Matrix form for the Kirchhoff prestack modelling/migration

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1. Introduction

We showed the possibility of applying Kirchhoff migration/ modelling to the seismic data/model as the matrix-vector multiplication instead of using the corresponding operators.

A visual method of construction modelling and migration operators in the matrix form, without going to the mathematical background of the Kirchhoff migration or Green functions, is presented.

2. Kirchhoff modelling, migration, and inversion

If the reflectivity model is defined by $\boldsymbol{m}$, and the Kirchhoff modelling operator by $\boldsymbol{G}$, then the seismic data, $\boldsymbol{d}$, is given by,

$$\boldsymbol{d} = \boldsymbol{G}\boldsymbol{m}. \qquad (1)$$

The inverse of this calculation should return the reflectivity back,

$$\boldsymbol{m} = \boldsymbol{G}^{-1}\boldsymbol{d}. \qquad (2)$$

The transpose (adjoint) of $\boldsymbol{G}$ matrix is $\boldsymbol{G}^T$, and is easier to calculate than the exact inverse. In the case of Kirchhoff modelling, the transpose process is prestack Kirchhoff migration and is defined by

$$\widehat{\boldsymbol{m}} = \boldsymbol{G}^T\boldsymbol{d}, \qquad (3)$$

where $\widehat{\boldsymbol{m}}$ is the migrated image and $\boldsymbol{G}^T$ is the migration operator.

By defining Kirchhoff modelling (equation 1) as a forward operator and Kirchhoff migration (equation 3) as its adjoint, seismic imaging can be considered as an inversion problem.

In least squares migration, the difference between the observed and modeled data is minimized. As an example, the minimum norm solution is achieved by solving the following equation:

$$\boldsymbol{m}_{LS} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G})^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{d}. \qquad (4)$$

We show how to create $\boldsymbol{G}$, and consequently, $\boldsymbol{G}^T$, and $\boldsymbol{G}^T\boldsymbol{G}$.

3. Matrix form of Kirchhoff modelling

Assume a simple 2D earth model with one scatterpoint at the surface with the amplitude of 1. Let's consider a seismic source right on the top of this scatterpoint and eight receivers in both sides of the source, as shown in Figure 1a [Far right].

The assumed model may be showed by the following matrices:

$$\boldsymbol{m} = \begin{bmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix}, \text{ or } \quad \widetilde{\boldsymbol{m}}_{1,5} = [1]. \qquad (5)$$

Data resulted from this experiment should look likes:

$$\boldsymbol{d} = \begin{bmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&1&0&0\\ 0&0&1&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0 \end{bmatrix}. \qquad (6)$$

The modeling operator for this point, $\widetilde{\boldsymbol{G}}_{1,5}$, should have the same shape as the data. Multiplication of the modelling operator with the model should be mathematically possible and creates the data,

$$\widetilde{\boldsymbol{G}}_{1,5}\, \widetilde{\boldsymbol{m}}_{1,5} = \boldsymbol{d}. \qquad (7)$$

Multiplication in equation 7, becomes possible by vectorizing $\widetilde{\boldsymbol{G}}_{1,5}$ and $\boldsymbol{d}$ matrices. To vectorize $\widetilde{\boldsymbol{G}}_{1,5}$, we put column i, ($i > 1$) of $\widetilde{\boldsymbol{G}}_{1,5}$ matrix below column $i-1$ of the matrix for all columns,

$$\widetilde{\boldsymbol{G}}_{1,5} = \begin{bmatrix} 0&\cdots&0\\ 0&\cdots&0\\ 0&\cdots&0\\ 0&\cdots&1\\ 0&\cdots&0\\ 1&\cdots&0 \end{bmatrix}, \quad (8) \quad \widetilde{\boldsymbol{G}}_{v1,5} = \begin{bmatrix} 0\\0\\0\\0\\0\\1\\ \vdots\\ 1\\0\\0 \end{bmatrix}, \quad (9) \quad \boldsymbol{d}_v = \begin{bmatrix} 0\\0\\0\\0\\0\\1\\ \vdots\\ 1\\0\\0 \end{bmatrix}. \quad (10)$$

