Decomposition of acoustic/elastic R_P into contributions from 1-parameter reflection coefficients

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ABSTRACT

In AVO/AVA inversion, a linearized form of the Zoeppritz equations known as the Aki-Richards approximation and variants are used to model R_P. This approximation can be viewed as a linear decomposition of the full reflection coefficient into contributions from the reflectivities of individual medium parameters. A forward/inverse series framework leads to an alternative approach to this type of decomposition. The first order terms in the decomposition are qualitatively similar to the Aki-Richards approximation, with second- and third-order terms correcting the approximation at large angle and large contrast. We test the approach both for acoustic and elastic reflection coefficients. In the elastic case, where forward/inverse methods of the kind we use require both the incorporation of R_P and R_S, we proceed in an approximate fashion using R_P only. The elastic nonlinear corrections, in spite of the approximation, provide a significant increase in accuracy over the linear/Aki-Richards approximation in several large contrast/large angle model regimes. Separately determining individual reflectivities could provide useful input to bandlimited impedance inversion algorithms, or the ability to extrapolate data from small to large angle.

Introduction

Practical inversion of amplitude information in reflection seismic data is based on linear-approximate solutions of the Zoeppritz equations, in particular that of Aki & Richards (hereafter AR). Although the Zoeppritz equations can be solved numerically, the linearized solutions have historically won out over the more complex exact forms as practical tools. One of the reasons for this is that the linear approximations represent direct decompositions of the full R_P coefficient into weighted contributions from reflectivities due to individual parameter variations (e.g., Goodway et al., 2006), seen in the fractions $\Delta\rho/\rho$etc.:

$$R_P(\theta) = \frac{1}{2}\left(\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}\right) - 2\frac{V_S^2}{V_P^2}\sin^2\theta\left(2\frac{\Delta V_S}{V_S} + \frac{\Delta\rho}{\rho}\right) + \frac{1}{2}\tan^2\theta\frac{\Delta V_P}{V_P},$$

The power of this type of decomposition, beyond its simplicity and easy-to-analyze form, is that, with individual reflectivities in hand, well-developed methods for normal-incidence, single-parameter bandlimited impedance inversion may be straightforwardly employed to complete the inversion.

In this paper we will take another approach, using the tools of direct inversion, originally developed for the determination of parameter contrasts from reflection amplitudes, to perform the decomposition. We begin by considering various acoustic configurations, i.e., reflections from contrasts in sets of parameters with acoustic analogues (e.g., including P-wave velocity, density, Q, etc., but not S-wave velocity). We develop a formula for the linear and nonlinear reconstitution of the full acoustic multiparameter reflection coefficient in terms of the relevant individual reflectivities. Remarkably, within this multiparameter acoustic configuration, the same formula is found to approximate R, regardless of which parameters vary, how many of them vary, and regardless of which experimental variable(s) R varies over. We then proceed to the elastic problem. The resulting formulas are only approximate, since the full problem must be posed using contributions from both R_P and R_S reflectivities, but in many regimes of large contrast/angle the accuracies of the approximate reconstitutions of R_P greatly exceed that of the AR approximation. We end by discussing some of the consequences of this approach to AVO modeling and to inversion, and some potentially fruitful directions in which to push this research in the near future.

Acoustic case

Let R_P be the reflection coefficient associated with an interface across which N acoustic parameters, $\mu=(\mu_1, \mu_2, \ldots \mu_N)$, have varied, from μ^0 in the incidence medium, to μ^1 in the target medium. For instance, these μ might represent P-wave velocity and density, in which case $\mu= (c,\rho)$ varies from $\mu^0=(c^0,\rho^0)$ to $\mu^1=(c^1,\rho^1)$. We introduce N additional reflection coefficients $R_\mu = (R_{\mu 1},R_{\mu 2},\ldots R_{\mu N})$, where $R_{\mu i}$ is the reflection coefficient associated with an interface across which only μ_i has changed. For instance, R_ρ is the reflection coefficient associated with an interface across which density varied from ρ^0 to ρ^1, and all other parameters remained constant. The full R_P is decomposable into the individual reflectivities through, explicitly to third order,

$$R_P = \sum_{i=1}^{N} R_{\mu_i} - \frac{1}{3}\left[\left(\sum_{i=1}^{N} R_{\mu_i}\right)^3 - \left(\sum_{i=1}^{N} R_{\mu_i}^3\right)\right] + \ldots,$$

…with fifth-order and higher corrections available if desired.

Example I. If the P-wave velocity of an acoustic medium c_0 varies across a boundary to become c_1, and the medium takes on a finite Q_1, we have that

$$R(\omega) \approx R_c + R_Q(\omega) - \left[R_c^2 R_Q(\omega) + R_Q^2(\omega) R_c\right]$$

Where $R_c = (c_1-c_0)/(c_1+c_0)$ etc. are the simple 1-parameter reflectivities. In FIG. 1 the first, third, and fifth order reconstitutions of R are plotted against the exact value for a range of frequencies.

