A least-squares shot-profile application of time-lapse inverse scattering theory

Mostafa Naghizadeh[1] and Kris Innanen[1]

1. Consortium for Research in Elastic Wave Exploration Seismology (CREWES), University of Calgary

Goal

- Applying least-squares shot-profile migration for imaging time-lapse data sets.

Born approximation of acoustic inverse scattering theory

- The forward operator (de-migration) models the scattered seismic wave-field using the Born approximation under the assumption of an acoustic and constant velocity Green's function (Kaplan et al.,2010),

$$\psi_s(\mathbf{x}_g, z_g|\mathbf{x}_s, z_s;\omega) = f(\omega)\iint_{-\infty}^{\infty} G_0(\mathbf{x}_g, z_g|\mathbf{x}', z';\omega)\left(\frac{\omega}{c_0}\right)^2 \alpha(\mathbf{x}', z')G_0(\mathbf{x}', z'|\mathbf{x}_s, z_s;\omega)d\mathbf{x}'dz', \qquad (1)$$

where G_0 is a Green's function for constant acoustic wave-speed c_0, so,

$$G_0(\mathbf{x}_g, z_g|x', z';\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(-\frac{1}{i4k_{gz}}\right)e^{-i\mathbf{k}_{gx}\cdot(\mathbf{x}'-\mathbf{x}_g)}e^{ik_{gz}|z_g-z'|}d\mathbf{k}_{gx}. \qquad (2)$$

In equation 1, α is the first order approximation to the scattering potential. Within the context of least-squares migration and SPDR, α is the model (a migrated shot gather). The forward operator in equation 1 describes the mapping from the approximate scattering potential α (the model-space) to the scattered wave-field ψ_s (the data-space) recorded at geophone positions $(\mathbf{x}_g, z_g)$ where $\mathbf{x}_g = (x_g, y_g)$, and due to the seismic source $f(\omega)$ located at $(\mathbf{x}_s, z_s)$ where $\mathbf{x}_s = (x_s, y_s)$. Equation 1 integrates over all possible scattering points $(\mathbf{x}', z')$ where $\mathbf{x}' = (x', y')$. The vertical wave-number k_{gz} in equation 2 is given by the dispersion relation,

$$k_{gz} = sgn(\omega)\sqrt{\frac{\omega^2}{c_0^2} - \mathbf{k}_{gx}\cdot\mathbf{k}_{gx}}, \qquad (3)$$

where $\mathbf{k}_{gx} = (k_{gx}, k_{gy})$ are the lateral wave-numbers (Fourier conjugate variables of $\mathbf{x}_g = (x_g, y_g)$).

Forward (demigration) operator

- Let's assume that the earth model is partitioned into n_z layers of constant thickness Δz. First, we define $v_{s(l)}$ for $l = 1 \dots n_z$ such that (Kaplan et al.,2010),

$$\begin{aligned} v_{s(1)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) &= u_{p(1)}(\mathbf{k}_{gx}, \omega)g(\mathbf{k}_{gx}, \mathbf{x}_s, \omega) \\ v_{s(l)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) &= \Delta u_p(\mathbf{k}_{gx}, \omega)v_{s(l-1)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s), \end{aligned} \qquad (4)$$

where $u_{p(1)} = \exp(ik_{gz}(z_1 - z_0))/(i4k_{gz})$ and $\Delta u_p = \exp(ik_{gz}\Delta z)$ are phase shift operators and $g(\mathbf{k}_{gx}, \mathbf{x}_s, \omega) = 2\pi f(\omega)e^{-i\mathbf{k}_{gx}\cdot\mathbf{x}_s}$ is the synthetic source term. Second, we define $v_{r(l)}$ so that,

$$\begin{aligned} v_{r(1)}(\mathbf{k}_{gx}, \omega) &= u_{p(1)}(\mathbf{k}_{gx}, \omega) \\ v_{r(l)}(\mathbf{k}_{gx}, \omega) &= \Delta u_p(\mathbf{k}_{gx}, \omega)v_{r(l-1)}(\mathbf{k}_{gx}, \omega). \end{aligned} \qquad (5)$$

Then the forward operator becomes,

$$\psi_s(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) = \left(\frac{\omega}{c_0}\right)^2 \Delta z \sum_{l=1}^{n_z} v_{r(l)}(\mathbf{k}_{gx}, \omega)\mathcal{F}\left[\mathcal{F}^* v_{s(l)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s)\right]\alpha(\mathbf{x}_g, z_l; \mathbf{x}_s). \qquad (6)$$

The main load of computation is in the two two-dimensional Fourier transforms required per depth and frequency.

