Azimuthal AVO inversion by simulated annealing

David Cho* and Gary F. Margrave

dwhcho@ucalgary.ca

ABSTRACT

Fractures influence the permeability pathways and mechanical properties associated with a rock mass and therefore, are a crucial aspect in the characterization of the subsurface. In this study we develop an azimuthal AVO inversion algorithm using a simulated annealing optimization technique. The parameterization of the problem is in terms of an isotropic background with the inclusion of fractures through an addition of excess compliances to the medium. Preliminary inversion results demonstrate a reasonable estimate of the model parameters in addition to an excellent match between the data and synthetic data. Associated errors in the estimated model parameters are attributed to variable sensitivities of the model parameters to the objective function. Future work will attempt to address these issues through different parameterizations of the problem and additional constraints in the objective function.

INTRODUCTION

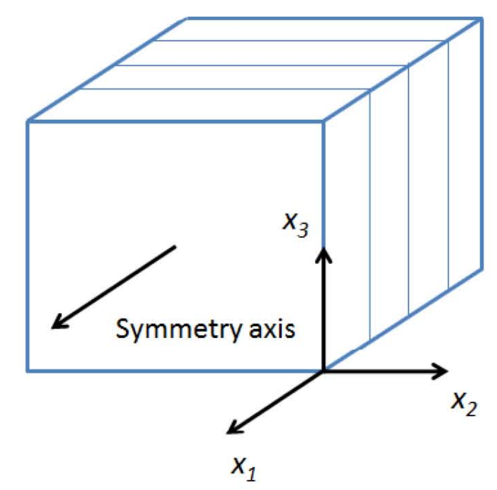
- Oriented fractures
  - Influence permeability pathways
  - Alters mechanical properties of rock mass
  - Differential stresses upon loading (deviations from regional stress field)
- Azimuthal AVO inversion
  - Optimization using simulated annealing
  - Fracture model used for parameterization (Downton and Roure, 2010)

FORWARD MODEL

- Assume Earth model consisting of transversely isotropic (TI) layers
- G(m)=d
  - d: Data – azimuthal angle gathers
  - m: Model – λ, μ, ρ, normal and tangential fracture compliance, fracture azimuth
  - G: Non-linear forward operator – Insert fractures to isotropic background using Linear slip theory (Schoenberg and Sayers, 1995), orient fractures using Bond transformations, calculate reflection coefficients, convolve with wavelet
- Reflection coefficient calculation
  - Rotated elastic stiffness matrix cannot be regarded as TI medium in given coordinate system
  - Rüger's (1998) formulation of reflection coefficients is insufficient
  - Vavryčuk and Pšenčík (1998)
    - Arbitrary anisotropy

$$R_{PP}(\theta,\phi)=R_{PP}^{iso}(\theta)+\frac{1}{2}\left[\Delta\left(\frac{A_{23}+2A_{44}-A_{33}}{A_{33}}\right)\sin^2\phi\right.$$
$$+\left(\Delta\left(\frac{A_{13}+2A_{55}-A_{33}}{A_{33}}\right)-8\Delta\left(\frac{A_{55}-A_{44}}{2A_{33}}\right)\right)\cos^2\phi$$
$$\left.+2\left(\Delta\left(\frac{A_{36}-A_{45}}{A_{33}}\right)-4\Delta\left(\frac{A_{45}}{A_{33}}\right)\right)\cos\phi\sin\phi\right]\sin^2\theta$$
$$+\frac{1}{2}\left[\Delta\left(\frac{A_{11}-A_{33}}{2A_{33}}\right)\cos^4\phi+\Delta\left(\frac{A_{22}-A_{33}}{2A_{33}}\right)\sin^4\phi\right.$$
$$+\Delta\left(\frac{A_{12}+2A_{66}-A_{33}}{A_{33}}\right)\cos^2\phi\sin^2\phi+2\Delta\left(\frac{A_{16}}{A_{33}}\right)\cos^3\phi\sin\phi$$
$$\left.+2\Delta\left(\frac{A_{26}}{A_{33}}\right)\sin^3\phi\cos\phi\right]\sin^2\theta\tan^2\theta$$

$$R_{PP}^{iso}(\theta)=\frac{1}{2}\frac{\Delta Z}{\overline{Z}}+\frac{1}{2}\left[\frac{\Delta\alpha}{\overline{\alpha}}-\left(\frac{2\overline{\beta}}{\overline{\alpha}}\right)^2\frac{\Delta G}{\overline{G}}\right]\sin^2\theta+\frac{1}{2}\frac{\Delta\alpha}{\overline{\alpha}}\sin^2\theta\tan^2\theta$$



SIMULATED ANNEALING

- Models a physical process in which a solid is slowly cooled until it reaches a state that minimizes its internal energy
- Original Metropolis algorithm (1953)
  - Random walk in solution space where an energy, *E* (objective function) is calculated at each step
  - Acceptance criterion $\Delta E < 0$
  - If $\Delta E > 0$, a new solution is accepted with a probability $exp(-\Delta E/T)$, where *T* is the temperature of the system
  - *T* is slowly lowered according to a defined annealing schedule until the system reaches a state of equilibrium
- Alternative implementation presented by Rothman (1985) and applied by Sen and Stoffa (1991) for a 1D FWI
  - Computes the probability of acceptance before a solution is selected
  - Gibbs probability density function

