Two-dimensional fast generalized Fourier interpolation of seismic records

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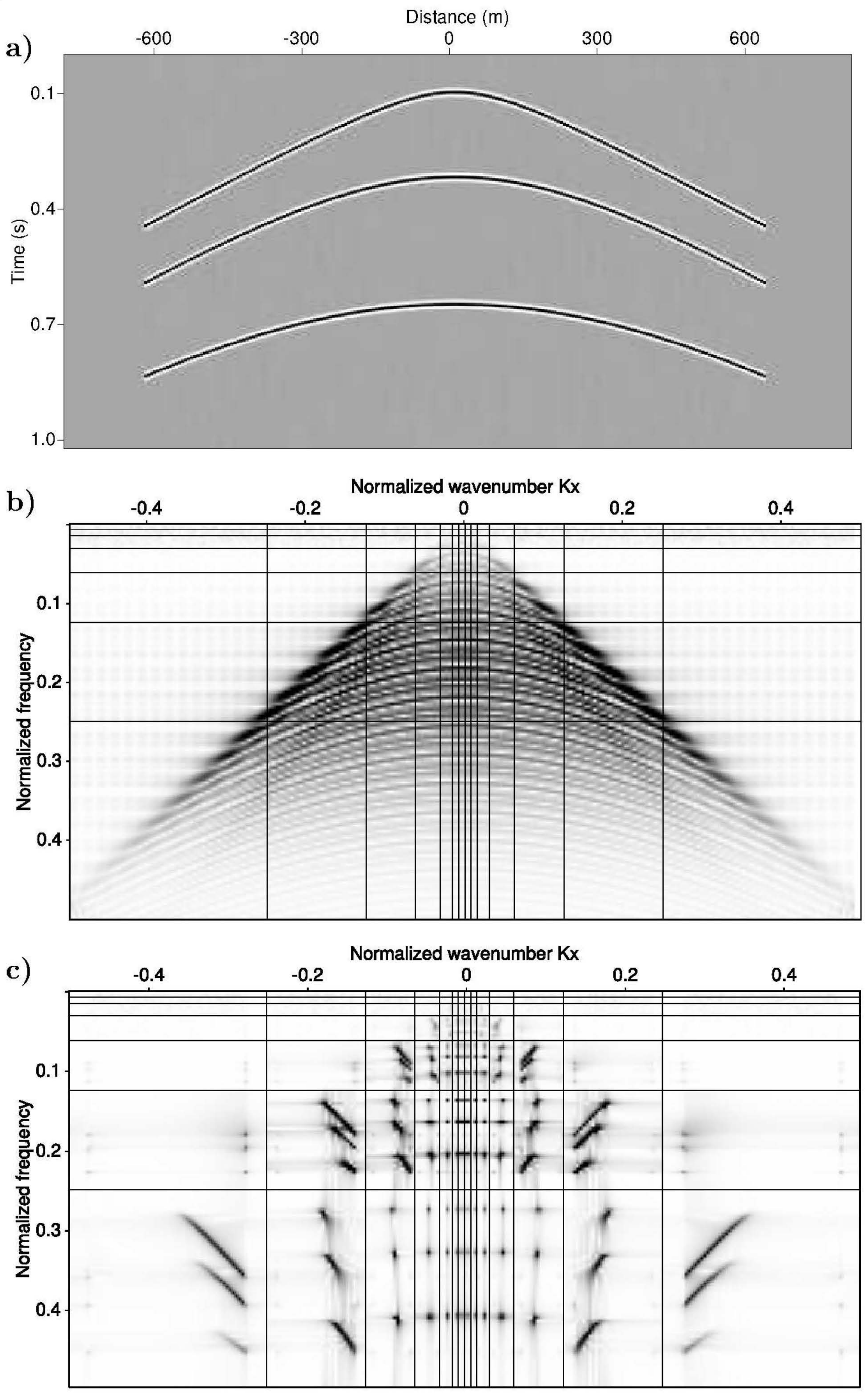
Goal

Using 2D FGFT algorithm as a fast and non-redundant algorithm for simultaneous time-frequency and space-wavenumber analysis of the seismic data and utilizing it for interpolation of curved (non-stationary) seismic events.

