Fermat's principle and ray tracing in anisotropic layered media

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Introduction

I consider the path followed by a seismic signal travelling through velocity models consisting of horizontal layers with VTI and TTI *velocity* anisotropy. I describe a ray-bending method based on Fermat's principle, which states that the raypath between the source and receiver must be the one with the least travel time.

An accurate approximation for quasi-P group velocities in VTI and TTI anisotropy is given by Byun et al. (1989) and Kumar et al. (2004):

$$V_p^{-2}(\phi) = a_0 + a_1 \cos^2(\phi+\psi) - a_2 \cos^4(\phi+\psi)$$

$$a_0 = V_h^{-2},$$

$$a_1 = 4V_{45}^{-2} - 3V_h^{-2} - V_v^{-2},$$

$$a_1 = 4V_{45}^{-2} - 2V_h^{-2} - 2V_v^{-2}.$$

The dip angle ϕ is measured from the VTI axis of symmetry. The tilt angle ψ between the VTI axis of symmetry and the vertical results in TTI anisotropy.

Fermat's principle and ray bending

Figure 3 shows a bent raypath between a source S and a receiver R intersecting two boundaries. Bending at the intersection points occurs in such a way that the total path must conform to Fermat's principle. Two-point ray tracing reduces to determining the coordinates x_1 and x_2 of the intersection points that minimize the total travel time t.

$$l_1 = \sqrt{(x_1 - x_S)^2 + (z_1 - z_S)^2},$$

$$l_2 = \sqrt{(x_2 - x_1)^2 + (z_2 - z_1)^2},$$

$$l_3 = \sqrt{(x_R - x_2)^2 + (z_R - z_2)^2},$$

$$t = u_1 l_1 + u_2 l_2 + u_3 l_3.$$

These equations can be extended to include more layers and more boundary intersection points.

Direct search minimization

A direct-search optimization routine used to find the points x_1 and x_2. For this problem, direct search is faster than a gradient based inversion scheme as there is no need to calculate and invert Jacobian matrices.

Summary

- Two-point ray tracing based on Fermat's principle together with direct search optimization allows us to quickly produce the raypaths and arrival times for sources and receivers embedded in layered media.
- By including the Byun/Kumar formulation for qP group velocities in VTI and TTI media, the method gives an efficient and flexible means for analyzing VSP, crosswell, and microseismic monitoring travel times in anisotropic layered media.
- Sloping straight boundaries and possibly gently curving boundaries can be accommodated.

Acknowledgement

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References

Byun, B.S., Corrigan, D., and Gaider, J.E., 1989. Anisotropic velocity analysis for lithology discrimination, Geophysics, **54**, 1566-1574.

Kumar, D., Sen, M.K., and Ferguson, R.J., 2004. Traveltime calculation and prestack depth migration in tilted transversely isotropic media, Geophysics, **69**, 37-44.

