Iteratively re-weighted least squares inversion for estimating density from well log

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ABSTRACT

A quantitative analysis of density log is established in this study by inverting of density log. The proposed IRLS inverse algorithms, to predict density log, utilize different constraints in the data and model spaces. The inverse algorithms have shown stable and fast convergence toward the final model with few numbers of iterations. The inverted density model has resolved different lithology layers, and successfully delineated gas-bearing sand reservoir of the Blackfoot.

The additional information incorporated into the weighted matrices used has enhanced the interpretation of inverted density log significantly; in particular, the sand base-line can be easily recognized as well as separation of sand and carbonate regions. Simultaneous inversion of Vp and Vs logs to predict density log showed some improvements in the final shape of density log.

IRLS INVERSION OF DENSITY LOG

The IRLS inverse equation of density log (Saeed et. al., 2010a & b) is written as:

$$(G^T W_d^T R_d W_d G + \lambda W_m^T R_m W_m) m = G^T W_d^T R_d W_d d$$

- $\boldsymbol{G}$ is the forward operator, $\boldsymbol{\lambda}$ is a damping factor, and $\boldsymbol{d}$ is measured data
- $\boldsymbol{R_d}$ and $\boldsymbol{R_m}$ are data misfit and model-structure weighting matrices. The optimal trade-off parameter,ϵ, used in $\boldsymbol{R_d}$ and $\boldsymbol{R_m}$ matrices is calculated from L-Curve.
- $\boldsymbol{W_d}$ is a diagonal weighted matrix in the data space, while $\boldsymbol{W_m}$ is weighted or roughness operator in model space.
- The damping factor , $\boldsymbol{\lambda}$ varies during IRLS inversion based on the ℓ_2-norm of the model space.
- IRLS procedure is repeated until the convergence between successive IRLS iterations becomes less than a pre-set tolerant value.

Model regularization operator W_m

- ***Smoothness constraint***: can be Identity or gradient or Laplacian matrices.
- ***Compactness constraint***: A stabilizing function that minimizes the area where strong variation in model parameter or discontinuity occur. The compactness constraint is non-linear operator and require the use of model-space in (IRLS) sense for effective solution. Modified total variation, minimum support (figure 2) and minimum gradient support constraints are investigated in this study.
- The weighted matrix, $\boldsymbol{W_d}$ was set as *identity* matrix during the inversion.

The weight matrix W_d

- Several methods in estimating the weighted matrix $\boldsymbol{W_d}$ have been tested in this study. $\boldsymbol{W_d}$ is incorporated in the data-misfit domain.
- Std. deviation , Robustness normalized by Std. deviation, Hybrid $\ell_1/\ \ell_2$ –norm, Huber & Tukey M-estimator, and the Annealing M-estimator constraints are investigated in this study. The Inverted density log using Huber constraint is showing in figure (3) along with RMS error during inversion.
- The weighted matrix, $\boldsymbol{W_m}$ was set as first-order difference during the inversion.

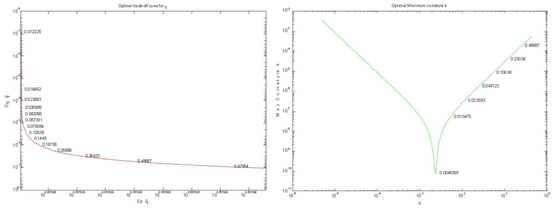

FIG. 1. *IRLS* inversion of Well08-08. Left: L-curve plot of logarithmic values of residual and solution norms. Right: The optimum trade-off parameter ϵ.

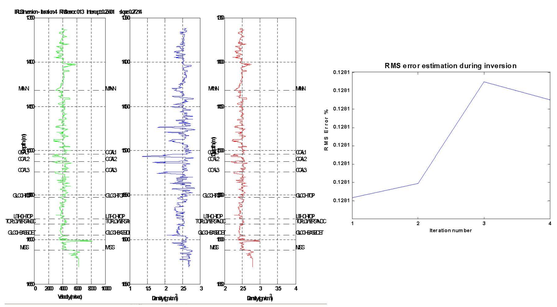

FIG. 2. *IRLS* inversion of Well08-08.Left:IRLS inversion of density log using minimum support constraint. P-wave velocity in green, measured density in blue, and inverted density in red colors. Right: RMS error during inversion.

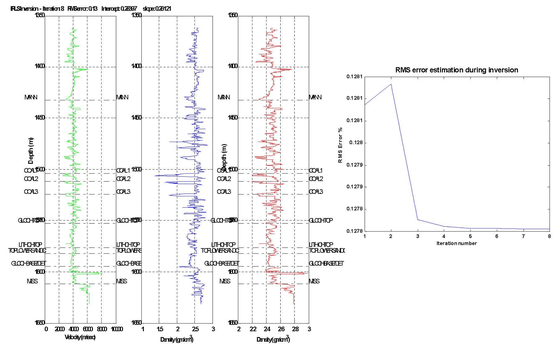

FIG. 3. *IRLS* inversion of Well08-08. Left: IRLS inversion of density log using Huber constraint. Right: RMS error during inversion.