2D fast generalized Fourier transform (FGFT)

The 2D FGFT is obtained by dividing the *f-k* domain of data into small rectangular windows and applying 2D inverse Fourier transforms inside each window. Window sizes are smaller for low frequencies/wavenumbers and bigger for high frequencies/wavenumbers. A synthetic example below shows the 2D FGFT scheme.

a) Original *t-x* data. b) The *f-k* spectra of data. the gridlines show the 2D FGFT windows. c) 2D FGFT domain after applying inverse 2D Fourier transform for each winndow in b.



Mask function for 2D FGFT interpolation

We define the natural number $n_a \geq 1$ as the alias severity factor (ASF). The normalized frequency on which aliasing starts, f_a, and the ASF are related by

$$0.5^{n_a+1} \leq f_a < 0.5^{n_a}, \quad (1)$$

or in other words

$$n_a = -(\lfloor log_2(f_a) \rfloor - 1), \qquad 0 < f_a < 0.5, \quad (2)$$

where $\lfloor . \rfloor$ means truncation to the nearest smaller integer number. The number of zero traces, n_{ztr}, needed to be interlaced between original grid of data is given by

$$n_{ztr} = 2^{n_a} - 1. \quad (3)$$

Notice that interlacing zero traces allows us to extract correct slope information from low frequencies in order to interpolate the high frequencies.

Next, we divide the frequency axis of the 2D FGFT domain into two groups: alias-free and alias-contaminated ranges. The weight function, **W**, for alias-free 2D FGFT coefficients, **g**, is derived by

$$\begin{array}{l} \text{For } i = n_a, n_a+1, n_a+2, \ldots \\ \quad f \in [0.5^{i+2}, 0.5^{i+1}) \\ \quad \text{If} \quad k \in [-(0.5^{i+1}), 0.5^{i+1}] \\ \qquad \mathbf{W}(f,k) = \begin{cases} 0 & \mathbf{g}(f,k) < \lambda_i \\ 1 & \mathbf{g}(f,k) \geq \lambda_i \end{cases}, \\ \quad \text{Else} \\ \qquad \mathbf{W}(f,k) = 0 \\ \quad \text{End} \\ \text{End} \end{array} \quad (4)$$

where λ_i represents the threshold values for each frequency range. For alias contaminated frequency ranges the weight function is computed from the weight function of alias-free frequency ranges via

$$\begin{array}{l} \text{For } i = na, na-1, \ldots, 1 \\ \quad f_h \in [0.5^{i+1}, 0.5^{i}) \\ \quad k_h \in [-(0.5^{i}), 0.5^{i}] \\ \quad f_l \in [0.5^{i+2}, 0.5^{i+1}) \\ \quad k_l \in [-(0.5^{i+1}), 0.5^{i+1}] \\ \quad \mathbf{W}(f_h, k_h) = \Omega(\mathbf{W}(f_l, k_l)), \\ \text{End} \end{array} \quad (5)$$

where Ω is a simple nearest neighbor interpolation operator which doubles the width and length of the input matrix.