Then, equation 7 can be written as:

$$\widetilde{\boldsymbol{G}}_{v1,5}\, \widetilde{\boldsymbol{m}}_{1,5} = \begin{bmatrix} 0\\0\\0\\0\\0\\1\\ \vdots\\ 1\\0\\0 \end{bmatrix} [1] = \begin{bmatrix} 0\\0\\0\\0\\0\\1\\ \vdots\\ 1\\0\\0 \end{bmatrix} = \boldsymbol{d}_v\,. \qquad (11)$$

If the model includes another scatterpoint right below the first receiver as shown in Figure 1b, then equation for modelling of this scatterpoint is:

$$\widetilde{\boldsymbol{G}}_{v1,1}\, \widetilde{\boldsymbol{m}}_{v1,:} = \boldsymbol{d}_v, \qquad (12)$$

where,

$$\widetilde{\boldsymbol{m}}_{v1,:} = \begin{bmatrix} 1\\0\\0\\0\\1\\0\\0\\0 \end{bmatrix}, \quad (13) \text{ and, } \quad \widetilde{\boldsymbol{G}}_{v1,1} = \begin{bmatrix} 0\\0\\0\\1\\0\\0\\ \vdots\\ 0\\0\\0 \end{bmatrix}. \qquad (14)$$

Equation 12 can be rewritten for the first row of the model as,

$$\widetilde{\boldsymbol{G}}_{1,:}\, \widetilde{\boldsymbol{m}}_{v1,:} = \begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ \vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix} \begin{bmatrix} 1\\0\\0\\0\\1\\0\\0\\0 \end{bmatrix} = \begin{bmatrix} 0\\0\\0\\1\\0\\1\\ \vdots\\ 1\\0\\0 \end{bmatrix} = \boldsymbol{d}_v, \qquad (15)$$

where,

$$\widetilde{\boldsymbol{G}}_{1,:} = [\widetilde{\boldsymbol{G}}_{v1,1}\ \widetilde{\boldsymbol{G}}_{v1,2}\ \cdots\ \widetilde{\boldsymbol{G}}_{v1,5}\ \ldots]. \qquad (16)$$

This method can be extended to include all points in the model by,

$$\boldsymbol{m}_v = \begin{bmatrix} \widetilde{\boldsymbol{m}}_{v1,:}\\ \widetilde{\boldsymbol{m}}_{v2,:}\\ \vdots\\ \widetilde{\boldsymbol{m}}_{vNz,:} \end{bmatrix}, \qquad (17)$$

and,

$$\boldsymbol{G} = [\widetilde{\boldsymbol{G}}_{v1,1}\ \widetilde{\boldsymbol{G}}_{v1,2}\ \ldots\ \widetilde{\boldsymbol{G}}_{v1,\mathrm{Nx}}\ \widetilde{\boldsymbol{G}}_{v2,1}\ \widetilde{\boldsymbol{G}}_{v2,2} \ldots \widetilde{\boldsymbol{G}}_{v\mathrm{Nz,Nx}}]. \qquad (18)$$

Then, the multiplication of $\boldsymbol{G}^T$ with $\boldsymbol{m}_v$ results the data in the vectorized form:

$$\boldsymbol{G}^T\boldsymbol{m}_v = \boldsymbol{d}_v\,. \qquad (19)$$

4. Example

Suppose the assumed geometry includes only one source, 15 receivers, 15 CMPs, 200 time samples and data has 300 samples per trace. $\boldsymbol{G}$ will be a 4500 by 3000 matrix. This matrix is shown in Figures 2a and 2b for an assumed velocity. Figures 2c and 2d show the corresponding $\boldsymbol{G}^T\boldsymbol{G}$ matrix.

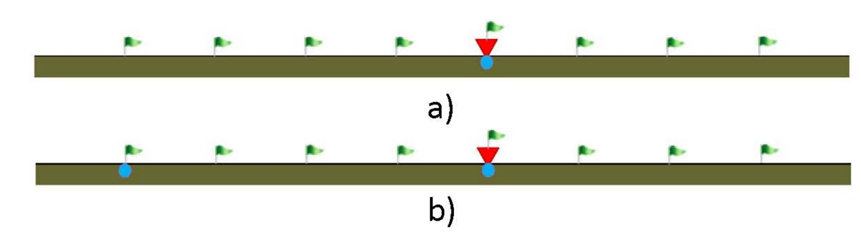

FIG. 1. a) A 2D model including one source (red triangle) on the top of a scatterpoint (blue circle) and eight receivers (flags) on the earth surface, b) Same model but with two scatterpoints.

