Full waveform inversion of anelastic reflection data: an analytic example

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ABSTRACT

Full waveform inversion is taking an increasing important role in exploration seismology. As this role continues to grow, we must develop our understanding of the basic nature of full waveform inversion. We develop an analytic example of anacoustic full waveform inversion for a simple attenuating Earth model. The first gradient of full waveform inversion is calculated for analytic, one-dimensional zero-offset data of the attenuating model. Analysis of the results yields that the gradient predicted an imaginary step function located at the depth of the attenuative target. This imaginary step function seems to be an intuitively correct result as introducing absorption into the wave equation usually involves allowing the wavespeed to have an imaginary component. The imaginary part appears to be moving well towards the right answer, whereas the dispersion and its effect on the real part has left much remaining work to be done to reconstruct the correct result.

Earth model and analytic data

The first gradient of full waveform inversion is calculated for analytic data of a very simple attenuating Earth model. This simple Earth model used to calculate the analytic data is shown in Figure 1. It consists of an elastic overburden overlaying an attenuative target with the interface between the layers occurring at a depth of 300m. The source and receiver are collocated at the surface and so the analytic data calculated is one dimensional and normal incidence. The analytic data for this simple model is used to calculate the first step of the gradient function for full waveform inversion. The gradient is analyzed and we attempt to draw insight into how full waveform inversion will reconstruct the proper model in this setting.