Adjoint (migration) operator

- The implementation of the adjoint operator is also derived using two iterations. First, we define $v^*_{s(l)}$ for $l = 1 \dots n_z$ so that,

$$\begin{aligned} v^*_{s(1)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) &= u^*_{p(1)}(\mathbf{k}_{gx}, \omega)g^*(\mathbf{k}_{gx}, \mathbf{x}_s, \omega) \\ v^*_{s(l)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) &= \Delta u^*_p(\mathbf{k}_{gx}, \omega)v^*_{s(l-1)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s). \end{aligned} \qquad (7)$$

Second, we define $v^*_{r(l)}$ for $l = 1 \dots n_z$ so that,

$$\begin{aligned} v^*_{r(1)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) &= u^*_{p(1)}(\mathbf{k}_{gx}, \omega)\psi_s(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) \\ v^*_{r(l)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) &= \Delta u^*_p(\mathbf{k}_{gx}, \omega)v^*_{r(l-1)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s). \end{aligned} \qquad (8)$$

Then, the adjoint operator becomes,

$$\alpha^\dagger(\mathbf{x}_g, z_l; \mathbf{x}_s) = \Delta\omega \sum_j \left(\frac{\omega_j}{c_0}\right)^2 \left[\mathcal{F}^* v^*_{s(l)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s)\right]\mathcal{F}^* v^*_{r(l)}(\mathbf{k}_{gx}, \omega; \mathbf{x}_s), \qquad (9)$$

again requiring two two dimensional Fourier transforms per depth and frequency.

Least-squares time-lapse shot-profile migration

- **Inversion of difference data:** The linear system of equations for the differnce data can be expressed as

$$\phi(\mathbf{m}_{diff}) = ||\mathbf{W}_d(\mathbf{d}_{diff} - \mathbf{L}\mathbf{m}_{diff})||_2^2 + \mu||\mathbf{W}_m\mathbf{m}_{diff}||_2^2, \qquad (10)$$

where $\mathbf{m}_{diff}$ and $\mathbf{d}_{diff}$ are the difference model and data difference sections, respectively.

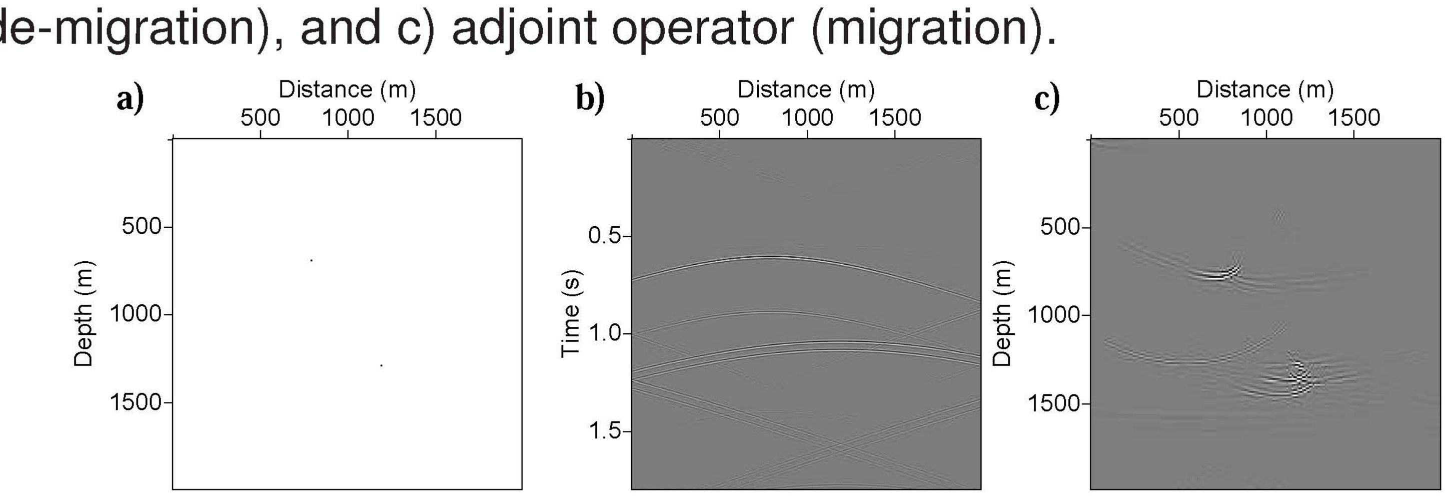
- **Joint Inversion of base and monitor data:** Let's $\mathbf{d}_b$ and $\mathbf{d}_m$ represent the data sets for base and monitor surveys, respectively. Similarly, $\mathbf{m}_b$ and $\mathbf{m}_m$ represent the subsurface scattering potential at base and monitor survey, respectively. A regularized solution can be found by minimizing the following cost function (Ayeni and Biondi, 2010)

