Numerical modeling for different types of fractures

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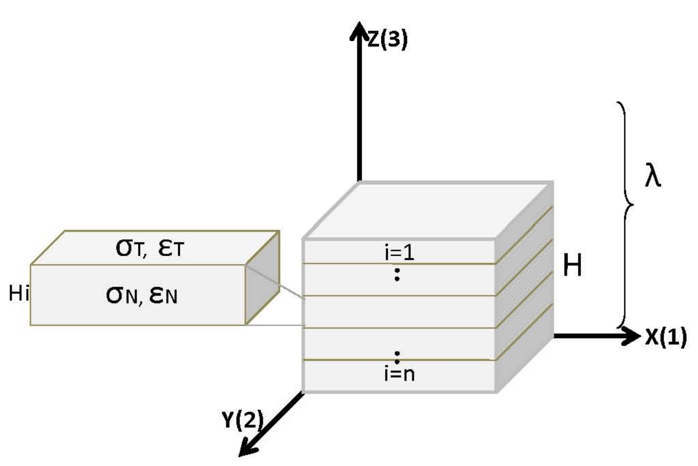
Summary

Fractures can be modeled as a non-welded contact linear slip interface (Schoenberg, 1980). Therefore, in the long wavelength limit, the fractured homogeneous isotropic media are equivalent to transversely isotropic media (TI) once impedance and thickness of a thin constituent approaches to zero in the layered and perfect bound media (Schoenberg and Muir, 1989). Horizontal, vertical and tilt fractures can be numerically modeled through an extended generalized homogenous approach (Korn and Stockl, 1982) with the fracture parameters such as tangential Zt and normal Zn compliances.

The elastic moduli for fractured media

Normal stresses σ_N (σ_3, σ_4, σ_5) acting on the layering plane are constant value in all layers as well as tangential strains ε_T (ε_1, ε_2, ε_6) acting in the layering plane in the layered media (Backus, 1962). Other stresses and strains are different from layer to layer, but an average value will be taken within a layer and named as tangential stresses σ_T (σ_1, σ_2, σ_6) and normal strains εT (ε_3, ε_4, ε_5).



In Hooke's law,

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\\ \sigma_3\\ \sigma_4\\ \sigma_5\end{bmatrix} = \begin{array}{c} \quad C_{TT} \qquad\qquad C_{TN} \\ \left[\begin{array}{ccc|ccc} c_{11} & c_{12} & 0 & c_{13} & 0 & 0\\ c_{21} & c_{22} & 0 & c_{23} & 0 & 0\\ 0 & 0 & c_{66} & 0 & 0 & 0\\ \hline c_{31} & c_{32} & 0 & c_{33} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{55}\end{array}\right] \\ \quad C_{NT} \qquad\qquad C_{NN}\end{array} \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\end{bmatrix}$$

$$\sigma_N = \overline{c_{NT}}\varepsilon_T + \overline{c_{NN}}\varepsilon_N$$

Once a layer is fractured, the normal strains will be enlarged, whereas the tangential strain relatively decreases into a value as same as the background medium. Then, $\sigma_{Nf} \approx \overline{c_{NNf}}\,\overline{\varepsilon_{Nf}}$

The normal and tangential compliances as fracture parameters are

$$\overline{C_{NNf}^{-1}} = z = \begin{bmatrix} z_N & 0 & 0\\ 0 & z_T & 0\\ 0 & 0 & z_T\end{bmatrix}$$

FD scheme of P-SV wave propagation in the fractures as non-welded contact slip interface

In the non-welded contact slip interface approach (Schoenberg, 1980), only stresses are continuituous across the interface, but the displacements are not. All displacement are linear function of the stresses.

$$U^{x+} - U^{x-} = \eta_T \delta^{zx+(-)} \qquad U^{z+} - U^{z-} = \eta_N \delta^{zz+(-)}$$

$$\delta^{zx+(-)} = \delta^{zx\text{-}(+)} \qquad \delta^{zz+(-)} = \delta^{zz\text{-}(+)}$$

