The Priddis pump-probe experiment and beyond: theories for seismic-seismic & seismic-radar interaction

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ABSTRACT

As we take closer and closer looks at the amplitudes in our geophysical data, we would be well-advised to prepare for the whole universe of nonlinearity that lurks in the smaller variations of the data.

I. In 2008, the Priddis pump-probe experiment, in which the Earth was simultaneously subjected to a vibrating source and a transient source, was carried out by CREWES with the hope of detecting nonlinear behaviour in an exploration seismology setting. Here we present a theoretical description of such behaviour, assuming an acoustic medium. Through a simple extension of the order arguments by which the fluid equations are manipulated to form a linear wave equation on the pressure p_1, we find a nonlinear equation for a corrective term, p_2, such that in total the field, $p=p_1+p_2$, is aware of the changes it itself makes in the medium through which it propagates.

II. A seismic disturbance alters the electrical properties of the Earth. This means, in principle, that within Earth volumes supporting both types of wave propagation, a radar wave field will tend to scatter from a seismic wave field. Given seismic disturbances with length-scales on the order of that of the radar pulse, such interaction may be detectable as back-scattering phenomena. Given seismic disturbances with length-scales much larger than that of the radar pulse, the interaction may be detectable as forward-scattering (e.g., anomalous traveltime) phenomena. Radar data with these characteristics would lend themselves readily to techniques of imaging-inversion, migration, and tomography, and would present the potential for providing "snapshot" images of a seismic wavefield during important stages of its evolution, e.g., as it propagates in poorly characterized and unconsolidated near surface structures.

I. Nonlinear seismology in an exploration setting

In 2008, CREWES researchers carried out and presented a discussion of the Priddis pump-probe experiment, in which an Earth volume was simultaneously subjected to a strong vibrating source and a transient source. At heart, this was an investigation into the presence or absence of nonlinear phenomena in exploration seismology as we carry it out today. Whether or not, that is, seismic waves influence themselves, scattering from each other and altering their own amplitudes (FIG. 1), and whether or not such phenomena could conceivably rise above the noise level in our records.

