Imaging of time-lapse structural changes with linearized inverse scattering

Shahin Jabbari*, **Kris Innanen** , and **Mostafa Naghizadeh**

sjabbari@ucalgary.ca

Introduction

This paper reproduces earlier work of Innanen and Naghizadeh (Innanen and Naghizadeh, 2010). In a time lapse seismic survey the baseline survey (the first seismic experiment) is compared with the monitoring survey (the following seismic survey at a particular interval of time). The difference data between a baseline and a monitor survey is categorized as the change in either the amplitude or location of the boundary (Lumley, 2001). The scattering theory is used to describe the difference data in a time- lapse problem through resembling the baseline survey as the reference medium , the monitoring survey as the perturbed medium and the difference data as the scattered wavefield data (Zhang, 2006). The study described here focuses on describing the difference data for structural change in a reservoir with scattering theory. A forward operator and adjoint operator are defined from Kaplan's thesis (Kaplan, 2010). A linear Born approximation is used to predict the difference data.

WAVE EQUATION AND BORN APPROXIMATION

The Helmholtz wave equation describes a wavefield propagating through an acoustic medium

$$\left[\left(\frac{\partial^2}{\partial x^2}\right)-\left\{\frac{1}{c^2(x,z)}\right\}\left\{\frac{\partial^2}{\partial t^2}\right\}\right]P(x,z|x_s,z_s;\omega)=f(\omega)\delta(z-z_s)\delta(x-x_s) \qquad \alpha(x)=1-\frac{c_0^2(x,z)}{c^2(x,z)}$$

where c(x,z) is wave speed with x = (x,y), and f(ω) is the frequency distribution of a point source at (x_s,z_s) with x_s = (x_s,y_s). This equation is solved using scattering theory

$$P(x_g,z_g|x_s,z_s;\omega)=P_d(x_g,z_g|x_s,z_s;\omega)+P_s(x_g,z_g|x_s,z_s;\omega)$$

$$P_d(x_g,z_g|x_s,z_s;\omega)=f(\omega)G_0(x_g,z_g|x_s,z_s;\omega)$$

G_0 is the Green's function satisfying

$$\left[\left(\frac{\partial^2}{\partial x^2}\right)-\left\{\frac{1}{c_0^2(x,z)}\right\}\left\{\frac{\partial^2}{\partial t^2}\right\}\right]G_0(x,z|x_s,z_s;\omega)=\delta(z-z_s)\delta(x-x_s)$$

P_d is a direct wavefield propagating from the source to the receiver and P_s is the scattered wavefield. P_s is calculated with the Born approximation

$$P_s(x_g,z_g|x_s,z_s;\omega)\approx f(\omega)\int\int_{-\infty}^{\infty}G_0(x_g,z_g|x',z';\omega)\left(\frac{\omega}{c_0(x',z')}\right)^2\alpha(x',z')G_0(x',z'|x_s,z_s;\omega)dx'dz'$$

$$G_0(k_{gx},z_g|x',z',\omega)=-\frac{1}{i4k_{gz}}e^{-ik_{gx}.x'}e^{ik_{gz}|z'-z_g|} \qquad G_0(x',z'|k_{sx},z_s,\omega)=-\frac{1}{i4k_{sz}}e^{ik_{sx}.x'}e^{ik_{sz}|z'-z_s|}$$

$$k_{gz}=\mathrm{sgn}(\omega)\sqrt{\left(\frac{\omega}{c_0}\right)^2-k_{gx}k_{gx}} \qquad k_{sz}=\mathrm{sgn}(\omega)\sqrt{\left(\frac{\omega}{c_0}\right)^2-k_{sx}k_{sx}}$$

Forward Operator

A forward operator is an operator which maps the earth model to the seismic data using the Born approximation. To drive the forward operator, the vertical and lateral variation in the velocity are treated separately applying the Gazdag and split-step modeling (Gazdag, 1978). In Gazdag modeling, the reference velocity is only a function of the depth which is divided into n_z layers

$$D_l=\{z\in R|0\le z_{l-1}<z\le z_l\} \qquad l=1\ldots n_z$$

The total scattered wavefield is the sum of wavefields within each layer, $P_{s(l)}$

$$P_s=P_{s(1)}+P_{s(2)}+\ldots+P_{s(n_z)}$$

$$P_{s(l)}(k_{gx},z_0|k_{sx},z_0;\omega)=f(\omega)u_{p(1)}u_{p(2)}\ldots u_{p(l-1)}\int_{z_{l-1}}^{z_l}u_{p(l)}(k_{gx},k_{sx},z',\omega)\frac{\omega^2}{c_{0(l)}^2}\alpha(k_{gx}-k_{sx},z')dz'$$

$$U_{p(l)}(k_{gx},k_{sx},z',\omega)=-\frac{e^{i(k_{gz(l)}+k_{sz(l)})(z'-z_{l-1})}}{16k_{gz(l)}k_{sz(l)}}$$

$$k_{gz(l)}=\mathrm{sgn}(\omega)\sqrt{\left(\frac{\omega}{c_{0(l)}}\right)^2-k_{gx}k_{gx}} \qquad k_{sz(l)}=\mathrm{sgn}(\omega)\sqrt{\left(\frac{\omega}{c_{(0)l}}\right)^2-k_{sx}k_{sx}}$$