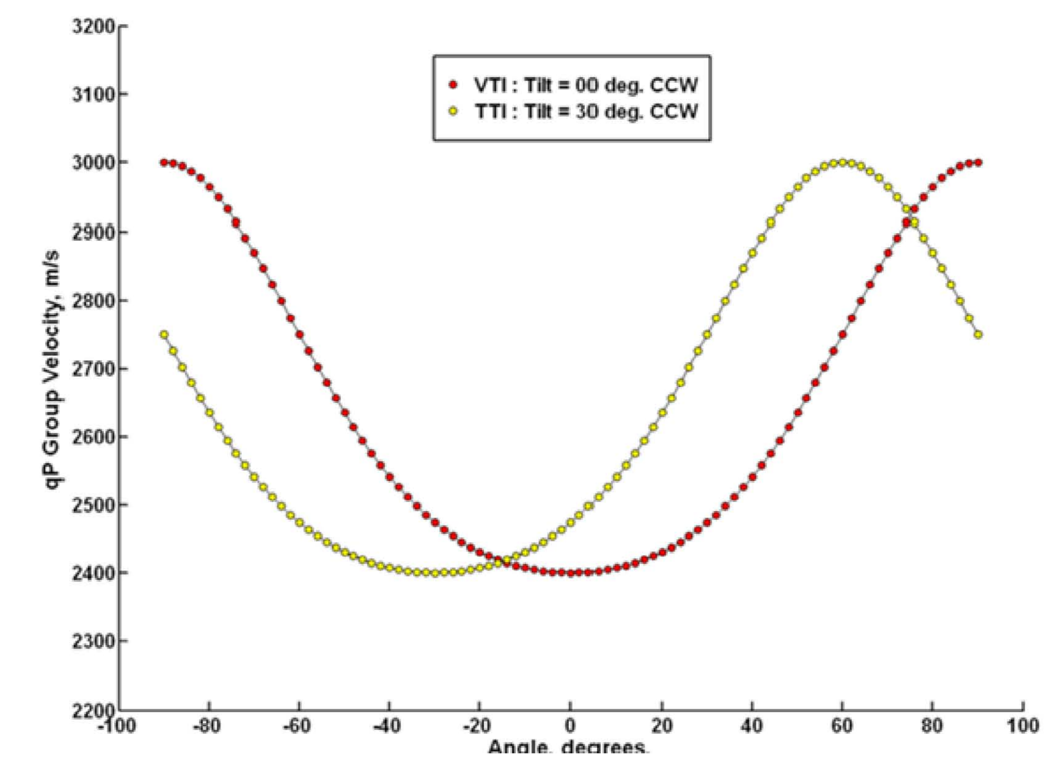

FIG. 1. qP group velocity based on Equation 6, with V_h =3000 m/s, V_v =2400 m/s, and V_{45} = 2585 m/s. Red dots are VTI velocities; yellow dots are TTI velocities.