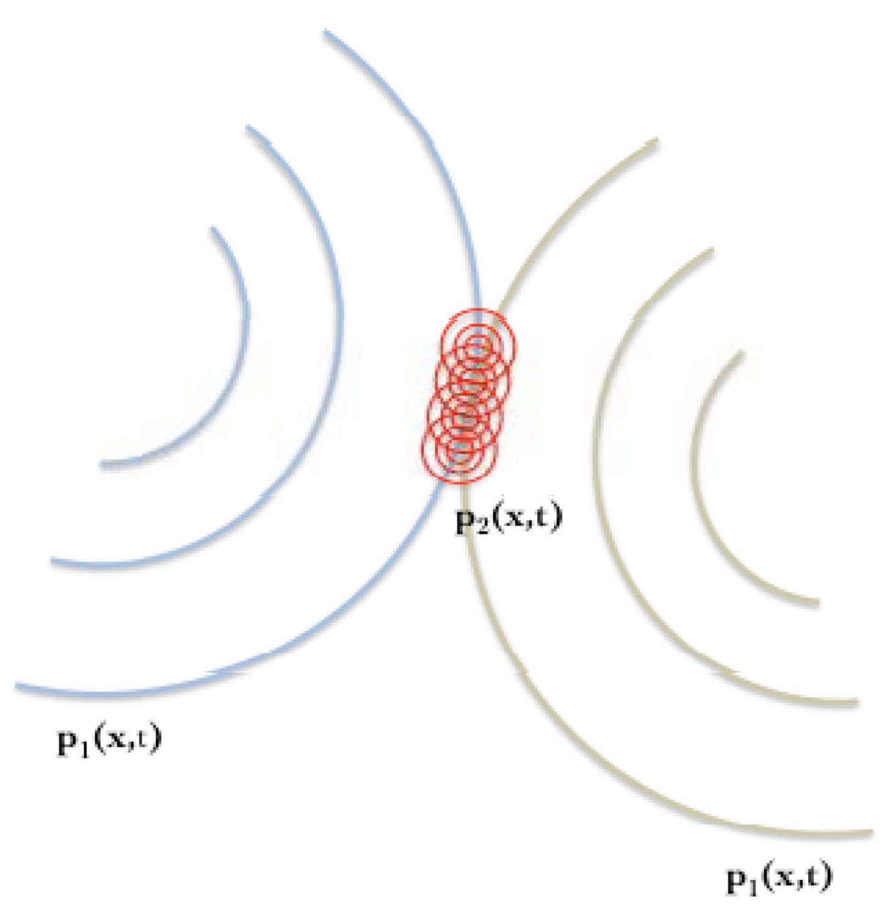

FIG. 1. Seismic waves alter the density and moduli of the Earth as they pass. In principle this means that two incident seismic fields might be seen to scatter from each other. Indeed the amplitude of a local portion of the seismic wave should be expected to influence itself, if its amplitudes are large enough. But if a wave, which depends on the properties of the medium it is in, changes those properties wherever it actually exists, how do you start the problem?

From a theoretical point of view, the idea is far from outlandish. Since normal seismic waves are predicted by equations of motion that have been explicitly linearized, it is simply the *low amplitude* of nonlinear seismic phenomena that best explains their current absence. This invisibility may not last, given ongoing advances in instrument sensitivity, survey design, and processing methods, in particular coupled with a newfound interest in recording seismic data from shots set off simultaneously.

We use no underlying equations that are not used in the development of linear wave theory. Following Landau & Lifshitz (1959), we begin with fluid equations

$$\frac{\partial p}{\partial x_j} = -\rho\left(\frac{\partial v_j}{\partial t} + v_i\frac{\partial v_j}{\partial x_i}\right), \qquad \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_j)}{\partial x_j} = 0,$$

but we consider not two, but three magnitude scales over which the three variables, density, pressure and fluid velocity vary:

$$\begin{aligned} p &= p_0 + \epsilon p_1 + \epsilon^2 p_2 + ..., \\ v_i &= v_{i_0} + \epsilon v_{i_1} + \epsilon^2 v_{i_2} + ..., \\ \rho &= \rho_0 + \epsilon\rho_1 + \epsilon^2\rho_2 + ... \; . \end{aligned}$$

Amongst other things this expression suggests nonlinear as well as nonlinear constitutive relationships

$$\rho_1 = \frac{1}{c_1^2}p_1, \qquad \rho_2 = \frac{1}{c_1^2}p_2 + \frac{1}{c_2^2}p_1^2$$

where

$$\begin{aligned} \frac{1}{c_1^2} &\equiv \left(\frac{\partial\rho_0}{\partial p_0}\right) \\ \frac{1}{c_2^2} &\equiv -\frac{1}{2}\left(\frac{\partial^2 p_0}{\partial\rho_0^2}\right)\left(\frac{\partial\rho_0}{\partial p_0}\right)^3 \end{aligned}$$

Following arguments that deviate only slightly from those that produce the linear equations, we derive a standard linear wave equation on first order variations in the pressure:

$$\left(\frac{\partial^2}{\partial x_j^2} - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\right)p_1 = 0$$

and a further wave equation on the second order variations in the pressure

$$\left(\frac{\partial^2}{\partial x_j^2} - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\right)p_2 = \mathcal{S}(p_1),$$

where

$$\mathcal{S}(p_1) \equiv \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}(p_1^2) + \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\left(\int_{-\infty}^{t}\frac{\partial p_1}{\partial x_i}dt\int_{-\infty}^{t}\frac{\partial p_1}{\partial x_j}dt\right).$$

A working interpretation is that as the main, linear wave propagates, it acts as a source $S(p_1)$ for a secondary wave p_2. The consequences of these equations, including the existence, tractability, and qualitative meaning of their solutions, and whether or not they predict measureable variability in seismic data that we can use to accept them as useful or reject them, is a matter of ongoing research.