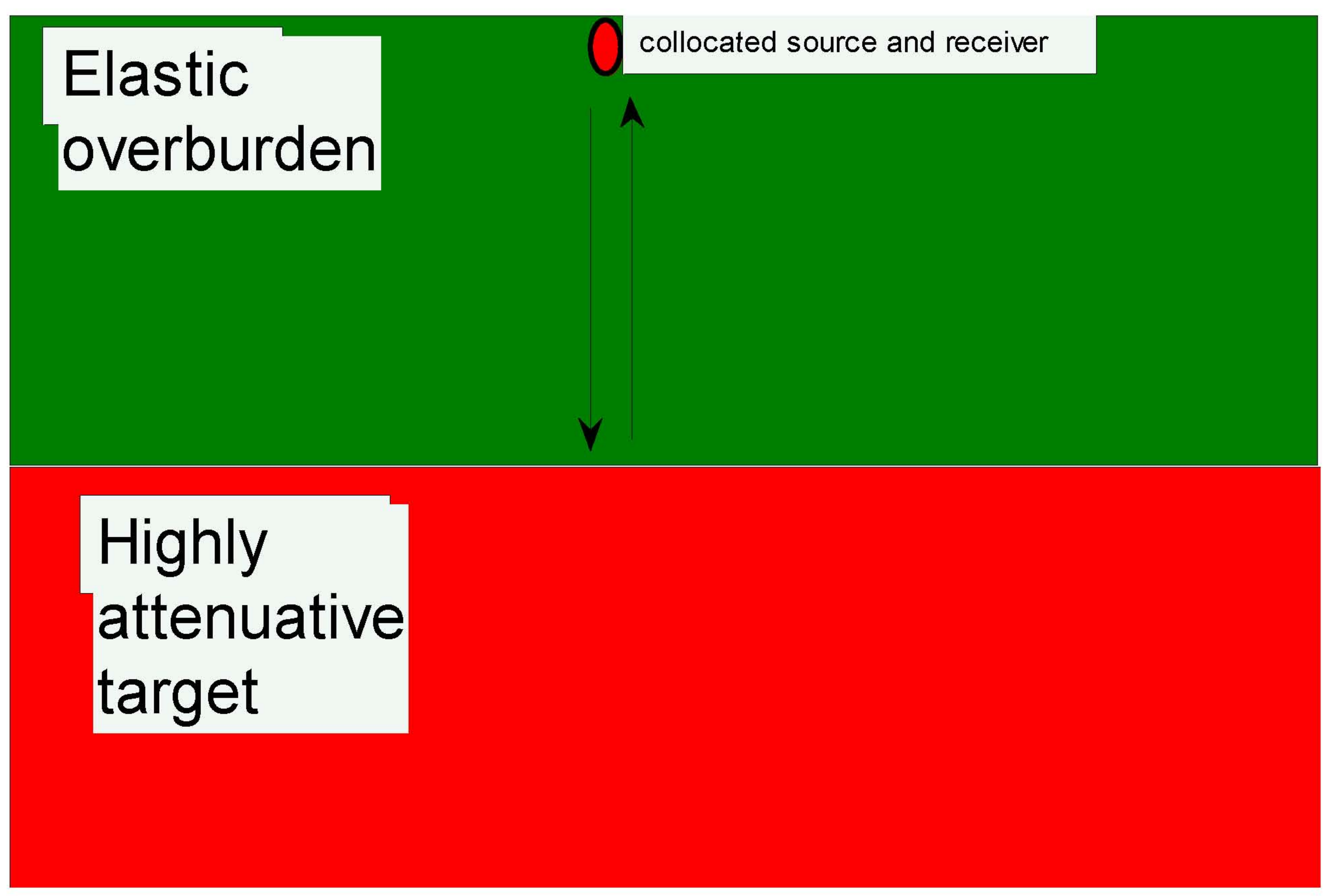

FIG. 1. Simple Earth model to calculate analytic data

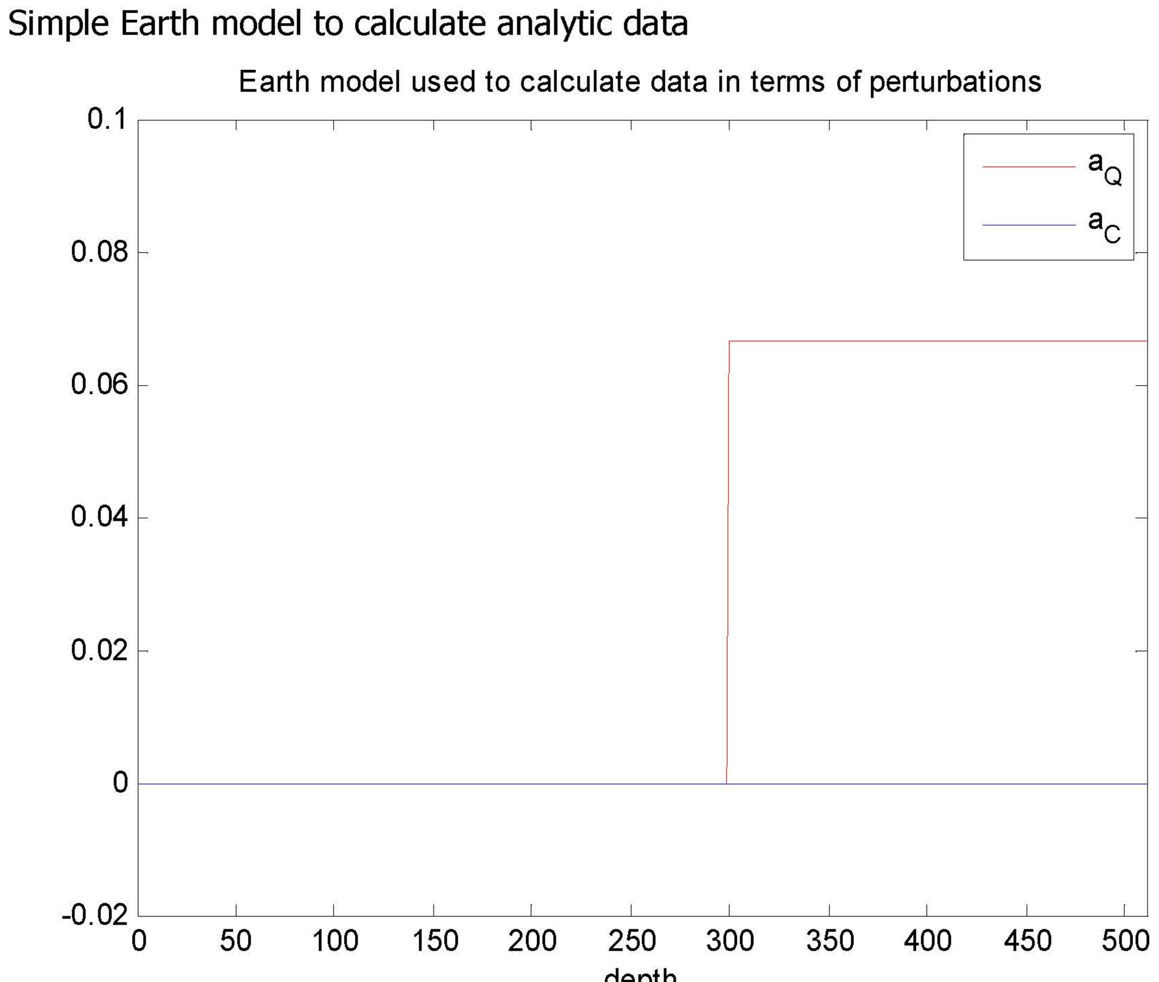

FIG. 2. Earth model in terms of perturbations in velocity (blue) and Q (red).

AVF inversion of anelastic reflectivity

Figure 2 shows the simple attenuating Earth model in terms of perturbations. The blue line in Figure 2 corresponds to the perturbation in velocity, ac, and the red line corresponds to the perturbation in Q, aQ. Notice that there is no contrast in the acoustic properties of the medium and so the profile of ac is constant at zero. However, the profile of aQ jumps at 300m, which corresponds to the contrast in Q at that depth. Also, the starting model to be used in the calculation of the gradient is that of a homogeneous acoustic model with velocity c0, which is shown in Figure 3. In Figure 3 ac and aQ are plotted as a function of depth.

