Least squares AVF inversion

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ABSTRACT

A frequency by frequency method (AVF) of inverting for Q exists which requires as input an estimate of the local spectrum of the absorptive reflection coefficient. We have a calibrated fast S-transform (FST) which we have demonstrated provides a high fidelity estimate of the local spectra of seismic reflection events and is suitable as input for AVF inversion. We formulate the AVF inverse problem in a least-squares formalism. The opportunity for optimization offered by a least squares approach my bring stability to the estimates of Q yeilded by AVF inversion. We also formalize this least-squares approach to use the estimate of a source wavelet, as opposed to removing the wavelet via deconvolution, which appears to offer stability to the estimates of Q and wavespeed. We also extend our least-squares approach to the broader problem of AVO by including angle of incidence. Finally, we use input from the FST as input into our least-squares AVF approach.

Using numerically modeled data we find that the inversion results are accurate up to angles of incidence up to roughly 35 degrees. Also we find that bringing an estimate of the wavelet into the inversion scheme stabilizes the inversion results of Q and wavespeed.

1. Normal Incidence and no wavelet

We start with the expression for the linearized absorptive reflection coefficient for an elastic overburden overlaying a highly attenuative target, given approximately by (Innanen, 2011; Bird et al., 2010)

$$R(\omega) \cong -\frac{1}{2} a_Q F(\omega) + \frac{1}{4} a_C \qquad (1)$$

Where R(ω) is the frequency dependent reflection coefficient, ac is the perturbation in seismic velocity and aQ is the perturbation in Q. We can discretize R(ω) in equation 1 into N samples and re-write as

$$\begin{bmatrix} R(\omega_1) \\ \vdots \\ R(\omega_N) \end{bmatrix} = \begin{bmatrix} 1/4 & -1/2\, F(\omega_1) \\ \vdots & \vdots \\ 1/4 & -1/2\, F(\omega_N) \end{bmatrix} \begin{bmatrix} a_C & a_Q \end{bmatrix} \qquad (2)$$

This highlights that solving for ac and aQ is an overdetermined problem which we can solve by casting as a least-squares minimization problem. Equation 2 is in the form

$$d = Gm$$

Where d=R(ω), and m=[ac aQ]. We can solve this system of equations for m in a least squares formalism with the equation

$$m = \left(G^T G\right)^{-1} G^T d \qquad (3)$$

This is the least squares solution for aC and aQ. To test the effectiveness of this approach modeling was performed for a series of absorptive reflections. The modeling is performed for an impulsive plane wave incident upon a planar boundary separating an elastic overburden with a highly absorptive target. Synthetic traces were generated for this experiment in which Q was allowed to vary from 10 to 105. The spectrum of the traces were then obtained using an FFT and equation (3) was implemented to solve for aC and aQ for each of these traces. Figure 1 and Figure 2 shows the accuracy of the inversion of aC and aQ for all of these experiments. Notice in Figure 1 that the inversion for aQ fails at low Q values. This is to be expected since we linearized the expression for the absorptive reflection coefficient by assuming small aQ, for example see Innanen, (2011); Bird et al., (2010). And as Q becomes small the assumption of a small aQ fails.

