Better Gradient Vectors for More Accurate Rays

Marcus R. Wilson*, Hassan Khaniani and John C. Bancroft
University of Calgary

Summary

It is generally accepted that a smooth velocity model is required in order to compute well-behaved ray paths. However, smoothing is a double-edged sword, as it also reduces the resolution of the model. One of the fundamental assumptions of ray theory is that the velocity of the medium varies slowly compared to the frequency of a travelling wave, so one can usually infer that any erratic ray bending can be attributed to insufficient smoothness of the velocity model. For this reason, other sources of error are often not considered.

We demonstrate that incorrect refraction of rays can be caused by poor estimation of velocity gradient vectors. In this paradigm, smoothing becomes one of many possible approaches for improving the accuracy of these vectors.

Concept and Strategy

Ray tracing is performed by solving this coupled system of first-degree differential equations:

$$\frac{d\vec{x}}{dt} = v^2\vec{p} \qquad \text{and} \qquad \frac{d\vec{p}}{dt} = -\frac{1}{v}\nabla v.$$

The change in the ray direction is governed by the direction and magnitude of the velocity model contrast, so it is important that the velocity gradient be computed accurately. This is a challenging problem, as it requires us to estimate partial derivatives of the continuous velocity function from information given at the grid points. We tested several approaches, and their behaviour is summarized as follows:

- **Smoothing -** gives better gradient vectors near the interface, but introduces artifacts.
- **Higher order finite difference -** computes poor gradient vectors and introduces artifacts.
- **Upsampling -** replaces the jagged edge with a smooth transition, which results in accurate gradients. Interpolation requires additional strong assumptions about the shape of the model, which are generally not available during inversion.
- **Proximal smoothing -** smooths only those gradient vectors close to the boundary. Gives promising results for blocky models but does not generalize to complex velocity structures.

Future Work

By casting the problem of velocity model smoothing in terms of the accurate computation of gradient vectors, we have a new strategy for producing more accurate ray paths and travel times. We have demonstrated the feasibility of improving ray tracing by computing better gradient vectors while minimizing the need for smoothing. We seek to develop more general methods of velocity gradient correction that can be used for more complex modelling. Ideally we would like to characterize the velocity model as a sampling from a sum of continuous basis functions from which accurate gradients can be easily computed. We will use these results to augment existing ray based modelling algorithms. Successful improvements will increase the resolution and continuity of reflectors on benchmark synthetic salt models.