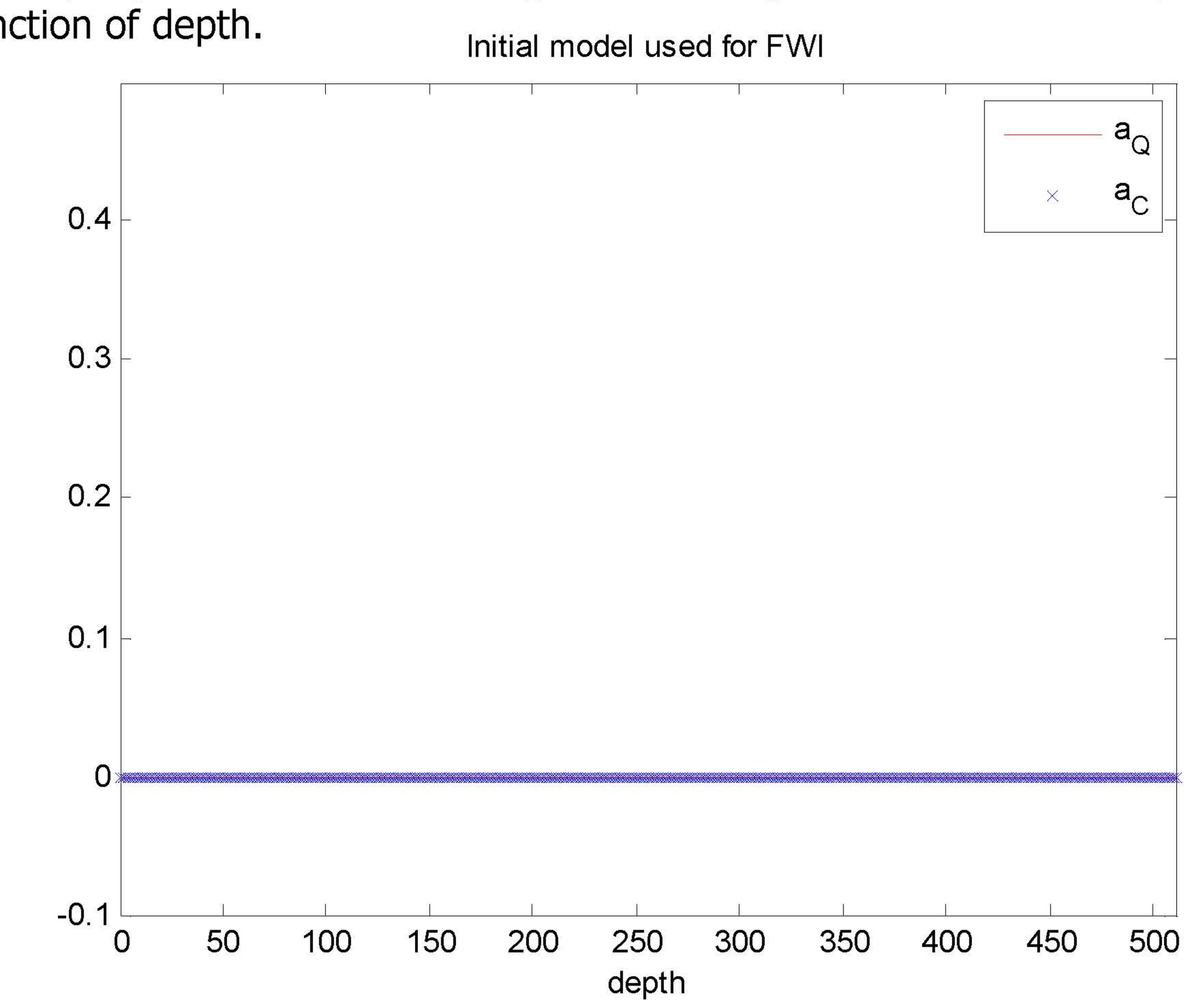

FIG 3. The initial starting model for FWI is homoegenous.

AVF inversion of anelastic reflectivity

For a source and receiver colocated at the surface the analytic, one-dimensional normal incidence data for this Earth model can be written as

$$D(\omega)=\frac{1}{i2K_0}+R(\omega)\frac{e^{i2K_0 z_1}}{i2K_0}$$

AVF inversion of anelastic reflectivity

where K_0 is the wavenumber in the incident medium, z_1 is the depth to the reflector and since there is no acoustic impedance contrast, the reflection coefficient is given approximately by (Bird et al., 2010; Innanen, 2011)

$$R(\omega)=-\frac{1}{2}a_Q\left(\frac{i}{2}-\frac{1}{\pi}\log\left(\omega/\omega_r\right)\right)$$

Full waveform inversion of analytic data

A well-known result of the theory of FWI is that the gradient is given by the equation (Tarantola, 1984; Pratt, 1999; Margrave, 2010)

$$g(z)=\int_{-\infty}^{\infty}\omega^2 G(0,z,\omega)G(z,0,\omega)\delta P^*(0,0,\omega)d\omega$$

where g(z) is the first step of the gradient, G(0,z,ω) is the Green's function for a wave traveling from source position zs=0 to depth z and G(z,0,ω) is the Green's function for a wave traveling from source position zs=z to a receiver at z=0. δP*(0,0,ω)is the complex conjugate of the data residuals. In order to calculate the data residuals, we need another Green's function, G(0,0,ω) which is the analytic data we obtain from our initial model. The needed Green's functions are:

$$G(0,0,\omega)=\frac{1}{i2K_0},G(z,0,\omega)=\frac{e^{iK_0 z}}{i2K_0},G(0,z,\omega)=\frac{e^{iK_0 z}}{i2K_0}$$

Calculation of data residuals and gradient

We can write the data residuals as

$$\delta P^*(0,0,\omega)=D(\omega)-G(0,0,\omega)=R(\omega)\frac{e^{i2K_0 z_1}}{i2K_0}$$

Now we can write the gradient (with a little math) as

$$g(z)=\frac{ic_0^3}{32}a_Q H(z-z_1)+\frac{c_0^2}{8\pi}\int_{-\infty}^{\infty}d\omega\frac{e^{i2K_0(z-z_1)}}{i2K_0}\log\left(\omega/\omega_r\right)$$

Where H(z-z1) is a Heaviside function. The first term in the gradient (call it g1) is a step function which turns on at the depth of the reflector z1, but notice that there is a complex i in front of the Heaviside and therefore the step is wholly imaginary. This seems intuitively correct as we know that in order to model attenuation we let wavenumber or velocity have an imaginary component. Also, the second term (call it g2) , contains a log(ω/ωr) which acts as filter and hence performing the integral should yield a step function which has been in some way filtered. This integral was evaluated numerically. Figure 4 shows the wholly imaginary step function vs depth. Figure 5 shows the absolute value of g2.

