A particle view of dispersive wave propagation

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Introduction

We have shown elsewhere that simple seismic wave phenomena may be modeled with sets of notional particles which drift freely and collide. To extend this modeling idea to incorporate attenuation, we merely replace each single particle with a large number of particles, each moving with a velocity drawn from a suitable distribution. Numerical examples demonstrate the qualitative correctness of this model; quantitatively it is supported by arguments due to Bickel (1993), who points out that the constant Q impulse response is equivalent to one of the (one-sided) probability density functions from which we have drawn particle velocities.

Review and approach

In last year's CREWES report we discussed the possibility of describing simple seismic wave experiments (such as zero-offset and walk-away VSP surveys) in terms of colliding particles, as opposed to classically propagating waves (Innanen, 2010). Each propagating waveform, or event, was identified as **a particle with a mass and a momentum**, and these properties were used to discuss scalar reflection, transmission, and propagation phenomena in layered media.

Here we describe how to extend the particle/collision view of a seismic experiment to incorporate another common seismic phenomenon: wave attenuation and dispersion.

The approach is to consider a waveform to be not one particle, but a large number of them, and to assign to each particle a probability of propagating with a given velocity. The waveform at a certain location and time will then be proportional to the number of particles which have reached that location in that time, given these probabilities.

Tracking these particles as locations and times change, we find that the totality of the particles and their distributions closely resembles attenuative and dispersive wave propagation in 1D.

Interestingly, the same probability density functions which supply causal/physical type attenuating pulse shapes in our particle model have already been associated with attenuation models. A constant-Q impulse response is identical to a Pareto-Levy probability distribution (Bickel, 1993). We sense, therefore, that a particle-based attenuation model may be establishing a useful link between a tangible (though notional) physical idea—particles that move with a range of velocities—and some of the formal mathematics of seismic attenuation.

Formulation

We consider a pulse propagating in 1D, i.e., in the direction of increasing distance z as time t increases. To begin, let the pulse be arranged such that it passes $z = 0$ at time $t = 0$, propagating in the direction of positive z. The standard wave interpretation of this pulse is that it represents a propagating disturbance in an otherwise quiet displacement, acceleration, or pressure field.

Let us instead interpret such a "spike" shaped function as a close clustering of a large number, say N, of particles of unit mass, all drifting freely to the right. The waveform could be considered an actual image of this cluster, or simply a plot of the relative number of particles found at all times with z fixed at $z = 0$.

Formulation continued

Now, if all N particles drift with the same speed, at greater depths the plots would illustrate the same "spike" shape shifted to the right by greater amounts. However, our intent is to assign different velocities to the particles. Some of the particles would then arrive at a given depth z earlier than others, and the spike shape would spread out as z increased.

Let c_R be the fastest speed any particle can take on. Then any one of the N particles might take on the speed

$$c = c_R - \Delta c, \quad \Delta c \geq 0. \tag{1}$$

Let the number of particles of the full N which deviate from c_R by Δc be

$$n_{\Delta c} = N \times p(\Delta c), \tag{2}$$

where p is a suitable probability density function.

Particles will arrive at locations z at many different times. At $t = t_1$, how many particles arrive at $z = z_1$? If a particle departs from $z = 0$ and $t = 0$ and arrives at z_1 at t_1, it must have moved at speed

$$c = \frac{z_1}{t_1}, \tag{3}$$

which, by equation (1), means it must have deviated from c_R by an amount Δc where

$$\Delta c = c_R - \frac{z_1}{t_1}. \tag{4}$$

So, the number of particles observed passing z_1 at t_1 is, by equation (2),

$$n_{\Delta c} = N \times p\left(c_R - \frac{z_1}{t_1}\right). \tag{5}$$

Equation (5) then is a prescription for plotting a distribution of particles in mid-drift, at a fixed time over all space z, or at a fixed z over all time. The distribution mimics attenuative wave propagation, inasmuch as it is a translation of a realization of the chosen probability density function, which, per Bickel (1993), if chosen properly is the impulse response of a constant Q medium.