Where

$$\delta^{zx} = \mu\left(\frac{\partial u^z}{\partial x} + \frac{\partial u^x}{\partial z}\right) \qquad \delta^{zz} = \lambda\frac{\partial u^x}{\partial x} + (\lambda + 2\mu)\frac{\partial u^z}{\partial z}$$

An extended generalized homogenous FD scheme is

$$\begin{aligned} U^{t+1}_{m,n} &= -U^{t-1}_{m,n} + 2U^{t}_{m,n} + \frac{1}{\rho}\left(\frac{\Delta t}{h}\right)^2 (F\hat{N}F(U^{t}_{m+1,n} - 2U^{t}_{m,n} + U^{t}_{m-1,n}) \\ &+ \hat{N}(U^{t}_{m,n+1} - 2U^{t}_{m,n} + U^{t}_{m,n-1}) \\ &+ \frac{1}{4}(F\hat{G}F + \hat{G})(U^{t}_{m+1,n+1} - U^{t}_{m+1,n-1} - U^{t}_{m-1,n+1} + U^{t}_{m-1,n-1}) \end{aligned}$$

Where

$$F = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\quad \hat{N} = \begin{bmatrix}\frac{\mu}{1+\delta} & 0\\ 0 & \frac{\lambda+2\mu}{1+\varphi}\end{bmatrix},\quad \hat{G} = \begin{bmatrix}0 & \frac{\mu}{1+\delta}\\ \frac{\lambda}{1+\varphi} & 0\end{bmatrix}$$

λ and μ are medium parameters. The extended generalized homogeneous FD scheme takes more physical insights into account to the fracture forward modeling because the medium and boundary conditions (BCs) are imposed explicitly. The equation of motion governs the displacements outside the discontinuity interface but non-welded contact boundary conditions are applied at the discontinuity interface.

Fictitious points are introduced as the same points as the real grid on both side of the interface. It makes possible to obtain BCs expressions for the displacements at the interface from either side of the medium. The BCs, $U^{z+}_{n+1/2} = U^{z-}_{n+1/2}$ where n+1/2 implies the interface at (n+1/2)z, the z^+ and z^-denote z-component displacements in the limit of the interface that approaches the upper edge and lower edge of the media (Figure 1).

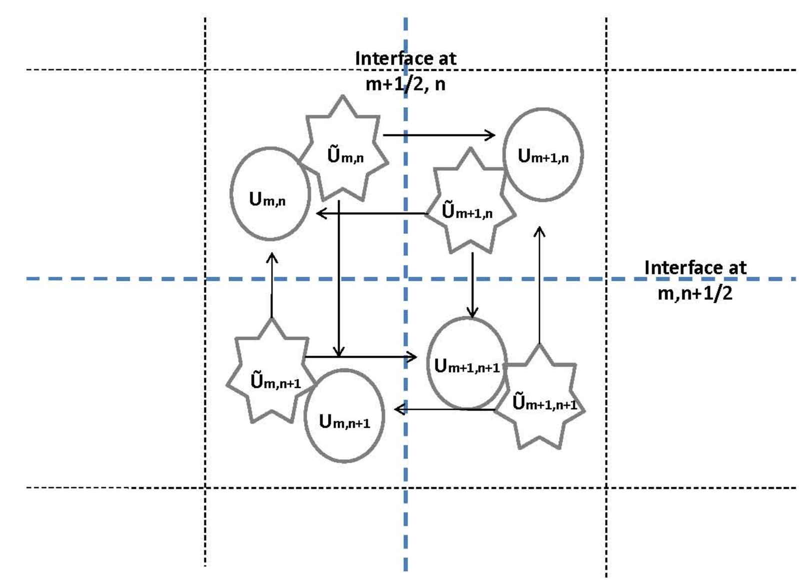


FIG. 1 The fictitious (flower) and real grid points (circle) are in the same grid. The non-welded interfaces as fractures (blue dash lines) is in between every two neighboring points