Example II. If the P-wave velocity of an acoustic medium c_0 and its density ρ_0 varies across a boundary to become c_1 and ρ_1, and the medium again takes on a finite Q_1, we have that

$$R = R_c + R_\rho + R_Q - R_c^2(R_\rho + R_Q) - R_\rho^2(R_c + R_Q) - R_Q^2(R_c + R_\rho) - 2R_c R_\rho R_Q + \ldots$$

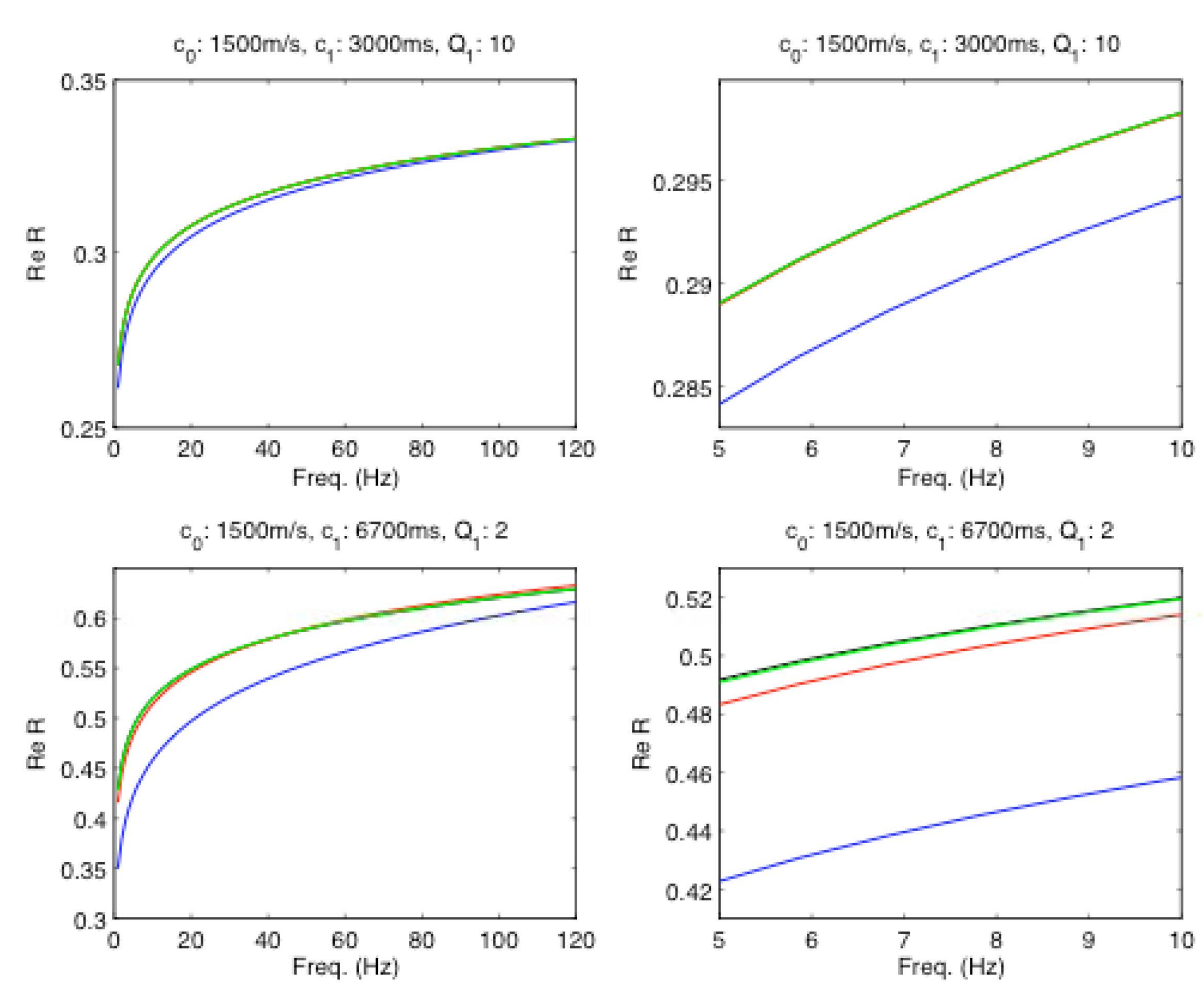

FIG. 1. Anacoustic reflection coefficient approximations. Black: exact; blue: linear; red: third order; green: fifth order. Top left: model 1; top right: detail. Bottom left: model 2; bottom right: detail.

Elastic case

Let us consider the extension of the previous methods to the three parameter elastic case. For an incident P-wave, there are two reflected modes, PP and PS. In order to correctly decompose either R_P or R_S using our approach, both data types must be invoked. We leave that for future research. Posing the inconsistent version of the problem, involving R_P only, in many important large contrast circumstances highly accurate approximations are produced. R_P is decomposed in terms of V_P, V_S and preflectivities through the formula

$$R_P(\theta) = R_1(\theta) + R_2(\theta) + R_3(\theta) + \ldots$$

where

$$R_1(\theta) = R_\alpha(\theta) + R_\beta(\theta) + R_\rho(\theta),$$

$$R_2(\theta) = W_1\, R_\beta^2(\theta) + W_2\, R_\beta(\theta) R_\rho(\theta),$$

etc. Higher orders and the factors W_i, which are simple functions of incidence angle θ, are included in the companion report. The reconstitution of $R_P(\theta)$ for 3 large contrast models are examined in FIG 2. The linear result (in blue) is often close to the AR approximation in relative error. Already at third order, R_P magnitudes out to the critical angle track very well with the exact values.