$$J = \left\|\begin{bmatrix}\mathbf{L}_b & 0 \\ 0 & \mathbf{L}_m\end{bmatrix}\begin{bmatrix}\mathbf{m}_b \\ \mathbf{m}_m\end{bmatrix} - \begin{bmatrix}\mathbf{d}_b \\ \mathbf{d}_m\end{bmatrix}\right\|^2 + \left\|\begin{bmatrix}\mu_b\mathbf{R}_b & 0 \\ 0 & \mu_m\mathbf{R}_m\end{bmatrix}\begin{bmatrix}\mathbf{m}_b \\ \mathbf{m}_m\end{bmatrix}\right\|^2 + \left\|\begin{bmatrix}\lambda_b\mathbf{D}_b & \lambda_m\mathbf{D}_m\end{bmatrix}\begin{bmatrix}\mathbf{m}_b \\ \mathbf{m}_m\end{bmatrix}\right\|^2, \qquad (11)$$

Where $\mathbf{R}_b$ and $\mathbf{R}_m$ are model regularization terms and $\mathbf{D}_b$ and $\mathbf{D}_m$ are temporal regularization terms. The model regularization can be smoothness or sparseness for spatial dimensions and the temporal regularization can be a derivative operator between the surveys.

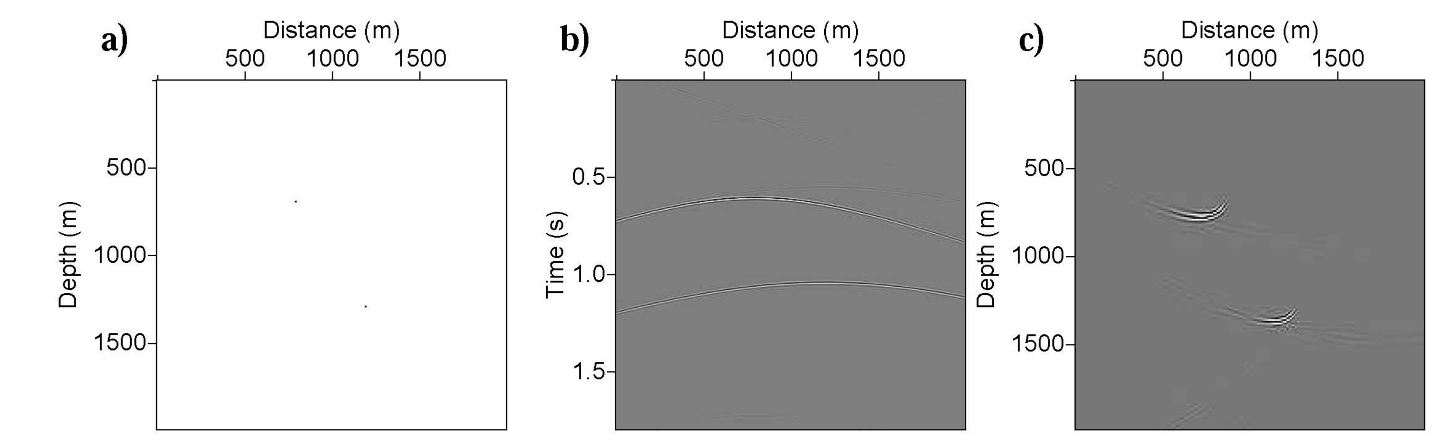
Examples

- **No zero-padding:** a) Scattering potential, b) forward operator (de-migration), and c) adjoint operator (migration).