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Examples

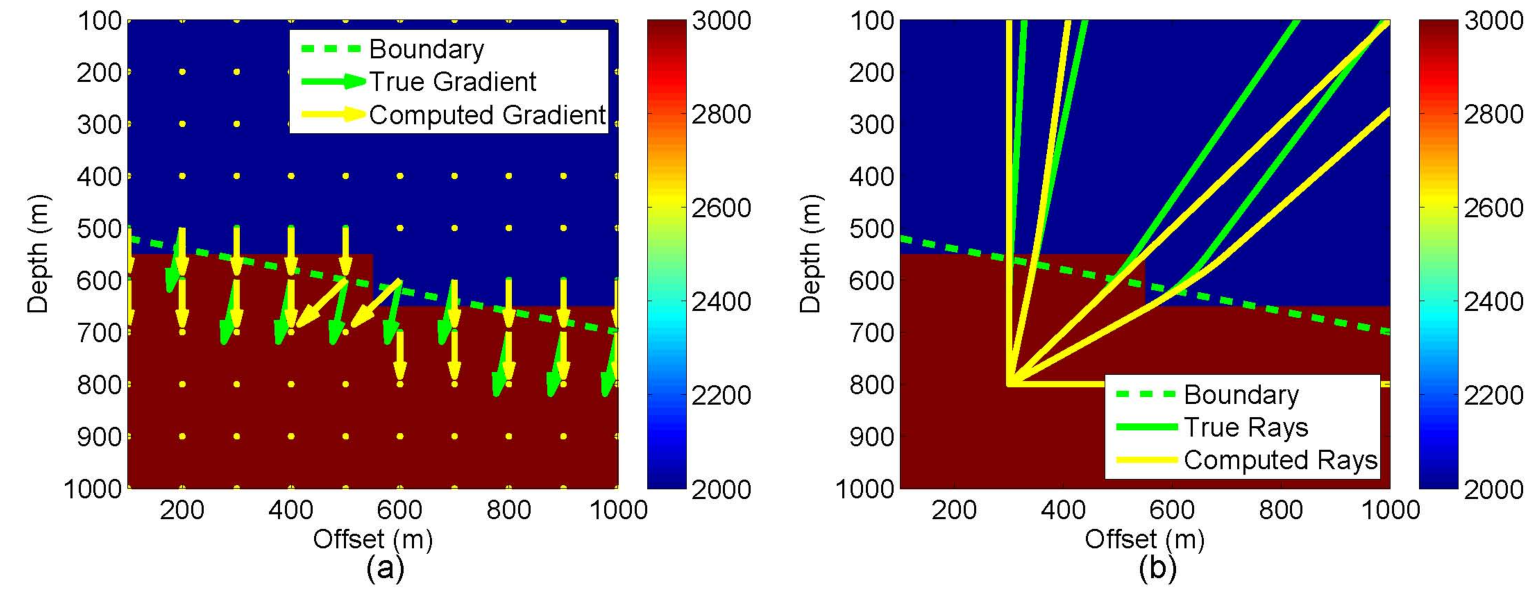

Figure: (a) Standard gradient vectors computed by finite difference across a dipping boundary, with vectors computed by Snell's Law for context. The finite difference method gives two different vectors at different points along the interface, neither of which are normal to it. (b) Rays computed using these gradient vectors do not obey Snell's Law, as they can not see the correct dip of the interface.