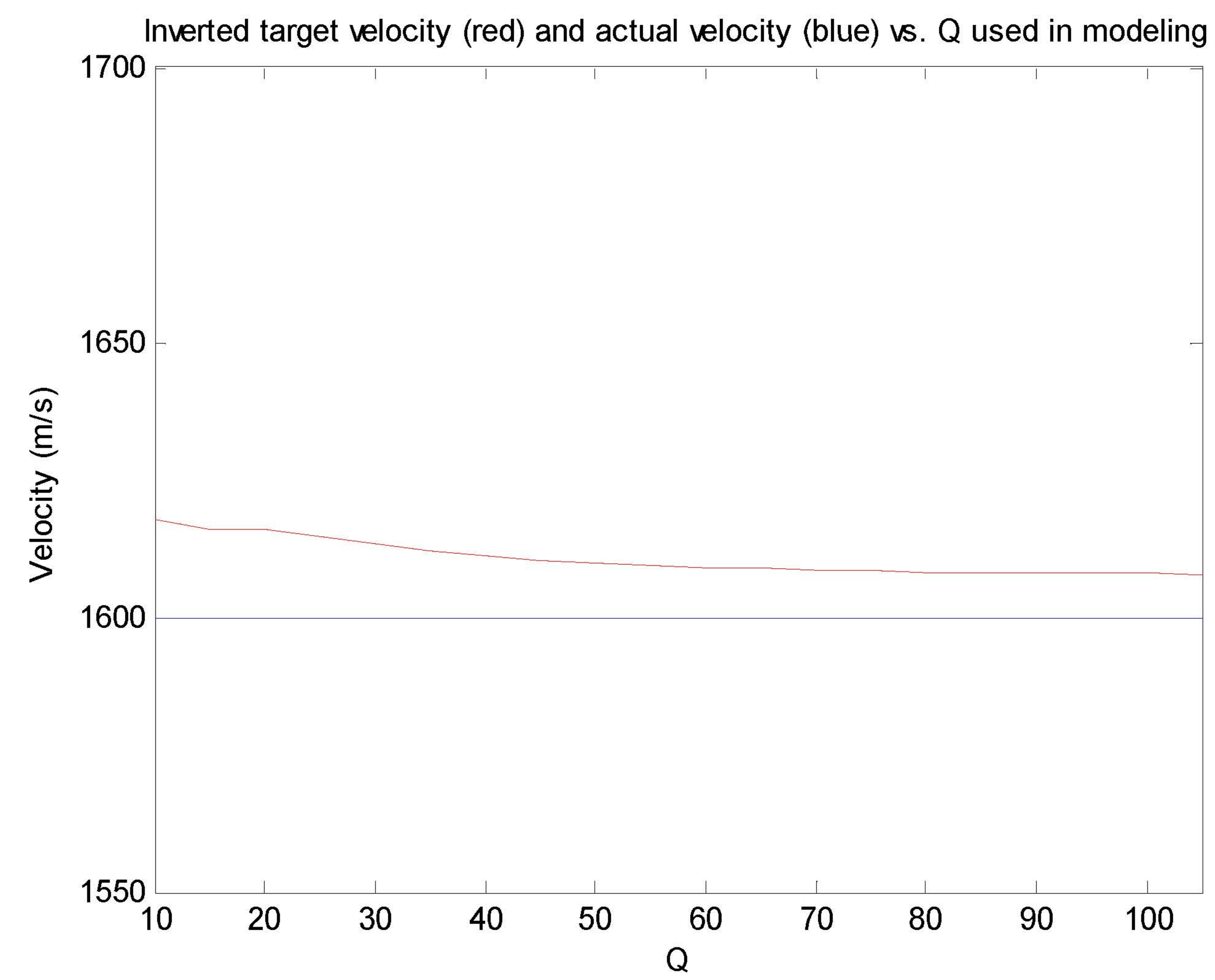

FIG. 1. Inversion results for seismic velocity versus the value of Q used to generate the synthetic traces. The blue curve is the actual velocity and the red curve is the inversion results. Notice that the inversion for velocity becomes more accurate as Q becomes large.