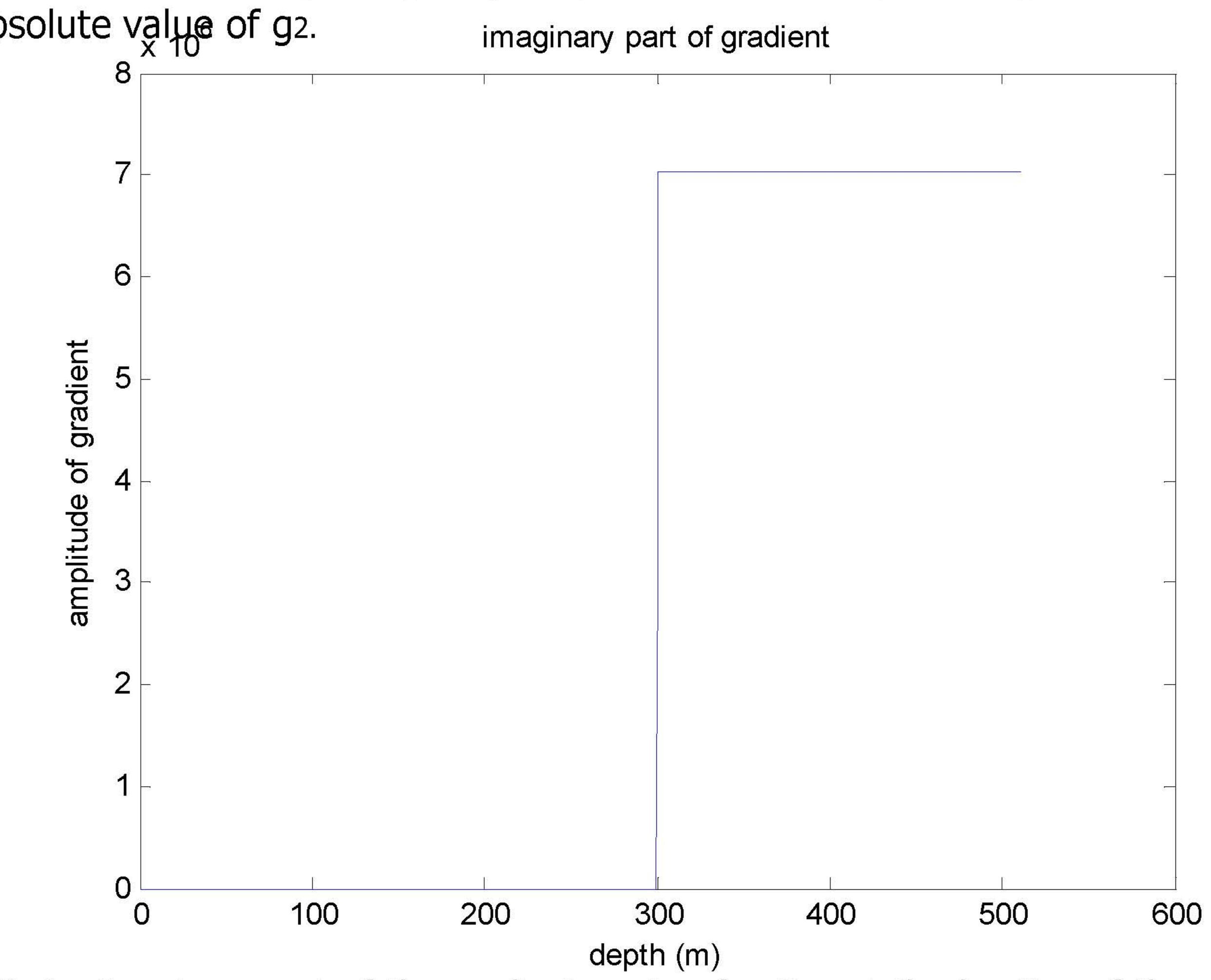

FIG 4. Imaginary part of the gradient, a step function at the location of the absorptive reflector.