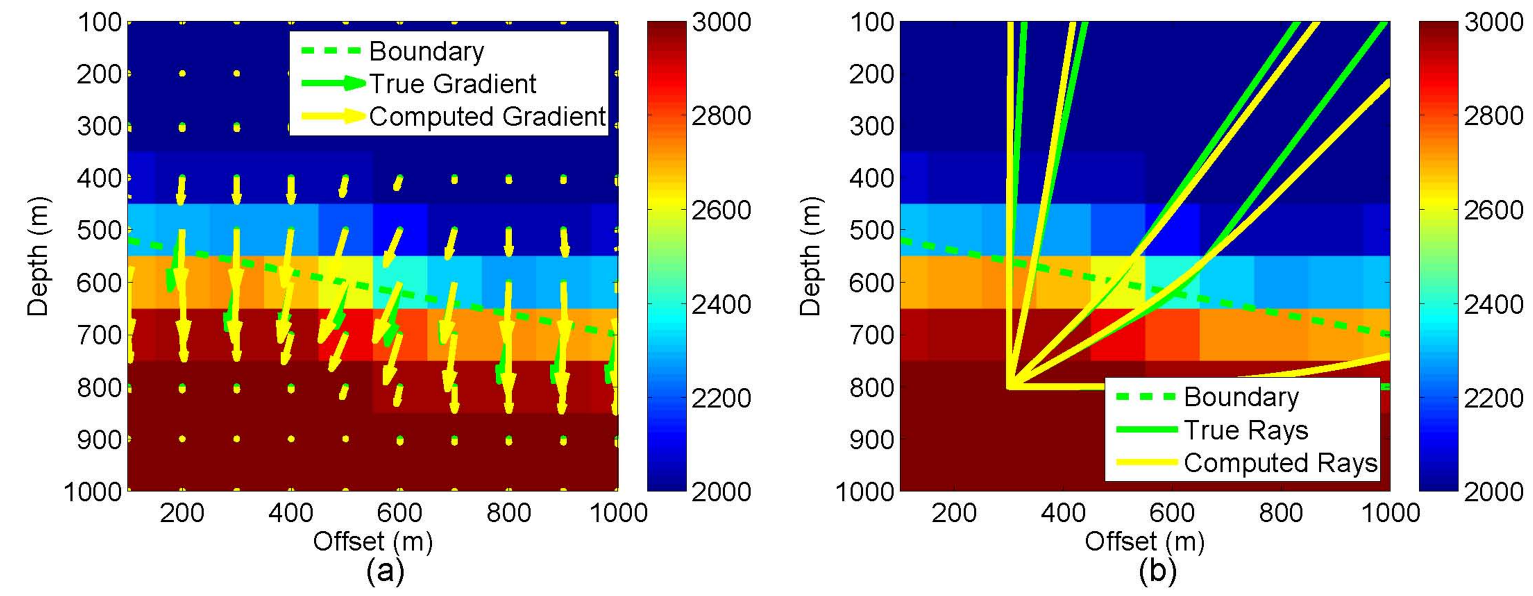

Figure: (a) Gradient vectors computed on a gently smoothed model. Vectors near the interface are closer to their true values, but some extra gradients are created, as the velocity contrast is smeared by the smoother. (b) The resulting ray paths are closer to the Snell's Law solution, but some rays have begun to bend erroneously.