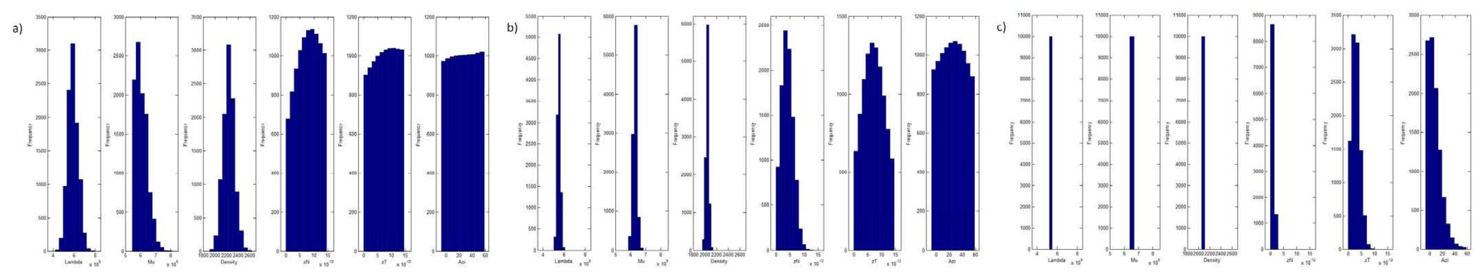
$$P(m_{ij})=\frac{\exp(-E(m_{ij})/T)}{\sum_j \exp(-E(m_{ij})/T)}$$

  - Two term objective function

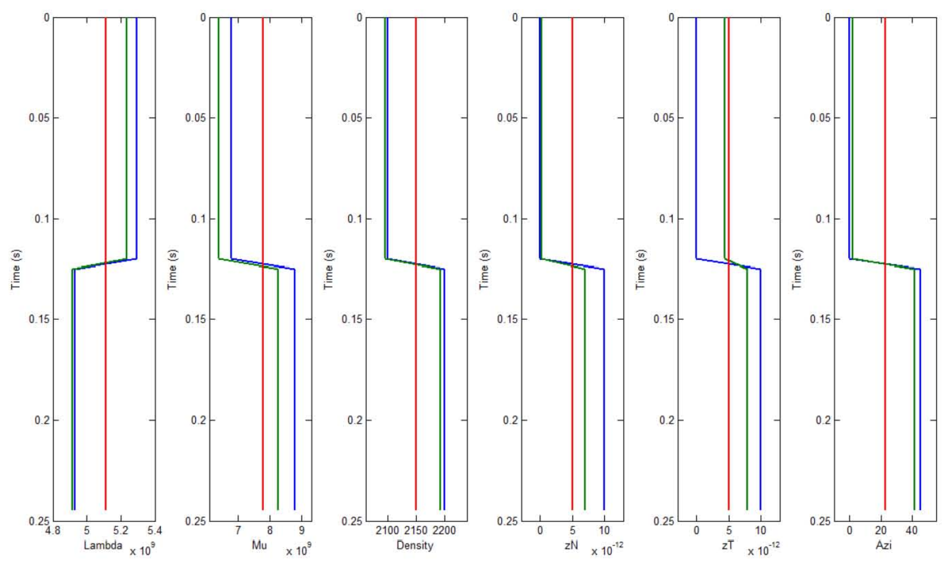
$$E(m_i)=\sum_i (Gm_i-d)^2+\sum_k w_k \sum_i \left(1-m_i/m_i^{(initial)}\right)^2$$

PRELIMINARY RESULTS

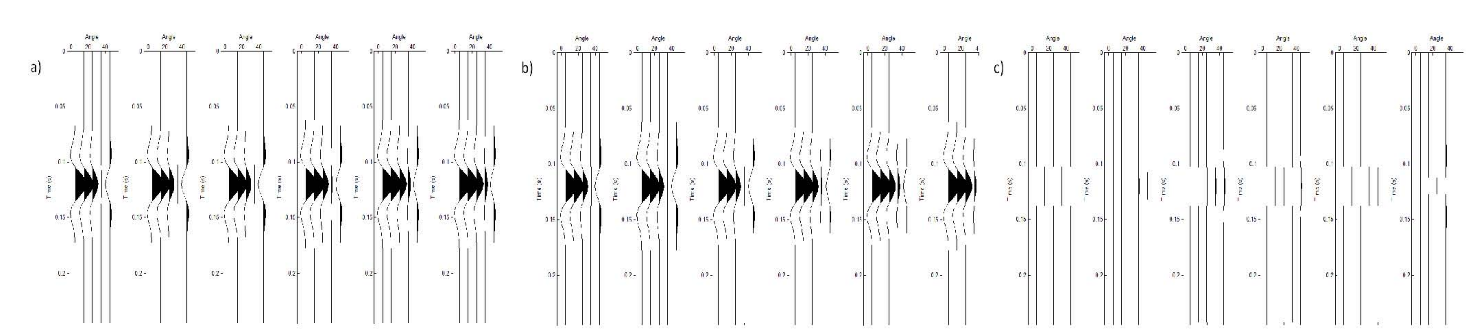
- Two layered Earth model with data generated using the forward modeling procedure outlined previously
- Annealing schedule
  - $T_n=T_0*0.6^n$ with $T_0=0.05$ for $n=1\text{-}20$
- Gibbs probability density functions for each model parameter for the first layer
  - a) after 5 iterations, b) 10 iterations and c) 15 iterations



- Inversion results
  - True model (blue), initial model (red) and inverted model (green)



- Seismograms (azimuthal angle gathers)
  - a) Data, b) synthetic data and c) data residuals



CONCLUSIONS

- Preliminary results demonstrate a reasonable estimate of the model parameters
- Excellent match between the data and synthetic data
- Errors are attributed to the parameterization of the problem
  - The sensitivity to the objective function for each model parameter varies
- Future work will explore alternative parameterizations and constraints in the objective function

ACKNOWLEDGEMENTS

- Many thanks to Faranak Mahmoudian for insightful discussions on the topic of anisotropy and the sponsors of the CREWES project for their support.