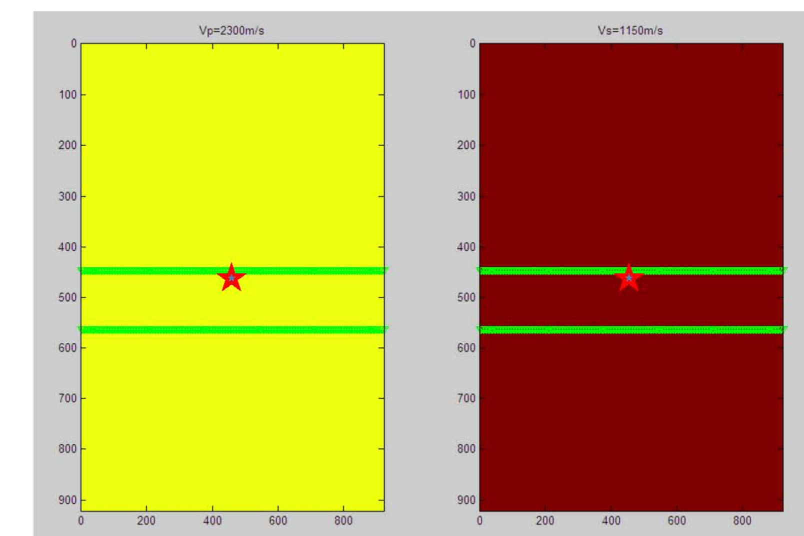


FIG. 2. Source (star point) and receivers (green points) map. Model parameters:
Vp=2300 m/s, Vs=1380m/s, ρ=2.37g/cm^3

Seismograms and discussions

Obviously, There are PP and PS reflections and amplitudes variation with offsets from the fractures even though there are no impedance contrasts in the homogenous medium (Figure3a-3c). The direction of the fracture strongly affect the amplitudes of the PP and PS wave that perpendicularly polarized to the fracture having stronger amplitudes.

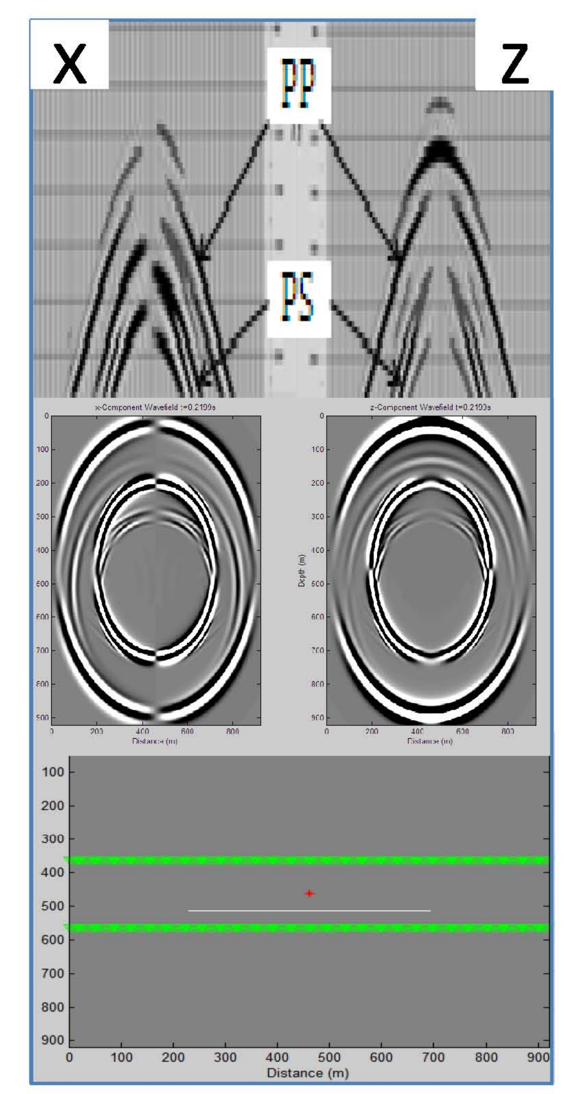


FIG. 3a Horizontal fracture model (bottom), wave field (middle) and seismograms from the fracture(top)