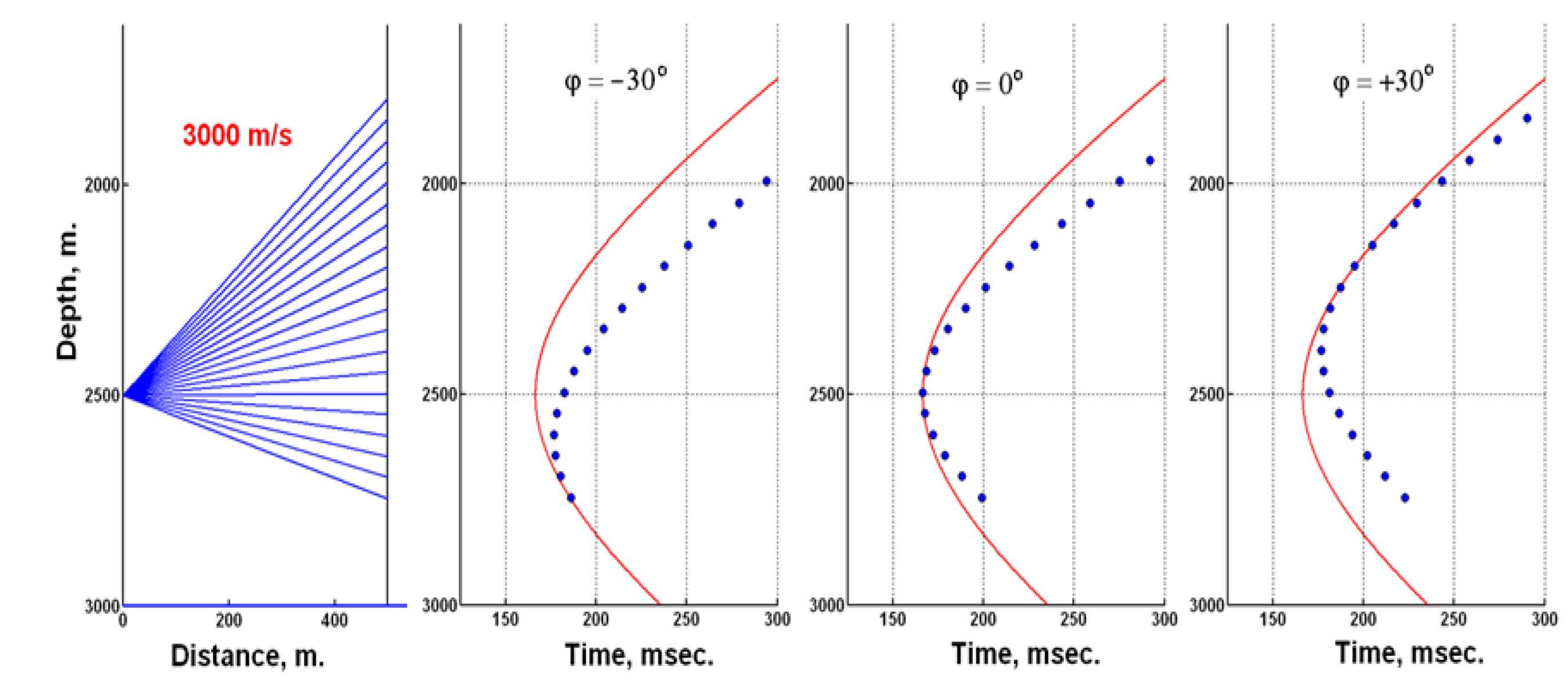

FIG. 2. Comparison of arrival times for homogeneous isotropic and tilted VTI media. The red curves are arrival times for the isotropic case. The blue dots are arrival times with tilt angles of -30, 0, and +30 for the VTI axis of symmetry..