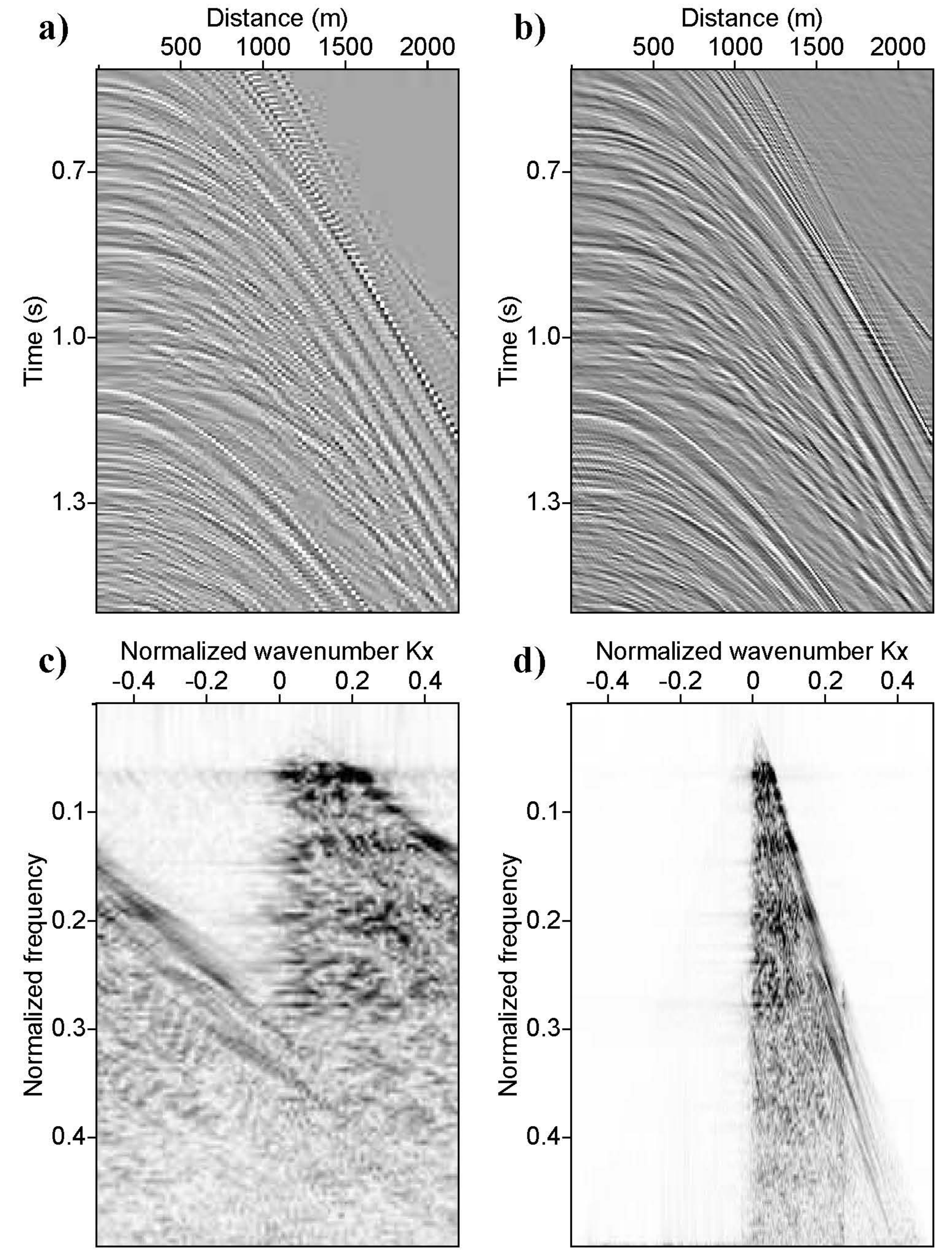
The 2D FGFT interpolation can be implemented by minimizing the following cost function

$$J \ = \ ||\mathbf{d}_{\text{obs}} - \mathbf{T}\,\mathbf{G}^T\mathbf{W}\mathbf{g}||_2^2 + \mu^2||\mathbf{g}||_2^2, \quad (6)$$