Causality and particle velocities

In equation (1) the condition $\Delta c \geq 0$ leads to a kind of causality. Since c_R is the fastest any particle can travel, it will define the arrival time of the wave, and the further condition then ensures that no particle will arrive earlier than that.

When we choose a distribution for Δc to follow, $p(\Delta c)$, we must choose it to conform to the condition $\Delta c \geq 0$. That is, the probability that any particle arrives before the true arrival time associated with c_R must be zero to ensure a causal pulse.

Symmetric distributions, like the Gaussian, are therefore not allowable. The best distributions for our purposes are those which track numbers of occurences (and so are defined over positive valued outcomes), such as Poisson and Gamma distributions, and of course the Pareto-Levy distribution mentioned above. The choice of pdf and its parameters is akin to choosing the particular Q model.

Examples

A Poisson distribution for particle velocities requires a pdf

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x \geq 0, \tag{6}$$

where λ is a parameter (Abramowitz, 1972). We may then choose values for λ and c_R, and a range of Δc values, and plot the resulting pdf; we do this in Figure 1a. We next apply equation (5). The arriving particles are plotted at three increasing depths in Figure 1b.

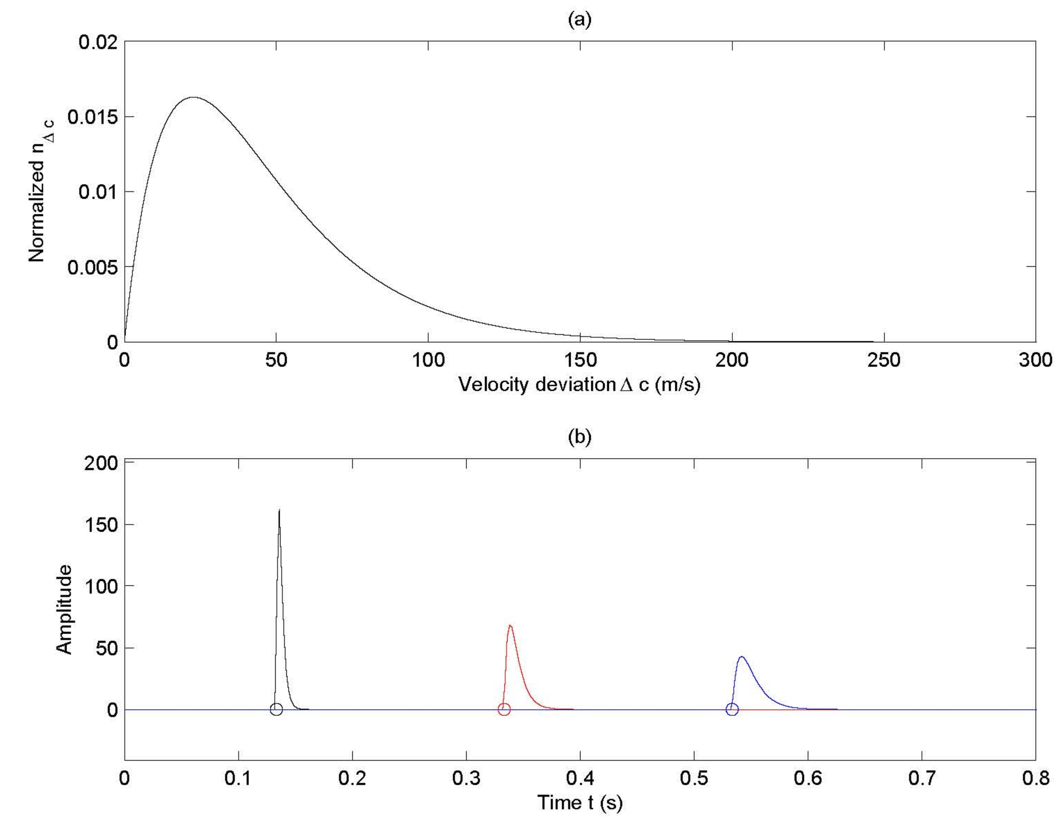

Figure: 1. (a) Normalized velocity distribution $Np(\Delta c)$: Poisson distribution, with $\lambda = 1.0 \times 10^{-10}$ and Δc, the deviation from $c_R = 1500$m/s, ranging from 0-300m/s. (b) Resulting traces at depth z values of 200m (black), 500m (red) and 800m (blue).