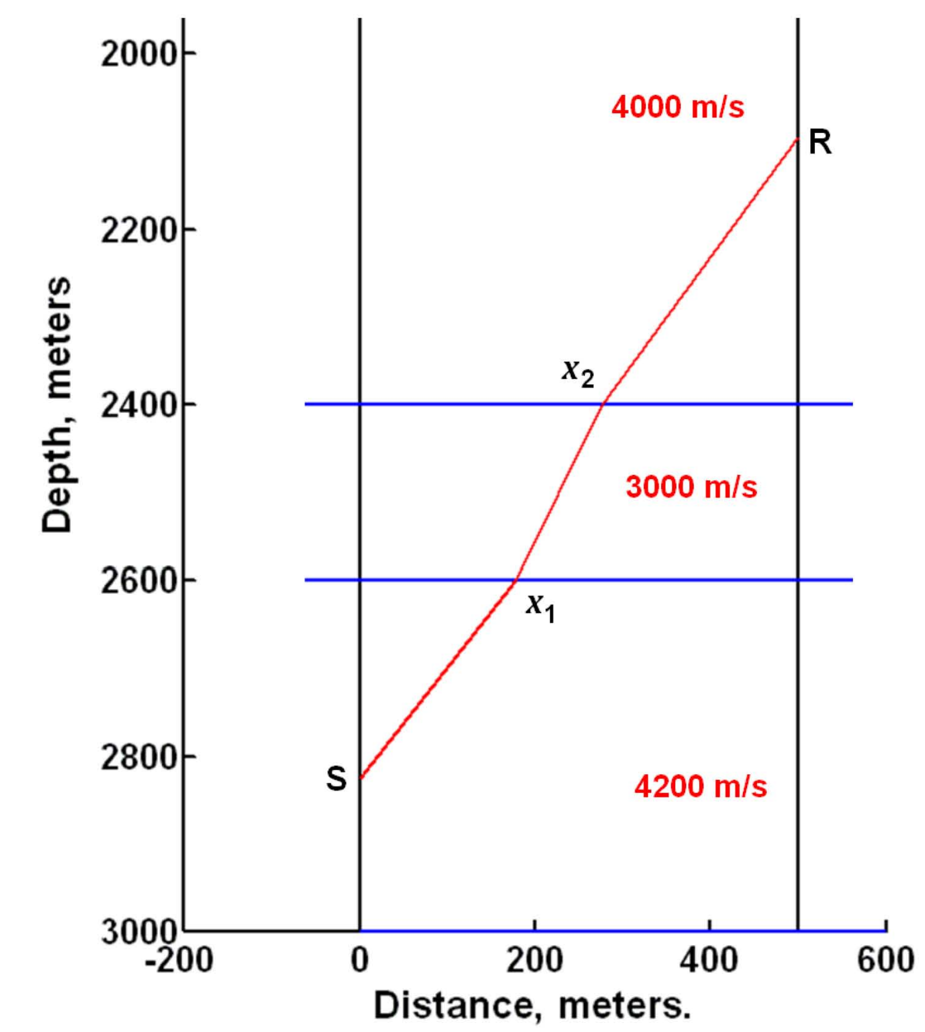

FIG. 3. Schematic of two-point ray tracing in layered media between a subsurface source S and a subsurface receiver R. The layer between 2400 m and 2600 m has VTI anisotropy with parameters of V_h =3000 m/s, V_v =2400 m/s, and V_{45} = 2585 m/s.