II. Scattering/diffraction of radar from a seismic disturbance

The investigation of mechanical waves with electromagnetic fields (in optics), has been extensively described. Here we consider a geophysical application of this idea. Why? The same near-surface Earth volumes within which we could conceive of interacting mechanical and radar waves, are where, in early and late stages of propagation, seismic wave fields often undergo interactions with a complex, unconsolidated environment. Radar scattering might render and help characterize such propagation.

Assuming an instantaneous, linear relationship between, e.g., volumetric strain and dielectric permittivity, and noting that a radar experiment occurs essentially instantaneously (i.e., within one seismic Δt), we can derive analytic and numeric models of backscattering and transmission responses of the radar field to a passing seismic wave (FIGS 2 & 3).

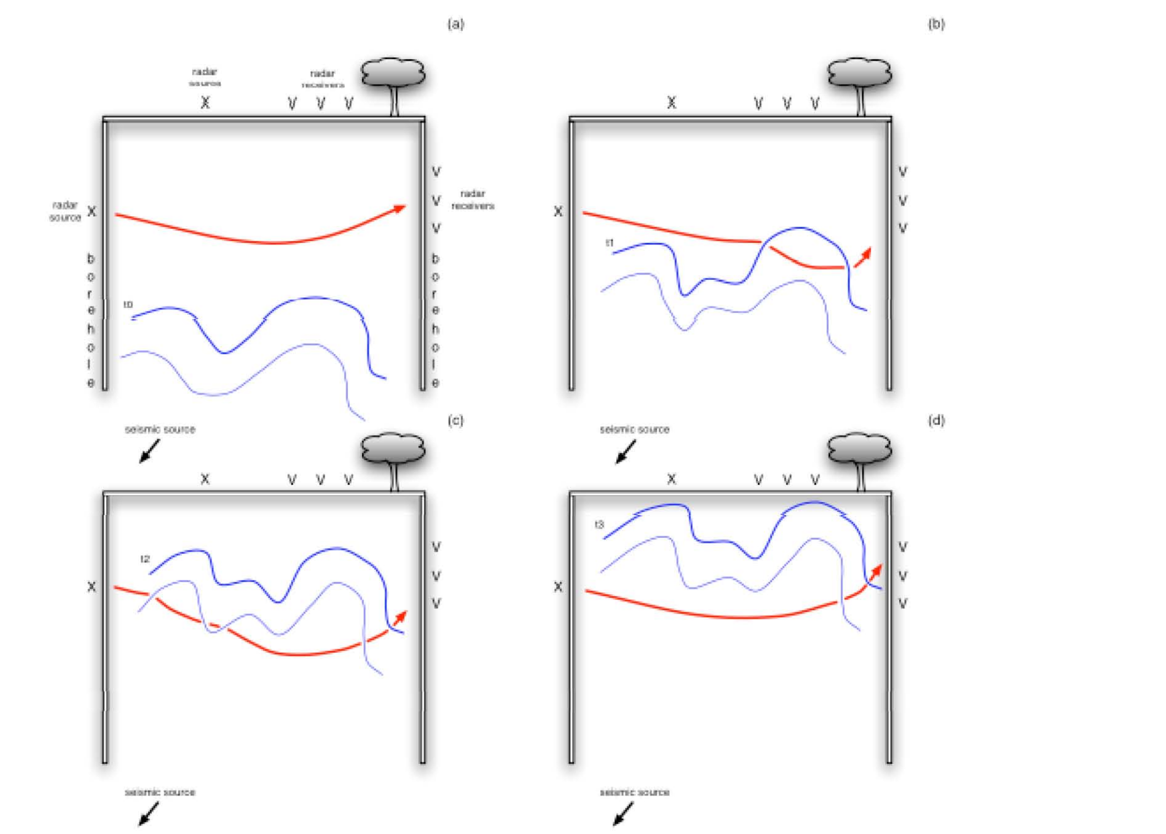

FIG. 2. Schematic diagram of radar transmitting through a seismic wavefield at four times during its propagation. (a)-(d) Times t_0-t_3 respectively. With a (relatively) slowly varying seismic waveform assumed to induce a perturbation in medium electrical properties, an incident radar field is illustrated undergoing a perturbed, forward scattering process.

