Nonlinear scattering terms in a seismic context

Shahin Jabbari*, and **Kris Innanen**

sjabbari@ucalgary.ca

Introduction

This paper is a review of the work reported by Matson (1996). In forward scattering theory, the wavefield in the earth is treated as a reference wavefield plus a perturbation. The nonlinear relationship between the perturbation and earth can be determined using the Lippmann-Schwinger equation which in turn is expanded into the Born series, an infinite terms of wave propagation (reflection and transmission). When the perturbation is small, the Born series can be truncated after the second term leading to the Born approximation, in which the data measured is linear in the model. Higher order terms in the Born series play an important role when the perturbation is lager (Matson, 1996). In seismology, the wavefield data are recorded to determine the model earth properties . This method is referred as inversion scattering problems and can be applied to predicts and remove multiples (Carvalho et al., 1991; Weglein et al., 2003; Innanen, 2009). In this project Born series terms have been derived for a simple one dimensional model.

Theory: The Born Series

We begin with the one dimensional constant density acoustic wave equation

$$\left[\left(\frac{\partial^2}{\partial x^2}\right)-\left\{\frac{1}{c^2(x)}\right\}\left\{\frac{\partial^2}{\partial t^2}\right\}\right]P(x|x_s;t)=\delta(x-x_s)\delta(t) \qquad \frac{1}{c^2(x)}=\left(\frac{1}{c_0^2}\right)[1-\alpha(x)]$$

Fourier transforming over time gives

$$\left[\left(\frac{\partial^2}{\partial x^2}\right)+\left\{\frac{\omega^2}{c_0^2(x)}\right\}\right]\widetilde{P}(x|x_s;k)=\delta(x-x_s)+\left(\frac{\omega^2}{c_0^2}\right)\alpha(x)\widetilde{P}(x|x_s;k) \qquad \alpha(x)=1-\frac{c_0^2}{c^2(x)}$$

$$\left[\left(\frac{\partial^2}{\partial x^2}\right)+\left\{\frac{\omega^2}{c_0^2}\right\}\right]\widetilde{P}_0(x|x_s;k)=\delta(x-x_s)$$

where $\tilde{P}_0$ is the Green's function. Using this Green's function and taking integral equation gives Lippmann-Schwinger equation

$$\widetilde{P}(x|x_s;k)=\widetilde{P}_0(x|x_s;k)+\int_{-\infty}^{\infty}\widetilde{P}_0(x|x';k)k^2\alpha(x')\widetilde{P}(x'|x_s;k)dx'$$

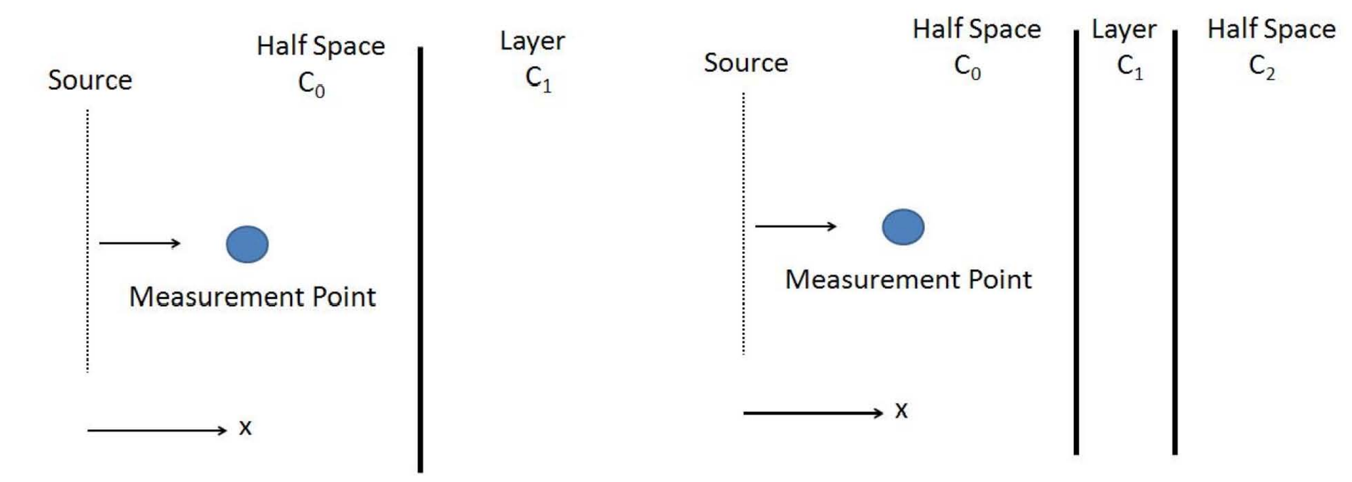
Iterating the Lippmann-Schwinger equation back into itself generates the Born series