In split-step modeling, we let $k_{gz(l})$ and $k_{sz(l)}$ be the functions of slowness $(c_{0(l)})^{-1}$ which is allowed to vary in the lateral dimension. Taking the Taylor expansions of $k_{gz(l)}$ and $k_{sz(l)}$ about a constant $(c_{1(l)})^{-1}$ which is the average of $(c_{0(l)})^{-1}$, and truncating it to the first term gives

$$k_{gz(l)}(c_{0(l)}^{-1})\approx k_{gz(l)}(c_{1(l)}^{-1})+\omega\left[1-|c_{0(l)}(x_g)k_{gx}/\omega|^2\right]^{-\frac{1}{2}}(c_{0(l)}^{-1}(x_g)-c_{1(l)}^{-1})$$
$$\approx k_{gz(l)}(c_{1(l)}^{-1})+\omega(c_{0(l)}^{-1}(x_g)-c_{1(l)}^{-1})$$

$$P_{s(l)}(x_g,z_g|x_s,z_s;\omega)=\left(\frac{1}{2\pi}\right)^{4l}f(\omega)(u_{s(1)}F_{gs}^{*}u_{p(1)}F_{gs})\ldots(u_{s(l-1)}F_{gs}^{*}u_{p(l-1)}F_{gs})$$
$$\times\int_{z_{l-1}}^{z_l}(u_{s(l)}(x_g,x_s,z',\omega)F_{gs}^{*}u_{p(l)}(k_{gx},k_{sx},z',\omega)F_{gs})\frac{\omega^2}{c_{1(l)}^2}\alpha(x_g,x_s,z')dz'$$

$$u_{s(l)}(x_g,x_s,z',\omega)=e^{i\omega(c_{0(l)}^{-1}(x_g)+c_{0(l)}^{-1}(x_s)-2c_{1(l)}^{-1})(z'-z_l)}$$

Adjoint Operator

In inversion, the adjoint operator maps the measured data to the perturbation and is defined as

$$P_s(x_g,z_0|x_s,z_0;\omega_j)=\sum_l u(x_g,x_s;\omega_j,z_l)\alpha(x_g,x_s;z_l)$$
$$\alpha^{*}(x_g,x_s;z_l)=\sum_l u^{*}(x_g,x_s;\omega_j,z_l)P_s(x_g,z_0|x_s,z_0;\omega_j)$$

where

$$u^{*}(x_g,x_s;\omega,z_l)=\left(\frac{1}{2\pi}\right)^{4l}f^{*}(\omega)\left(u_{s(l)}^{*}F_{gs}^{*}\frac{\omega^2}{c_{1(l)}^2}u_{p(l)}^{*}F_{gs}\right)\ldots(u_{s(2)}^{*}F_{gs}^{*}u_{p(2)}^{*}F_{gs})(u_{s(1)}^{*}F_{gs}^{*}u_{p(1)}^{*}F_{gs})$$

A structural perturbed time-lapse problem

In this one dimensional time-lapse problem, the depth of the reflector varies from a baseline survey to a monitoring survey. The wavefield for the baseline and monitoring surveys are defined by the Green's function and perturbed medium respectively

$$\left[\frac{d^2}{dz^2}+\frac{\omega^2}{c_I^2(z)}\right]G_0(z,z_s;\omega)=\delta(z-z_s) \qquad \frac{1}{c_I^2(z)}=\begin{cases}c_I^2(z), z>z_I\\ c_0^2(z), z<z_I\end{cases}$$

$$\left[\frac{d^2}{dz^2}+\frac{\omega^2}{c_F^2(z)}\right]P(z,z_s;\omega)=\delta(z-z_s) \qquad \frac{1}{c_F^2(z)}=\begin{cases}c_I^2(z), z>z_F\\ c_0^2(z), z<z_F\end{cases}$$

The solution for the perturbed medium is calculated as

$$P=\frac{1}{i2k_0}+R_I\frac{e^{i2k_0z_I}}{i2k_0}+\frac{\alpha_{TL}}{4}\left[\frac{e^{i2k_0z_F}}{i2k_0}-\frac{e^{i2k_0z_I}}{i2k_0}\right]+\ldots \qquad k_0=\frac{\omega}{c_0} \qquad R_I=\frac{(c_I-c_0)}{(c_I+c_0)}$$

$$\alpha_{TL}=1-\frac{c_I^2(z)}{c_F^2(z)}=\begin{cases}0, & z<z_F\\ 1-\frac{c_0^2(z)}{c_I^2(z)}, & z_F<z<z_I\\ 0, & z_I<z\end{cases}$$
$$=\alpha_{TL}[H(z-z_F)-H(z-z_I)]$$

Conclusion

Time-lapse measurements provide a tool to monitor the dynamic changes in subsurface properties. A forward and adjoint operator for the difference data based on linear Born approximation can be derived for a time lapse problem. One of the main obstacle on using linear scattering theory in the time-lapse problem is producing spurious events due to the complexity of the reference medium which is the baseline survey. A possible solution to this, is calculating the higher order terms in the Born series and investigating if these events can be removed through the involving higher order terms.

ACKNOWLEDGEMENT

The authors wish to thank the sponsors, faculty, and staff of the Consortium for Research in Elastic Wave Exploration Seismology (CREWES) for their support of this work, as well as Sam Kaplan for his comprehensive thesis which was helpful in understanding the basic for this work.

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