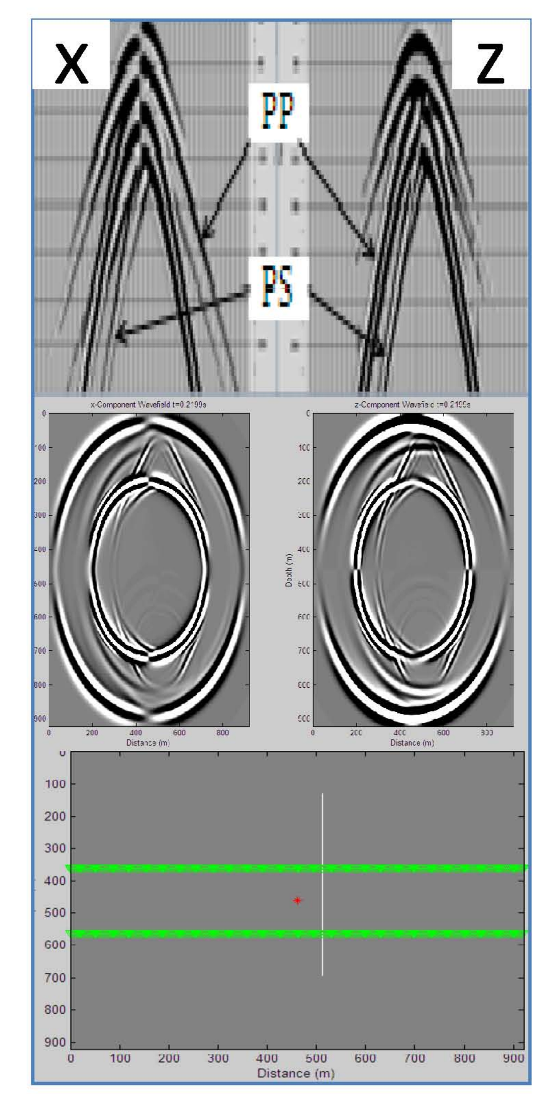


FIG. 3b Vertical fracture model (bottom), wave field (middle) and seismograms from the fracture(top)

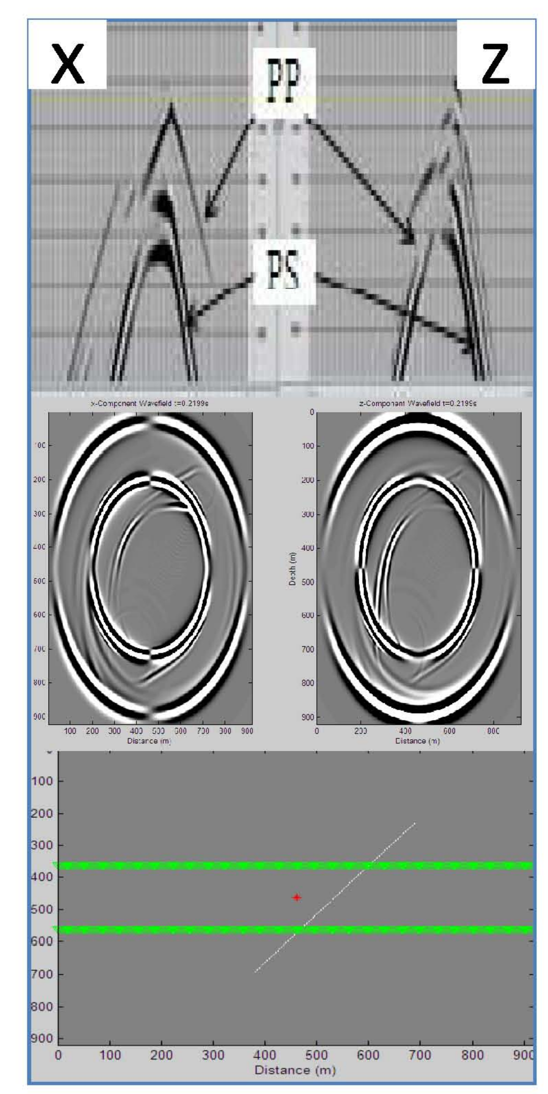


FIG. 3c Tile fracture model (bottom), wave field (middle) and seismograms from the fracture(top)

Conclusions

The fracture causes discontinuous displacement across two side medium, therefore PP and PS waves are reflected in the homogeneous medium without impedance contrast. The fractures can be indicated from the seismic records because the direction of the fracture affects the seismic amplitudes .

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