$$\widetilde{P}(x|x_s;k)=\widetilde{P}_0(x|x_s;k)+\int_{-\infty}^{\infty}\widetilde{P}_0(x|x';k)k^2\alpha(x')\widetilde{P}_0(x'|x_s;k)dx'$$
$$+\int_{-\infty}^{\infty}\widetilde{P}_0(x|x';k)k^2\alpha(x')\left[\int_{-\infty}^{\infty}\widetilde{P}_0(x'|x'';k)k^2\alpha(x'')\widetilde{P}_0(x''|x_s;k)\right]dx''+\ldots$$
$$=\widetilde{P}_0+\widetilde{P}_1+\widetilde{P}_2+\ldots$$

Scattering by a single interface

Two semi-infinite half-spaces with wave velocities c_0 and c_1 are considered (Fig 1. A). The Born approximation is calculated as

$$\widetilde{P}_{Born}(x<x_1|0;k)\approx\frac{e^{ikx}}{2ik}+\left(\frac{\alpha_0}{8ik}\right)e^{ik(2x_1-x)} \qquad \widetilde{P}_{Born}(x>x_1|0;k)=-\frac{e^{ikx}}{2ik}\left[\frac{\alpha_0}{4}\{2ik(x-x_1)-1\}-1\right]$$



(A) **(B)**

FIG. 1. A) Plane source of acoustic waves incident on a planer interface, B) Plane source of acoustic wave incident on a layer between two half-space.

The reflection and transmission coefficients calculated are not in agreement with the expected ones. For smaller values of α_0, these coefficients approach to the true values (FIG 2).

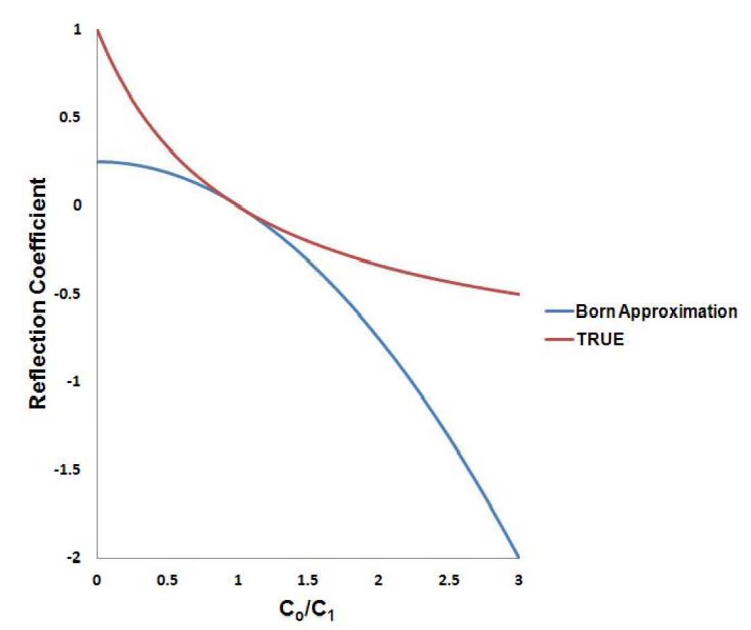


FIG. 2. A comparison of the true reflection coefficient to the Born approximate coefficient