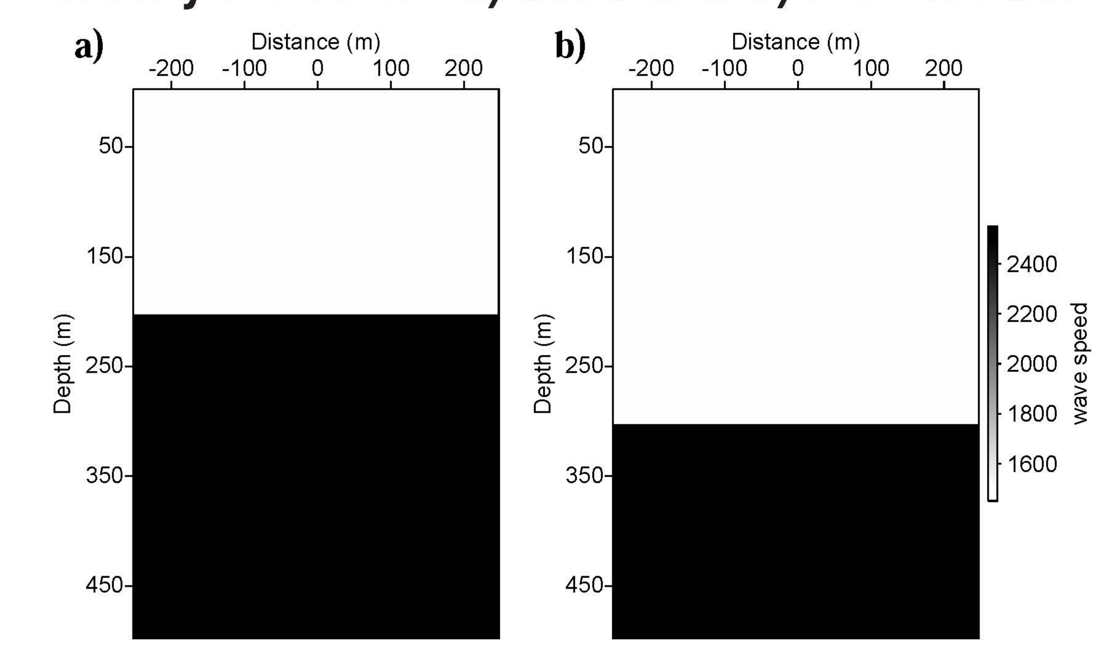


Examples

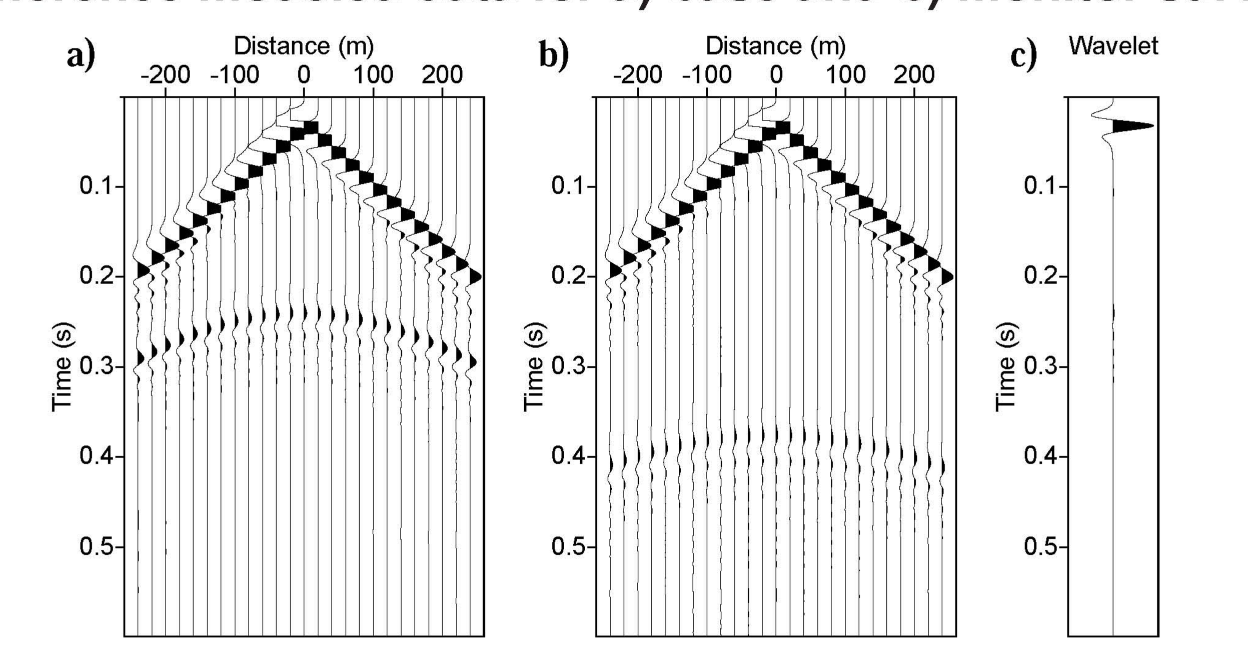
- **Previous example with proper zero-padding.**