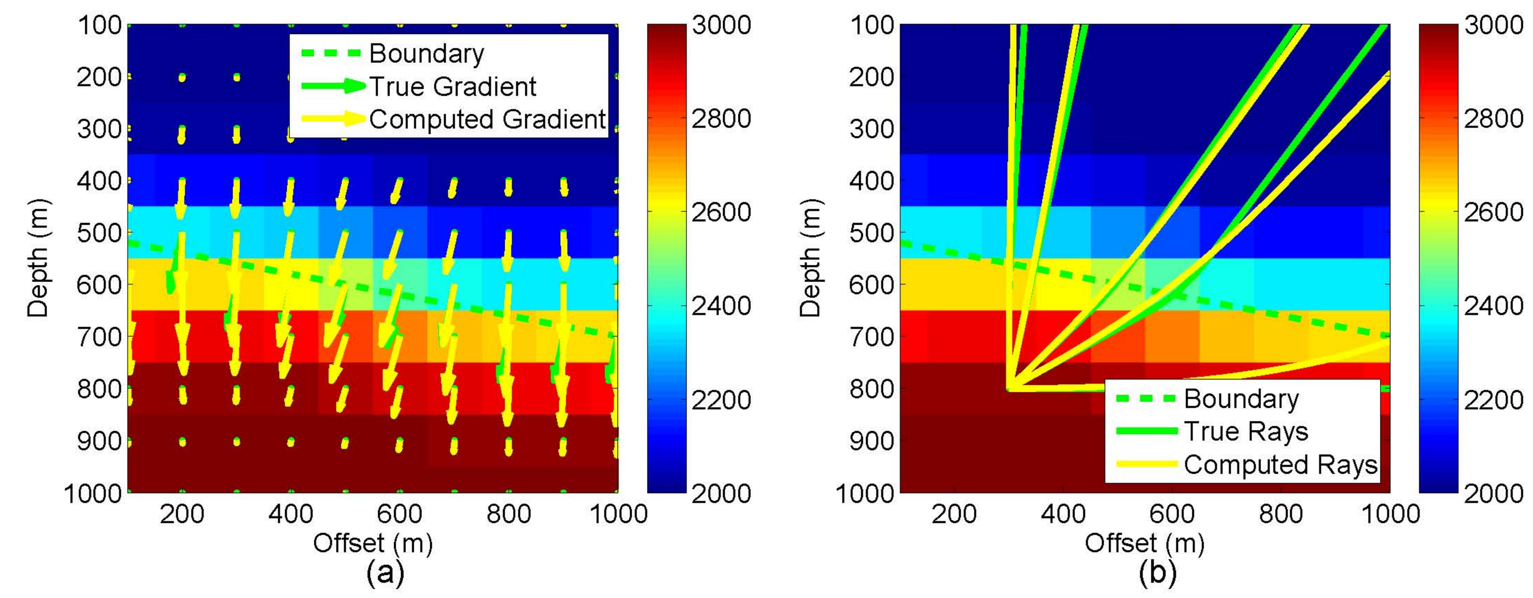

Figure: (a) Gradient vectors computed on a smoother model. There is some improvement over the gently smoothed case above, but returns are diminishing. (b) Most ray paths are a little better, but others are getting caught and deflected by the wider interface.

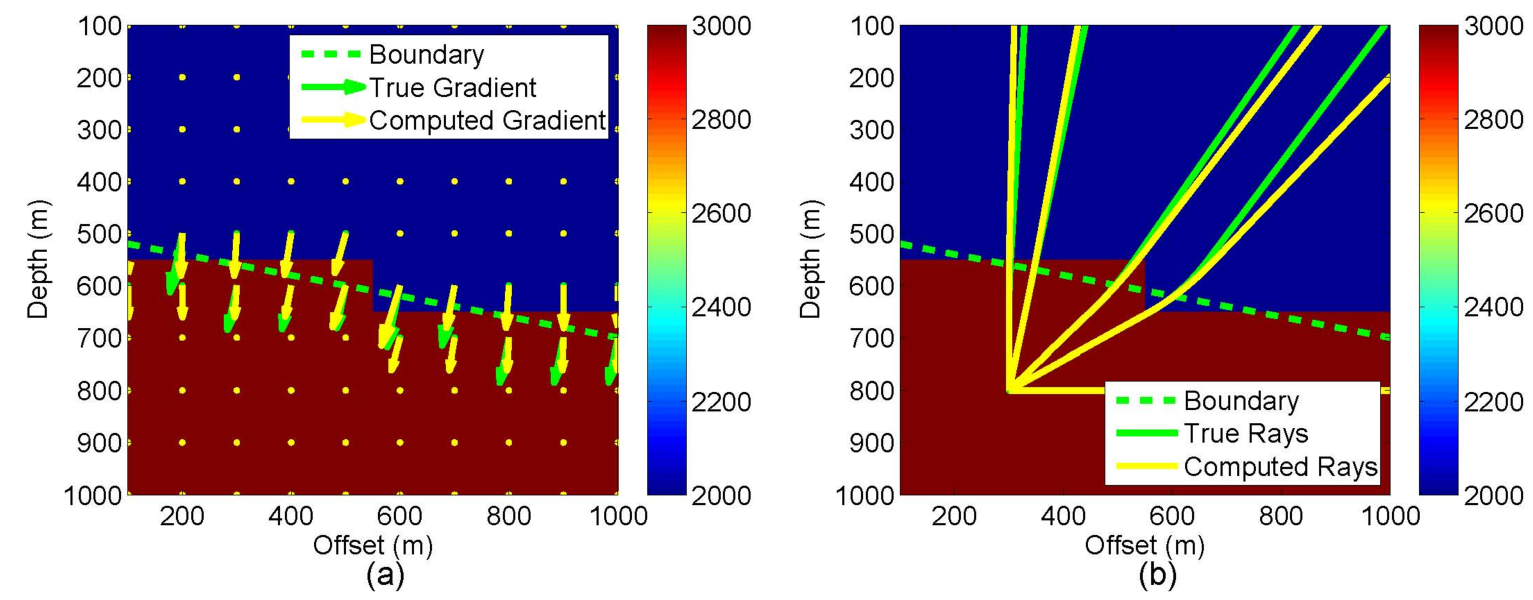

Figure: (a) Gradient vectors computed using a the proximal smoothing method. (b) We are able to retain the improvement that we get from smoothing, without losing detail in our model or negatively affecting any ray paths.

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