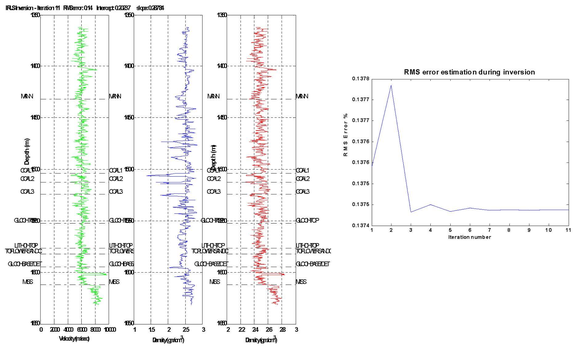

FIG. 4. *IRLS* inversion of Well08-08. Left:IRLS inversion of noisy data log using Huber constraint. Right: RMS error during inversion.

INVERSION OF NOISY DATA

- 5% random noise with zero-mean and variance not equal to 1 were added to the sonic Vp log. The Huber weight method is used during the inversion of the noisy data
- Inverted density log (figure 4) shows that major events corresponded to lithology change are mapped well, but with less resolution compared to the noise-free data.

CROSS-PLOT OF DENSITY VERSUS Vp/Vs

- Figure(5) shows measured & inverted density cross-plotted against Vp/Vs.
- Sand and carbonate regions can be differentiated from inverted density cross-plotted against Vp/Vs.

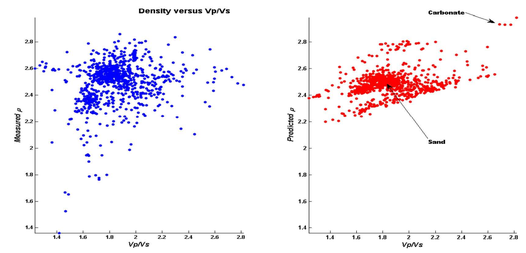

FIG. 5. Cross-plot of measured density (left panel) and predicted density (right panel) versus Vp/Vs.

IRLS OF JOINT INVERSION MULTI- LOGS

The IRLS joint-inversion of multi-logs to predict density log (Saeed et. al., 2010b) is written as:

$$\left[(G_1^T W_{d_1}^T W_{d_1} G_1 + \lambda_1 W_m^T W_m) + (G_2^T W_{d_2}^T W_{d_2} G_2 + \lambda_2 W_m^T W_m)\right] m = \left[(G_1^T W_{d_1}^T W_{d_1} d) + (G_2^T W_{d_2}^T W_{d_2} d)\right]$$

$$(G^T W_d^T R_d W_d G + \lambda W_m^T R_m W_m) m = G^T W_d^T R_d W_d d$$

- The predict density log using Vp and total logs show very good resemblances.
- There is a gap between measured density and predict density when using Vs log in calculating predict density log. This can be attributed to the different physical properties that were used in inverse operators, knowing that Vp magnitude is almost double Vs values. Also density is inversely proportional to S-wave.

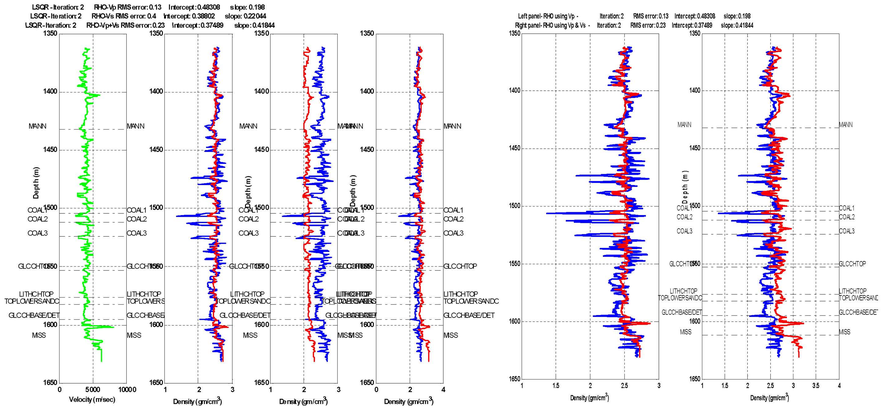

FIG. 6. *IRLS* joint inversion of Vp and Vs to predict density log using Huber constraint. Left: P-wave and inverted density calculated from Vp, Vs and sums of total logs. Right: inverted density calculated from Vp alone, and from sums of total logs.

ACKNOWLEDGMENT

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