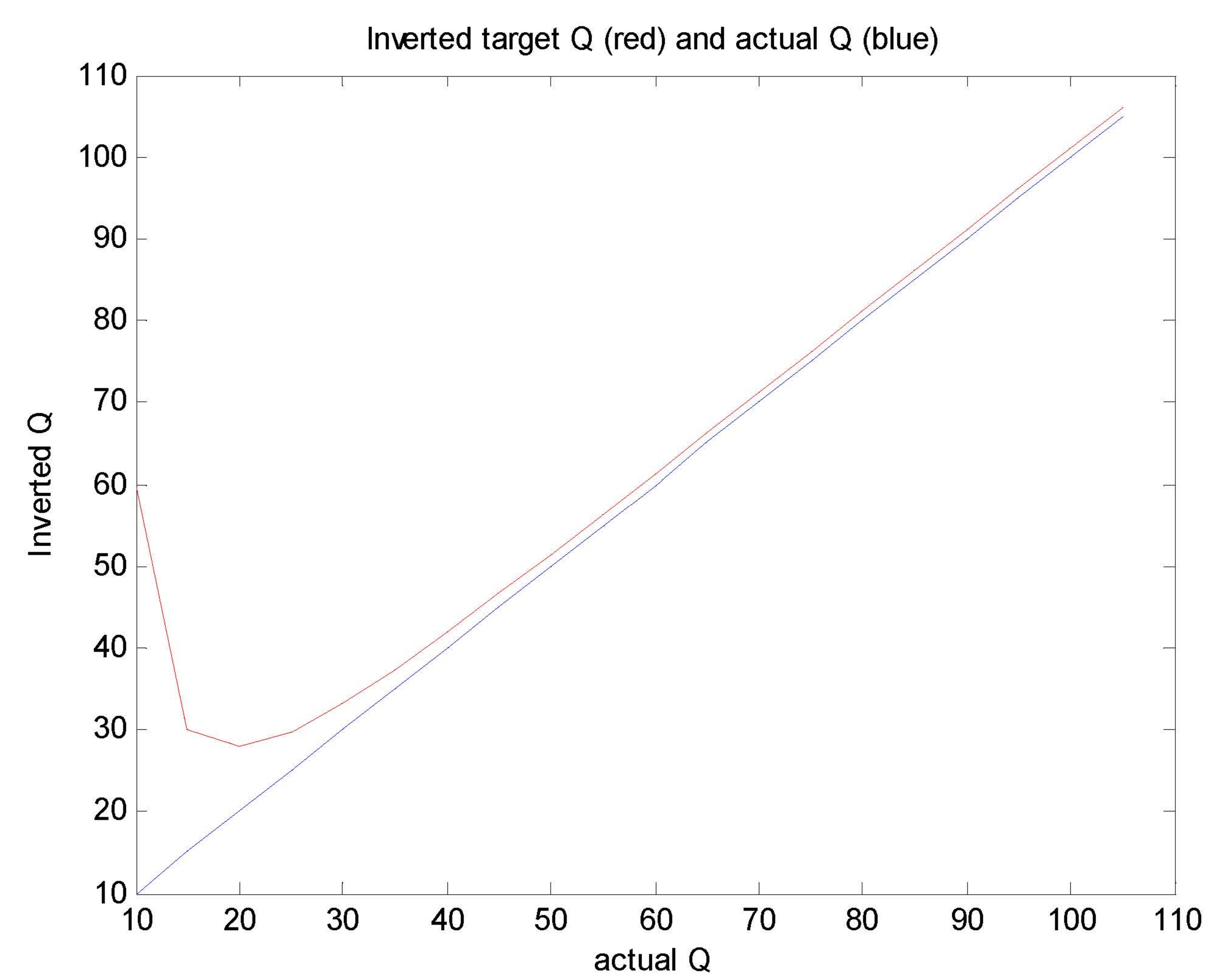

FIG. 2. Inversion results for Q. The blue curve is the actual Q and the red curve is the inversion results. Notice that the inversion for velocity becomes more accurate as Q becomes large. The failure of the inversion at low Q is a result of linearization error.

2. Non-normal Incidence and a source wavelet

We can extend this idea to include angle dependence and a source wavelet. We first extend the normal incidence formula in equation 1 to oblique incidence , for derivation see Innanen, (2011). At the same time we can introduce the effect of a seismic wavelet S(ω) with the expression

$$\tilde{R}(\omega,\theta) \cong \left(-\frac{1}{2} a_Q F(\omega) + \frac{1}{4} a_C\right)\left(1+\sin^2(\theta)\right) S(\omega) \qquad (4)$$

In equation 4, θ is the angle of incidence. For a given angle of incidence, θm, we can solve equation 4 with the same least squares approach as we solved equation 1. To test this least-squares AVF inversion framework, angle dependent reflection coefficients were calculated and a minimum phase wavelet was convolved with the reflection coefficients. Of course on real seismic data we will not have the luxury of knowing the exact wavelet. Therefore a standard weiner deconvolution code from the CREWES matlab toolbox was used to obtain an estimate of the wavelet and that was implemented in equation 4, prior to inversion. Figure 3 shows the inversion result for the wavespeed, the red line corresponds to the inversion result and the blue line is the actual wavespeed. Notice that the inversion fails at high angles of incidence, this is due to the fact that equation 4 is a linearized expression. Figure 4 shows the inversion result for Q in red and the actual Q in blue, again the failure of the inversion at high angles of incidence is due to the linearization error. Figure 3 and Figure 4 show that this least squares approach has the ability to invert for the wavespeed and Q of the target for angles of incidence up to about 40 degrees. This is a promising result as it shows that an estimate of the wavelet using standard deconvolution codes can be used to obtain reliable estimates of target Q.

