Unphysical negative values of the anelastic SH plane wave energy-based transmission coefficient

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Summary

Computing reflection and transmission coefficients in viscoelastic media is complicated since the proper signs have to be chosen for the vertical slowness of the scattered waves. Research has shown that the commonly used methods for deciding the sign of the vertical slowness in the elastic media do not produce satisfactory results in viscoelastic case. New methods have been suggested by researchers to solve the problem, but none of them are quite perfect since some undesirable results still exist in these methods. ERC or the Extended Radiation Condition suggested by Krebes and Daley (2007) is one of the methods that give reasonable results. It is found that when ERC is used, in some cases, the energy-based transmission coefficient becomes negative in a specific range of supercritical angles. In this work the conditions under which T becomes negative is determined and examined numerically. It will be shown that only for the case with increasing velocity and decreasing absorption, under specific conditions, T is negative for specific range of supercritical incidence angles.

R/T Coefficients

Consider a plane SH wave striking a plane boundary separating two viscoelastic media. According to The "elastic-viscoelastic correspondence principle" one could obtain the same equations as in the elastic case for viscoelastic media if the velocities and angles are replaced by the complex ones. The continuity of the normal energy flux across the boundary can be expressed as follows (Borcherdt, 1977):

$$R+T+I=1 \tag{1}$$

where

$$R=\left|C_R\right|^2 , \qquad T=\left|C_T\right|^2\left[\frac{Re(M_2\eta_2)}{Re(M_1\eta_1)}\right] \tag{2}$$

And

$$I=-2Im\ C_R\left[\frac{Im\ M_1\eta_1}{Re\ M_1\eta_1}\right] \tag{3}$$

R, T and *I* are energy based reflection, transmission and *interaction* coefficients. *I* is a normal energy flux caused by interaction between stress and velocity fields of the incident and reflected waves (Aki and Richards, 1980). C_R and C_T are displacement reflection and transmission Coefficients, and $M=\rho\beta^2$ is the complex shear modulus. η is the vertical component of the slowness vector, which is complex in viscoelastic media :

$$\eta_n=\pm\sqrt{\frac{1}{\beta_n^2}-p^2} \tag{4}$$

The sign of the root square determines whether the Wave is downgoing or upgoing. Since in the viscoelastic case the term under the root square is complex the choice of the sign becomes complicated (Ruud, 2006; Krebes and Daley, 2007; Krebes, 1983).

Extended Radiation Condition(ERC)

This method which was introduced by Krebes and Daley (2007) represents a good criterion for choosing the sign of the vertical slowness and it is implied as follows:

If $Im\ \eta_2^2<0$ and $Re\ \eta_2^2<0$ then choose the negative square root , otherwise choose the positive root square.

This method gives reasonable results in comparison with recently suggested methods. Numerical experimentations show that when ERC is used the energy-based transmission coefficient becomes negative in a specific range of incidence angles. In this work we determine the conditions under which T is negative for a incidence SH wave.

Results

After some mathematical calculations it is found that only for the case in which $V_2>V_1$ and $Q_2>Q_1$, T becomes negative for a range of incidence angles. Q is the Quality factor inversely proportional to the loss energy in the corresponding medium.

Assuming $V_2>V_1$ and $Q_2>Q_1$ for an incident plane SH wave, if ERC is used then:

$$\text{If}\quad 1<\frac{Q_2}{Q_1}\le 2-\frac{V_1^2}{V_2^2}\quad\text{then}\quad T<0\quad\text{for}\quad j_{c1}<j_1<j_1{}' \tag{5}$$

$$\text{and if}\quad \frac{Q_2}{Q_1}>2-\left(\frac{V_1}{V_2}\right)^2\quad\text{then}\quad T<0\quad\text{for}\quad j_{c1}<j_1<90^\circ \tag{6}$$

$$\text{where}\ \sin j_{c1}=\frac{V_1}{V_2}\quad\text{and}\quad \sin j_1{}'=\left(\frac{V_1}{V_2}\right)\Big/\sqrt{2-\frac{Q_2}{Q_1}} \tag{7}$$

Numerical Examples

The numerical computations for the earth model presented in table below are preformed, and confirm the analysis above.

Medium	ρ (g cm^{-3})	V_{HP} (km s^{-1})	V_{HS} (km s^{-1})	Q_P	Q_S
1	2.1	2.5	1.0	25	15
2	2.2	5.0	2.0	40	20

Using the analysis above for this model equation (5) become $1<1.33<1.75$ which states T becomes negative for $30°<j_1<38°$. This result is obtained when the energy-based R/T coefficients are calculated for this model. Figure1 shows these coefficients for this example and confirms that T becomes negative for $30°<j_1<38°$. A more complete presentation of the results of this report can be found in Krebes and Moradi (2011).