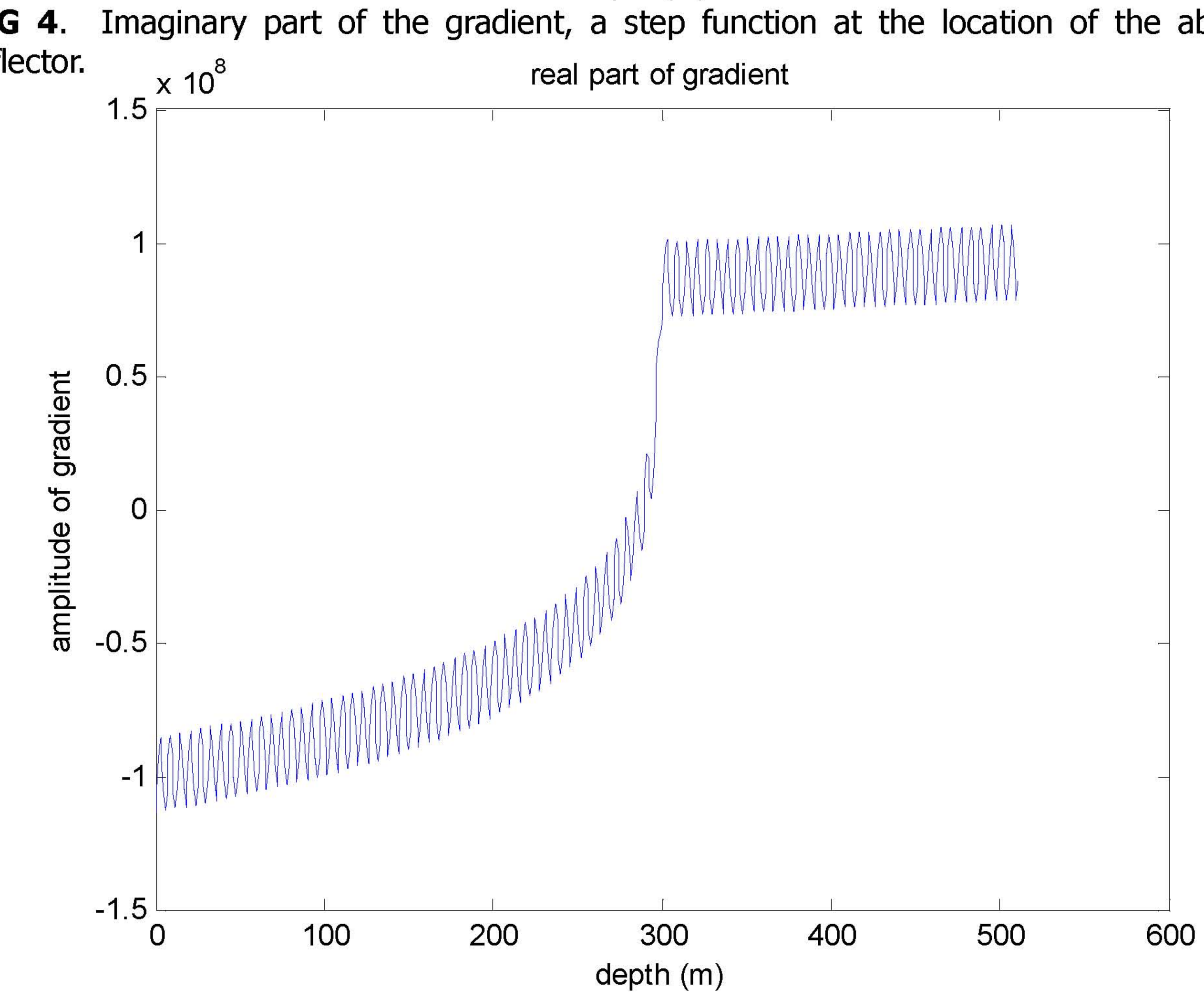

FIG 5. real part of the gradient. This is a step function filtered by the log(ω/ωr) in the integral. Future steps of the gradient will have to remove this character.

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