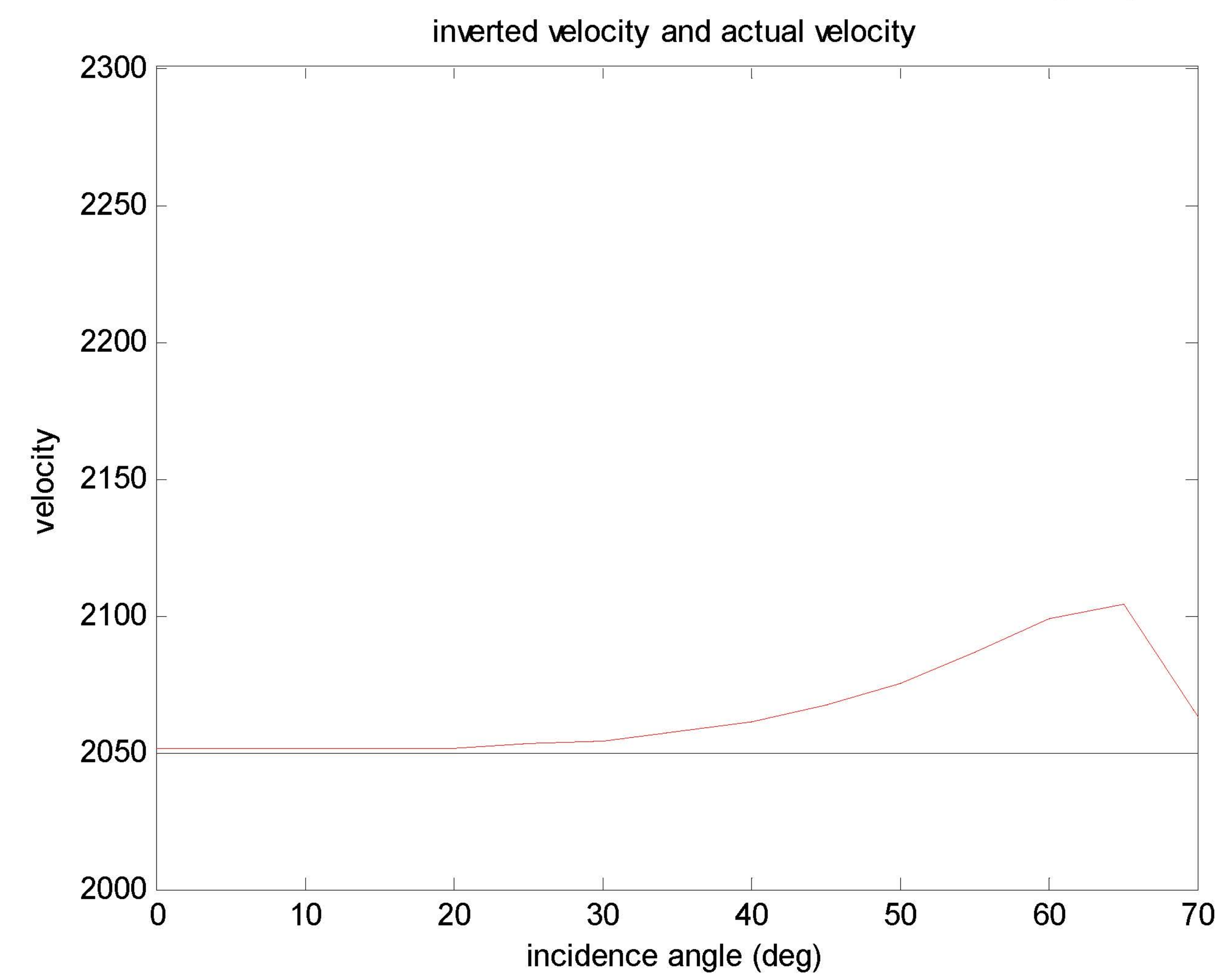

FIG. 3. True velocity (black) and inverted velocity (red) vs. incidence angle

