Feasibility of least-squares nonstationary phase shift

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Summary

To optimize irregular nonstationary phase shift, we introduce implicit and explicit preconditioned conjugate gradient frameworks. The implicit scheme gives faster convergence, resulting in decreased runtime over the standard inversion. This speedup comes at the cost of accuracy, as deriving the implicit scheme involves approximating the evanescent filter portion of the operator. An explicit scheme is proposed to mitigate this error, but it fails to perform as expected, resulting in slower convergence than the standard scheme.

The speed of the implicit scheme suggests that a preconditioning operator exists that reduces the runtime of the method without sacrificing accuracy, although this operator has yet to be determined. Therefore the implicit scheme can be used as an ideally fast algorithm to estimate the asymptotic complexity of the method. A sample of runtimes is collected for trace gathers of up to 2^{15} traces, and polynomial regression estimates that for n traces, the number of conjugate gradient iterations required is $\mathcal{O}(n^{0.0158})$, and the total runtime is $\mathcal{O}(n^{1.082})$. This is a promising result, as an accurate preconditioned scheme that approaches this runtime would be feasible for use on large 3D surveys.

Irregular nonstationary phase shift

- The phase shift method quickly extrapolates a regularly sampled wavefield through a homogeneous medium.
- The method acts on plane waves, multiplying the Fourier coordinates by a complex exponential in the wavelike region, and a real negative exponential in the evanescent region.
- When the medium velocity varies with depth, we can phase shift iteratively through a series of constant velocity depth steps.
- For a laterally variable medium, we use nonstationary phase shift operators to estimate the wavefield at each depth step.
- If the wavefield is sampled irregularly, but the samples are on a regular grid with some samples missing, we can use weighted-damped least squares to extrapolate the wavefield by computing matrix inverses on normal equations.
- Computing linear inverses directly is much more computationally expensive than a forward phase shift, and is infeasible for large 3D surveys.

Preconditioned Conjugate Gradients

- The conjugate gradient method reduces the computational effort to that of the nonstationary phase shift operator, multiplied by some unknown iterations function.
- Without preconditioning, this inversion converges quickly for high frequencies and poorly for low frequencies.
- Poor convergence in the low frequencies is caused by the action of the phase shift in the evanescent region.
- An approximation can be made that will allow the evanescent filter portion of the operator to be factored out of the normal equations.
- This new linear system can be solved in fewer iterations, but the approximation affects the quality of the output image.
- This is an implicit form of preconditioning, so we should be able to do the same thing explicitly, and achieve a similar speedup with no loss in image quality, as no approximation is required.
- Performing the same inversion on the original system, with the factored portion as the preconditioner, results in slower convergence than the unconditioned system.
- The implicit system suggests that there exists a preconditioner that will achieve a comparable speedup without sacrificing accuracy.

