Picking microseismic first arrival times by Kalman filter and wavelet transform

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ABSTRACT

Due to the high energy content of the ambient noise, microseismic monitoring system records considerable erroneous data. To pick up the first arrival times, special techniques must be applied due to the very low signal to noise ratio data. This paper presents three techniques: wavelet transform is applied to de-noising the noisy data; Kalman filter and modified STA/LTA method are implemented to pick up the first arrival times. The results show that the first arrival times are picked up accurately even in very noisy data by incorporating these techniques.

▪ **Noise**. The noises are built by Gauss-Markov process, its discrete form can be written as:

$$n_k = a_w n_{k-1} + b_w w_k,$$
$$a_w = e^{-\beta\Delta}, b_w = \sigma\sqrt{1 - e^{-2\beta\Delta}}.$$

β is the reciprocal of the time constant, σ is noise variance, Δ is the sampling rate, and wk is white Gaussian noise.

▪ **Kalman filter (KF).** KF addresses the problem of trying to estimate the state x of a discrete-time controlled process that is governed by the linear stochastic different equation:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$$
$$z_k = Hx_k + v_{k-1}$$

Here x_k is state vector, A is $n\times n$ state transition matrix, B is $n\times r$ optional control matrix, u_k is control vector, z_k is measurement vector, and H is $m\times n$ measurement matrix. The random variables w_k and v_k represent the process and measurement noise respectively.

The discrete form of the KF governing equation is:

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} 1 & \Delta\omega\cos(\Delta\omega(k-1)) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\beta\Delta} \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ q(t)\Delta & 0 \\ 0 & \sigma\sqrt{1 - e^{-2\beta\Delta}} \end{bmatrix} \begin{bmatrix} w_1(k-1) \\ w_2(k-1) \end{bmatrix}$$

x is the state vector, and $w_1(k\text{-}1)$ and $w_2(k\text{-}1)$ are zero mean, unity variance, Gaussian white noise.

▪ **Wavelet transform.** In this paper we use wavelet *db4* and decompose signals up to level 6.

▪ **ModifiedSTA/LTA.** To obtain the first arrival times automatically and accurately, a modified STA/LTA was applied to Kalman filtering results. The formula is shown in Fig. 1, L_1 and L_2 were set to 40 and 10 sampling points respectively.

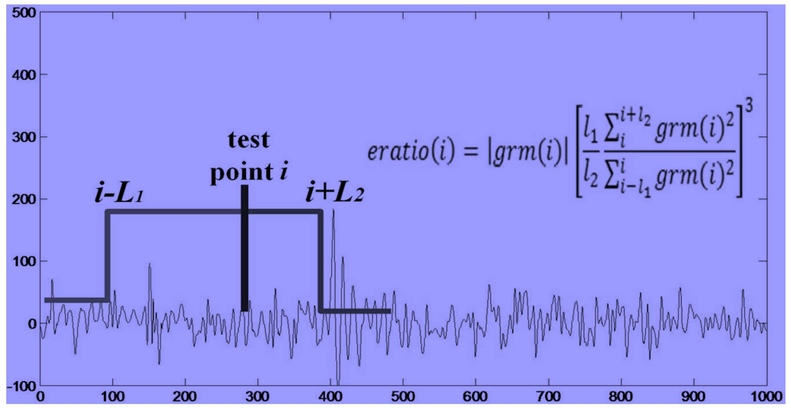

FIG. 1. Diagram of the modified STA/LTA。

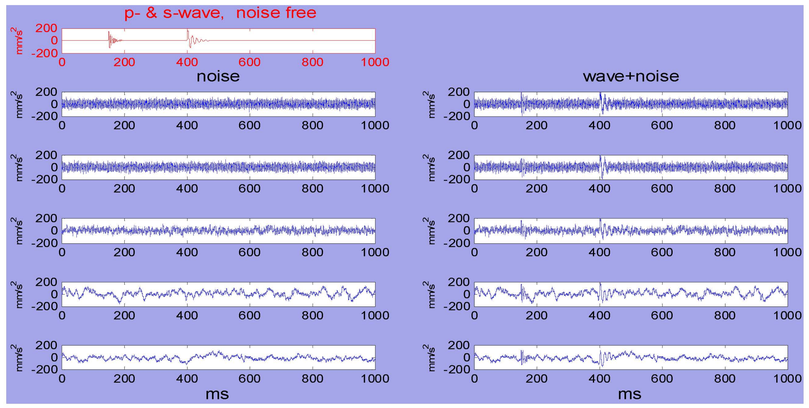

FIG. 2. Signals and the noises. The wavelet and noises are shown on the left side, the signals are shown on the right side. Noises are simulated by Gauss-Markov processes with different parameters, the purpose is to test our techniques on different levels of noisy data. We can see that it is difficult to pick up the first arrival times on the noisy data if nothing is done beforehand.