A Pareto-Levy distribution for particle velocities requires a pdf

$$p(x) = \left(\frac{1}{2\pi}\right) \int d\omega e^{i\omega x} \tilde{p}(\omega), \tag{7}$$

where $\tilde{p}(\omega) = e^{K(i\omega)^\alpha}$. Here K and α are parameters (Bickel, 1993). See Figure 2.

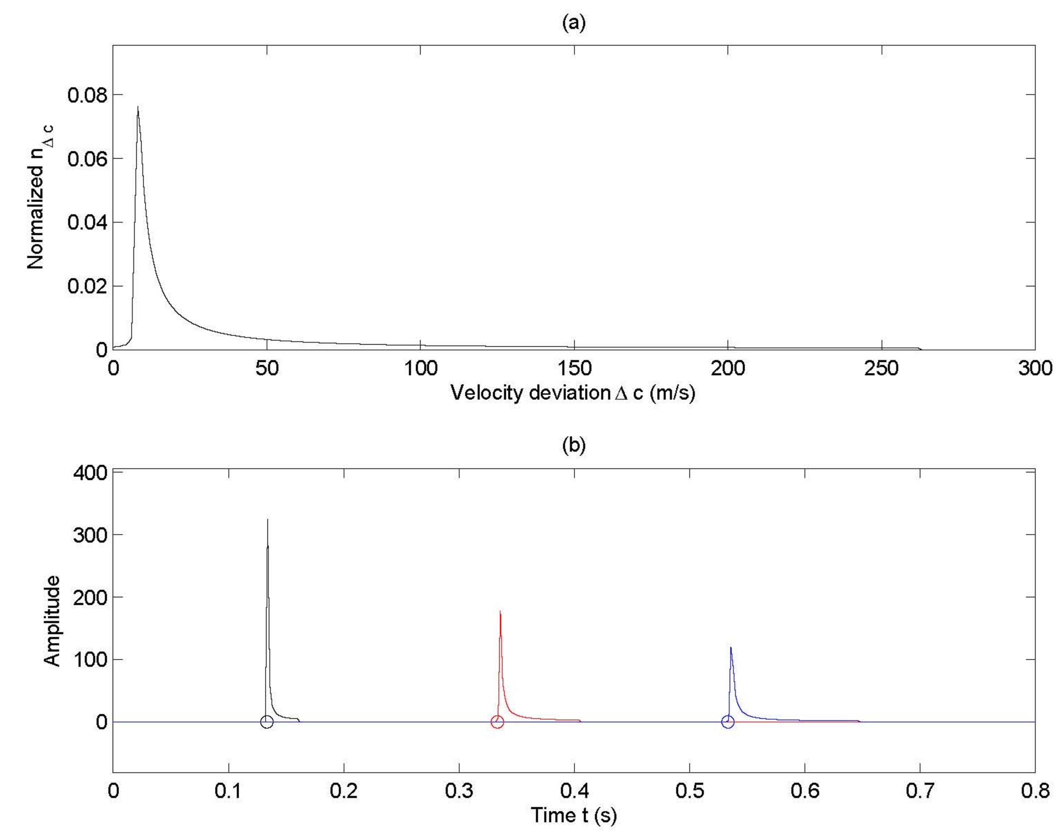

Figure: 2. (a) Normalized velocity distribution $Np(\Delta c)$: Pareto-Levy distribution. (b) Resulting traces at depth z values of 200m (black), 500m (red) and 800m (blue).

Bibliography

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