A POCS algorithm for spectral extrapolation

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Introduction

Projection onto convex sets, or POCS, algorithms, are simple and robust methods for completion of data sets. They have been widely used in seismic data processing for interpolation of missing traces (e.g., Abma and Kabir, 2006; Galloway and Sacchi, 2007), and their promise has led to attempts to extend their application to problems such as time-lapse data differencing (Naghizadeh and Innanen, 2011).

Missing bandwidth in seismic data, particularly in the low end, is a critical obstacle for seismic inversion. Measurement of these low frequencies is of course ideal, and research towards providing—via sources and sensors in combination—the lowest possible spectral cutoff has been a large thrust of CREWES research this year (Margrave et al., 2011). Still, in the absence of measurement down to 0 Hz, spectral extrapolation methods (e.g., Ulrych and Walker, 1984) may be extremely useful, if only to “finish the job” begun by an appropriate experiment. Together with well logs, it may provide a bridge for practical seismic inversion (Lloyd and Margrave, 2011).

In this paper we examine the potential POCS-type algorithms have for extrapolation of low frequencies in seismic data. The approach relies on a particular view of seismic signals. We assume a trace is sparse in the time domain, meaning that the signal, in its pure state, has a small number of large coefficients. Something like a true reflectivity. When low frequencies are missing, the effect on the time domain is that a larger number of nonzero coefficients appear, in the form of sidelobes etc., but with amplitudes significantly smaller than the ones at and around the spike maxima.

Algorithm

With these conditions in place, the POCS algorithm can be carried out in a simple iterative fashion. We begin with a measured trace $x_0(t)$, which is deficient in frequencies below f_0. A threshold Υ_0 operator is formed, which generates $y_0(t) = \Upsilon_0 x_0(t)$, a trace which is equal to $x_0(t)$ for all values above the threshold, and zero everywhere else. This trace $y_0(t)$ is subject to a Fourier transform, and so is the original data trace, creating $X_0(f)$ and $Y_0(f)$ respectively.

A new spectrum is now generated, equal to $X_0(f)$ within the signal band, and equal to $Y_0(f)$ elsewhere:

$$X_1(f) = \Theta\, Y_0(f) + [1-\Theta] X_0(f), \tag{1}$$

where $\Theta = H(f - f_0) - H(f + f_0)$ and H is the Heaviside or step function. This spectrum is inverse Fourier transformed to the time domain, forming $x_1(t)$. The process is then begun again, with a new threshold Υ_1 being chosen, and thus a $y_1(t)$ formed, etc.

The main input to the algorithm is the sequence of thresholds. If, for instance, two iterations are to be carried out, as an input a vector $\upsilon = [\upsilon_0, \upsilon_1]^T$ must be provided in order to construct the operators Υ_0 and Υ_1 etc.

In total then, using the symbol FT to denote the Fourier transform operator, the updated trace $x_{n+1}(t)$ is given in terms of $x_n(t)$ by

$$x_{n+1}(t) = \mathrm{FT}^{-1}\left\{\Theta\, \mathrm{FT}\left[\Upsilon_n x_n(t)\right] + (1-\Theta)\, \mathrm{FT}\left[x_n(t)\right]\right\}. \tag{2}$$

Example I: simple test case

We begin with the simplest of our synthetic examples. In Figure 1 we illustrate the input, with the lowest 10 Hz of the spectrum missing. We next iterate POCS. In Figure 2 each row represents an iteration, with the top row being the input. Most importantly, on the right panel is the integral of the bandlimited trace (red) overlain on the exact integral (black). The difference between red and black in this top right panel is a clear illustration of the need for low frequencies.

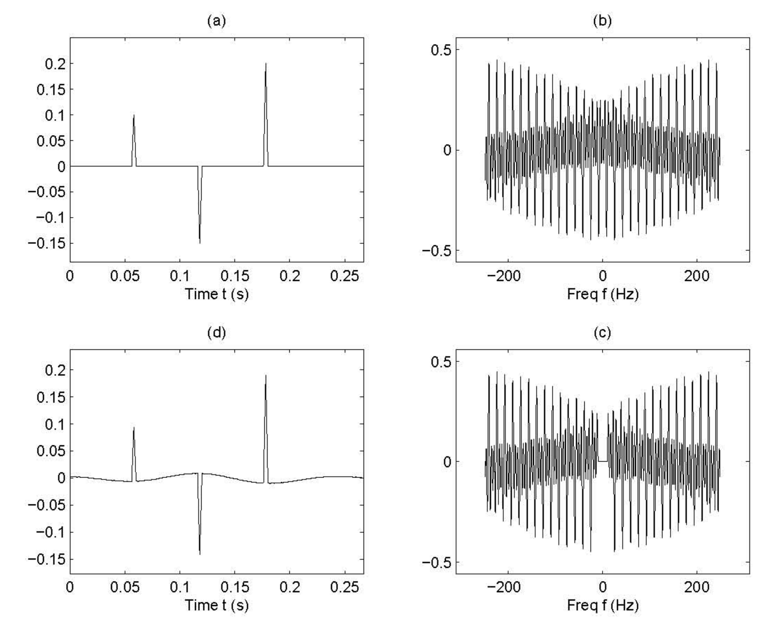

Figure: 1. (a) Reflectivity at full bandwidth. (b) Spectrum of reflectivity. (c) Spectrum w/o lowest 10 Hz. (d) Bandlimited trace.