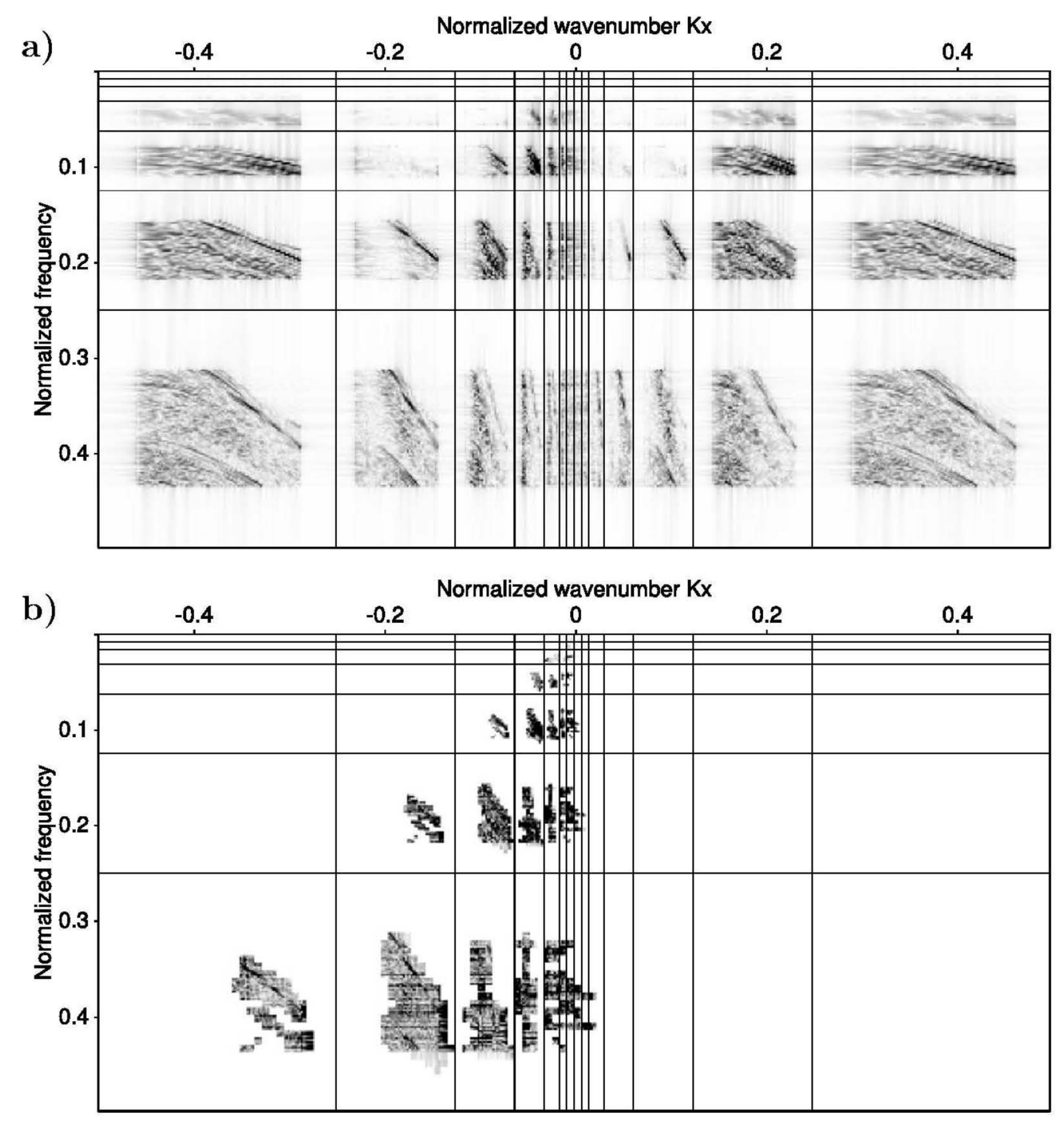
where $\mathbf{d}_{\text{obs}}$, **T**, and **G** represent the observed data in *t-x* domain, sampling function, and 2D forward FGFT operator, respectively.

Real data example

a) Original shot gather from Gulf of Mexico. b) 2D FGFT interpolated data. c) and d) are the *f-k* spectra of a and b, respectively. monitor survey.



a) 2D FGFT representation of original data ($n_{ztr} = 3$). b) 2D FGFT representation of interpolated data.



Acknowledgment

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