Adding higher terms in the Born series solves this problem.

$$\widetilde{P}_R(x<x_1|0;k)=\frac{e^{ik2x_1}e^{-ikx}}{ik}\left[\left(\frac{1}{\alpha_0}\right)\left\{1-\frac{\alpha_0}{2}-(1-\alpha_0)^{\frac{1}{2}}\right\}\right]$$

$$R=\frac{\widetilde{P}_R}{\widetilde{P}_0}=\left(\frac{2}{\alpha_0}\right)\left\{1-\frac{\alpha_0}{2}-(1-\alpha_0)^{\frac{1}{2}}\right\}=\frac{c_1-c_0}{c_1+c_0}$$

$$\widetilde{P}_T(x>x_1|0;k)=\left[\frac{c_1}{c_1+c_0}\right]\frac{e^{ik_1(x-x_1)}e^{ikx_1}}{ik} \qquad k_1=\frac{\omega}{c_1}$$

$$T=\frac{\widetilde{P}_T}{\widetilde{P}_0}=\left[\frac{2c_1}{c_1+c_0}\right]=1+R$$

Scattering by a layer between two half-spaces

In this model, shown in Figure 1.B., a single layer is between two semi-infinite half spaces. The perturbation is

$$\alpha(x)=\alpha_1H(x-x_1)+(\alpha_2-\alpha_1)H(x-x_2) \qquad \alpha_1=1-\left(\frac{c_0}{c_1}\right)^2 \qquad \alpha_2=1-\left(\frac{c_0}{c_2}\right)^2$$

where x1 and x2 are the location of the first and second interfaces.

The reflected data from the first and second interfaces plus the transmitted data are calculated using the same approach. For reflection data this is written as

$$\widetilde{P}(x<x_1|0;k)=\widetilde{P}_0(x<x_1|0;k)+\widetilde{P}^{pr1}(x<x_1|0;k)+\widetilde{P}^{pr2}(x<x_1|0;k)$$
$$+\widetilde{P}^{mlt1}(x<x_1|0;k)+\widetilde{P}^{mlt2}(x<x_1|0;k)+\text{higher order multiples}$$

where the subscripts pr1, pr2, mlt1, mlt2 represent the first and second primary reflections, and the first and second order multiple reflections, respectively.

$$\widetilde{P}^{pr1}(x<x_1;k)=R_1\left[\frac{e^{-ikx}e^{-ik2x_1}}{2ik}\right] \qquad \widetilde{P}^{pr2}(x;k)=T_{01}T_{10}R_2\left[\frac{e^{-ikx}e^{-ik2x_1}e^{-ik2(x_2-x_1)}}{2ik}\right]$$

$$R_1=\frac{\left[2-2(1-\alpha_1)^{\frac{1}{2}}-\alpha_1\right]}{\alpha_1}=\frac{c_1-c_0}{c_1+c_0} \qquad R_2=\frac{\left[2-\alpha_2-\alpha_1-2(1-\alpha_1)^{\frac{1}{2}}(1-\alpha_2)^{\frac{1}{2}}\right]}{\alpha_2-\alpha_1}=\frac{c_2-c_1}{c_2+c_1}$$

$$\widetilde{P}^{mlt1}(x;k)=-R_1(1-R_1^2)R_2^2\left[\frac{e^{-ikx}e^{ik2x_1}e^{ik_14(x_2-x_1)}}{2ik}\right]$$

$$\widetilde{P}^{mlt2}(x;k)=-R_1^2(1-R_1^2)R_2^3\left[\frac{e^{-ikx}e^{ik2x_1}e^{ik_16(x_2-x_1)}}{2ik}\right]$$

Conclusion

Scattering theory could simplify a complex wave field in a complex medium into a reference wavefield, Green's function, which is perturbed by medium. The calculation in this project have not only confirmed that the role of the higher order terms is to alter the amplitude of reflected and transmitted wave, adjust the propagation velocity of transmitted wave, and describe inter layer multiples, but have also shown the details of how this occurs.

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