Preconditioning Operators

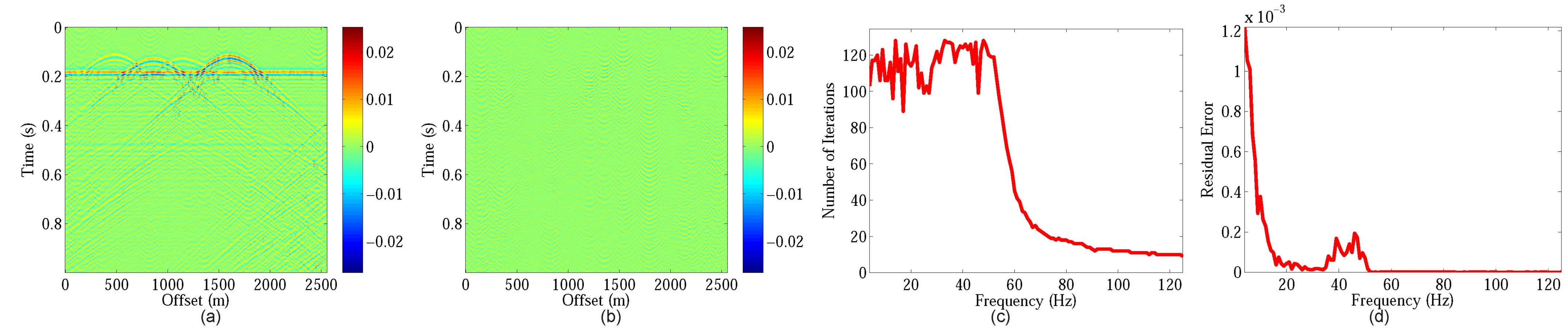

Figure 1: Output for the nonpreconditioned scheme. (a) The recovered wavefield. (b) The difference between the recovered wavefield and the source. (c) The number of CG iterations required at each frequency. (d) The residual error of the output at each frequency.

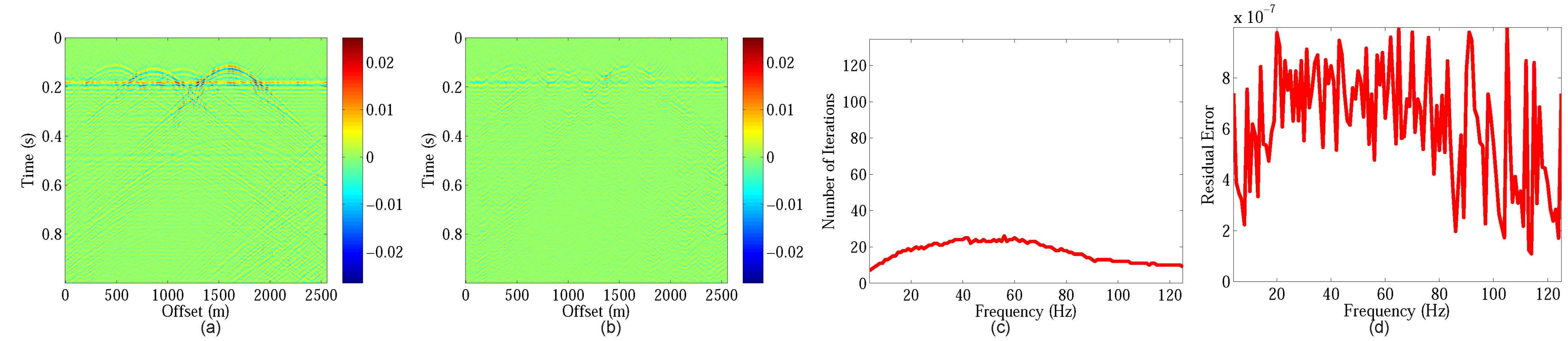

Figure 2: Output for the implicit preconditioning scheme. (a) The recovered wavefield. (b) The difference between the recovered wavefield and the source. (c) The number of CG iterations required at each frequency. (d) The residual error of the output at each frequency.