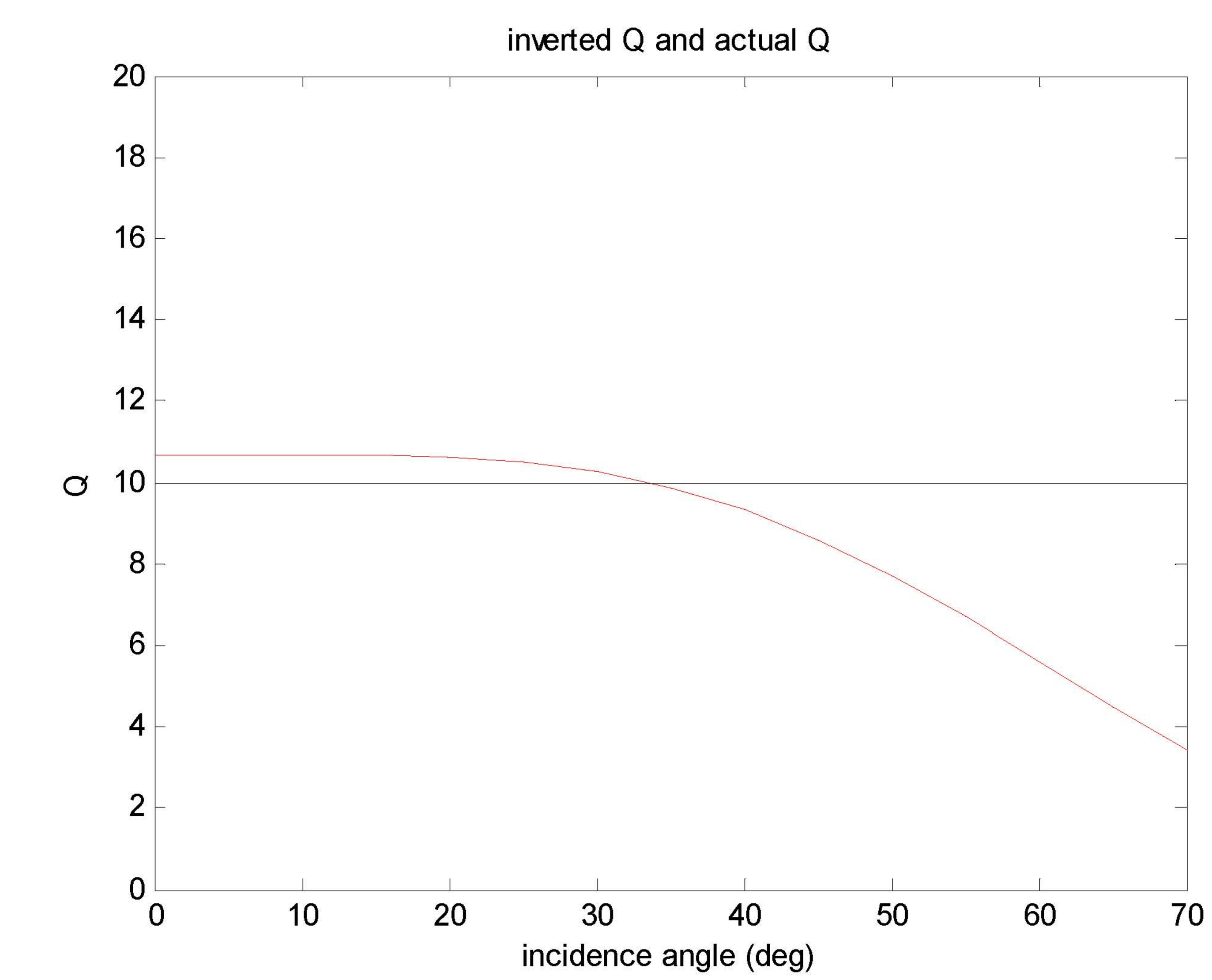

FIG. 3. True Q(black) and inverted Q (red) vs. incidence angle

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