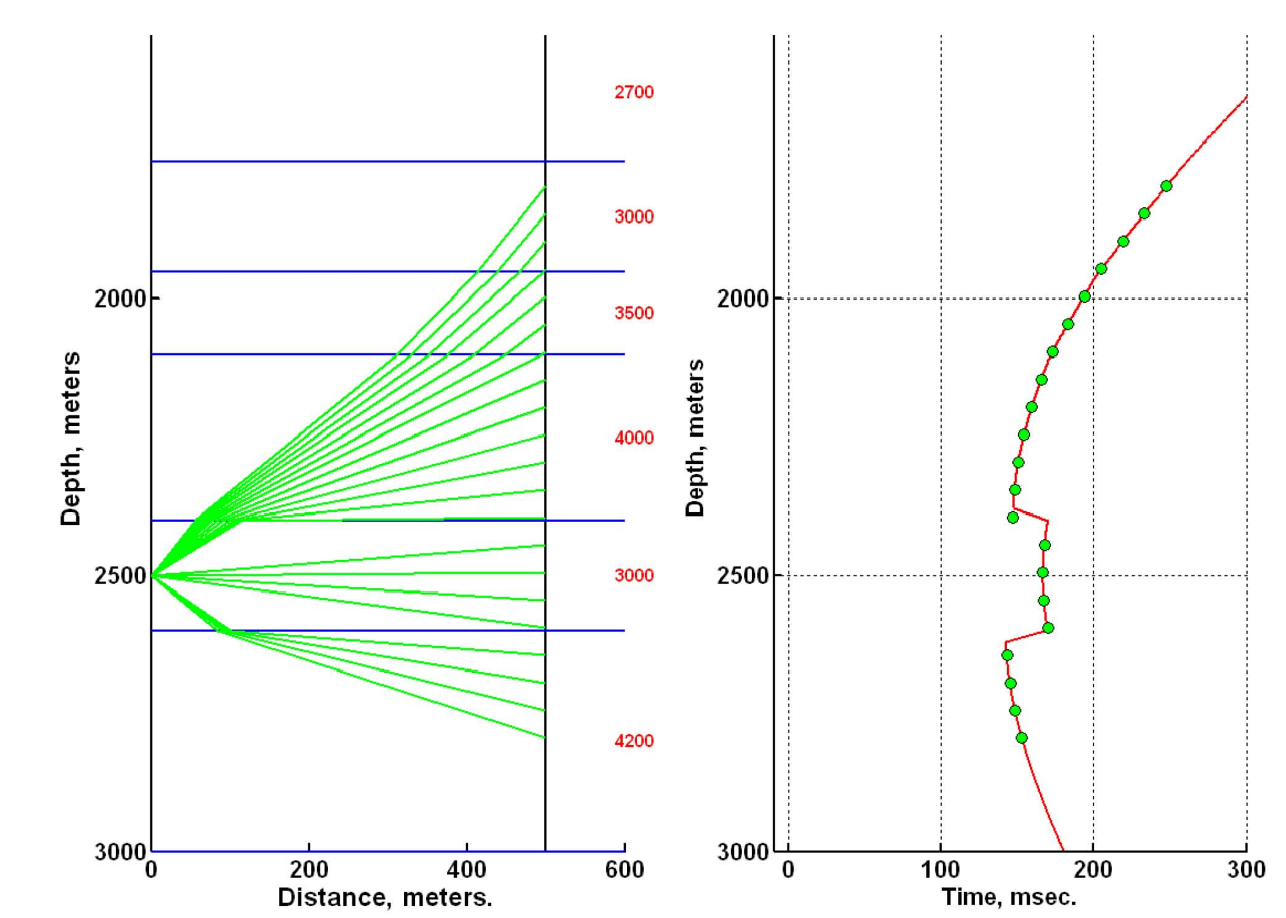

FIG 4. Left: rays traced from a source and 20 receivers embedded in a layered isotropic velocity model. Right: comparison of travel times from two-point ray tracing (green points) with those from shooting (red curve).

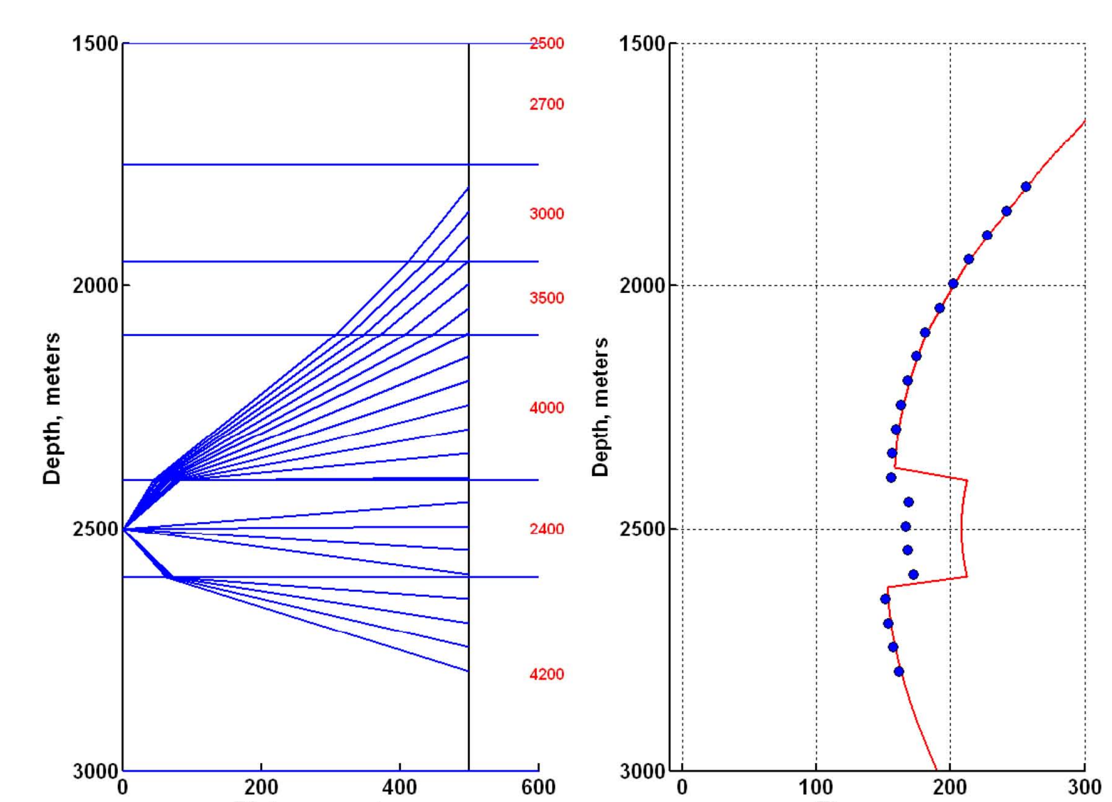

FIG. 5. Raypaths and arrival times for a source in the VTI velocity layer. The red curve plots arrival times for the isotropic model. Blue dots are the arrival times for the layer being VTI with V_h =3000 m/s, V_v =2400 m/s, and V_{45} = 2585 m/s.