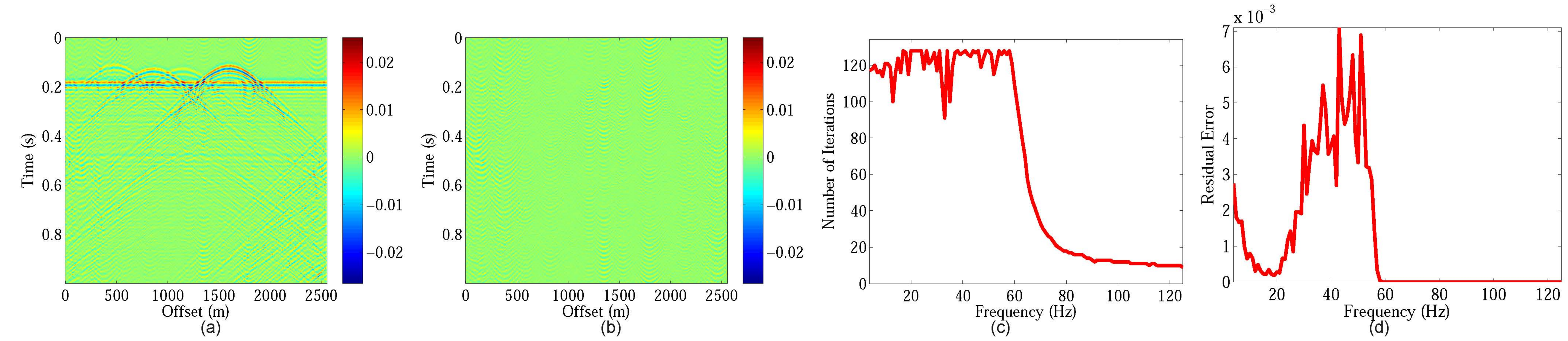

Figure 3: Output for the explicit preconditioning scheme. (a) The recovered wavefield. (b) The difference between the recovered wavefield and the source. (c) The number of CG iterations required at each frequency. (d) The residual error of the output at each frequency.

Asymptotic Complexity

The implicit scheme is much faster than the unconditioned scheme, so we can use it to estimate a lower bound on the asymptotic complexity of the method. We run the inversion on a random sample of survey sizes, and record the number of iterations required as well as the total runtime. The number of iterations is almost constant, so the total runtime is roughly equivalent to that of the forward operator.

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Runtimes

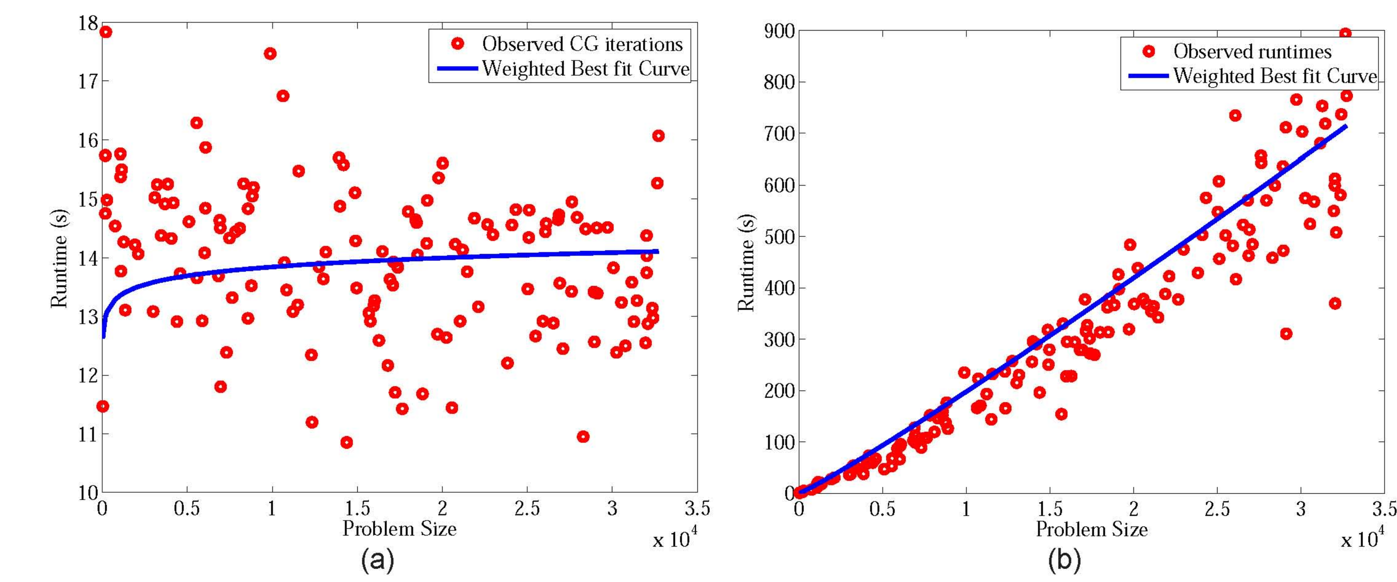

Figure 4: (a) The number of CG iterations vs the number of traces for the implicit preconditioning scheme. (b) The total runtime of the inversion vs the number of traces. Regression estimates that the iterations function is $\mathcal{O}(n^{0.0158})$, and the total runtime function is $\mathcal{O}(n^{1.082})$.