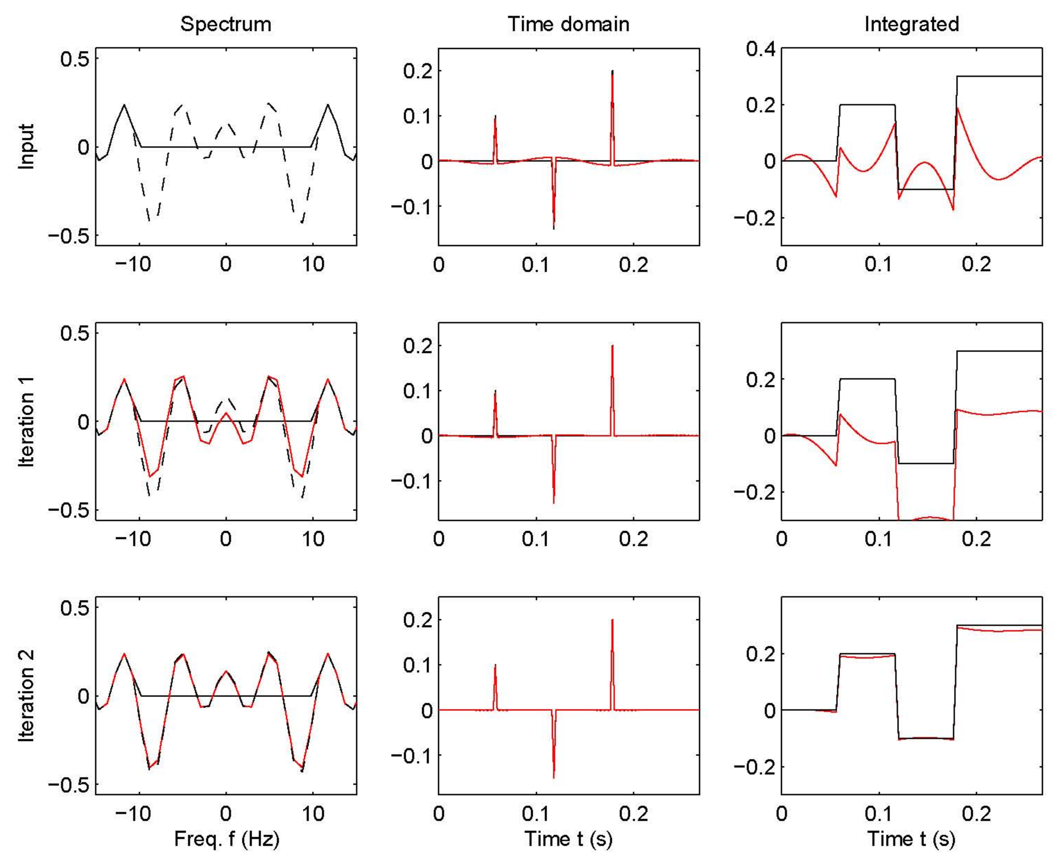

Figure: 2. Iterations of POCS. Top row: left; input trace (black) vs. idealized (dashed); middle; input trace (red) vs. idealized trace (black); right; integrated traces, input (red) vs. idealized (black). Middle row, first it. of POCS; bottom row, second it. of POCS.

Example II: more events and noise

Here we add more events and %1 uncorrelated noise. Figures 3-4 demonstrate the method’s basic insensitivity to data inaccuracy.