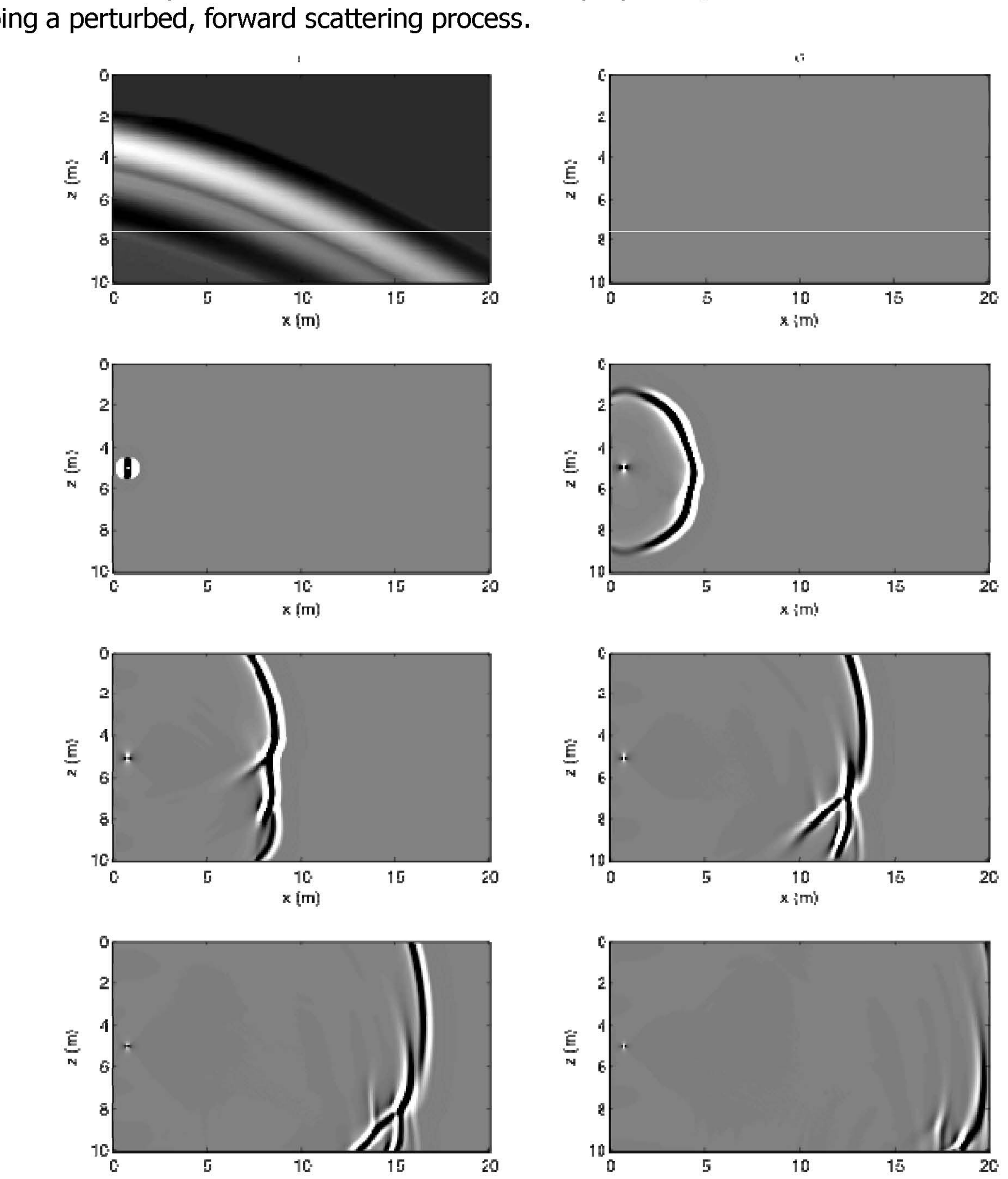

FIG. 3. Numerical model of radar propagating through a seismic disturbance in the case of a linear instantaneous relationship between volume strain and permittivity. Top panels: spatial distributions of σand ε; here σ is held constant. Lower panels: the radar wave at six times during propagation.