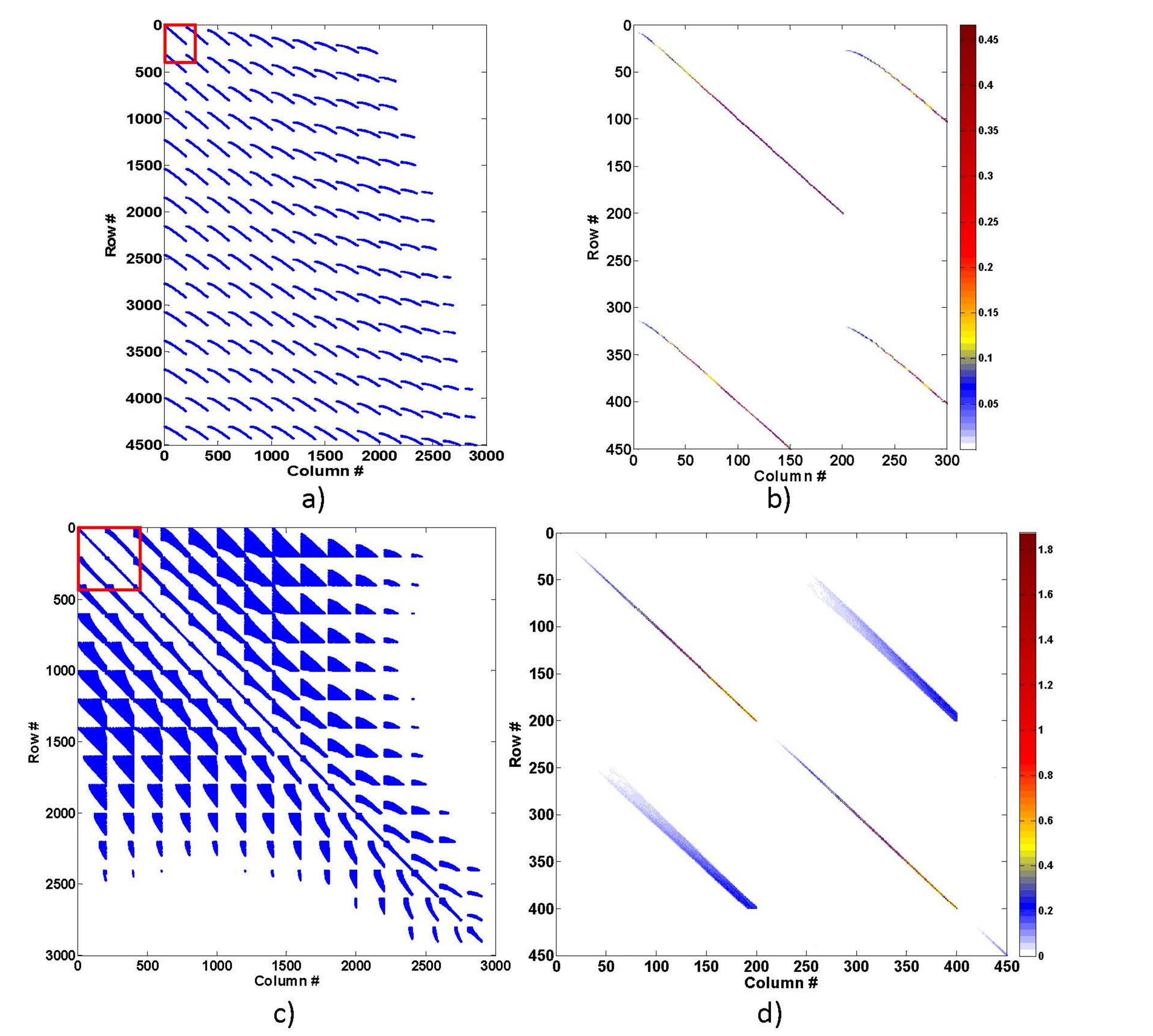

FIG. 2. a) Non-zero elements of matrix $\boldsymbol{G}$, b) Close up of the first 450 rows and 300 columns, c) Non-zero elements of matrix $\boldsymbol{G}^T\boldsymbol{G}$, d) Close up of the first 450 rows and columns.

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