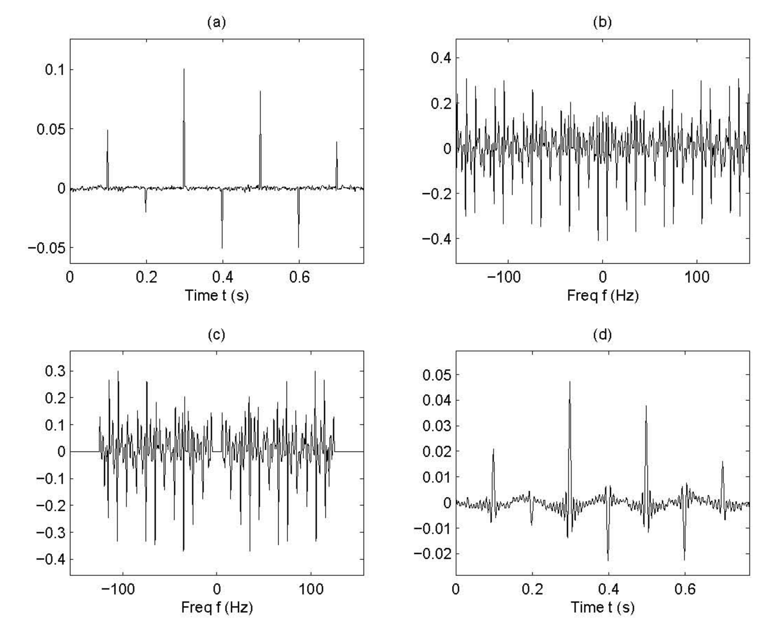

Figure: 3. Input data with %1 noise drawn from a Gaussian distribution.

Example II: more events and noise (continued)

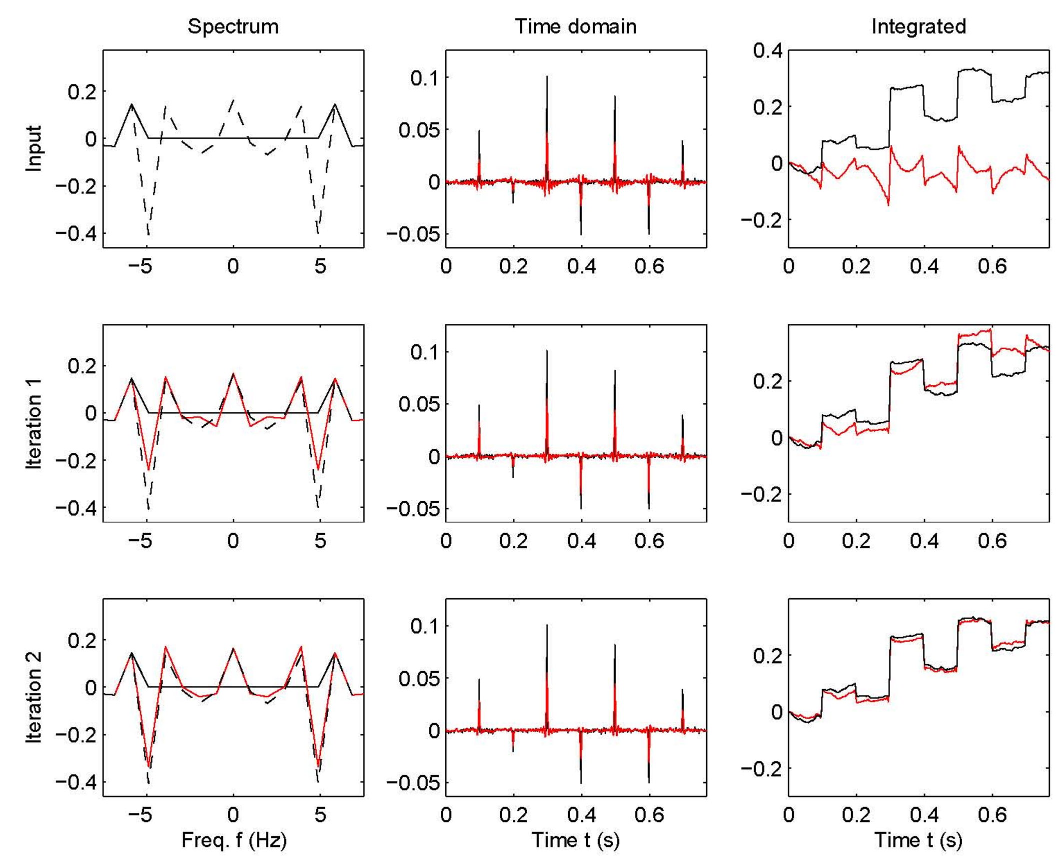

Figure: 4. Iterations of POCS with %1 noise drawn from a Gaussian distribution. Top row: left; input trace (black) vs. idealized (dashed); middle; input trace (red) vs. idealized trace (black); right; integrated traces, input (red) vs. idealized (black). Middle row, first iteration of POCS; bottom row, second iteration of POCS.

Conclusions

POCS based algorithms have a record of completing seismic data in a robust manner, mostly for multidimensional interpolation. It is natural to ask whether such an algorithm might complete the low end of the frequency spectrum, under the assumption that data events are “spike like”.

Synthetic testing appears to confirm the basic applicability of the idea. Mild stressing of the problem by (1) limiting the number of input data points, (2) increasing the number of events, and (3) adding uncorrelated noise with amplitudes of up to %5 of the signal maxima, does not appear to obstruct its use.

Clearly, systematic testing on field data with comparison to well control is the next step. If successful, POCS spectral extrapolation could be seen as a useful preprocessing step prior to various types of seismic inversion.

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