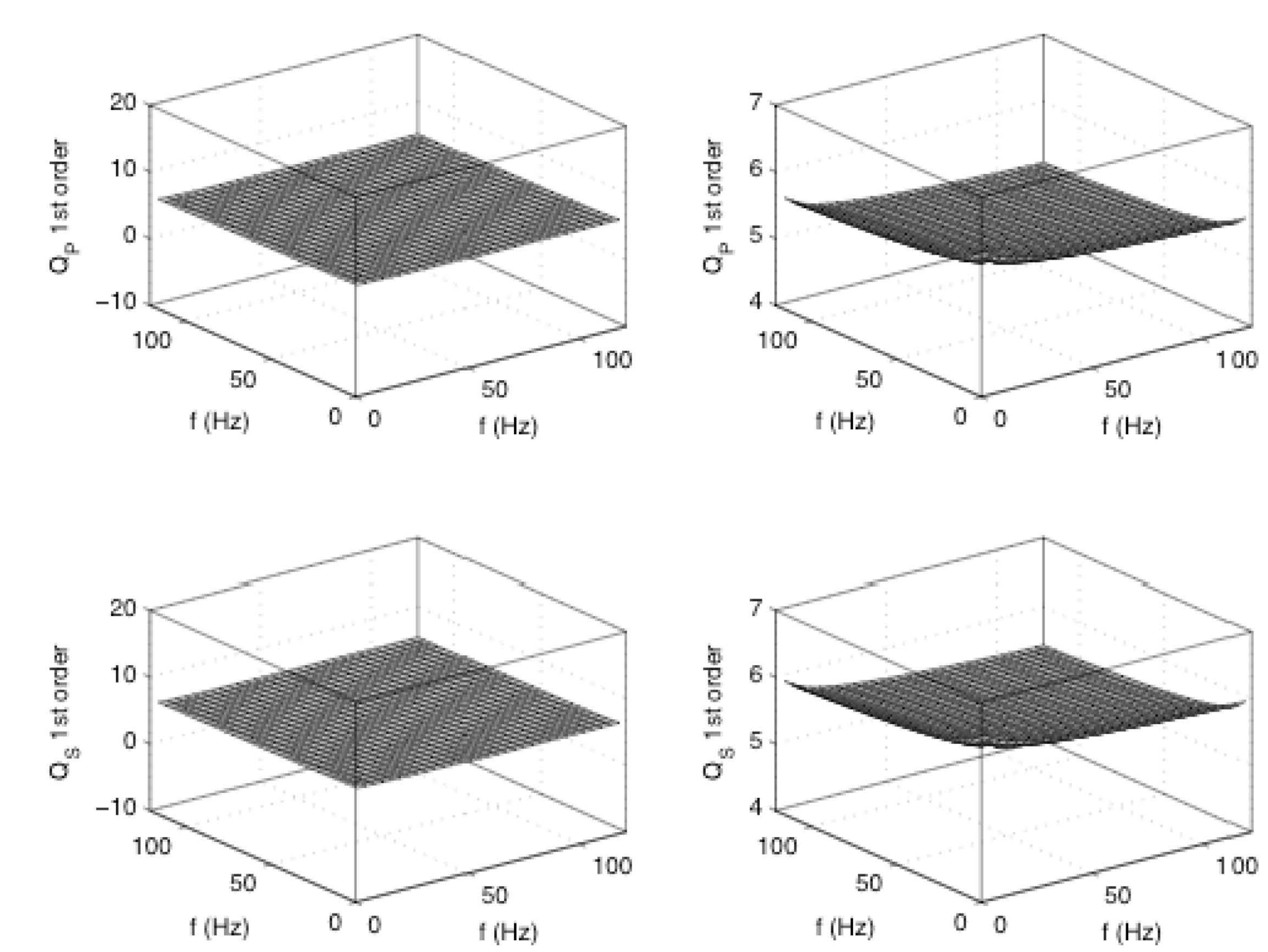

FIG. 4. Linear AVF recovery of Qp and Qs from reflectivity measurements over a range of frequency pairs. Left column: actual Qp=Qs=5; right column: detail of same.

CONCLUSIONS

In addition to continuing with the basic theory, Bird et al. (this report) are designing a set of procedures for extraction of these reflectivities from the seismic trace. In the coming year we will incorporated the lessons of the reflectivity study into the more general anelastic scattering problem.