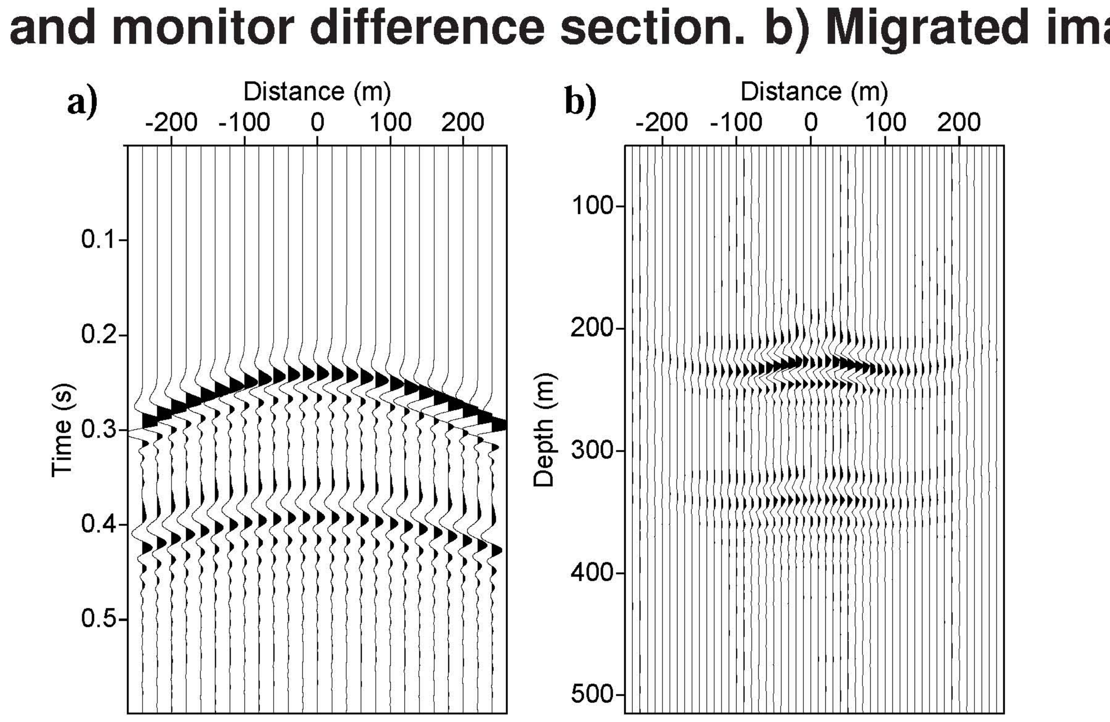
- **A synthetic velocity model for a) base and b) monitor surveys.**



- **Finite-difference modeled data for a) base and b) monitor surveys.**



- **a) The base and monitor difference section. b) Migrated image.**



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