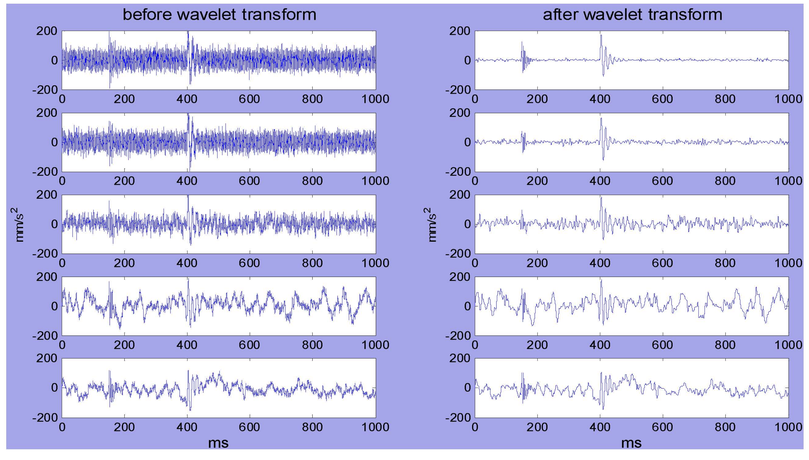

FIG. 3. Wavelet transform results. Comparing the original signal (left) and the wavelet transform results (right), we can see that SNR is improved especially for the top two signals. We use *db4* as the wavelet and decompose signals up to level 6.

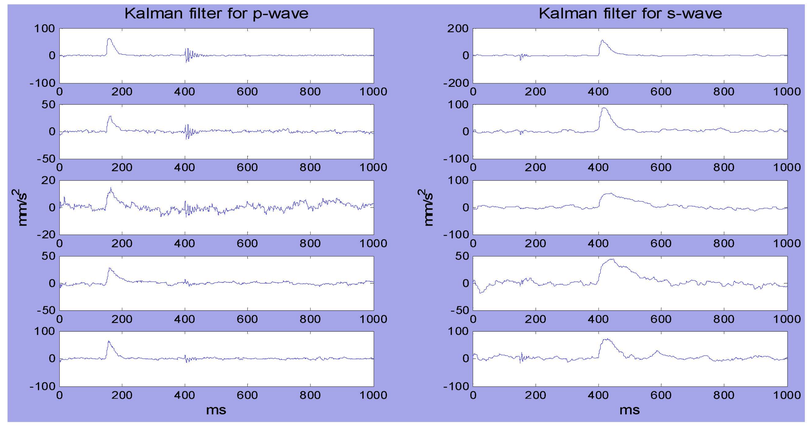

FIG. 4. Kalman filtering results. Using the wavelet transform data, we apply KF and plot the state x_2. We can see that there is a dramatic improvement in the SNR when comparing these results to the initial seismic time series in Fig. 1.

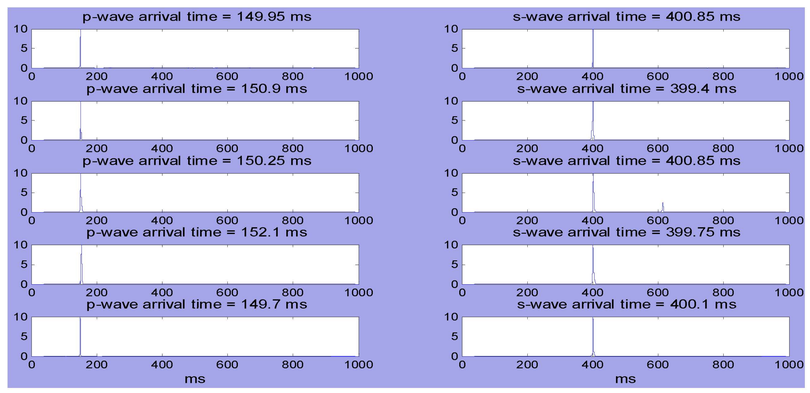

FIG. 5. Modified STA/LTA calculating results. Based on the Kalman filtering results, in order to obtain the first arrival times automatically and accurately, a modified STA/LTA method was applied. We can see that the calculating results is very close to the real first arrival times (150ms and 400ms).

Table-1. The error between real and estimation of the first arrival times when using modified STA/LTA method. average error is 0.72 ms for *p*-wave and 0.53 ms for *s*-wave.

	noise1	noise2	Noise3	noise4	noise5
p-wave error (ms)	-0.05	0.9	0.25	2.1	-0.3
s-wave error (ms)	0.85	-0.6	0.85	-0.25	0.1

CONCLUSION

By testing our techniques on synthetic data, it shows that wavelet transform can attenuate considerable microseismic ambient noise given appropriate wavelet and decomposing level. The test also shows that the first arrival times can be picked up accurately by combining Kalman filter and modified STA/LTA method.