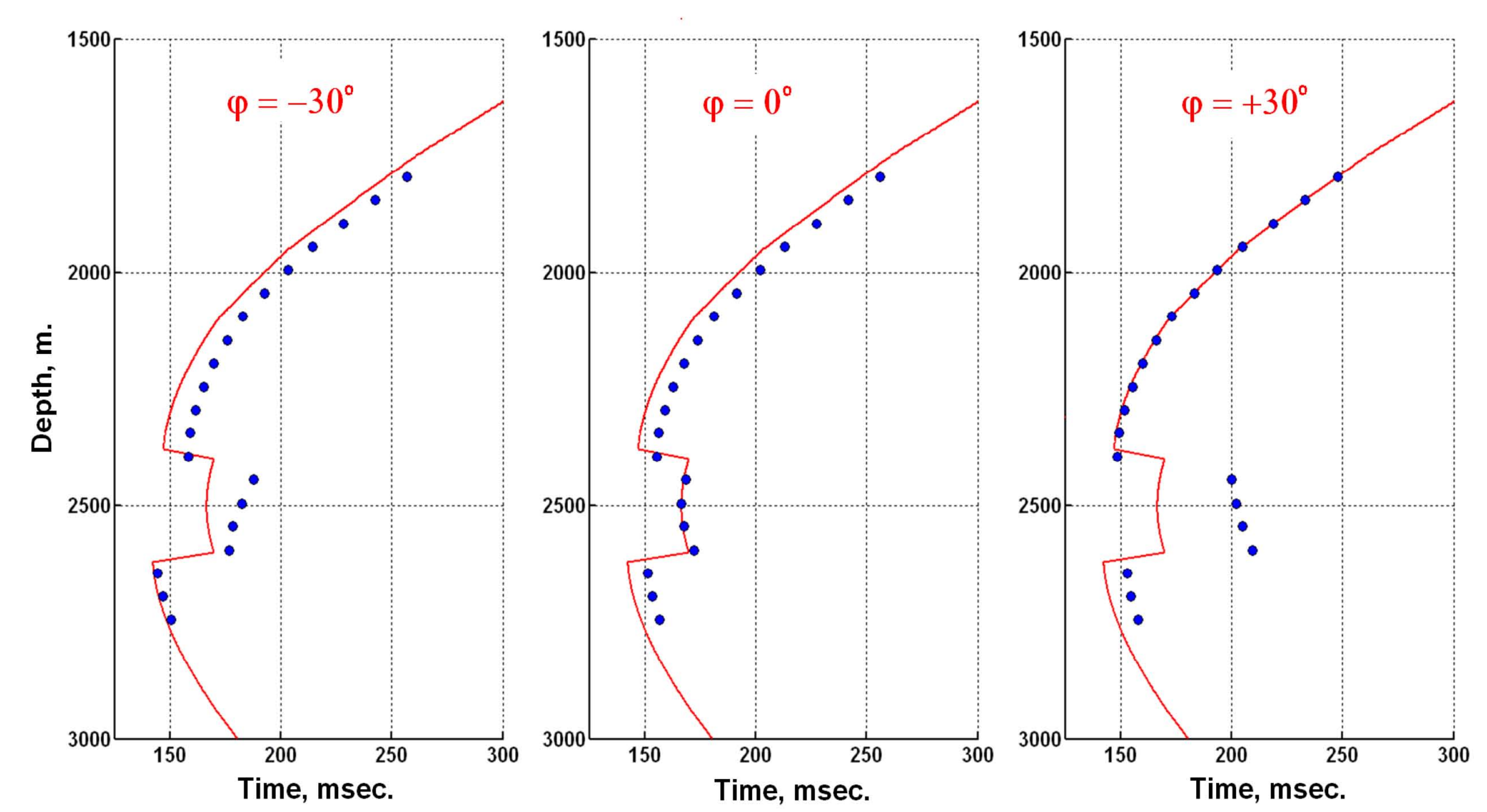

FIG. 6. Arrival times for transverse anisotropy in the shale layer for different tilt angles of the symmetry axis with V_h =3000 m/s, V_v =2400 m/s, and V_{45} = 2585 m/s. The red curve represents the isotropic model for which the isotropic shale velocity is 3000 m/s.