Conclusions

We have investigated the conditions under which the SH viscoelastic energy-based transmission coefficient becomes negative when ERC is used for choosing the sign of the vertical slowness. It is found that only for the case that $V_2>V_1$ and $Q_2>Q_1$, if $\frac{Q_2}{Q_1}>2-\left(\frac{V_1}{V_2}\right)^2$ then SH-wave energy-based transmission coefficient becomes negative for all supercritical incidence angles, and if $1<\frac{Q_2}{Q_1}\le 2-\frac{V_1^2}{V_2^2}$ it becomes negative for only part of the supercritical zone.

using these conditions we are able to predict negative T values, but the interpretation of these unphysical values remains uncertain.

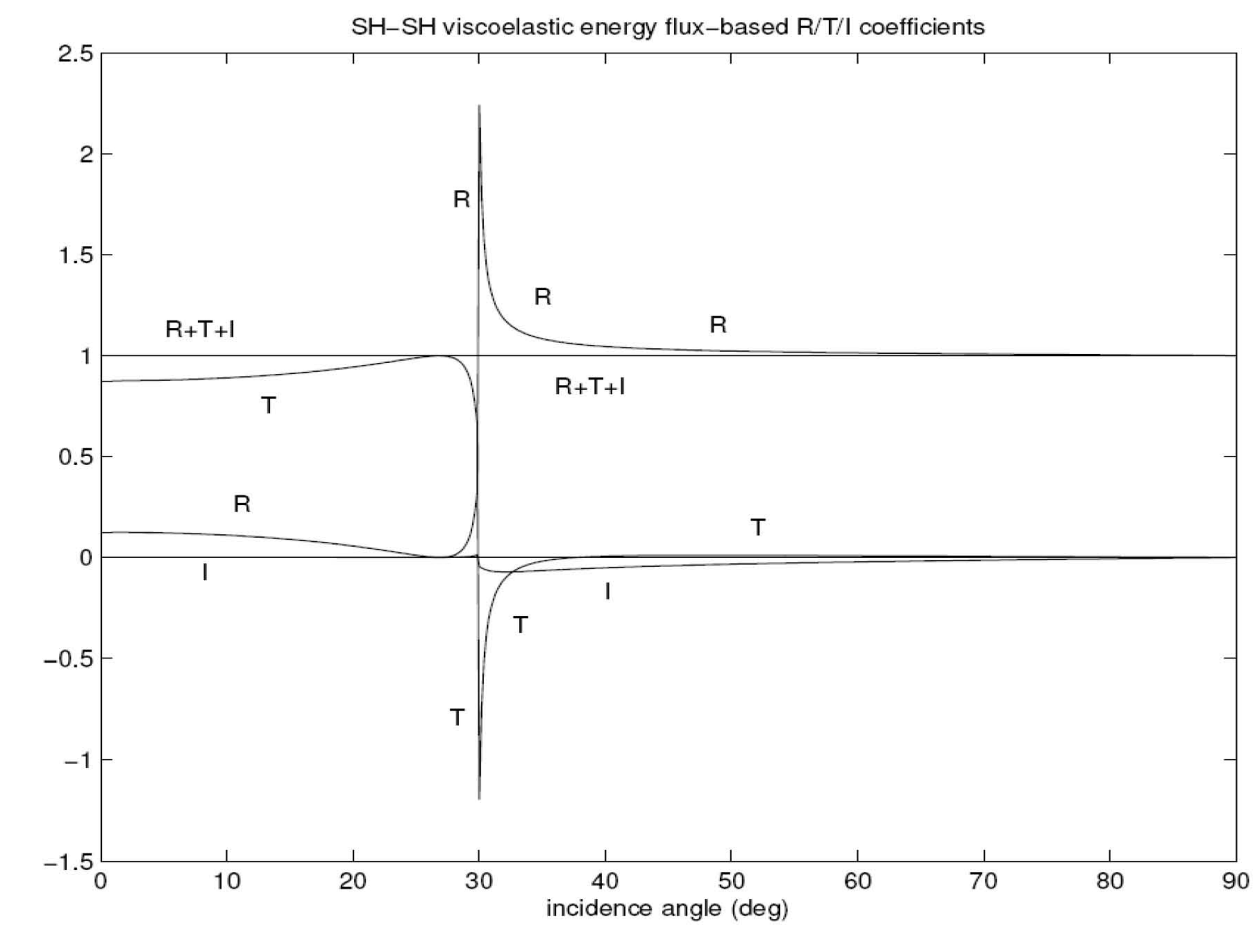

FIG. 1. SH viscoelastic energy based coefficients R, T and I for the model in presented in the table. As predicted ,T is negative for $30°<j_1<38°$ (from Krebes and Moradi, 2011).

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