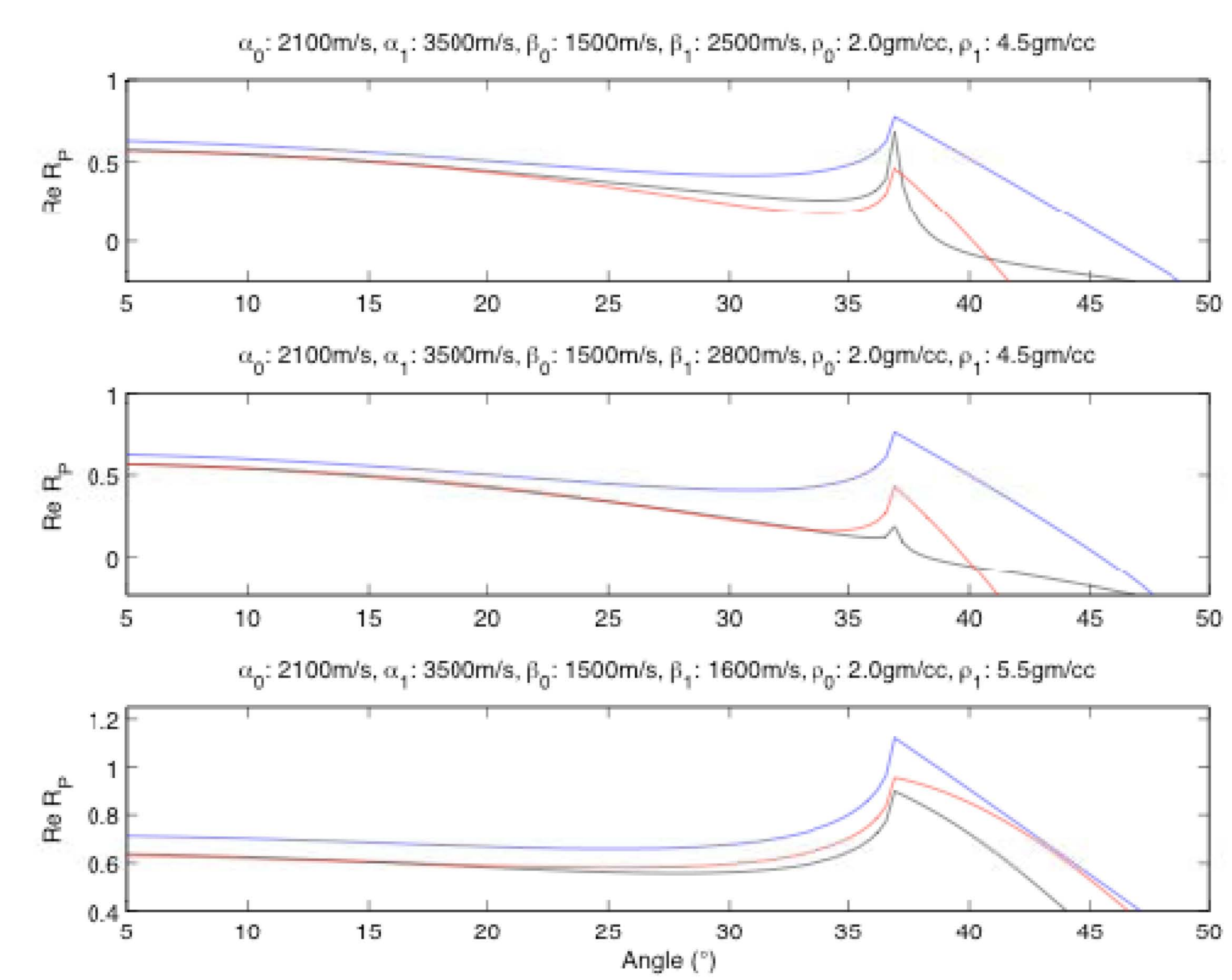

FIG. 2. Decomposition of elastic RP into 1-parameter reflectivities. Black: exact RP; blue: linear decomposition; red: third order. This approximation seems to perform particularly well in comparison to the AR approximation when either all three parameters undergo large contrasts, or VP and pundergo large contrasts.

CONCLUSIONS

There are several ways these relationships could be used. Each constitutive reflectivity could be estimated and evaluated at θ=0. Then, impedance inversion could determine multiple parameter profiles. Also, extrapolation from limited offsets becomes more stable given increased modeling accuracy. Finally, there is no reason to limit ourselves to $V_P/V_S/\rho$. Standard impedance, or Lamé impedance reflectivities could be used as the underlying "basis functions" if desired.